Introduction to Accounting and Finance for Civil Engineers

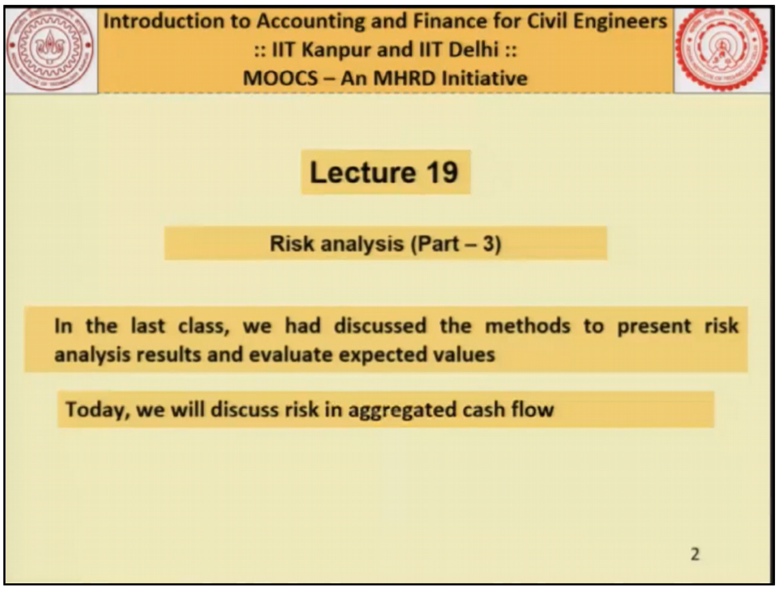
Professor. Sudhir Misra
Department of Civil Engineering, IIT Kanpur
Professor. Kumar Neeraj Jha
Department of Civil Engineering, IIT Delhi

Module No. #04
Lecture No. #19
Risk Analysis (Part-3)

Good morning, Namaskar, and Welcome to the course, once again. In the last lecture, if you remember, we discussed few concepts related to, Risk Analysis. We discussed, how to draw Investment Risk Profile. We discussed, how to draw AID, which is Acceptable Investment Diagram.

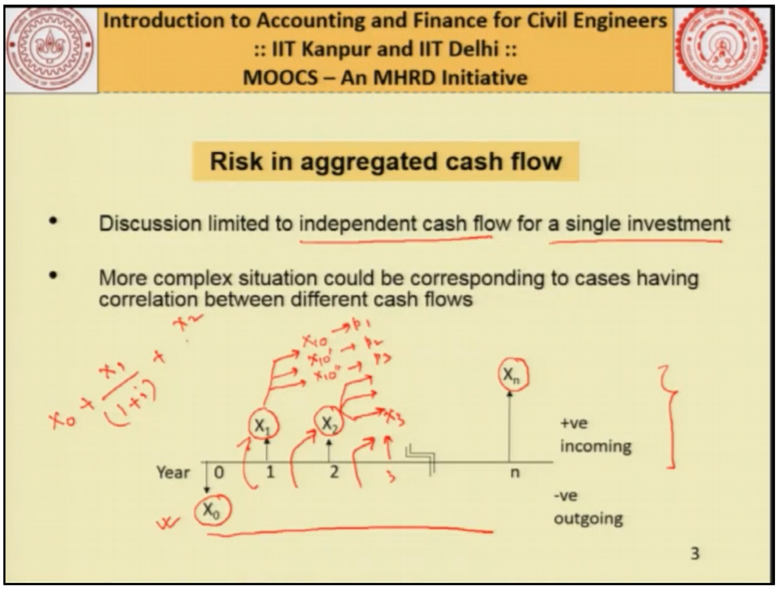
We also learnt, few concepts related to, Auxiliary Investment Criteria, or rather, Auxiliary Decision Criteria. Under that, we discussed, the criteria pertaining to minimisation of Variance, Probable Future Criterion, and Aspiration Level. In this lecture, we are going to learn, one very important aspect under Risk Analysis, that is to consider the risk in, Aggregated Cash-flow.

(Refer Slide Time: 01:03)



So, when I say, Aggregated Cash-flow, if you remember, we have been drawing, Cash-flow Diagrams, something like this.

(Refer Slide Time: 01:05)



Now, this is what we are saying, Aggregated Cash-flow. Because, here at time T is equal to 0, you are having some Cash-flow, at time T is equal to 1, some Cash-flow. Likewise, at different points of time, we are having different Cash-flows. Now, so far, what we have been doing was, to assume that, this X 0, X 1, X 2, all are constant. Subsequently, if you remember, we said no, X 1 cannot be constant, they can vary. So, I started taking different values for, these X 1.

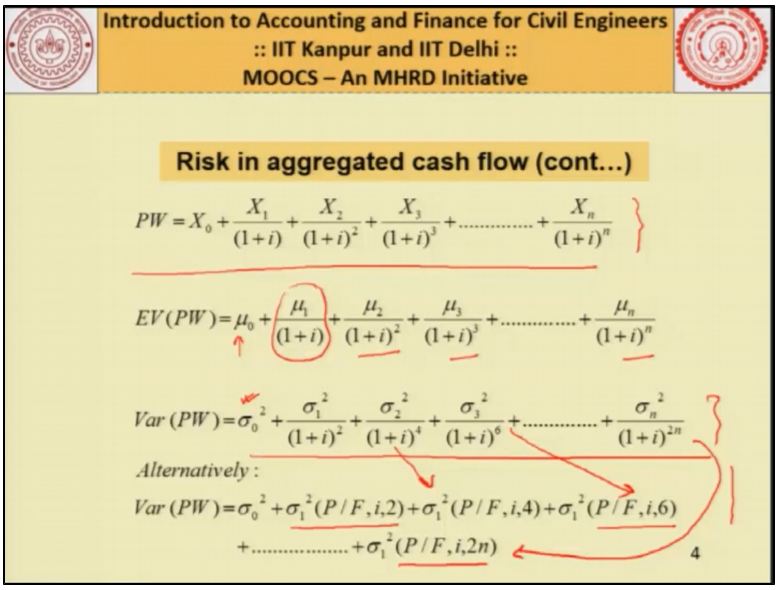
Likewise, I took different values for this X 2, and then I understood, how to calculate the net present worth, and thereafter, how to take the decisions. Subsequently, in the last lecture, if you remember, for each of these Cash-flows, I also assigned certain probability values. For example, corresponding to this X 1, let us say, the values could be X 10, X 10 Dash, let us say, X 10 Double Dash. And, associated probability, could be P1, could be P2, could be P3, and so on.

And, under this situation, we also found the net present value. We call that net present value as the expected value. You understand the concept of expected value, from one of the previous lectures. It is nothing but, the weighted average of different Cash-flows. And, how do we get this. You multiply different Cash-flows, with their probability values, and thereby, you get the expected value. We have also learnt, how to calculate the variance, and thereby, the associated standard deviation.

Now, the analysis, what we are going to do right now, is assuming that, the Cash-flows are independent. When I say independent, it means, the Cash-flow that has occurred in Year-1, is in no way influencing, the Cash-flow in Year-2, and so on. It is not in any way, connected to Cash-flow in Year-3. Let us say, X 3, and so on. So, we are assuming that, X 1 is independent of X 2, and same is independent with X 3, and so on.

So, this is what, we are going to learn. In higher classes, you will be doing the analysis, in which you will assume that, Cash-flow in one year, is dependent on the Cash-flow in the next year, and so on. But, that is beyond the scope of this particular lecture. We are concentrating right now, on the independent Cash-flow, that too, for a single investment. So, for this example, let us assume, we are investing X 0, at time T is equal to 0. And, it is giving me a return of X 1, X 2, X 3 and X N, at different time periods, 1, 2, 3 and N respectively.

(Refer Slide Time: 04:18)



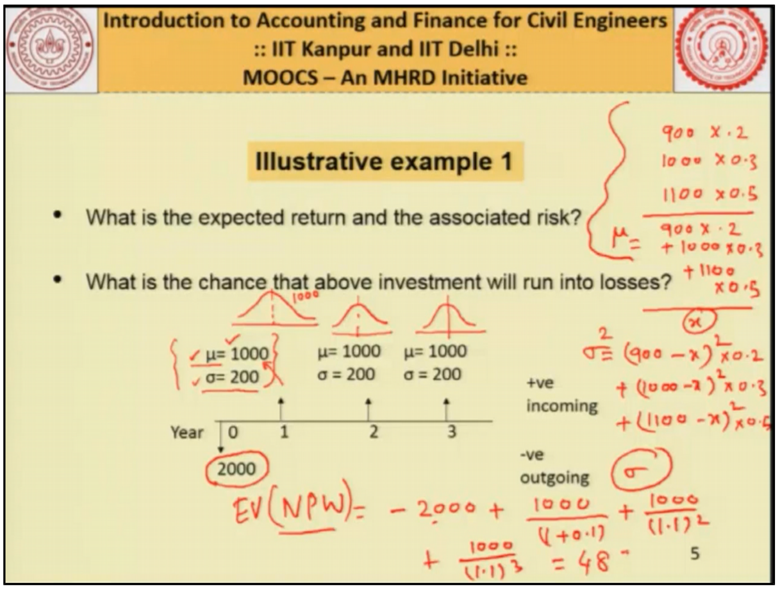
So, I will quickly give you some of the formulas, that we often use, in the context of finding the risk in Aggregated Cash-flows. So, as far as the present worth formula is concerned, it is nothing new to you. What you do is, it is simple. X 0, you write it with proper sign, and add X 1 divided by 1, + I + X 2 divided by 1, + I raised to power 2, and so on. So, this is how, you can derive. And, this is how, it has been written here.

So, present worth is equal to X 0 + X 1 upon 1, + I + X 2 upon 1, + I raised to power 2, and so on. Now, the expected value of present worth, you have already been calculating this. I am just introducing the symbol Mu 0 here, at time T is equal to 0. That is, the mean value there for all my X 0's for Year-1, it is given by Mu 1 upon 1, + I Mu 2 upon 1, + I raised to power 2, for the second year. This is for the third year. And, this for the Nth year.

Variance is given by this formula, which is Sigma 0 square, + Sigma 1 square upon 1 + I raised to power 2, + Sigma 2 square upon 1 + I raised to power 4, + Sigma 3 square upon 1 + I raised to power 6. And finally, Sigma N square divided by 1 + I raised to power 2N. Now, this also, we can write it in terms of P given F factor. So, for this expression, 1 upon 1 + I raised to power 2, I can as well write, P given F, for an interest rate of I, for 2 years.

This will be converted like, P given F, for an interest rate of I, and for a time period of 4. This one, you can convert it like this, P given F, for an interest rate I, for a time period 6. And finally, this one, you can convert it in terms of, P given F, for an interest rate I, and for time period 2N. So, using the concept of these formulas, we will try to evaluate the risk. And, as you know, risk, we are measuring in terms of, variances and standard deviation.

(Refer Slide Time: 06:40)



So, let us move to one small question in which, we are given that, there is an investment proposal in which, you are investing 2,000 Rupees, at time T is equal to 0. And, this is likely to give me a return of 1,000 Rupees, at the end of Year-1, with standard deviation of 200. So, that means, the distribution at the Year-1, is something like this. Mean value of return is 1,000, and its standard deviation is 200.

In some of the previous problems, if you remember, you were not directly given the Mu value, and the Sigma value. In this problem, in order to simplify and to save time, I have directly given you the expected value, which is Mu, and the standard deviation for each year's Cash-flow. So, here also, this is 1,000, this is 200. And, here also, this is 1,200. So now, you must be wondering, how from a simple assumption, we have come to a very complicated situation.

Now, what is happening here. Earlier, in all our cases, we were assuming that, this Mu is fixed, there is no standard deviation. That was a constant value of Cash-flow. That, we assume that, was certain to be occurring. Now subsequently, we started changing it. So, we said that, 1,000 could become 1,100, it could become 900, anything, but we did not assign any probability. In subsequent lectures, what we did for each of these state of occurrences, whether it is 900, whether it is 1,000, whether it is 1,100, we started assigning probabilities.

So, we said okay, corresponding to 900, probability is 0.2, corresponding to 1,000, probability is 0.3, and corresponding to 1,100, let us say probability is 0.5. Now, from there, we were able to calculate the expected value. For example, if I give you for Year-1, let us say, the probable values are 900, 1,000, and 1,100, you can easily find out its expected value, if you know the probability. Suppose, this probability is 0.2 here, this probability is 0.3 here, and this probability is 0.5, you can easily calculate the Mu value from here.

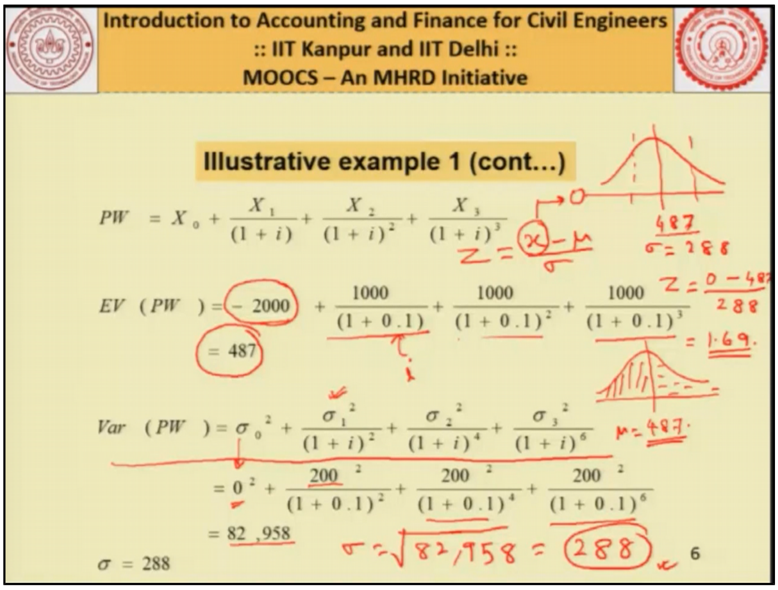
So, Mu will be given by, 900 into 0.2, + 1,000 into 0.3, + 1,100 into 0.5. Suppose, this is coming to be some value X. Now, knowing this X, we can very easily calculate, the Sigma square, and thereby Sigma. So, how do we calculate Sigma square. For this, it would be 900 - the mean value square of this, multiplied by 0.2 + 1,000, - X square of this, multiplied by 0.3 + 1,100 - X square of this, multiplied by 0.5. Now, you take the under-root of this, and you get Sigma.

So, to avoid all these calculations in the problem, I have directly given you these two values. So, what is the expected value at the end of Year-1, what is the probable standard deviation for Year-1, all these values are given to you. Not only for Year-1, but for Year-2, Year-3, as well. Now, for finding the risk out of this proposal, what we need to do is, we need to find the Net Present Worth for this option.

And, you remember, I had given you this formula, to calculate the net present worth. It is something like this. So, expected value of present worth is given by, Mu-Not + Mu 1 upon 1 + I. So, in this case, Mu 1 is one single value, which is coming to be 2,000. So, you can write - 2,000 here, + 1,000, which is the mean value here, divided by, let us assume, I is equal to 10%.

So, I write, 1 + 0.1. For Year-1, this becomes, 1,000 by 1 0.1 raised to power 2. For Year-2, and + 1,000 divided by 1 0.1 raised to power 3. So, this is the expected value of Net Present Worth. You can write expected value of Net Present Worth, using the formula, that we derived earlier. And, this value is going to be 487, this is coming to be 487. So, this calculation has been performed here.

(Refer Slide Time: 11:36)



You can see here, - 2,000 + 1,000 divided by 1 + 0.1. This 0.1 is nothing but, your interest rate. Here, for second year, 1,000 divided by 1 + 0.1 raised to power 2. And here, this is 1,000 divided by 1 + 0.1 raised to power 3. So, you are getting 487. Now remember, this is expected value. In earlier case, we were assuming that, it is going to be certain. So, there was no probability associated with anything, that was bound to happen.

But, now here, we say that, this expected value, when you say expected value, you know that, it is only a mean value. That means, there is every possibility, that the expected value could be, either more than 487, or less than 487. So, there is a 50% chance that, the expected value of Net Present Worth is likely to be less than 487. And, there is again 50% chance that, expected value is going to be more than 487.

So, you can draw a normal distribution like this also. And, here is your mean, 487. So, there is a possibility that, expected value can be below 487, and there is possibility that, expected value can be more than 487 also. Now, we also need to calculate the variance, and thereby the standard deviation. So, if you go back to the variance formula, it is Sigma zero square +

Sigma 1 square upon 1 + I raised to power 2 + Sigma 2 raised to power 2 upon 1 + I raised to power 4, Sigma 3 square upon 1 + I raised to power 6. Because, my duration is only up to 3 years.

Now, you will find that, Sigma-Not square is zero. Why because, this 2,000 is a fixed value. It is not going to change. So, if there is no variation in any amount, the variance is going to be zero. This is a constant. So, the variance associated with that, is going to be zero. But, it is not the case, with other values, at different points of time. There, if you remember, the variance for Year-1 was given to be, 200.

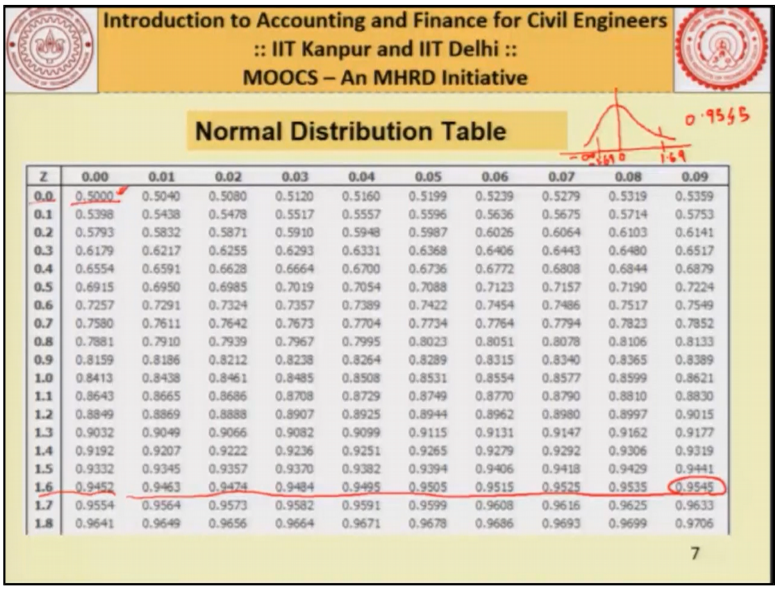
So, this is 200 square upon 1 + 0.1 raised to power 2, 200 square upon 1 + 0.1 raised to power 4, and finally 200 square upon 1 0.1 raised to power 6, and this is coming to be 82,958. If you take the route of this, so Sigma will become under-root 82,958. And, if you calculate, this is coming to be 288. So, what you find here is that, Mu is 487, for overall expected value of Net Present Worth, and the standard deviation is associated with that is coming out to be 288.

So, that means, as you know, when you draw this, standard deviation is 288 here. That means, there is a likelihood that, the value of Net Present Worth might go below zero also. Now, that is what is the risk associated with this particular proposal. I want to find, what is the probability that, my Net Present Worth becomes less than zero. In those cases, I am going to make losses because, as long as my Net Present Worth is more than zero, there is no issue.

But here, we got that Sigma value as 288, and mean value is only 487. So, there is every possibility that, my Net Present Worth may fall below zero. So, that is what, I am interested in finding. And, that is what, I am defining it as risk. So, what is the risk of making losses, when I say, I am interested in finding the value of probability in which, the Net Present Worth is going to be below zero. So, you know the concept of standard variant, or normal variant, we define it as Z.

So, Z is given as, X - Mu upon Sigma X, is the desired value. Mu is the mean value, which is 487, in this case. And, Sigma is 288. Now, I am interested in finding the probability, corresponding to a Net Present Worth of zero. So, X I keep it as zero, so my Z becomes zero - 487 divided by 288, if you calculate this, you will find, it is coming very close to 1.7, it is about 1.69. Now, all of you are familiar with normal distribution table.

(Refer Slide Time: 16:40)



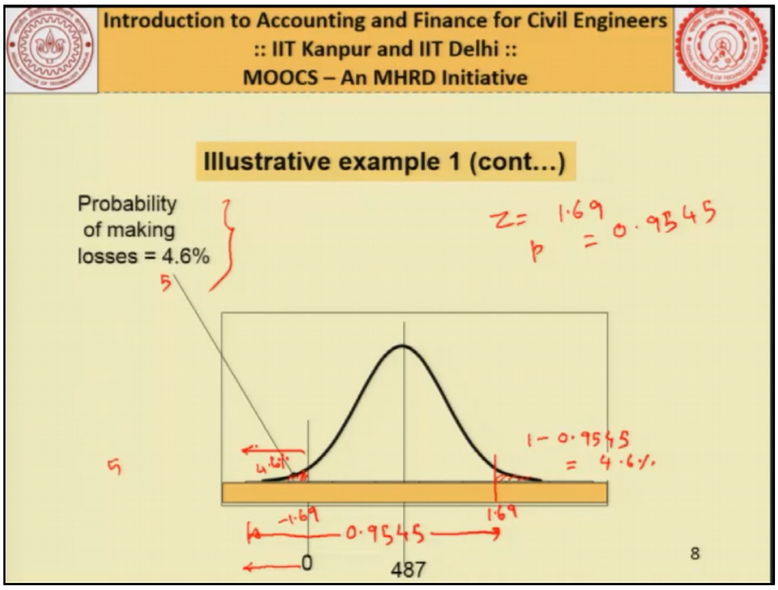
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706

The table is very much like this. There are two types of table. Rather in this table, if you look, these values carefully, the values are given from minus infinity onwards. So, that is why, if you find the value of Z is equal to 0, it is coming to be 0.5. Z is equal to 0, would be somewhere here. Right. And, corresponding to this, the value given in this particular table is coming to be 0.5.

It means, this table is measuring the value from minus infinity onwards. Now, we have to locate a point, you will find, this is coming to be - 1.69. You also know that, the normal distribution graph is symmetrical. So, I can as well find a value corresponding to Z is equal to 1.69. So, I will see, where this 1.69 comes. So 1.6 is here. And, I go on moving here.

And, I am in the last column, and I find. This is the value. So, corresponding to Z is equal to 1.69, so it would be somewhere here, 1.69 is somewhere here. So, 1.69 will be, - 1.69 would be, somewhere here. So, corresponding to 1.69, my probability is 0.9545. Right. Now, I will draw the figure, once again.

(Refer Slide Time: 18:06)



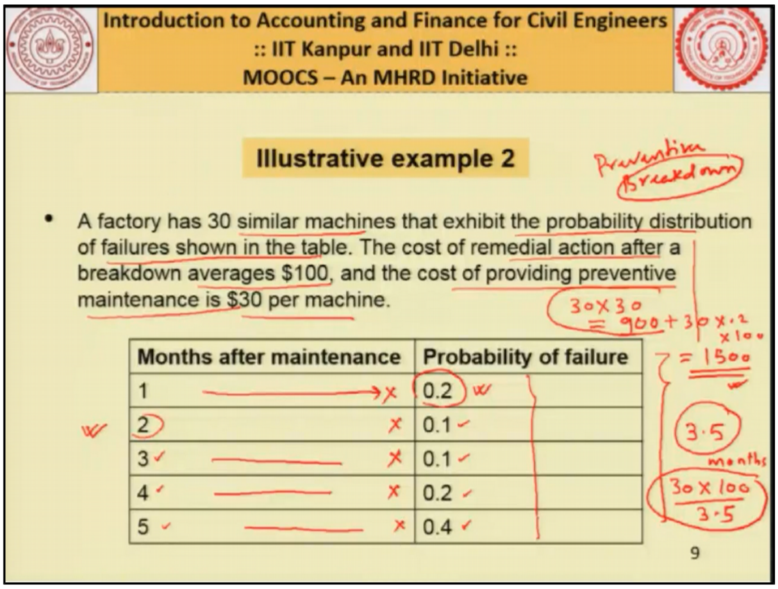
Now, that I have got corresponding to 1.69, my value is corresponding to Z is equal to 1.69, my probability is 0.95. How much was it? 9545. So, corresponding to this, it is this whole value is coming to be, 0.9545. That means, the area under this curve, up to here, all these areas are equal to, 0.9545. But remember, I am interested in, - 1.69. So, it is symmetrical. So, it would be somewhere here. So, I am interested in this area. Because, less than zero, area is represented by this.

I want, what is the probability of Net Present Worth, going below zero. So, if this whole area is 0.9545, this area is going to be, how much? It is going to be, 1 - 0.9545, which is roughly about 4.6%. So, this area is also 4.6%. So, what you find, the probability of making losses, in this investment proposal is, 4.6%. So, let us say, it is very close to 5. So, that means, 5 out of 100. If you invest in such proposals, you are likely to make losses.

Now, we would not have understood the losses, had we not carried out the Risk Analysis, as we have done now. Because, if you remember, in earlier cases, we were not at all bothered, to even look beyond the Net Present Worth. If we were getting a Net Present Worth in positive, here also, we would not have look beyond. But, just because we notice that, Sigma associated with this project is very high, it was coming to be 288, so even though my net present worth's mean value is 487, I am still likely to go for losses.

Of course, not very big losses, but still, there are chances. Now, this we would not have known, had we not carried out the Risk Analysis. So, this is the power of carrying out Risk Analysis. And, you know that, knowing this now, I can take a very wise decision, whether to go for this alternative or not. So, Risk Analysis in a sense is helping us, go into more detail, and to analyse whether, this particular option is worth pursuing or not. So, this is one very good application of Risk Analysis, we have discussed, in the context of Aggregated Cash-flows.

(Refer Slide Time: 21:05)



Now, connected to this, we take one another example in which, let us say, we have to decide, for some kind of a maintenance policy. So, what happens in Civil Engineering projects, if you look, there are large number of equipment. There would be equipment for excavation. There would be equipment for concreting. There would be equipment for hoisting. A large number of equipment. Now, you also have a set-up, to have repair work done, to have maintenance work done, for all this equipment.

In fact, you will have a separate department itself. We call it as, Plant and Machinery department. Now, one of the decisions, that they have to take is, whether to go in for preventive maintenance, or whether to go in for breakdown maintenance. Now, we have to understand these two terms, very carefully. What happens in breakdown maintenance, you are not at all bothered about, going for preventive maintenance. So, as and when the machine breaks down, you go and repair it.

Obviously, it requires more money, compared to the preventive maintenance. In preventive maintenance, you have a schedule for each of these equipment. Okay. This is a tower crane. This is a mobile crane. This is a batching plant. This will go for maintenance on, 1st of every month. This will go for maintenance on, 10th of every month. So, you have a quite a big schedule for each of these equipment already made. And, your plant and machinery in-charge, will be sticking to those schedule.

Now, what is done is, let us say, we are having a situation in which, we have to make a choice between, whether to go for a preventive maintenance, or whether to go for a breakdown maintenance. So, there is a particular organisation, which has collected this data, in the form of this particular table. Now, that organisation has got 30 similar machines. And, that exhibit the probability distribution of failures, as shown in the following table. It is also notices that, the cost of remedial action after a breakdown is averaging 100 Dollars.

That means, suppose some equipment has failed, it is under breakdown, and if you want to repair it, on an average, it is costing you 100 Dollar. And, if you are doing the preventive maintenance, the cost is very low, it is only 30 Dollar per machine. Now, the organisation has collected this data. And, based on that data, what they are saying is, there is a probability of failure of 0.2, that the equipment will go under breakdown, 1 month after the maintenance.

Likewise, 2 months after the maintenance, the probability of failure is 0.1. 3 months after maintenance, the probability of failure is 0.1. 4 months after the maintenance, probability is 0.2. 5 months after maintenance, the probability is 0.4. These data have been captured, based on past experiences. Now, as I have been telling you repeatedly, solving a problem is not a big deal. The formulation of problem, requires your time and effort, and it comes to you only with experience.

So, although this data has been captured, very crisply in a very small table, but believe me, collecting such data, takes lot of time and effort. Now, in this particular problem, we have to make a choice, whether to go for preventive maintenance, or whether to go for breakdown maintenance. Suppose, I go for breakdown maintenance, then what is the frequency, on an average of the breakdown for these equipment, so that I can calculate again, based on the concept of expected value.

So, you can find, 1 month after maintenance, probability is 0.2. So, 1 into 0.2. 2 months after maintenance, probability is 0.1. So, 2 into 0.1. 3 months after, this probability is 0.1. So, 3 into 0.1. Likewise, 4 into 0.2. And, this is 5 into 0.4. So, if you go and calculate it like this, you will find, you are getting this value as, 3.5 months. So, that means, 3.5 month is basically the frequency in which, machines are breaking down.

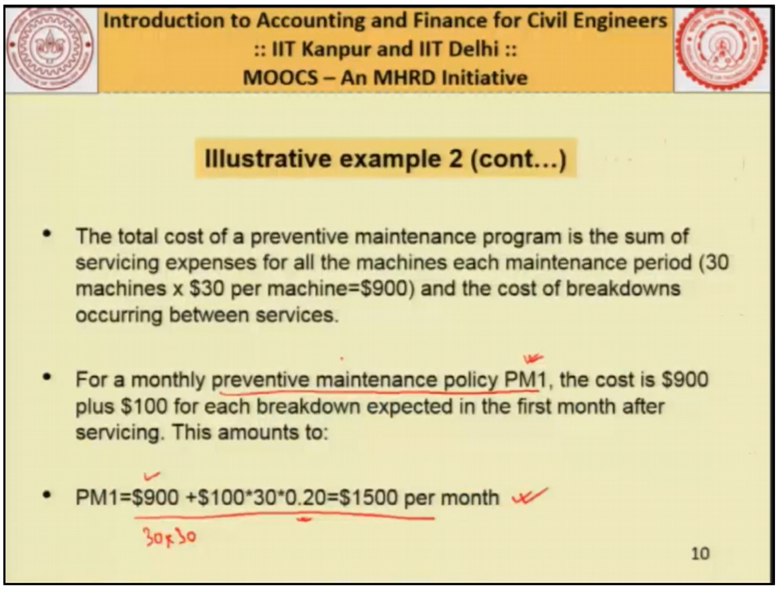
So, let us say, you have 30 machines. And, you are incurring 100 Dollar as breakdown maintenance cost. So, this will get distributed over a period of 3.5 months. So, this is the cost, that you have to incur, in case, you are going in for breakdown maintenance. Now, you have other strategies also available. For example, you can make a choice of having a preventive maintenance every month, or for that matter, every 2 months, or every 3 months, every 4 months, every 5 months.

Now, we would like to calculate, how much is going to be my cost implication. Should I go in for adopting, 1month period of preventive maintenance. So, how do I calculate it. So, looking at this table, 1 month after maintenance probability of failure is 0.2. So, 30 machines we are maintaining it at a cost of 30 Dollars, so 900 is the fixed cost. This is what, you have to pay, in case, you are going in for breakdown maintenance.

Now, even after this maintenance, what this table says, there is a 20% equipment, which is going to fail. So, that would require again, breakdown maintenance. So, 900 is your cost of going in for preventive maintenance. How? 30 machines at a rate of 30 Dollars. So, it is 900 Dollars. Now, out of this 30, you will find, 20% will still go for breakdown. And, for that, how much additional cost is needed.

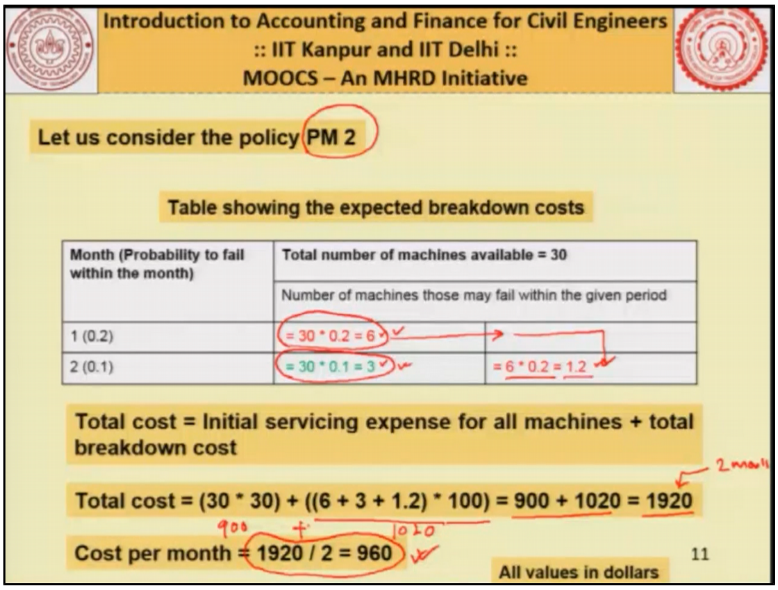
It would be, 30 multiplied by 0.2, multiplied by 100. So, what is going to be your monthly cost? 900 + 30 multiplied by 0.2 into 100. So, this is going to be 1,500. So, that means, if you are going in for monthly prevention maintenance, in that case, your cost liability is coming to be 1,500. Now, in the same manner, let us assume, I am going in for bi-monthly preventive maintenance.

(Refer Slide Time: 27:59)



So, how does cost come to? You will have to find it like this. This is what we did it for month 1. You can see, 900 Dollars. How it is coming? This is coming by multiplying 30 with 30. 100 Dollars per breakdown, 30 machines, and probability of failure 20%, so we are getting 1,500 Dollars per month, in case, you are going in for preventive maintenance policy every month. So, PM 1 we are denoting it by.

(Refer Slide Time: 28:27)

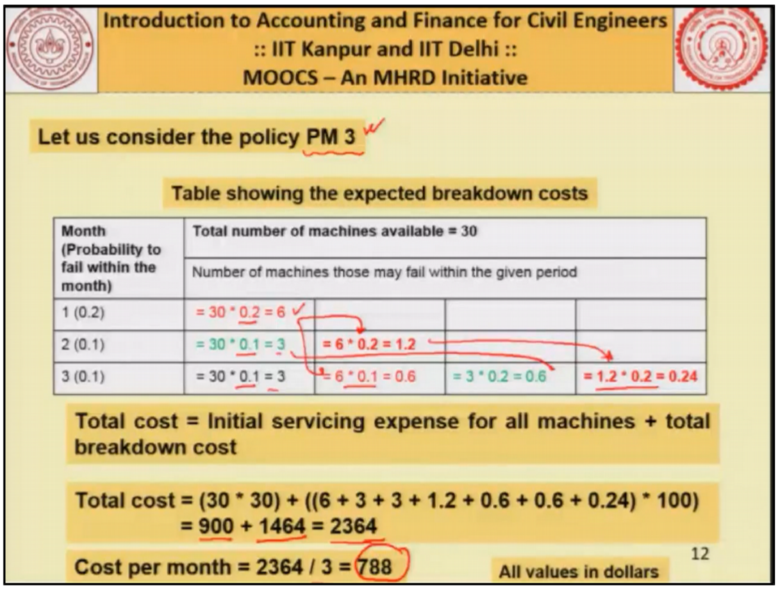


Now, in case of PM 2 policy, let us say, that means, every 2 months, you have decided to maintain the equipment, using preventive maintenance policy. So, in that case, the cost will be calculated like this. First month, out of 30 equipment, 20% will go under breakdown. So, you can see, 30 into 0.2, it is 6. Next month again, 30 into 0.1. If you go back to the table, it says, first month probability is 0.2, second month probability is 0.1.

So, I calculate this as, 30 into 0.1, which is 3. Now, one thing you have to notice here is that, these 6 equipment, which you have maintained in this month, 20% of that again may turn out to be under break down. So, you can see here, it is 6 into 0.2, the probability again is 0.2 for failure in case you have maintained it and it may fail even after 1 month. So, this value is coming to be 1.2. So, if you add, how many times you have to pay for your breakdown maintenance, it is 6 + 3, which is 9 + 1.2, so 10.2 here. And, the breakdown maintenance cost is 100 Dollars.

So, this is 1,020 here. And, 30 into 30, anyway you know because, every 2 months, we are maintaining 30 equipment, at a cost of 30 Dollars, so 900. So, total cost is coming to be, 900 + 1,020, it is 1,920. So, this is the cost implication for 2 months. So, per month, if you have to calculate, it is 1,920 by 2, it is 960. So, if we were doing preventive maintenance on a monthly basis, we were incurring 1,500. When we change it to bi-monthly maintenance policy, we are getting a cost implication of 960.

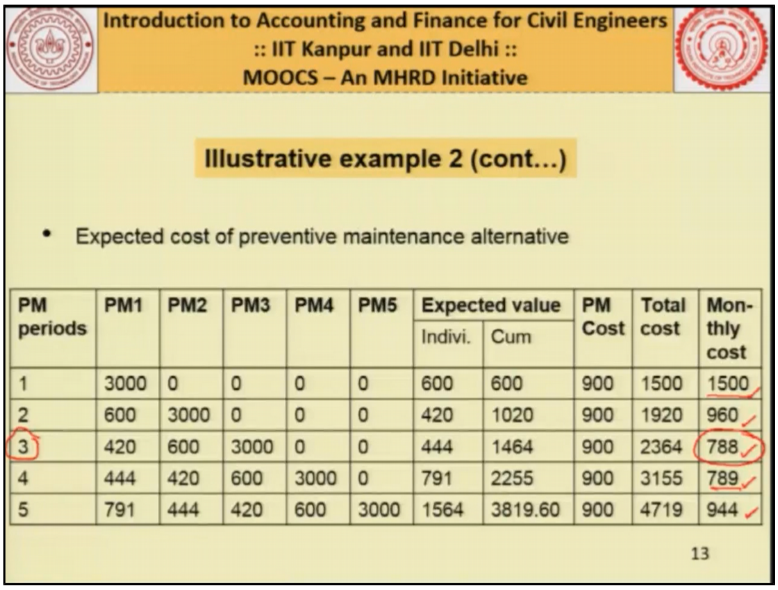
(Refer Slide Time: 30:30)



Now, in the same manner, you can do it by assuming that, you are going to maintain your equipment, every 3 months. We are calling this as, PM 3 policy. So, here also, as before, this is 30 into 0.2. Because, whatever equipment you maintained, there is a 20% chance of failure, in second month, 10% chance of failure, in third month, 10% chance of failure. So, 6 + 3 + 3 is here. Now, out of this 6, again 20%, out of this 6, again 10%, at the end of 2 months. So, this is it. Then, when it comes to this 3, here you have 3 into 0.2, from the previous table.

And, finally here is 1.2, here also 20% may go under breakdown, so 0.24. So, if you add all this, 6 + 3 + 3, which is 12 + 1.2, 13.2, 13.9, and this is finally, you are getting 900 here, + 1,464, you are getting 2,364. Right. And, this is for a period of 3 months. So, when you divide it for calculating the monthly cost, you are getting 788. Now, all these values, whatever we have calculated, I can put it in a separate table.

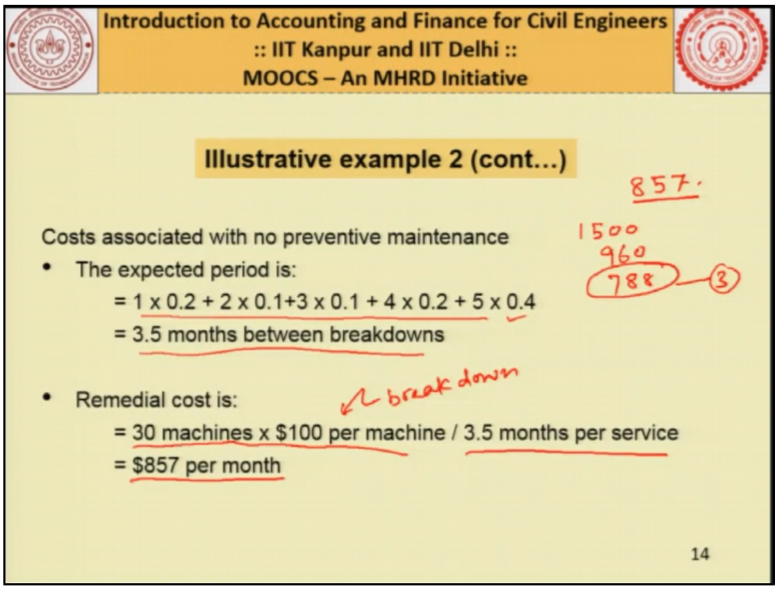
(Refer Slide Time: 31:46)



PM periods	PM1	PM2	PM3	PM4	PM5	Expected value		PM Cost	Total cost	Mon-thly cost
						Indivi.	Cum			
1	3000	0	0	0	0	600	600	900	1500	1500
2	600	3000	0	0	0	420	1020	900	1920	960
3	420	600	3000	0	0	444	1464	900	2364	788
4	444	420	600	3000	0	791	2255	900	3155	789
5	791	444	420	600	3000	1564	3819.60	900	4719	944

So, here you can see, if you look at the last column, if you are following a policy of preventive maintenance on a monthly basis, your cost is 1,500, bi-monthly 960, tri-monthly 788, 4 months 789, 5 months 944. So, you find your least cost is coming, when you are going in for a policy, which involves preventive maintenance, every 3 months' period. Now, you have already seen, the cost implication of breakdown policy.

(Refer Slide Time: 32:21)



So, this is how you remember, you got it. So, the time period between breakdowns is nothing but, the expected value 1 into 0.2, + 2 in to 0.1, + 3 into 0.1, + 4 into 0.2, + 5 into 0.4. So, it is 3.5 months. So, the cost is going to be distributed over 3.5 months like this. 30 machines, 100 Dollar per machine. This is the breakdown cost. Breakdown cost and divided by 3.5 months. So, this is coming to be 857.

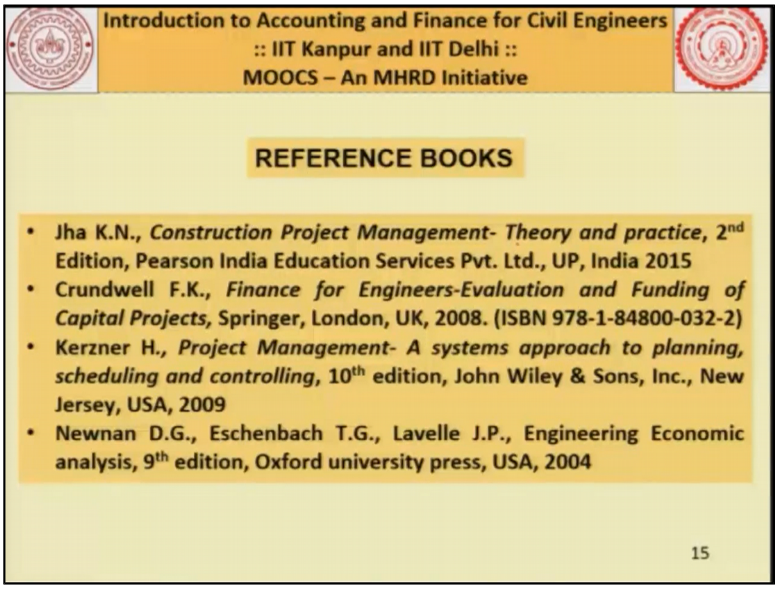
So, if you are going in for breakdown maintenance, it is costing 857. If you are going in for preventive maintenance monthly basis, 1,500. Bi-monthly basis, we are getting 960. And then, 788. So, this is 960, and 788. You find that, this is coming to be the cheapest one. So, what is the conclusion? We can go in for, the preventive maintenance policy, every 3 months.

So, these are some of the examples, which we can think of, in the context of Civil Engineering, wherein you can apply the concept of expected value, you can apply the concept of finding the risk, and so on. So, in this lecture, I have given you an overview of finding the risk, in Aggregated Cash-flows. I have given you the formulas, to calculate the expected value. I have given you the formulas, to derive the variance, and thereby the standard deviation.

Using the concepts of these two terms, and using the standard deviation, rather the normal distribution tables, you can easily find out, the risk associated with any proposal. So, if you remember, from the very beginning, we started with our assumption that, Cash-flows are certain. Then, we started changing them, and then we try to find out, what is the impact of the changes, on my overall decision.

Subsequently, we started assigning probability values also. And then, I am in a position to find out the risk associated with, all these investment proposals. Now, this would not have been possible, if we would not have involved, the concept of Risk Analysis. So, as before, I will just give you the reference books, quickly.

(Refer Slide Time: 34:40)



These chapters are covered in detail, in my book on, Construction Project Management Theory and Practice, published by Pearson. There are other books also, which are mentioned here. You can refer anyone of them, and try solving few similar examples, to get more clarity. So, with this, we stop this lecture, and see you some time, in other lecture. Thank you, very much.