Introduction to Accounting and Finance for Civil Engineers

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Module No. #04
Lecture No. #17
Risk Analysis (Part-1)

Good morning, Namaskar, and welcome to the course, once again. In the last lecture, we discussed about Replacement Analysis, in which we learnt, two methods of replacement of a particular asset. In the first case, we compared, a Defender with a Challenger. And, in the second case, we found out the ideal time, to replace an asset. In this class, we are going to start a new topic, which is called, Risk Analysis.

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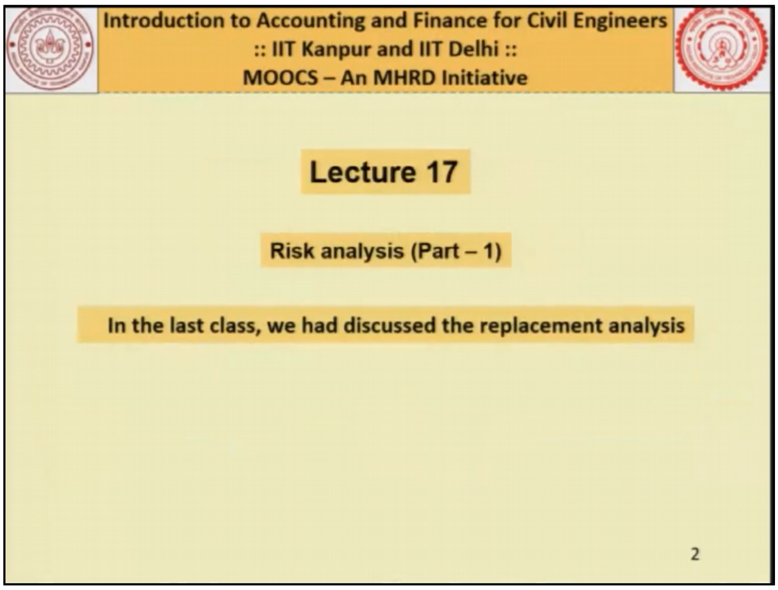

Now, Risk Analysis is part of the process of, Risk Management.

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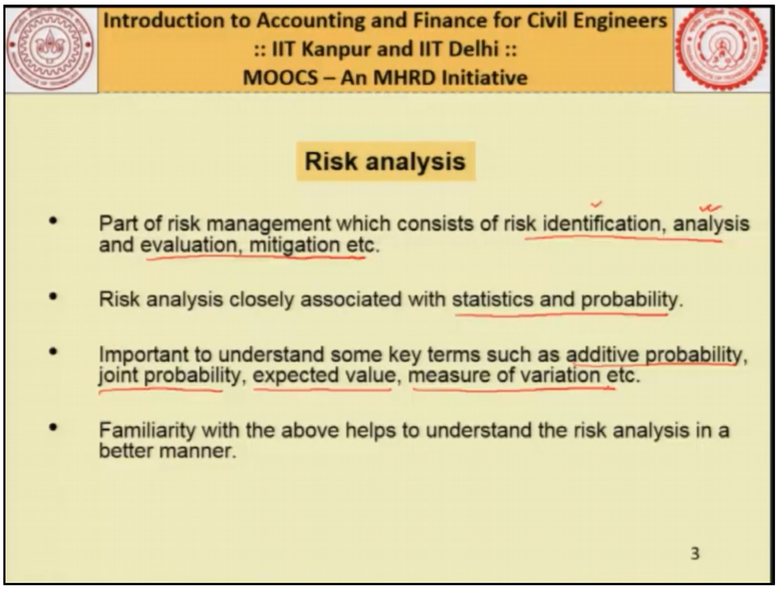

Now, Risk Management, if you see, it consists of, Risk Identification, Risk Analysis and Evaluation, and subsequently, we also have Risk Mitigation. Right now, we will be involved, only with Risk Analysis. We are not touching, the process of risk identification, neither we are touching, risk mitigation. Risk Analysis is very closely associated with, statistics and probability. So, we must understand, few concepts of statistics and probability, before we move into the details of, carrying out this particular Risk Analysis.

We need to understand, some key terms, such as additive probability. We need to understand, what is meant by joint probability, or multiplicative probability. We need to understand, what is meant by, expected value. And, we also need to understand, various measures of variation. These terms, if you understand in a clear manner, will help us to understand the Risk Analysis, in the context of a project management.

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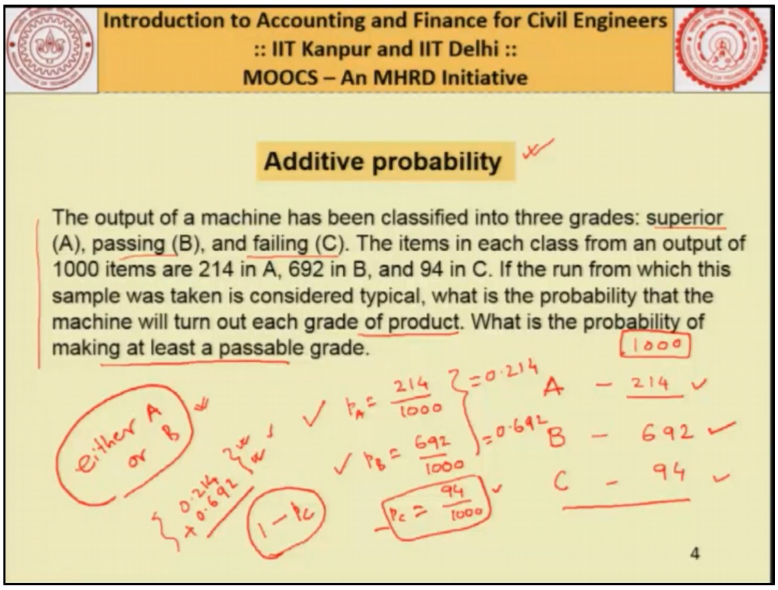

Now, first we take a small example, to understand the concept of additive probability. Now, in this example, it is told to us that, the output of a machine, has been classified into 3 grades, superior grade, passing grade, and failing. So, there is a machine, which is producing these products. And, these products can be classified into 3 grades. Superior grade, we are calling it as A. Passing grade, we are calling it as B. And, failing grade, we are calling it as C.

The items in each class, from an output of 1,000 items are, 214. So, out of 1,000 products, that we are producing, 214 can be classified in category A, 692 in category B, and 94 in category C. Now, if the run from which, this sample was taken, is considered typical, what is the probability, that the machine will turn out, each grade of product? So, each grade of product, what is the probability. So, out of 1,000, 214 we are getting A, 692 we are getting B, and 94 we are getting C grade.

So, the probability for each one of them, you know, is simple. For A, it is 214 by 1,000. For B, it is 692 by 1,000. And, for C, it is 94 divided by 1,000. So, these are the probability, probability of getting A, probability of getting B grade, probability of getting C grade. This is how, we calculate. Now, this also question that, what is the probability of making, at least a possible grade. Just read this line, very carefully, at least a possible grade. Now, possible grade is nothing but, either you produce A grade, or you produce B grade.

So, at least a possible grade is meaning, either A or B. So, whenever you find, you are getting an expression like this, either A or B, you have to add the probability. So, you will say, okay, probability of making at least a possible grade is, probability of making A grade + probability of making B grade. So, this becomes 0.214. This is for, A grade. For B grade, it was 0.692. So, + 0.692. So, you add it up. This is what, is your finding the probability of, at least a possible grade.

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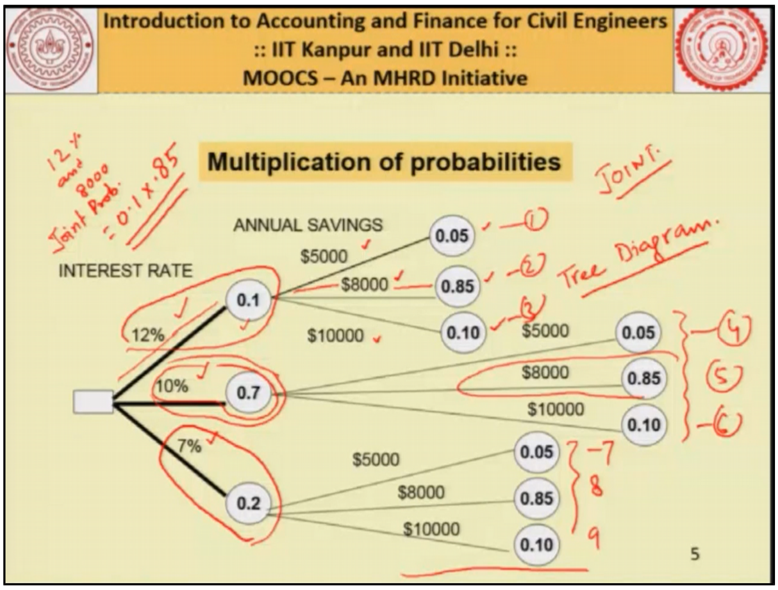

Now, we take another example, to understand the multiplication of probabilities. But, before that, I would also like to tell you, I could have solved this problem, in a slightly better manner also, or slightly different manner also. I could have found out, what is the probability of a failure grade, which is this. So, if I subtract this particular probability, 1 - PC, this is same as probability of making, at least a possible grade.

So, either you do, add these two probability, or subtract probability of a failing grade, from one, both will give you the same answer. But, since we are into additive probability, you just understand this concept that, if the expression is, either A or B, you do it like this. So, for this example, it is simple probability addition of 0.214 and 0.692. Now, we move to another problem, to understand the application of multiplication of probability.

Sometimes, this is also known as, joint probabilities. So, let us say, you have an investment proposal. And, in that proposal, the interest rate could be, either 12%, or it could be 10%, or it could be 7%. Remember, in our earlier problem solving, we were assuming, all these values to be constant. Then, when we understood sensitivity analysis, we started changing these variables. But, in sensitivity analysis also, we never considered probability, for each of these state of occurrence.

Here, what we are doing is, we are considering different probability values, corresponding to different interest rates. So, for example, there is a 10% probability, that the interest rate could be 12%. So, you can see there. There is a 70% probability, that interest rate could be 10%. And, there is a 20% probability, that interest rate is going to be 7%. Likewise, the annual saving values also can change.

It could be, either 5,000, or it could be 8,000, or it could be 10,000 dollars. The associated probability is 0.05. That is, 5%. 85%, corresponding to 8,000. And, 10%, corresponding to 10,000. Likewise, for different interest rate, the different possibilities of annual savings are there, and corresponding probabilities are also given. So, this is given, in the form of a Tree Diagram.

So, this figure, you also sometimes referred to it as, Tree Diagram. Now, suppose, somebody tells me, what is the probability that, interest rate is 12%, and annual saving is 8,000. So, we are saying, interest rate 12%, and annual saving 8,000. So, I follow, this path. I go along this 12%, the probability is 0.1. And then, I moved to this path, the probability is 0.85, corresponding to 8,000 dollars. So, the joint probability, or the multiplication of probability, will be done like this.

Joint probability of what, interest rate being 12%, and annual saving of 8,000, is given by, 0.1 multiplied by 0.85. So, this is how, you calculate joint probability. You can calculate, the joint probability, along each of these path. So, you can see, in this particular example, you have a total of 9 path. So, Path number 1 here, 2 here, 3 here. Likewise, Path-4 here, 5, 6, 7, 8 and 9. So, for each of these paths, we can calculate the joint probability, and we can solve the problem there onwards.

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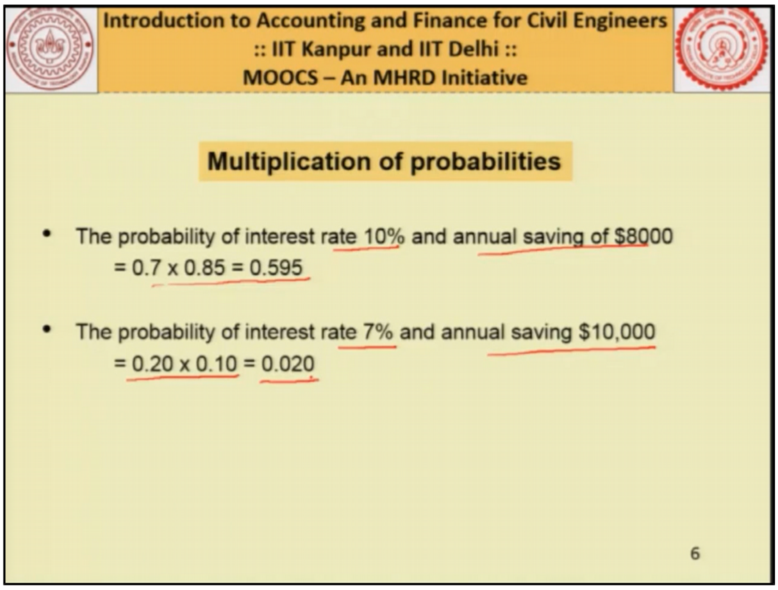

So, this is what, it says. The probability of interest rate, let us say 10%, and annual saving 8,000, is given by, let us say 0.7. So, it is 10%. So, this you can see. 10%, the probability is 0.7. 8,000, the probability is 0.85. So, the probability that, interest rate becomes 10%, and annual saving becomes 8,000, is given by, 0.7 in to 0.85, which is 0.95.

Probability of interest rate 7%, and annual saving 10,000. So, rate 7%, and annual saving 10,000, let us see, go back to the previous problem. So, 7% the probability is 0.2, and 10,000 the probability is 0.1 so, you multiply these two value, 0.2 and 0.1, and you are getting a joint probability of, 0.02. So, we have seen, how to add probability values, and how to multiply probability values.

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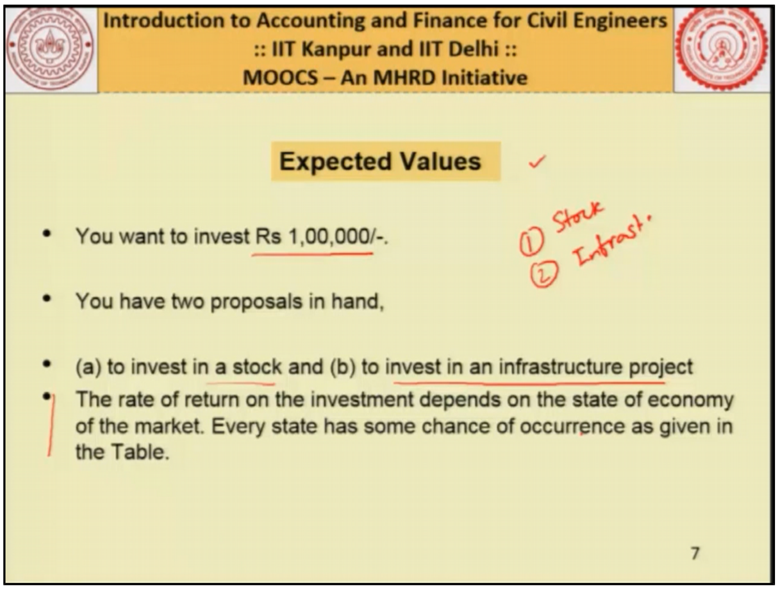

Now, we will see, one very important concept, that is, expected value. This is very important to understand, in the context of, Risk Analysis. And, this also, we would like to go with the, small problem. Let us say, you have two investment proposals, for investing Rupees 100,000. First, you can invest in a stock. So, stock option is the first one.

And, second, you can invest in an infrastructure project. So, second, you can understand, there are different types of infrastructure bond, you can invest therein. Now, the rate of return on the investment, depends on the state of economy of the market. And, every state has some chance of occurrence, as given in the table. We will show you the table.

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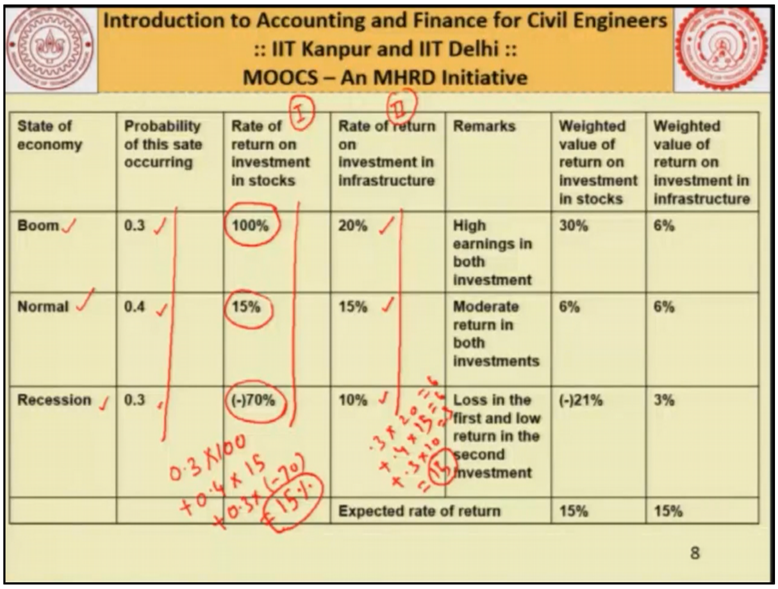

State of economy	Probability of this sate occurring	Rate of return on investment in stocks	Rate of return on investment in infrastructure	Remarks	Weighted value of return on investment in stocks	Weighted value of return on investment in infrastructure
Boom	0.3	100%	20%	High earnings in both investment	30%	6%
Normal	0.4	15%	15%	Moderate return in both investments	6%	6%
Recession	0.3	(-)70%	10%	Loss in the first and low return in the second investment	(-)21%	3%
			Expected rate of return		15%	15%

So, look at the table. The state of economy could be, either booming, or it could be normal, or it could be recession. The probability that, booming economy prevails is 0.3. The probability that the normal economy prevails is, 0.4. And, there is a 0.3 probability, that recession prevails. So, the probability values for, each one of these state of economy, is also known. Now, for the two option, the first option is stock option, and the second one is, investment in infrastructure bond.

So, if you have invested in stocks, and if there is a booming economy, you are likely to get 100% return, on your investment. If it would be normal, you are getting 15%. And, if it is a recession, you are likely to lose money. So, that means, your investment may give you, a return of - 70%. So, this is, as far as, stock option is concerned. So, we can see, depending on the state of economy, my rate of returns is changing.

In the booming market, I am expected to get a, very high return. On the other hand, if there is a recession, I am going to get a, very negative return. On the other hand, for the infrastructure option, booming economy, you are likely to get 20%. Normal economy, you are likely to get 15%. And, recession, you are likely to get 10%. So, all of the returns, are not very high. But, in all the three cases, you are still likely to get, a positive rate of return. Now, the question is, in which option, you will invest your money.

Whether you would invest your money, in the stock option, or whether you would like to invest your money, in the infrastructure option. So, here comes, the concept of expected value, that will be very useful to you, for such analysis. So, if you want to calculate the expected value, which is nothing but, the weighted average. So, what I do? I multiply the probability, with the return value, expected return value, for all the states of economy.

So, for example, for a stock option, if you find, it we will be like this. 0.3 multiplied by 100, + 0.4 multiplied by 15, + 0.3 multiplied by - 70. So, this is 30, this is 6. So, 36 - 21. So, this is 15%. So, the average value, the expected value, likely for the stock option is, 15%. On the other hand, if you take the infrastructure option, you can do the same calculation, again.

You multiply, this 0.3, with 20%. So, it is 0.3 into 20, + 0.4 into 15, + 0.3 into 10. So, this is 6 + 6 + 3. So, again, you are getting, 15%. So, as far as, the expected value is concerned, both are equal, and that is 15%. So, can you tell me, which option will you go. Whether, you would like to invest in stock option, or whether you would like to go for infrastructure bond option. In both the cases, we are getting a return of, 15%.

Now, you have to understand, expected value like this, it is not that, if you are investing only once, you are likely to get 15%. But, when we talk of expected value, the number of times you have to invest, is quite large. So, maybe, you have to invest, may be 100 times, 200 times, in the similar manner, then only, you are likely to get 15% return, both from the stock option, as well as from the infrastructure bond option.

So, whenever we talk of expected value, the number of times, is quite important. It is not for only once, or only twice, it is for a large number of times, you continue to play the same game, again and again. And then, you expect that, you are likely to get a return of 15%.

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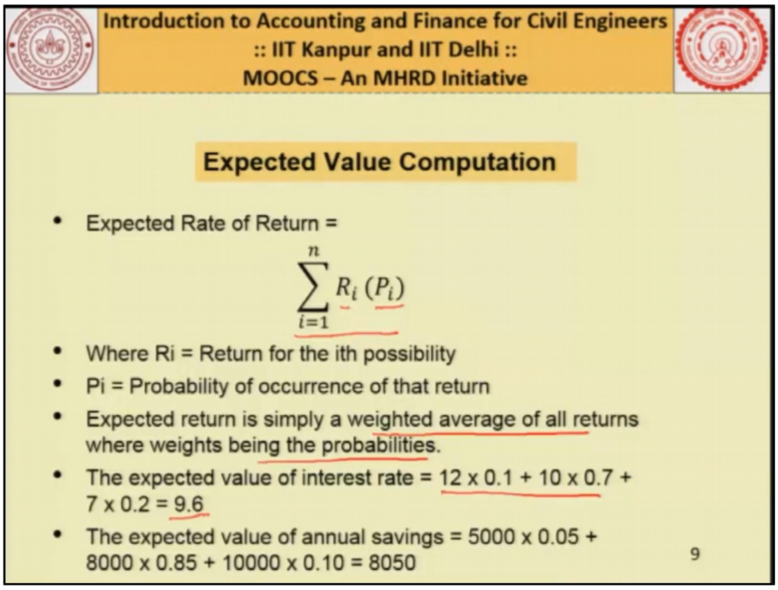

So, this is how, if you want to derive a formula, you can derive it, like this. Expected rate of return, you can calculate it, by multiplying this return, for the Ith possibility, with the probability of occurrence of that return. And, as you now understand, expected return is simply a weighted average, of all returns, where weights, are nothing but the probabilities.

So, as I told you, the expected value of, let us say, if we calculate the interest rate for the previous problem, if you want to find the expected interest rate, for this particular problem, you can see, the probability that, 12% interest rate would prevail is, 0.1, 10% would prevail is 0.7, and 7% would prevail is 0.2. So, the expected value of interest rate will be, 12% multiplied by 0.1, + 10 multiplied by 0.7, + 7 multiplied by 0.2. So, this is how, you calculate the expected value of interest.

Here also, you can calculate, the expected value of annual saving. It is nothing but, let us say, for this particular example, you can calculate, 5,000 multiplied by 0.05, + 8,000 multiplied by 0.85, + 10,000 multiplied by 0.10. So, this is the expected value of annual saving, corresponding to 12%. So, this is how, you calculate the expected value. In this case, you can see, the value is 9.6, for the interest rate part, and annual saving, you are getting 8050, as expected value.

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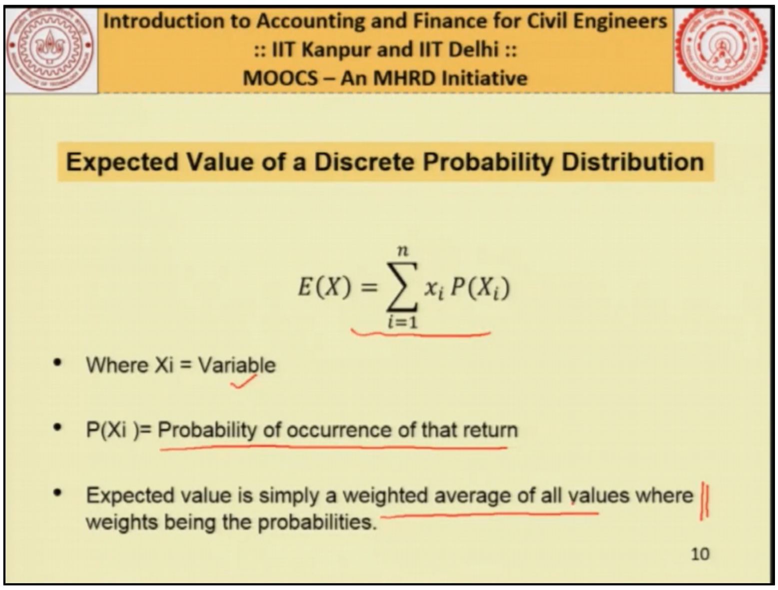

Sometimes, you also express, the expected value of a discrete probability distribution, in this manner, it is XI multiplied by, probability of XI. So, XI is your variable, and probability of XI is, probability of occurrence, of that particular return. And, now you understand, I mean, expected value is simply a weighted average, of all values, where weights being the probabilities.

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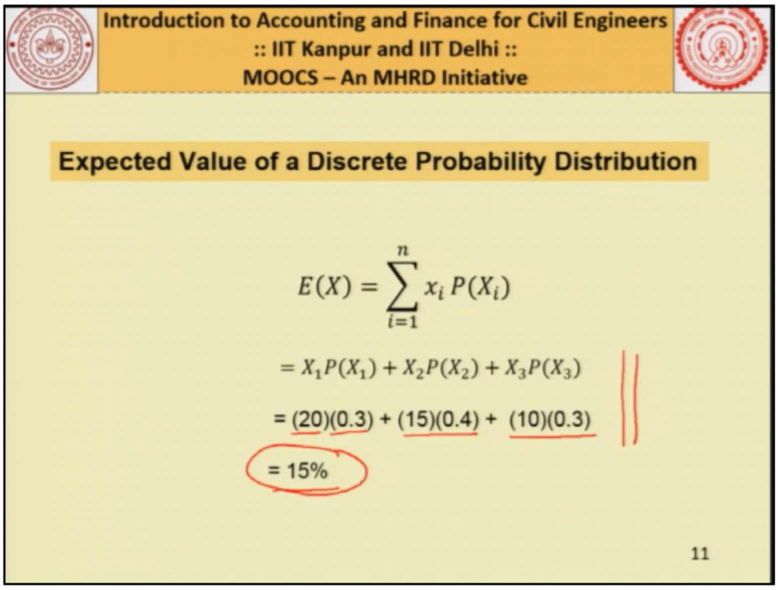

So, coming back to this particular problem of, infrastructure bond option. So, you can see, 20% is corresponding to booming economy. So, probability is 0.3, for booming economy. So, 20 multiplied by 0.3, + 15 multiplied by 0.4, + 10 multiplied by 0.3. Total, 15%. So, this is the expected value, from the infrastructure bond option. In the same way, you could have calculated the expected value, for the case of stock option. Now, we illustrate the concept of expected value, in the context of, evaluation of alternative.

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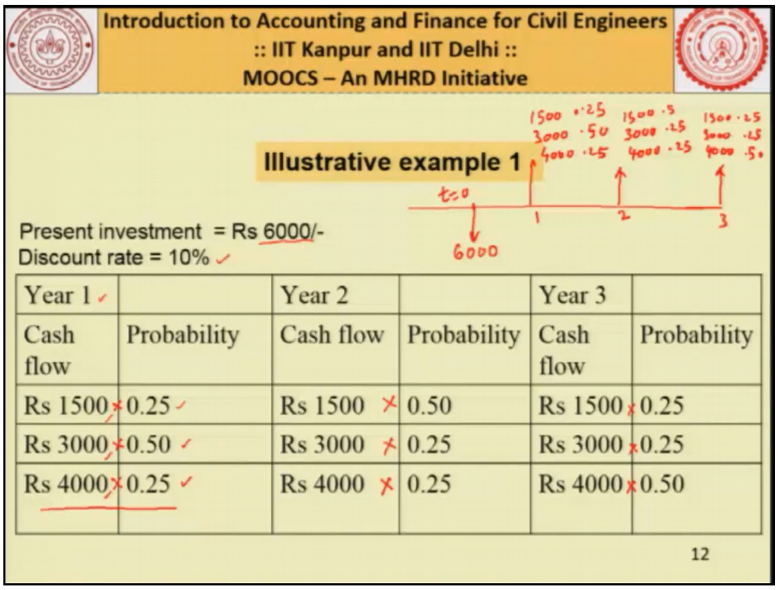

Year 1		Year 2		Year 3	
Cash flow	Probability	Cash flow	Probability	Cash flow	Probability
Rs 1500	0.25	Rs 1500	0.50	Rs 1500	0.25
Rs 3000	0.50	Rs 3000	0.25	Rs 3000	0.25
Rs 4000	0.25	Rs 4000	0.25	Rs 4000	0.50

For this, let us take this small example, in which, we assume that, the investment at time T is equal to 0, is Rupees 6,000. Let us consider, the discount rate to be 10%. Now, you can see, in the table, which I have given you. I have given you, the cash flows for, year-1, year-2, and year-3. For year-1, the cash flows could be, 1500, 3,000 or 4,000. And, the associated probability values, are also given. It is, 0.25 here, 0.5 here, and 0.25.

So, this is the difference between, the earlier cases, and the current case. In the earlier cases, if you remember, we were taking the cash flows as fixed and certain. They were not changing. But, when we started studying the concept of sensitivity analysis, we change these values. But again, we did not assign any probability values, corresponding to those values. But, here in the context of Risk Analysis, we are assigning probability values also, for the different cash flows.

So, for year-1, if I draw the cash flow diagram, it would be something like this. At, time T is equal to 0, I am investing 6,000 Rupees. For year-1, I have got, three probability values given. Corresponding to 1500, the probability is 0.25. It could be 3,000, the probability is 0.5. It could be 4,000, the probability here is 0.25. So, this for year-1. Likewise, for year-2 also, my cash flows are, 1500, 3,000 and 4,000. And, probability values are also known.

It is 0.5, 0.25, and 0.25. For year-3, the values are also given, 1500, 3,000, and 4,000. And, here also, the probabilities are, 0.25, 0.25, and 0.5. So, this is for year-1. This is for year-2. And, this is for year-3. This is, at time T is equal to 0. So, this is how, your cash flow diagram looks like. Now, what I do, in the first step is to, find the expected value, for each of these cash flows, for year-1, year-2, and year-3.

So, how do I calculate it? I multiply, this 1500 with 0.25, 3,000 with 0.5, and 4,000 with 0.25, and I get certain value. Likewise, for year-2, I multiply, these two values. For, year-3 also, I multiply them, like this. So, when you multiply this, you will get a cash flow diagram, something like this.

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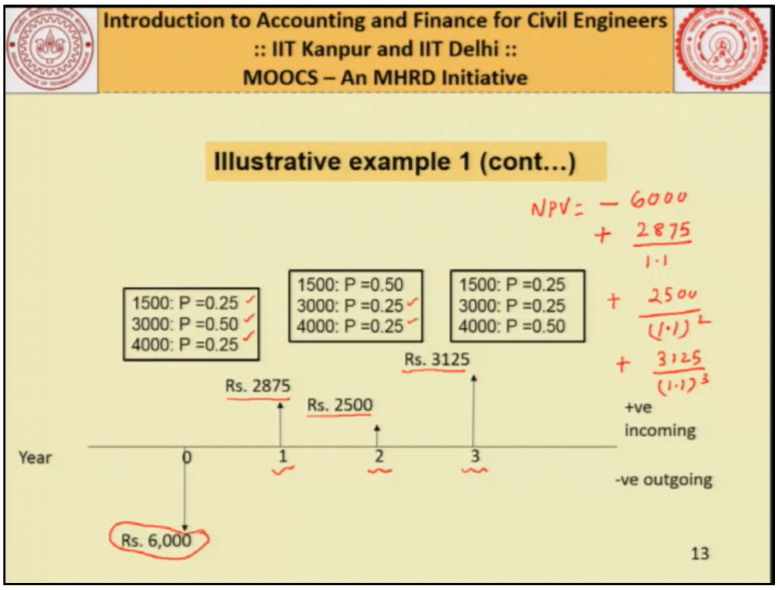

So, at time T is equal to 0, there is no change. This 6,000, at the time T is equal to 0, in the negative side. When you multiply this, 1500 with 0.25, 3,000 with 0.5, and 4,000 with 0.25, you will get a value of 2,875, which will be the expected cash flow, for year-1. For year-2, it is 1500 multiplied by 0.5, + 3,000 multiplied by 0.25, + 4,000 multiplied by 0.25, we will get a value of Rupees 2,500.

That is the expected cash flow, for year-2 end. And likewise, I can get the expected value for year-3 end also. So, you now understand the distinction between, the earlier cash flow diagrams, and the current cash flow diagram. In earlier cash flow diagrams, we never considered, the probability values. But here, we are considering probability values, corresponding to each of the expected cash flows.

Now, how do I calculate the net present value, for such situation. It would be given by, net present value is equal to, - 6,000 + 2,875, divided by 1.1, + 2,500 divided by 1.1 raised to power 2, + 3,125 divided by 1.1 raised to power 3.

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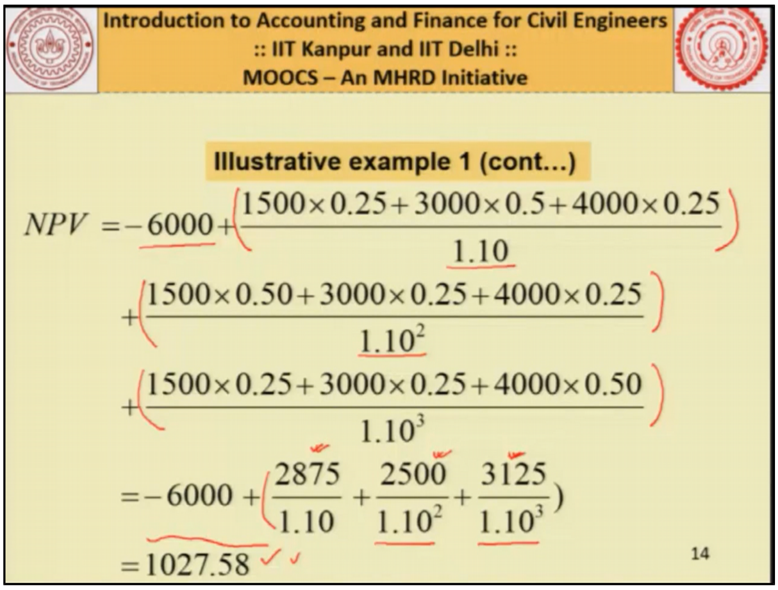

So, this, you could have also done, in a systematic manner, as shown in this particular slide. So, - 6,000, there is no issue here, + 1500 multiplied by 0.25, + 3,000 in to 0.5, + 4,000 in to 0.25, divided by 1.10. Plus, for the second year, this is going to be the case. 1500 multiplied by 0.5, + 3,000 multiplied by 0.5, + 4,000 multiplied by 0.5, divided by 1.1 raised to power 2. And, for the third year, this is how, it is done.

So, if you see, it will be, - 6,000 + 2,875, which you have already seen, + 2,500 divided by 1.1 raised to power 2, 3,125 divided by 1.1 raised to power 3. So, if you sum it up, you will get a value of, 1027.58. So, despite these probability values, you find that, the net present value, the expected value of net present value is, 1027.58.

In earlier cases, we were assuming, all these values to be constant. But now, we are assigning probabilities. And still, as far as the expected value is concerned, we are getting 1027.58. Now, we will reinforce this understanding, with the help of one separate example.

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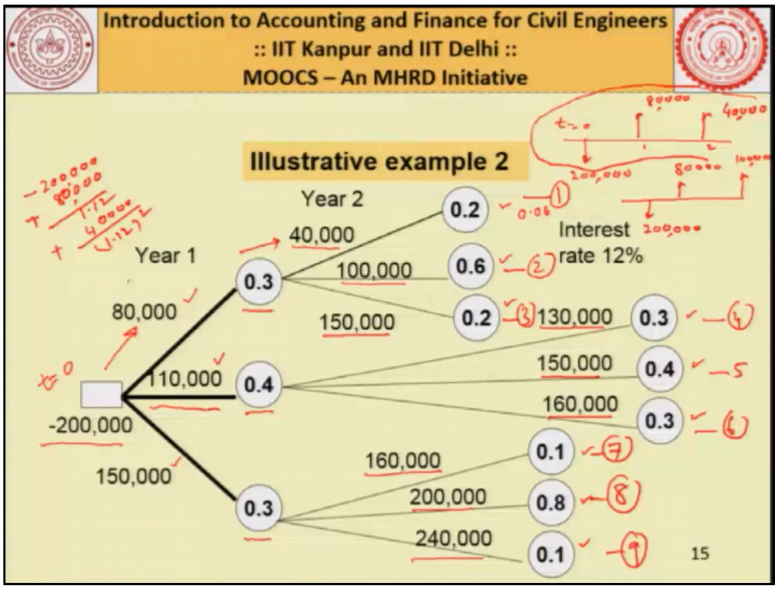

Let this example be, like this. I have an investment proposal in which, I am investing 200,000, at time T is equal to 0. I am likely to get a return of 80,000, or 110,000, or 150,000, in year-1. The probability values are also known. It could be, 0.3 corresponding to 80,000, 0.4 corresponding to 110,000, and 0.3 corresponding to 150,000. The return for the year-2, are also expected.

It could be, either 40,000 with associated probability of 0.2, 100,000 with associated probability of 0.6, and 150,000 with associated probability of 0.2. Corresponding to the 110,000 values, the expected return for year-2 could be, 130,000, 150,000, 160,000. And, associated probabilities are, 0.3, 0.4, and 0.3. Likewise, this value could be, 160,000, 200,000, 240,000, with associated probability, 0.1, 0.8, and 0.1.

Now, so far, what we have been doing, we have been drawing a cash flow diagram, like this. So, let us say, at time T is equal to 0, it is 200,000. And, year-1, 80,000. We are following, this path. Year-2, it is 40,000. This is not to scale, year-1, year-2. So, this is corresponding to, first path. We are calling this as, Path-1. Likewise, my cash flow diagram for the second path, would have been like this, 200,000, 80,000, and 100,000.

So, knowing the concept of, either present worth or annual cost, I can find out the corresponding values. Let us try to do this, using Present Worth Method of Analysis. So, for this path, I can calculate the present worth. For Path-1, It could be given by, - 200,000 + 80,000, divided by, let us say, what is the interest rate, 12%, 1.12 + 40,000, divided by 1.12 raised to power 2.

So, this will be the net present worth, for Path-1. Now, you already understand, the concept of joint probability. So, the probability that, 80,000 would be the return in year-1, and 40,000 would be the return in year-2, would be given by, multiplication of 0.3 and 0.2. So, corresponding to Path-1, my joint probability is 0.06.

Likewise, I can calculate, the net present worth, and the joint probability, for each of these paths. Path-2, Path-3. Likewise, for Path-4, 5, 6, 7, 8 and 9. So, in total, I have 9 paths. So, for each of these 9 paths, I can calculate the joint probability, and I can calculate the net present value, as shown for Path-1. I can draw the cash flow diagram, like this. And, I can very well calculate the, net present value.

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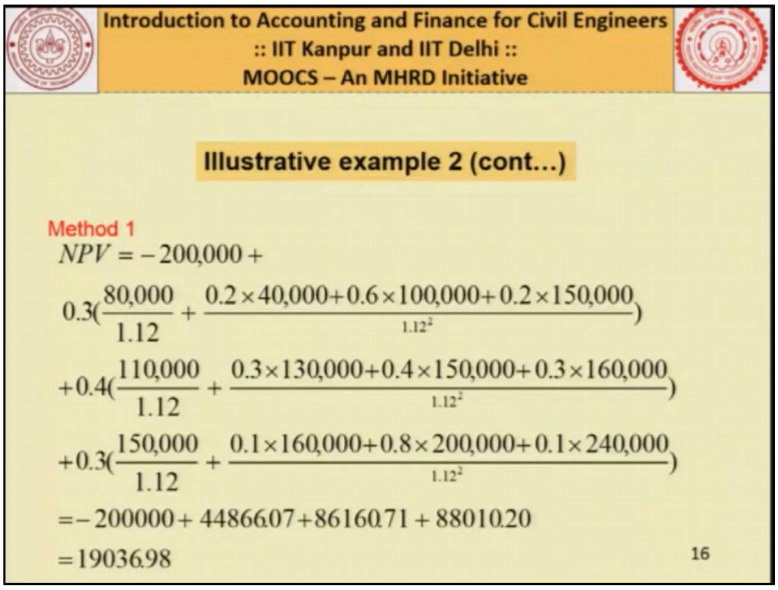

So, I can draw a table, something like this.

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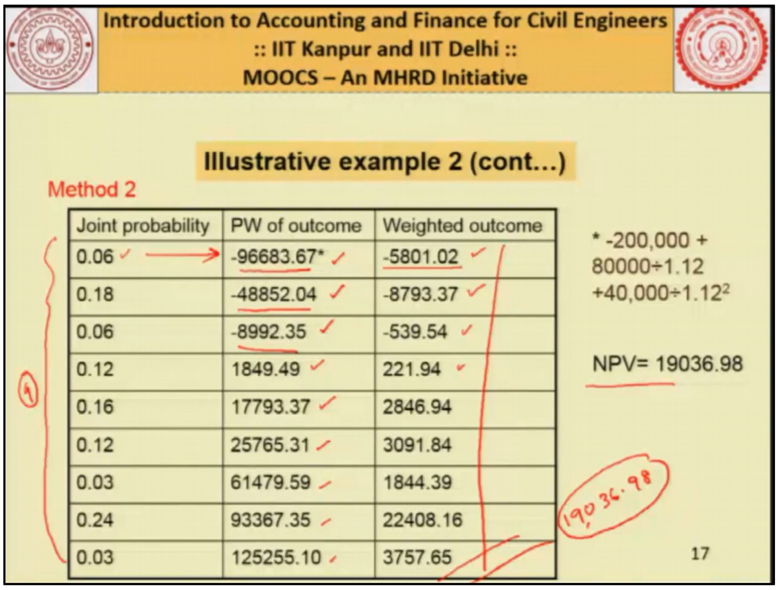

So, you can see, corresponding to different paths, I have calculated the joint probability. So, these are, in total, 9 paths. 1, 2, 3, 4, 5, 6, 7, 8, 9. So, total of 9 paths. Now, for each of these 9 paths, I can calculate the present worth. For example, for Path-1, the present worth is coming in negative. For path, this, again negative. For this path, again negative. Then, we are getting positive net present value, corresponding to this path, this, this, this, and this.

Now, what I do? I multiply this joint probability, with this net present worth, and I get the expected outcome. So, if you multiply 0.06 with - 96,683.67, you are getting a value of, -5,801. Likewise, for this path, we are getting, this weighted outcome. For, all these 9 paths, I can very well calculate. And, in order to find the net present value, what I am doing, I am adding all these weighted outcomes. So, you will find, you are getting a value of, 19,036.98.

You can crosscheck this calculation. So, the idea is, using the concepts of joint probability, and using the concept of net present value, we can always find, what is the expected value of net present worth. Now, another thing, that is of importance to us is, to understand the measure of variation. If you remember, the problem in which, we had two options, stock options versus infrastructure bond option.

In both the cases, we got an expected value of 15%. So, how to take decisions, under those circumstances. If I had to categorise, the riskier option, out of the two, how do I categorise? How do I measure, the term, risk? So, you will find, there are different ways in which, I measure these terms, risk.

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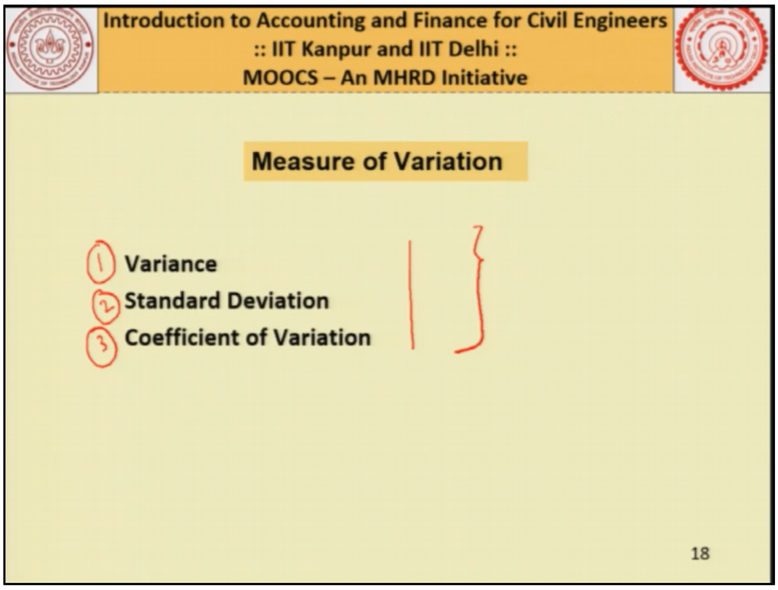

And, one of the terms, that we use for measuring risk is, variance. Then, we have a standard deviation. And, in some cases, we find, we also use the term, coefficient of variation. So, all these terms, again we are going to learn, with the help of small example. Some of these examples, have already been done. So, we just quickly find out, how to calculate the variance, how to calculate the standard deviation, and under what circumstances, we use the term coefficient of variation.

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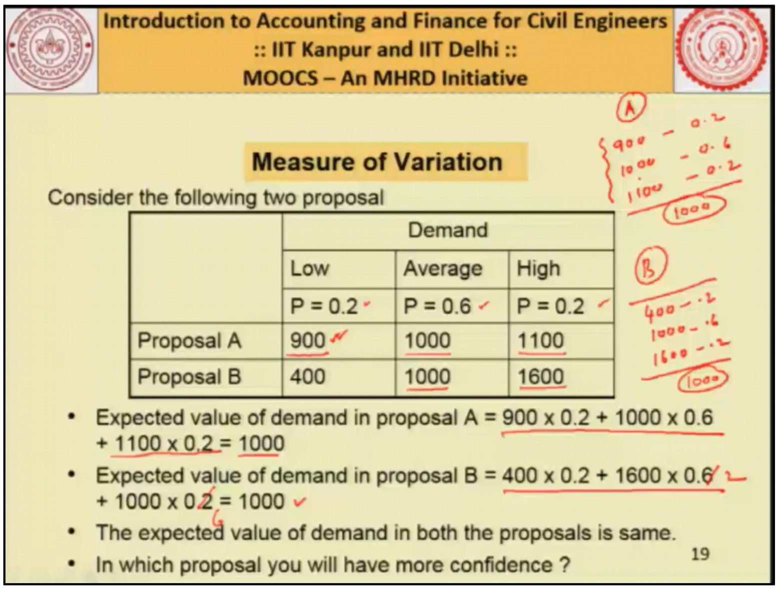

So, let us try to, go for a new problem, to understand the concept of measure of variation. For this, let us consider, two proposals in which, the demands are being forecasted. In proposal-A, the demand is forecasted to be 900, and probability of its occurrence is 0.2. 1,000, the demand probability is 0.6. 1,100, the probability is 0.2. In the second proposal, corresponding to 400 demand, the probability is 0.2. Corresponding to 1,000 demand, probability is 0.6. And, corresponding to 1,600 demand, probability is 0.2. Right.

So, for Proposal-A, you have 900, 1,000, and 1,100. So, these are the three forecasts of the demand. The probabilities are, 0.2, 0.6, and 0.2 again. This is for Proposal-A. For Proposal-B, we are given, the likelihood of demand values as, 400, 1,000, and 1,600. The probabilities are, 0.2, 0.6, and 0.2 again. So, now you understand, how to calculate, the expected value of the demand. So, it would be simple multiplication, 900 multiplied by 0.2, + 1,000 multiplied by 0.6, + 1,100 multiplied by 0.2.

So, the expected value of demand in Proposal-A is, 1,000. Now, for the second case also, if you calculate, 400 multiplied by 0.2, + 1,000 multiplied by 0.6, + 1,600 multiplied by 0.2, you will find this, 1,600 multiplied by 0.2, and 1,000 multiplied by 0.6, we will get a value of 1,000, again. So, here also, you find, the expected value, in both the cases, are same. Now, the question is, which one out of this, is a riskier one, as far as demand forecast is concerned.

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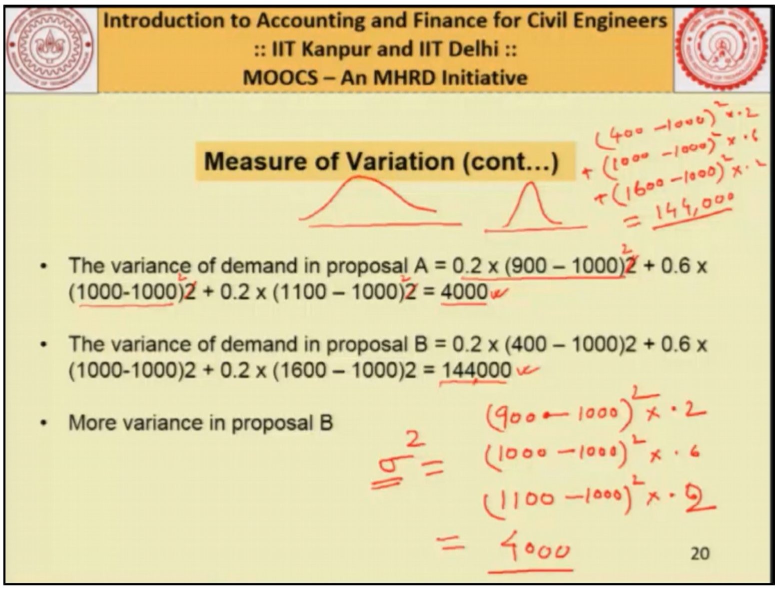

For that, we calculate a term called, variance. And, variance is calculated like this. For the first case, it would be 0.2, which is the probability corresponding to 900, and average demand, you have already calculated it to be 1,000, this power 2, + 0.6 into 1,000, - 1,000 raised to power 2. So, these are not multiplication. These are rather, raised to power 2. So, you get a value of 4,000.

So, if you see, 900 was the value, then another value was 1,000, then another value was 1,100, you got average value of, 1,000. So, 900 - 1,000 whole square, and we are multiplying with probability value. Likewise, 1,000 - 1,000 whole square, we are multiplying it with 0.6. And, 1,100 - 1,000 whole square, we are multiplying it with 0.2. So this is how, you calculate variance. Variance is denoted by, sigma square.

And, if you calculate, you are getting 4,000. Same manner, we can calculate for option B also. So, it we will be 400 - 1,000 whole square, multiplied by 0.2. Then, you have for 1,000 - 1,000, + 1,000 - 1,000 whole square, multiplied by, how much was the probability, it was 0.6 + what was for the last case, 1,600 - 1,000 square, multiplied by 0.2. If you add it up, you will find, you are getting a value, 144,000.

So, for the first case, the variance is 4,000. Whereas, for the second case, the variance is 144,000. So, this variance is a major of spread. So, how far, your demand is a spread, that is what is being captured, by your variance. And, there is another term called, standard deviation, which is nothing but, sigma. So, if you take the under-root of this, you get the sigma value. So, these two terms captures, how far, your values are spread, from the mean.

So, if you remember, the concepts of normal distribution, you will find that, if for a distribution, the variance is very large, its spread is very large. If the variance is small, you will find, the spread is very small. So, variance is also sometimes, giving you an idea about the spread. How far, scattered your values are, from the mean values. Right. The more scattered your values are, the lesser would be your confidence, in that particular Proposal. And, the more-riskier, would be the proposal.

In the first case, if you see the values, we were getting, 900, 1,000, and 1,100. So, the spread is very low. 1,100, and 900, these are the range. 1,100 - 900, only 200. But, in the second case, if you see, the values were, 400, then 1,000, and 1,600. So, the values have spread, too much.

So, if you see the difference, 1,600 - 400, you are getting a spread of 1,200. So, the variance is very large here. So, we do not have, that much confidence, in this particular demand forecast. And, that is why, we call that, the second proposal, that is Proposal-B, is the riskier one, compared to the first one.

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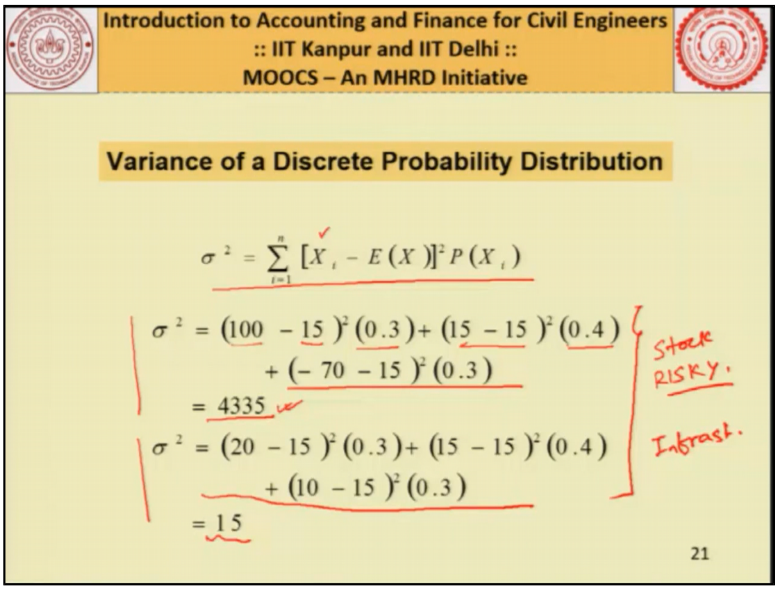

So, this is how, you calculate the variance. XI - expected value of X raised to power 2, multiplied by probability of XI. So, using this concept, we can calculate the variances, for any situation. This was the case, when we considered two alternatives, one was the stock option, and another one was the infrastructure option.

So, if you remember, in the first case, if we were going in for stock option, and it was booming, you were getting a return of 100. The expected value of return was 15. So, I multiplied with 0.3, square of this. In the second case, corresponding to normal economy, the return was 15%. So, 15 - 15 whole square, multiplied by 0.4. And, when it was recession, we were getting a negative return of, - 70.

So, - 70 - 15, when you multiply it with 0.3, you are getting a value of, 4,335. So, stock option, you see, the variance is 4,335. On the other hand, if you calculate the variance, for the infrastructure option, you are getting it, only 15. So, you find that, the stock option is very risky. So, this sigma square is a measure of risk, associated with any particular proposal.

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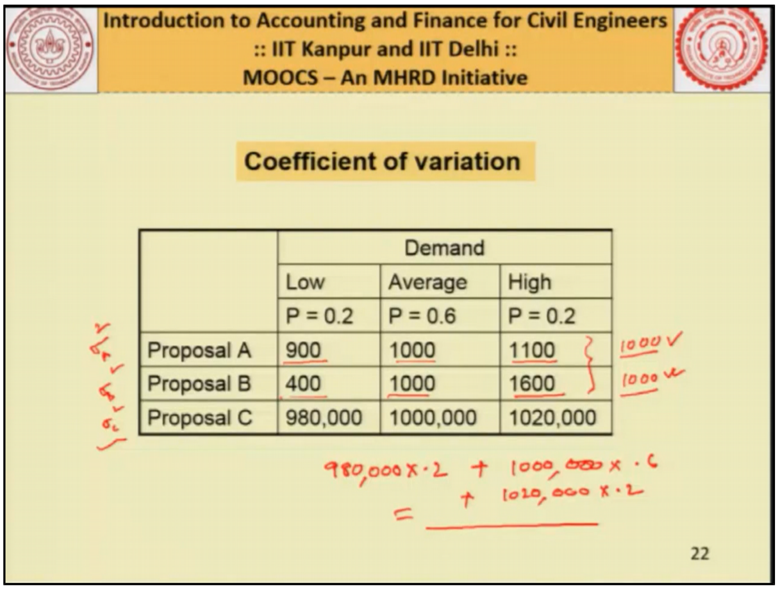

Now, there is another term called, coefficient of variation, which is also sometimes used. Because, in some cases, variances may not give you a correct picture. Now, let us take Proposal-C, and superimpose it with another two proposals, Proposal-A and B. So, for Proposal-A, already you know, the demand forecast is 900, 1,000, and 1,100. For Proposal-B, it is 400, 1,000, and 1,600.

So, the mean value for this, we had got 1,000, and for this also, we had got 1,000. So, this was the expected value, or the mean value. For this case, the mean value can be calculated, like this. 980,000 multiplied by 0.2, + 1,000,000 multiplied by 0.6, + 1020,000 multiplied by 0.2. So, you will find, the expected value is very high here, compared to this, 1,000 and 1,000 here.

So, under such situation, if you try to calculate, the variance for A, for B, you will find, the variance value for C, would be coming to be very high. Now, that will not give you a very correct picture. Because, here, the expected value of demand itself, is quite high. So, whenever you find that, the expected values are changing, by a large extent, in such cases, you should measure the risk, with the help of a term called, coefficient of variation.

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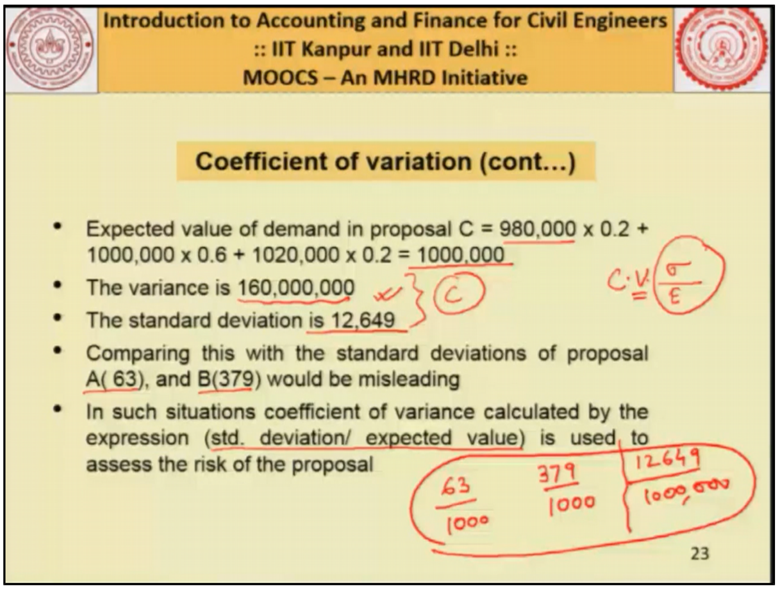

Now, this is defined as, standard deviation divided by expected value. So, you calculate the sigma, corresponding to each one of them, and divide it by the expected value E. So, this is how you calculate the, coefficient of variation. So, in this case, as you can see, for Proposal-C, the expected value of demand, if you calculate, you are getting 1,000,000. And, variance if you calculate, it is very large number.

Standard deviation, if you calculate, it is 12,649 for Proposal-C. These two values are, for Proposal-C. For Proposal-A and B, already the data is with you. For A, it is 63, and for B, it is 379. So, what you can do is, now you can calculate, the value of coefficient of variation, corresponding to each one of them. So, for the first case, it is going to be, 63 divided by 1,000, for the second one, 379 divided by 1,000, and for the third one, 12,649 divided by 1,000,000.

So, you calculate, these three values. And, whichever is giving you the maximum value, that is having higher risk. Now, this would not have been possible, if you would have compared, based on sigma value, alone. Because, looking at the sigma value, you will find, sigma is very high for this. But, you have to look at the sigma value, in the context of, the corresponding expected value. So, this is how, you have to use, these three terms, variance, standard deviation, and coefficient of variation.

Coefficient of variation has to be used, in those situation, where the expected values are changing, to a large extent. As long as, they are of comparable magnitude, you should go in for, either the sigma square, or sigma. But, when they vary to a large extent, it is always better to measure, the coefficient of variation, which is nothing but, standard deviation divided by the expected value. So, what I do now, is to summarise, whatever we studied in this particular lecture.

If you remember, we have studied, the concepts of Risk Analysis, in this lecture. And, for this, as I told you, we need to understand, our concepts such as, additive probability, joint probability, expected value, and measure of variation. Additive probability, we solve one problem. Joint probability, we also did, one or two problems. And then, subsequently, we learnt the concept of expected value. Expected value is nothing but, the weighted average value.

And, we also understand, the concept of variation. There, we learnt three concepts, sigma square, sigma, and coefficient of variation. Sigma square and sigma, major you can use, when the expected values are not changing, to a large extent. But, coefficient of variation has to be used, when your expected values are changing, by a large margin. So, we stop at this particular point. And, next class, when we meet, we further discuss, some more concepts of Risk Analysis. So, till then, thank you very much, and see you, some other time.