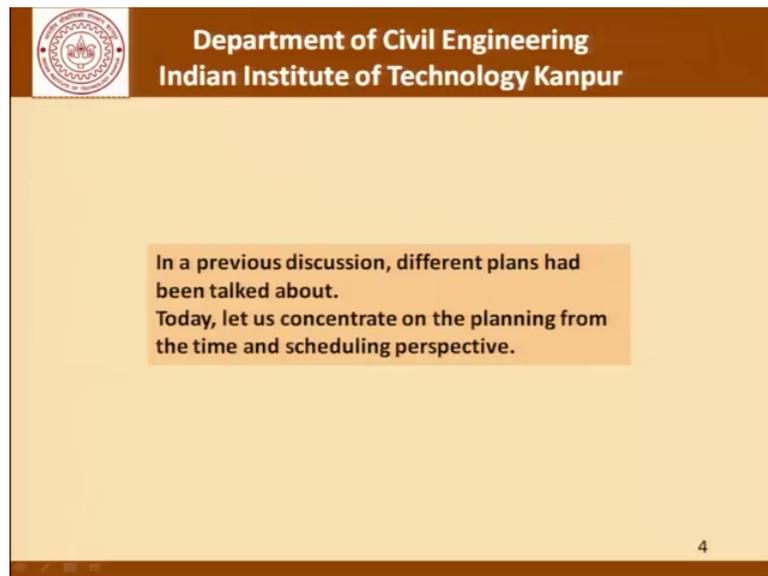


Principles of Construction Management
Prof. Sudhir Misra
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture - 16
Project Scheduling

[FL] and welcome to this series of lectures on principles of construction management once again. And in the last few lectures we have been talking about project planning and scheduling, and today we will concentrate on bar charts and critical path. On the previous discussions we have talked about different plans. Safety plan, quality plan, materials plan, time schedule, a finance plan and so on, which tell us that all these resources and points need to be foreseen. And today what we will do is concentrate on planning from the point of view of time and scheduling.

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Construction schedules *Project → Act → Cam*

Schedules can be either time oriented or resource oriented

- Project schedules are mainly intended to assign start and finish dates to various activities in the project.
- They serve as a basis for matching resource requirement with various activities over time.
- Detailed construction schedules often serve as means to monitor and control the progress of work
- This also serves as a basis for mid-course correction in strategy, should that be required !!

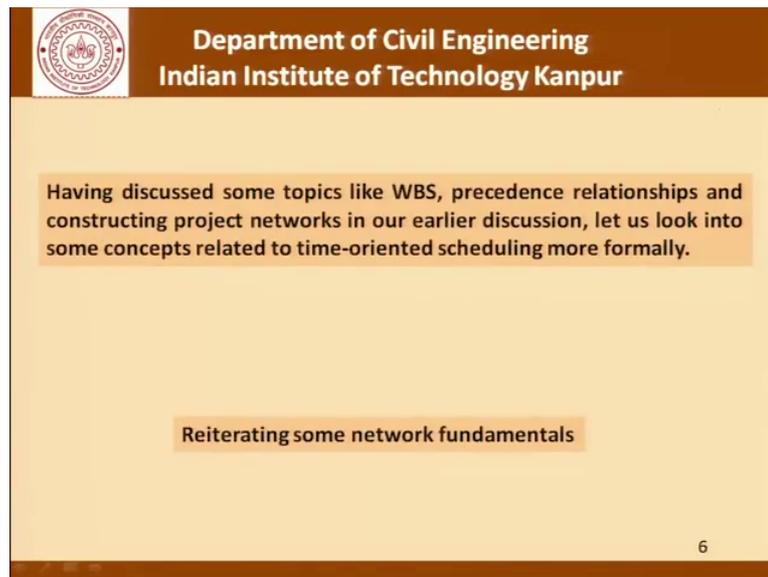
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Now, coming to construction schedules what is the focus of the discussion today. These can be drawn up either with the time orientation or a resource orientation, and what is the interest in drawing up these schedules? The basic interest is to find out and assign start and finish dates to various activities in the project.

We have talked about the fact that a project can be broken up into activities, and we must know that if we have so many activities in the project each of these activities when they can start and when they must start. Can start depends on whenever the preceding activities has been completed and must start is basically the point where it should be ensured that if they do not start by that time the project is likely to be delayed. So, how do we arrive at these numbers or how do we arrive at the earliest start and the latest start days for each activity is what the focus of our discussion today is.

These schedules also serve as a basis for matching and developing resource requirements with various activities overtime. So, we try to figure out if this activity is going to be carried out and this particular point in time. We must ensure that all the resources required are available at that point in time detailed construction schedules also serve as a means to monitor and control the progress of projects. And of course, as a corollary to that they would also serve as a basis for any mid course correction in strategy should that be required. If there is an activity which is likely to be delayed the top management has to be informed that this is the situation and if a resource reallocation additional resources have been employed all that has to be done and that is done on the basis of these construction schedules.

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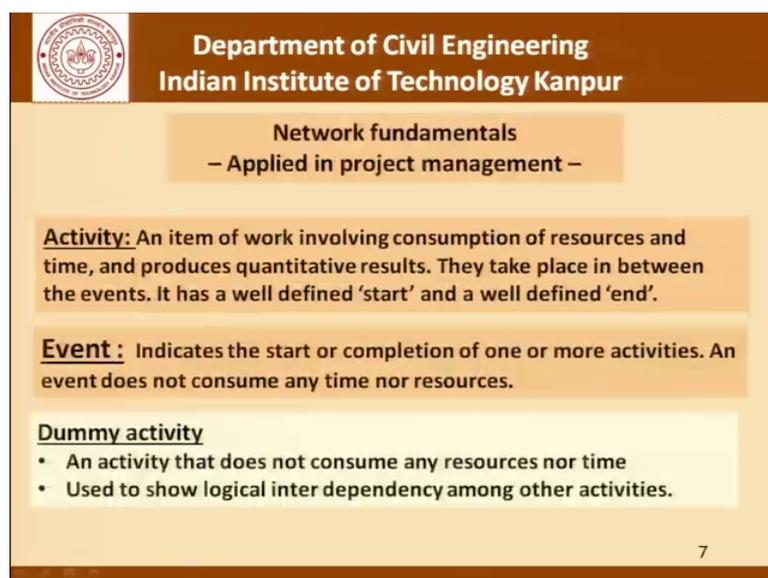
Having discussed some topics like WBS, precedence relationships and constructing project networks in our earlier discussion, let us look into some concepts related to time-oriented scheduling more formally.

Reiterating some network fundamentals

6

We have already discussed topics like the work breakdown structures precedence relationships and constructing project networks in our earlier discussion for simple cases and today the discussion is largely to look into some concepts relating to time oriented scheduling more formally. Before we get started let us reiterate some of the networks fundamentals once again as they are applied in construction project management.

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Network fundamentals
– Applied in project management –

Activity: An item of work involving consumption of resources and time, and produces quantitative results. They take place in between the events. It has a well defined 'start' and a well defined 'end'.

Event : Indicates the start or completion of one or more activities. An event does not consume any time nor resources.

Dummy activity

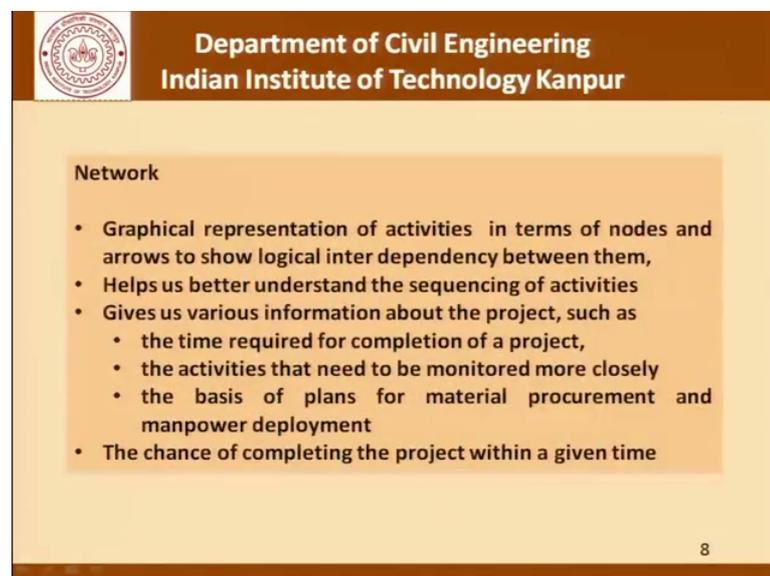
- An activity that does not consume any resources nor time
- Used to show logical inter dependency among other activities.

7

An activity is an item of work involving consumption of resources and time and produces quantitative results. They take place in between events and have a very well defined start and a well defined end.

Events indicate the start or time of one or more activities; an event does not consume any time nor resource. Then we had talked about dummy activities is an activity which does not consume any resource or time and is used to show logical inter dependency among different activities. And a network is graphical representation of activities in terms of nodes and arrows to show logical interdependency between them.

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Network

- Graphical representation of activities in terms of nodes and arrows to show logical inter dependency between them,
- Helps us better understand the sequencing of activities
- Gives us various information about the project, such as
 - the time required for completion of a project,
 - the activities that need to be monitored more closely
 - the basis of plans for material procurement and manpower deployment
- The chance of completing the project within a given time

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The networks help us better understand the sequencing of activities give us various informations about the project such as the time required for completion the activities that need to be monitored more closely the basis of plans for material procurement and manpower deployment and as we will see later on the chance or the probability of completing projects within a given time.

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Types of networks

AOA : Activity on arrow

Node - i: start event
Node - j: end event

AON : Activity on node

A, B are activities

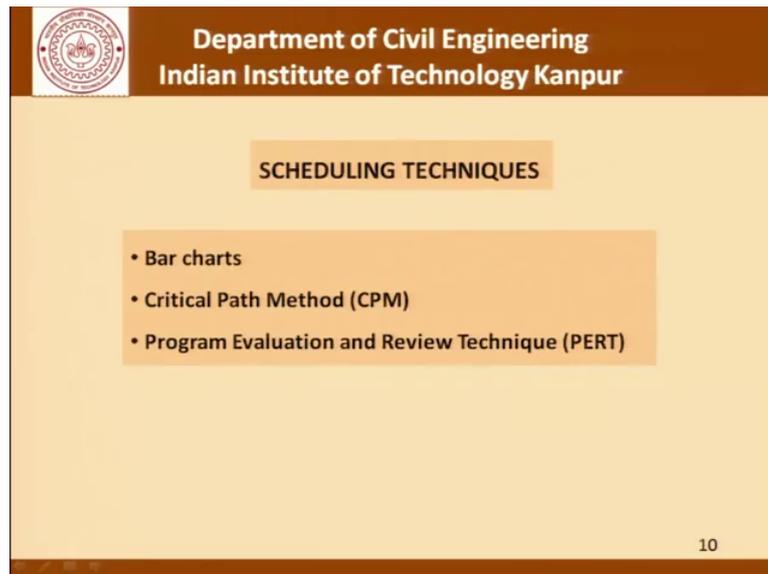
In this course, AOA representation will be used to represent networks

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Now, we also talked about the 2 representations of activities as far as network is concerned, one is the activity on node which is shown on here where activities A and B are shown at these nodes. And this arrow basically says that B can be done only after A has been completed and then there is this representation, which is activity on arrow which basically serves that an activity A is represented as activity A i j and i being the starting point of the activity and j being the end point of this activity.

When we were talking about the example I had added the activity B here to say and explain that now B is being represented by the activity B j k where j represents also the starting point of B as it does the end point of A, and B off course can be undertaken only after a has been completed. So, we have done all this before and in this course we will basically follow the activity on arrow representation in our discussions.

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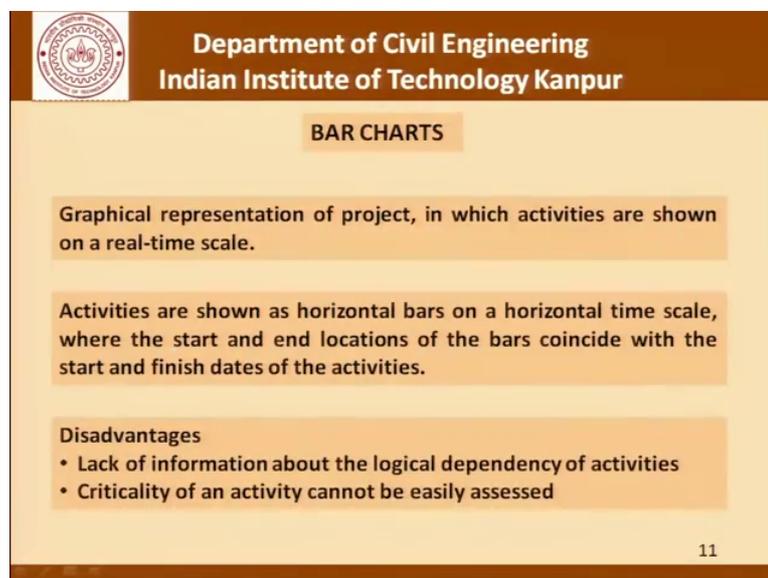
SCHEDULING TECHNIQUES

- Bar charts
- Critical Path Method (CPM)
- Program Evaluation and Review Technique (PERT)

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So now coming to the scheduling techniques, there are 3 techniques which we will talk about as far as this course is concerned. Bar charts, the critical path method and the program evaluation and review technique. Some time it is also called as project evaluation review technique it is the pert.

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BAR CHARTS

Graphical representation of project, in which activities are shown on a real-time scale.

Activities are shown as horizontal bars on a horizontal time scale, where the start and end locations of the bars coincide with the start and finish dates of the activities.

Disadvantages

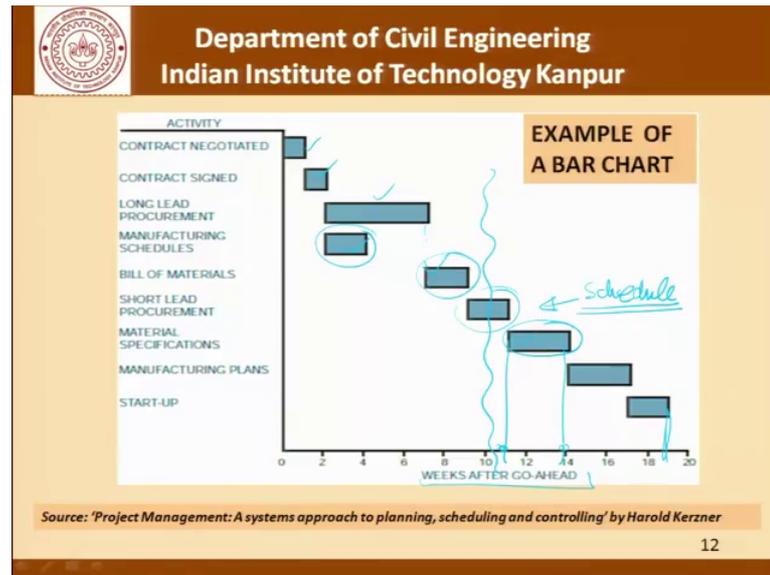
- Lack of information about the logical dependency of activities
- Criticality of an activity cannot be easily assessed

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So now coming to bar chart which is the first and most simple concept to understand, these are graphical representation of a project in which activities are shown on a real time scale.

These are shown as horizontal bars on a horizontal time scale where the start and end locations of the bar coincide with the start and finish dates of the activities. The disadvantages we will see of bar charts is that they lack informations about the logical dependence of activities. And therefore, it sometimes become very difficult to figure out the criticality of an activity based on a bar charts.

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Or Let us look at example of a bar chart. Here we have activities like the contract negotiated contract signed manufacturing schedules bill of materials short leave procurement material specifications manufacturing plans and startup. And this is the time on the horizontal axis which is given in weeks after the go head and we find that the project can be completed or it scheduled to be completed say 19 weeks.

What this does not show is whether for example, this activity can be actually started only after this activity is completed. Or for that matter whether this activity also has to be completed before this activity can be started or not. So, this is the major disadvantage of this representation except that yes, it gives a very clear understanding that at a given point in time if you want to know what are the activities that should have been completed it is very clear that as far as this line that I have drawn is concerned at is point in time. These activities should all have been completed and this activity should be let us say 50 percent complete.

Please also remember that this representation is actually a schedule that emerges out of our planning, in order for somebody to be able to really ensure the deployment of the right kind of the resources. So, if we know in the plan that this activity has been scheduled from this point to this point, regardless of it is interdependencies and so on we know that whatever resources required for this particular activity should be available at this point in time and may be free after this point in time.

So, this bar chart representation has its own merits, but of course, it has its demerits. So, continuing with our discussion.

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Earliest start time (EST) of an activity:
The earliest time at which the activity can start, given that all its preceding activities are completed first.

Earliest finish time (EFT) of an activity:
This is the earliest time at which an activity can be completed.

$EFT(i, j) = EST(i, j) + D(i, j)$ where, $D(i, j)$ is the duration of the activity A

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What is the earliest starting time that is the EST of an activity? This is the time when the activity can logically start given that all its preceding activities have been completed. The earliest finish time this is the earliest time at which the activity can be completed. The earliest finish time of an activity $i j$ is basically the sum of the earliest starting time of that activity and the duration that the activity takes.

So now if we take this as an example here A is my activity $A i j$ and $D i j$ is the duration associated with that activity. So, the earliest finish time of this activity $i j$ is nothing but the sum of the earliest starting time of the activity $i j$ and the duration of the activity $i j$ as it is shown in this equation.

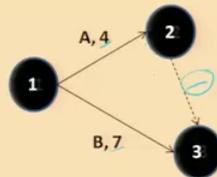
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Illustration-1

A project consists of two activities A and B, which can start at the same time. The duration for activities A and B are 4 weeks and 7 weeks respectively.

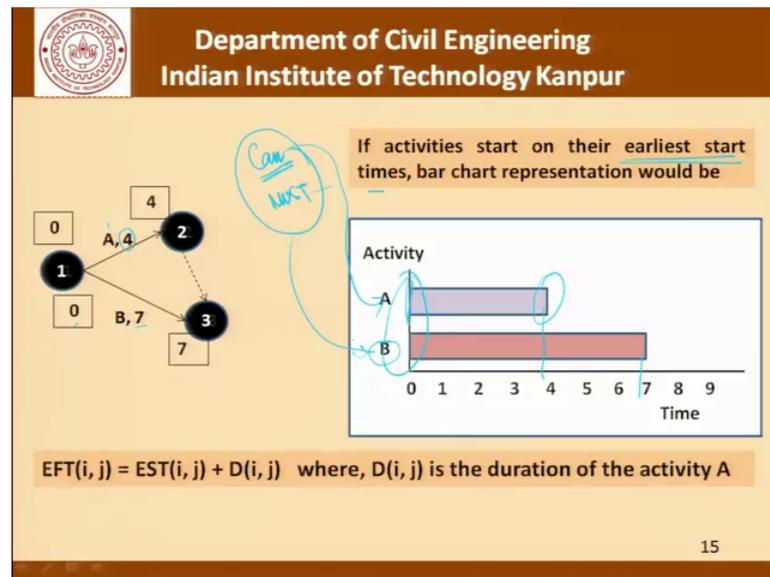
- Construct the network of the project
- Prepare bar chart if the activities are to start on their EARLIEST start times



As an illustrative example Let us consider a project consisting of 2 activities A and B which can start at the same time. The durations of the activities A and B are 4 weeks and 7 weeks respectively. Now if you are required to construct a network of the project and prepare the bar chart if the activities are to start at the earliest times.

Now, coming to the solution of this problem we create an activity 1 2 which is a and on top of that we write duration of this activity let us say 4 weeks. And then we create an activity B which is 1 3 and write the duration 7, which is also the duration in weeks. So, and as far as completion of the project is concerned we create this dummy activity 2 3 and we will complete this project when we have reached node 3. Node 3 represents the completion of activity B and also activity A through this dummy activity.

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Now, let us try to carry out the calculations that we said earlier. This 0 here represents the starting point of the activity A which is the earliest that can start the project, we have seen that the earliest finishing time of an activity $i j$ is EST of $i j$ plus $D i j$ and therefore, the earliest that we can reach node 2 and that is the completion of the activity A is 4. Because as far as a is concerned it is earliest starting point is 0 the duration of the activity is 4 and this becomes 4.

Similarly, for B it can starts at 0 because it is an independent activity. And we can reach node 3 at 7 that is 0 plus 7. So, what is the minimum time with the project can be completed? Intimately we know that this project can now be completed in 7 weeks. So, what becomes the bar chart representation, this is how it can be represented. We start both these activities A and B at 0 a gets completed at 4 and B gets completed at 7.

So obviously, since I have said that the activities will start at their earliest starting point a has been started at 0. We will very soon see that a need not start at 0, and that is why we have to be careful an activity can start and it must start. And that is the difference which I had pointed out even at the outset of today's discussion.

So, it is very clear that though a can start at 0 it need not start at 0; however, B must start at 0, in order that the project is not delayed. So now, let us continue this discussion little further, this is the latest time at which the activity must be completed to avoid the delay

in the project completion. And latest start time of the activity is the latest time when an activity must be started in order to avoid delays.

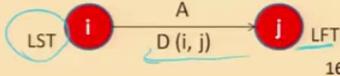
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Latest finish time (LFT) of an activity:
This is the latest time at which an activity must be completed to avoid delay in the project completion.

Latest start time (LST) of an activity:
This is the latest time when an activity must be started, in order to avoid delay.

$LST(i, j) = LFT(i, j) - D(i, j)$ where, $D(i, j)$ is the duration of the activity



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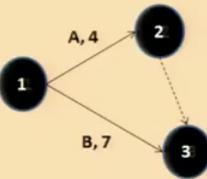
Mathematically speaking the latest starting time of an activity i, j is represented as latest finishing time of that activity minus the duration of that activity. So, if we look at this example once again the latest starting time of an activity i, j is the latest finishing time of this activity minus the duration of this activity.

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Illustration-1 (continued..)

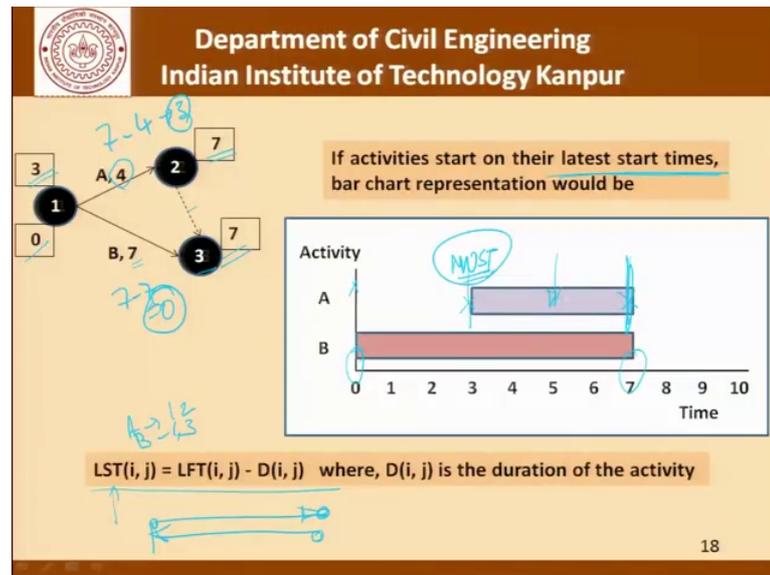
For the network shown below, prepare bar chart if the activities are to start on their LATEST start times



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So, continuing with our illustrative example where we had these 2 activities A and B taking 4 weeks and 7 weeks respectively A and B being independent to start with, this is the network we had drawn and now what we are trying to do is to determine or is to see how the situation changes when it comes to the determination of the latest start times.

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We know that the project can be completed in 7 weeks, B being the activity which is governing it and that duration is given to be 7 weeks. Now because of the dummy activity node 2 will also be 7. And if we use this equation try to find out what is the latest start times for these activities i j which is in this case is A and B, A being 1 2 and B being 1 3. So, as far as a is concerned the latest finish time of A is 7, the duration is 4 and therefore, the latest A must start is 3.

Similarly as far as B is concerned it must start at 0 because the latest finishing time for B is 7 and it is duration is also 7. So, 7 minus 7 gives you 0 here and 7 minus 4 gives you 3. So, this is how we evaluate the latest starting times of an activity. So, please note that as far as the first discussion is concerned basically we went from left to right. And this part now is gone from right to left. So, this understanding is going to come in hand even we go for the down in our discussion today. And if we represent this as a bar chart with the activities being shown at the latest start times what it will show is like this that yes B will start at 0 finish at 7, but a can be started at 7 now to ensure that a also gets completed at 7, it must be ensured that a must start at 3.

So, in the previous discussion we had said that a can start at 0 yes, but it must start at 3 given that its duration is 4 and the fact that its finishing time cannot exceed 7. So now, coming formally to what is called a forward pass.

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Forward pass

The *forward pass* moves from the 'start' node towards the 'finish' node, and basically calculates the EARLIEST occurrence times of all EVENTS.

Forward pass is done to calculate

- Earliest project completion time
- When an activity can start

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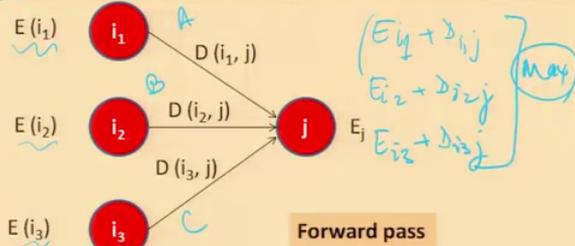
The forward pass moves from the start node towards the finish node and basically calculates the earliest occurrences of all the events. We have illustrated this with a very small network of 3 nodes and we take up some more illustrative examples later on. And a forward pass is done to calculate the earliest project completion and when an activity can start.

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While carrying out the forward pass, at a given node j , we need to check all the arrows those meet at j , and take the **MAXIMUM** of the early finish times of all those activities (meeting at j) as the earliest occurrence time of EVENT j .

$E_j = \text{Max} [E_i + D(i, j)]$, where the maximization is for all nodes i those meet at j



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While carrying out the forward pass at a given node j we need to check all the arrows which meet at j and take the maximum of the early finish times of all those activities meeting at j as the earliest occurrence time of the event j . This is something which must be clearly understood, that if there is a node j where several activities are coming together. So, what we are talking of as far as the node j is concerned there is an activity let us say A B and C which is $i_1 j$, $i_2 j$ and $i_3 j$ the earliest that the node j can be reached is basically the maximum of E_i depending on whether we are talking of i_1 , i_2 or i_3 plus that respective durations of that activities.

So, in this case it will be the maximum of $E_{i_1} + D_{i_1 j}$, $E_{i_2} + D_{i_2 j}$, $E_{i_3} + D_{i_3 j}$. So, the maximum of this will be the E of j .

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Illustrative example for evaluating minimum project duration

Based on the information given in the table about the activities of a project, determine the minimum time required to complete the project.

Activity	Duration (days)	Predecessors
A	5	---
B	6	---
C	4	A, B
D	3	B

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So now if we take an illustrative example based on the information given in the following table about the activities in the project determine the minimum time required to complete the project. You will recall that this example has also have been done earlier we have 4 activities in the project with durations being 5 6 4 and 3 the predecessors. That is A and B do not require any preceding activity they can be started on their own C requires both A and B to be completed D requires only B to be completed.

So, if we want to draw the network first we draw this network 2 3 being a dummy activity as discussed earlier.

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Illustrative example for evaluating minimum project duration

Minimum time required to complete the project = 10 days

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Now, coming to evaluating the minimum project duration through the forward pass, we look at this network once again A B C and D with the respective durations plotted. We start at 0 reach node 2 at 0 plus 6, we reach node 3 from this route at 0 plus 5, but please note that this node 3 also has an incoming arrow from 2. So, we have to see when can we reach node 3 from 2. And that is 6 plus 0 because this being a dummy activity does not consume any resource and therefore, this becomes 6. So, the value itself is also 6 because this is the maximum of 0 plus 5 and 0 plus 6, which in this case is 5 and 6 the maximum of this is 6. So, node 3 is reached at 6.

Similarly, coming to node 4 from this side that is completion of activity C the time where it can be reached is 6 plus 4 which is equal to 10 and from this side it is 6 plus 3 which is equal to 9. So, the maximum of these 2 values is 10. So, the minimum time required to complete this project is 10 days.

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Illustrative example for evaluating minimum project duration

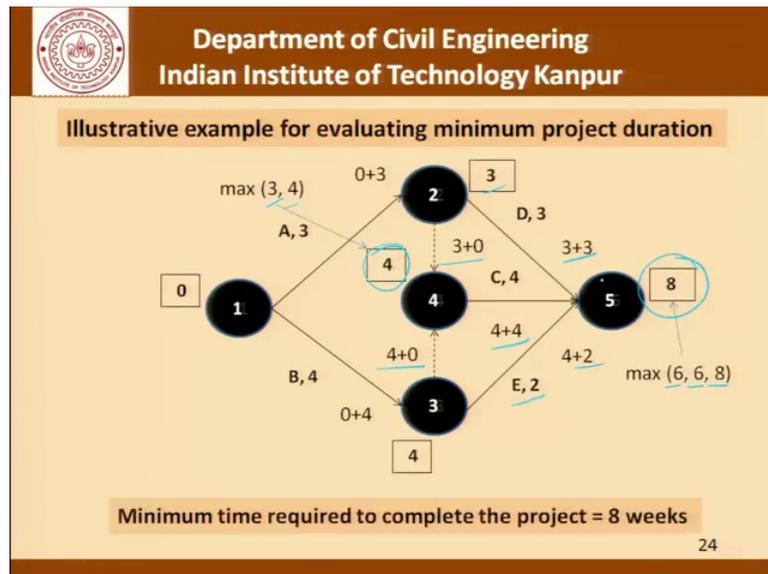
Based on the information given in the table about the activities of a project, determine the minimum time required to complete the project.

Activity	Duration (weeks)	Predecessors
A	3	---
B	4	---
C	4	A, B
D	3	A
E	2	B

23

Now, let us take another illustrative example where the project has 5 activities which is given in weeks 3 4 4 3 and 2 are durations of the activities A B C D and E. And given these interdependences, we have already drawn this network diagram.

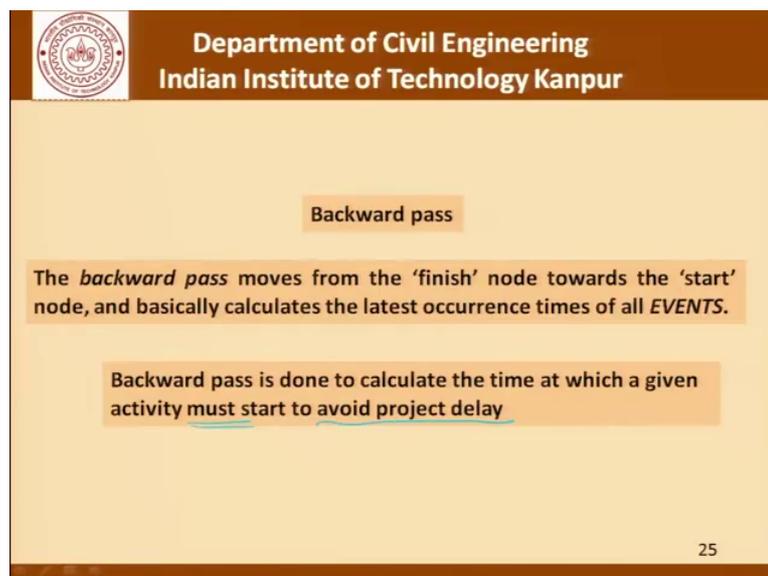
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And now proceed to evaluate the minimum project duration for this project. So, given this information, we obviously start at 0. What is the earliest that we can reach? Node 2 is 0 plus 3, what is the earliest we reach? Node 3 is 0 plus 4 and as far as the node 4 is concerned it is the maximum of 4 plus 0 and 3 plus 0 which is the maximum of 3 and 4 and this node is therefore, 4.

Similarly, when we try to do the node 5 we are looking at 3 plus 3 4 plus 4 and 4 plus 2, that is we are looking at the maximum of 6 6 and 8 and this is the 8. So, the minimum time required to complete this project is 8 weeks.

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So now let us try to find out what are the latest starting point or the latest starting times of each of these activities through the process called the backward pass. Now the backward pass moves from the finish node towards the start node and basically calculates the latest occurrence times of all these events. The backward pass is done to calculate the time at which a given activity must start to avoid project delay like we did in the case of a forward pass, where we tried to find out earliest that we can reach a node now we are finding out the latest that we should reach a node.

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While carrying out the backward pass, at a given node i , we need to check all the arrows those emerge from i , and take the MINIMUM of the latest start times of all those activities (succeeding i) as the latest occurrence time of EVENT i .

$L_i = \text{Min} [L_j - D(i, j)]$, where the minimization is for all nodes j those succeed i

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While carrying out the backward pass at a given node i , we must check all the arrows that emerge from i and take the minimum of the latest starting times of all those activities succeeding i as the latest occurrence for that event i . Mathematically speaking the L_i can be written as the minimum of the L_j minus all the D_{ij} 's. Where D_{ij} 's are the durations of these activities i and j . So, as is given in this example given that 3 activities let us say A , B and C which are $i j_1$, $i j_2$ and $i j_3$. Then the latest that we are reaching let us say nodes j_1 , j_2 , j_3 is L_{j_1} , L_{j_2} and L_{j_3} , the latest we must reach here is basically the minimum of $L_{j_1} - D_{ij_1}$, $L_{j_2} - D_{ij_2}$ and $L_{j_3} - D_{ij_3}$.

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Illustrative example for backward pass

Based on the information given in the table about the activities of a project, perform the backward pass

Activity	Duration (days)	Predecessors
A	5	---
B	6	---
C	4	A, B
D	3	B

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Now, let us do an illustrative example performing this backward pass. We will use the same example that we did earlier where we did the forward pass, and there is a network that we have drawn and we had found that this project can be completed in 10 days.

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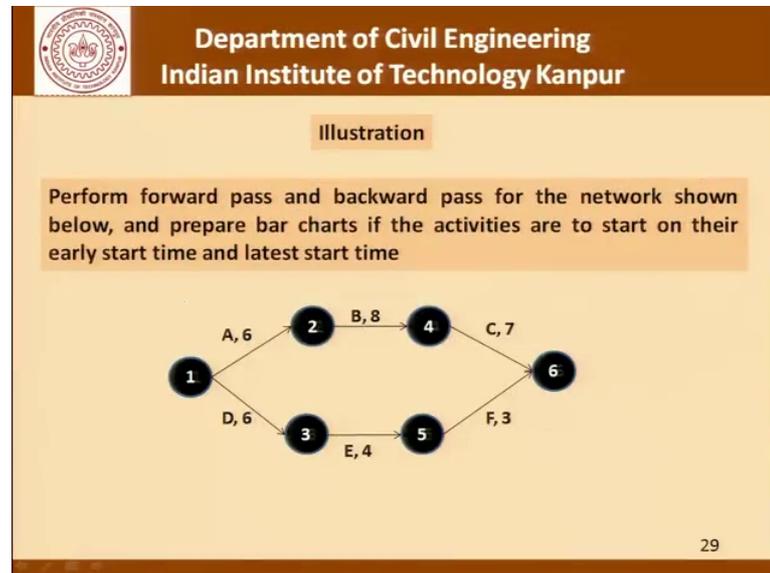
Illustrative example for backward pass

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Now, we are trying to find out what is the latest time that these activities A B C and D must start. Now going back from this we find that as far as the activity C is concerned this must start at 10 minus 4 which is 6, similarly as for as B is concerned it must start at 10 minus 3 which is 7 and 6 minus 0 which is 6. So, the minimum of this gives us 6. So, we go back and find out the latest we must be at one that is 6 minus 6 and 6 minus 5 and that; obviously, is the minimum of 0 and 1 and that gives us 0. We must be at node one at

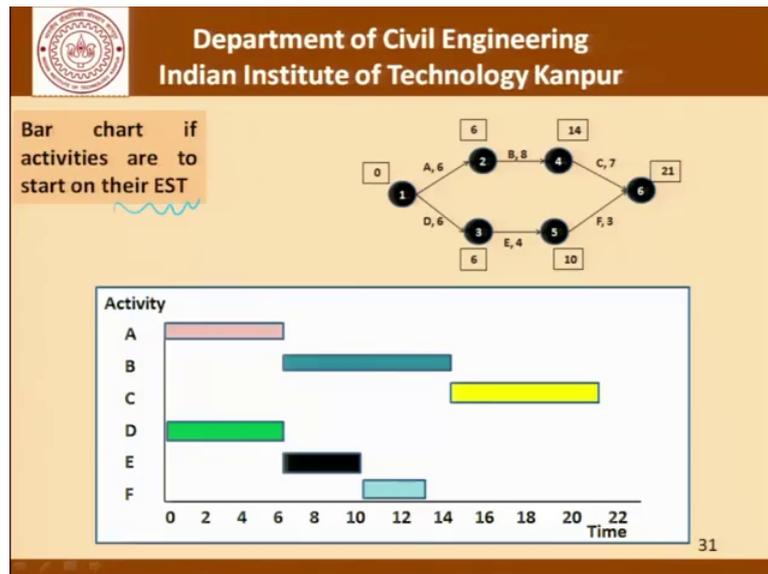
0 we must be at node 2 at 6 node 3 is also at 6 and we will reach node 4 at 10. So, this is how we now calculated the latest starting times for each of those nodes.

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Let us do another example. A project which has 6 activities A B C D E F which are in a manner that A B and C are interdependent and inter related to the extent that C can be done if B is completed, and B can be done, if A is completed on an independent route F can be done when E is completed and E can be done when D is completed and A and D can be started independently. So, if we have this information given to us with the duration of the activities also listed here on the arrows, let us try to find out the earliest start time and the latest start times.

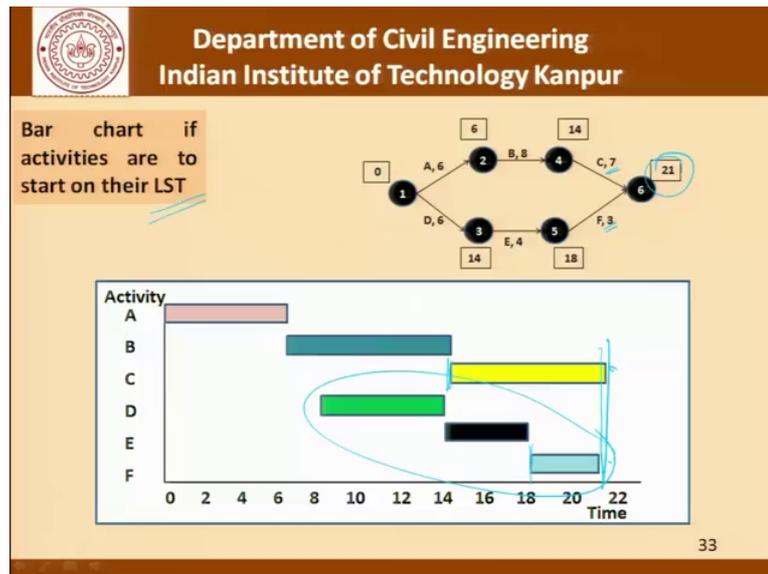
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We start with 0 here; here also we reach 6 as far as nodes 2 and 3 are concerned because both these activities are taking 6 units of time. And we reach 4 at 14 which is 6 plus 8 and we reach node 5 at 10 which is 6 plus 4, and we reach node 6 which is 21, because that is the maximum of 14 plus 7 and 10 plus 3. So, this is 21 this is 13 and the maximum of this being 21. We reach node 6 at 21 which now becomes our minimum project duration.

So, if we want to plot this in a bar chart with the activities starting at their earliest starting times, we have activity A which can start at 0 and goes up to 6. Activity B which can start only after a has been completed it starts at 6 and goes up to 14. Activity C which can start after B has completed starting at 14 going up to 21. And on the independent route the other route which is D E and F the activities are D E and F, we see that these activities D E and F have been positioned in a manner that they will start on their EST which is the earliest starting time that these activities can start.

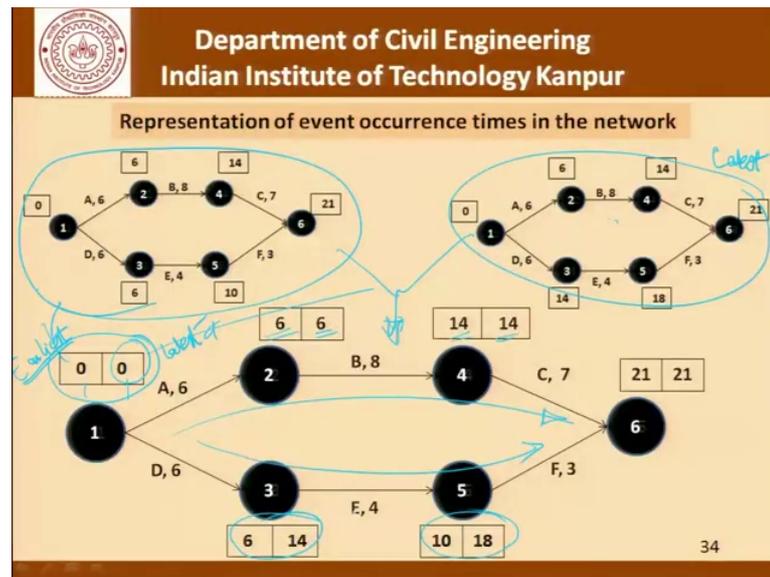
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So, continuing this example let us do the backward pass analysis now. We start with 21 we must reach node 5 at 18 because this 18 is 21 minus 3. So, nearly we must reach node 4 at 14 which is 21 minus 7. We must reach node 3 at 14 which is 18 minus 4, node 2 at 6 which is 14 minus 8 and node one at 0. Now given this information we can again plot the bar chart for these activities and we find that this is how our activities C and F both must be completed at 21 and therefore, as per the activity C is concerned given the fact that it takes 7 units of time it must start at 14. So, 14 becomes the latest starting point for activity C. Similarly for F 18 becomes the latest starting time because it takes 3 units of time and we do not want a delay beyond 21.

So, carrying this discussion forward we have the plot for activities B E A and D. So, this set here is now plotted with the condition that they will start at their latest starting times. So now, we obviously, have a difference between the earliest starting times and the latest starting times for some activities. In some activities there is no difference between the earliest that they can start and the earliest that they must start. And that is what we need to investigate little further and that is what we planned to do in this slide If we use this representation which is basically the earliest that these activities can be started against that the nodes can be reached and so on.

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And this representation which is the latest that these activities can be started finished the nodes that the node we are interested in and so on.

So now if we combine these 2 representations we get the representation which is shown at the bottom. Here what we have done and that is conventionally done also if we look at different books in the subject. We have created this box which has 2 entries. This entry is taken from here which is basically represents the earliest that this node can be reached. It also represents the earliest of that activities eliminating from that node can be started.

This digit; however, has been borrowed from the figure on the right here. And this shows the latest this node must be reached and also the latest starting time of the activities eliminating from that node. So now, if we see this route the A B C route, we find that the earliest and the latest starting times are all the same. Where as if we go through this route there is a difference in these values. And this is something which we investigate a little more in subsequent slides.

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Bar chart if activities are to start on their EST

Bar chart if activities are to start on their LST

- From the bar charts, it is clearly evident that there exists some flexibility to activities D, E and F in their starting and finishing times, whereas activities A, B and C do not have any such flexibility
- The project duration is primarily depending on activities A, B and C

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Now, before we do that let us look at this bar chart representation side by side, this has been drawn on the assumption that the activities are started at the earliest that they can be started whereas, this has been drawn on the basis that the activities are started at the last movement. That is they must be started at that time less the project is delayed. What we see is that there is some flexibility in activities D E and F in starting and finishing times whereas, activities A B and C do not have any such flexibility and that is what we have discussed in the previous slide. And what we also conclude from that is that the project duration is primarily governed by activities A B and C. So, as far as this route is concerned maybe we can do D as it is, but shift E a little bit F a little bit. So, long as the finishing dates of these activities are preserved.

Taking this concept formally we have the concept of total float.

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TOTAL FLOAT

Total float of an activity is the amount of time by which the start of an activity may be delayed without causing a delay in project completion.

$$TF(i, j) = LST(i, j) - EST(i, j)$$
 or

$$TF(i, j) = LFT(i, j) - EFT(i, j)$$
 or

$$TF(i, j) = L_j - E_i - D(i, j)$$



Thus mathematically, float can be evaluated either using occurrence times of activities or events

If the total float of an activity is zero, then the activity is known as critical activity for the network.

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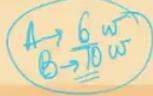
The total float of an activity is the amount of time by which that start of an activity may be delayed without causing a delay in the project. So, what we see for example, here is that this activity can be started here, but must be started here. So, there is this duration which is being called as float.

Similarly, this activity can be started here, but must be started here. So, there will be this duration which also floats. So, E the float is associated with an activity. To reiterate it is the amount of time by which the start of an activity may be delayed without causing a delayed to the project completion. Mathematically the total float of an activity $i j$ can be given as $L s t$ of $i j$ minus the EST of $i j$ or $L F t$ of $i j$ minus $E F t$ of $i j$ or L_j minus L_i minus $D I_j$. So, I am leaving it to you to ensure that the arithmetic suggested here is correct, but the principle is something which I have already explained fairly clearly I hope. That total float of an activity is the amount of time by which the start of that activity may be delayed without causing a delay in the project completion. So, moving forward float can be evaluated either using occurrence times of activities or events. And if the total float of an activity is 0 then the activity is known as the critical activity for the network. That is what does float 0 mean? Float 0 means that the activity must be started at the time when it can be started, there is absolutely no rule for a delay. There is no way that we can allow any delay in that activity. For other activity is yes it is possible that the can start and must start dates are slightly different.

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CRITICAL PATH METHOD



- Most widely used scheduling technique
- Critical path represents the series of activities, which should not be delayed for timely completion of project.
- Indeed, it is the longest path in the network (longest time of completion), representing the minimum time required to complete the project.
- The duration of activities is considered to be deterministic (fixed)

Activities on the critical path have their total float as zero.

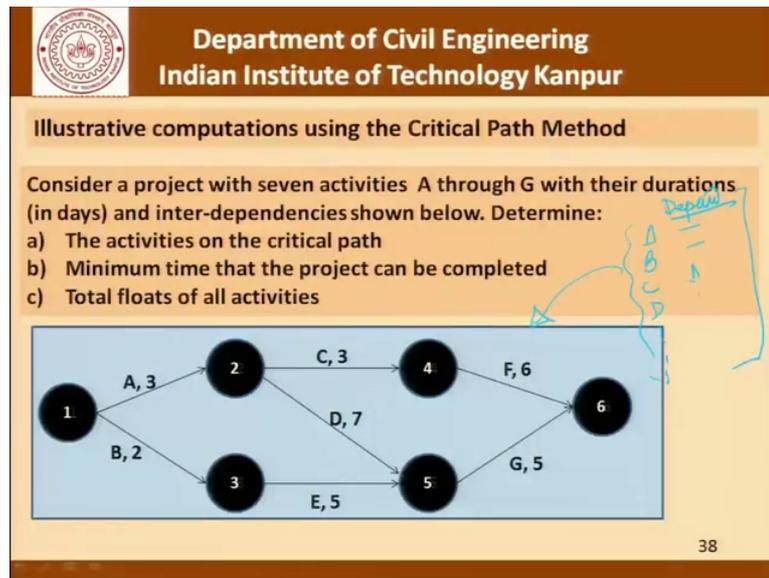
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So now coming to the critical path method, it is the most widely used scheduling technique and the path represents the series of activities, which should not be delayed for timely completion of project. We can imagine that a project or a network representing the project can have different paths through which the starting point and the end points are connected. Now each path has a set up activities. So, the critical path represents that series of activities which should not be delayed for timely accomplishment of the project. So, in the previous example A B C constituted one path, D E F constituted another path and we find that A B C is the critical path. Indeed it is the longest path in the network representing the minimum time required to complete the project. That is what we have illustrated in the example that we just completed. And the duration of activities is considered to be deterministic.

Now, what we have assumed in our discussion all the time is that the duration of each of these activities is clearly known. We know that an activity A will be completed in 6 weeks we know that the activity B is completed in 10 weeks. And we have already seen an example as to how these estimates are drawn. So, once this value is known to us we can use the critical path method the way we have illustrated in this discussion and find out what are the activities that need to be watched more carefully and which are the activities that determine the time of completion of the project and for which of the activities we can afford to be a little more relaxed. Or we can do resource allocation in a manner that they need not take the same amount of time.

We can possibly allow an extension of times on non critical activities because they do not affect the project duration. This is the luxury which we should not take with the activities lying on the critical path. So, as already stated activities on the critical path have total float as 0.

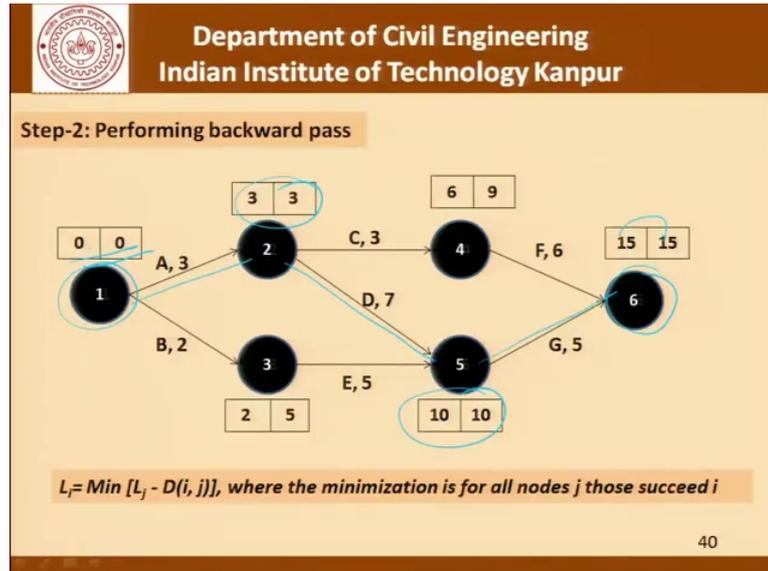
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So now let us do an illustrative example computing the critical path for a project which is shown here. So, in this case there are activities A B C D E F and G. And the interdependence is also on along with the durations is given here, and what we are required to find out is the activities in the critical paths the minimum time that the project can be completed and the total floats of all these activities.

What I have not done here is tried to make the table where we are said that there are activity A B C D E and so on, and what do they depend on. So, I am leaving to you to complete this table. Because this table is actually the starting point for creating this network. So, because it is an illustrative example and we have done some of these things before I have not done this, but in this case we find a does not depend on anything B does not depend on anything C depends on a and so on. This is something that which you can do as a homework and as far as we have concerned let us move forward and do the computations.

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So now the first step is to find out the earliest that we can reach the different nodes using this equation that we already discussed in detailed before. We start 0 here, we get to 3 which is 0 plus 3 for activity A, node 3 0 plus 2, node 4 is 3 plus 3 node 5 is the maximum of 2 plus 5 and 3 plus 7 which is 10. And node 6 is the maximum of 6 plus 6 and 10 plus 5. So, this 15 represents the earliest that the project can be completed in whatever units it is weeks month's days whatever.

So, I am leaving this computation of the latest starting times as an assignment. Now taking this information of 15 days or weeks units of time being the minimum duration of the project, and translating it through the backward pass to the latest starting times of activities, we are not doing the calculations here and I am leaving it out to you to verify these if these are numbers that we get what we see is that in the case of nodes 2 nodes 5 and of course, node 6 and node one these numbers are the same. So, what is our critical paths? It is basically A D and G.

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CRITICAL PATH METHOD

Step-3: Tabulating the earliest and latest EVENT times i → j

Activity	D (i, j)	E _i	L _i	E _j	L _j	TF	Remarks
A	3	0	0	3	3	0	Critical
B	2	0	0	2	5	3	
C	3	3	3	6	9	3	
D	7	3	3	10	10	0	Critical
E	5	2	5	10	10	3	
F	6	6	9	15	15	3	
G	5	10	10	15	15	0	Critical

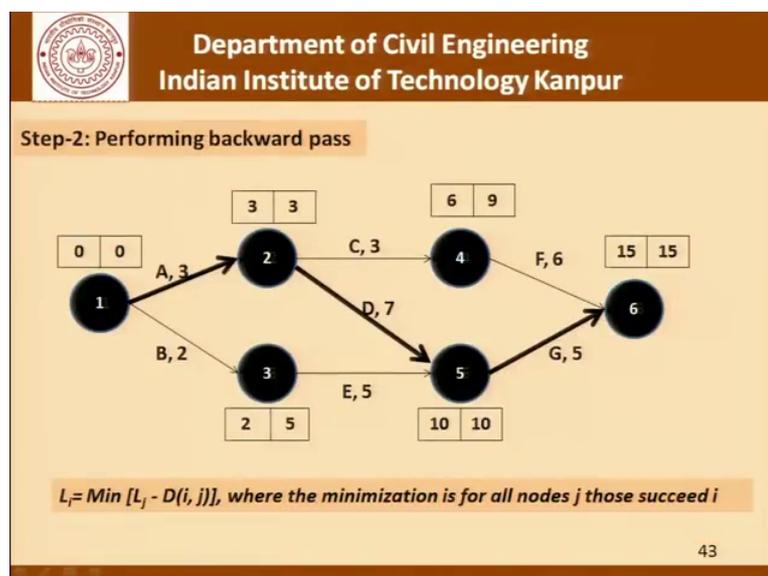
$TF(i, j) = L_j - E_i - D(i, j)$

Critical path: A → D → G Project completion time: 3+7+5 = 15 days

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Now, what we do is we calculate or tabulate the earliest and the latest event times for all the events all these activities and the D i j's are given here earliest starting time (Refer Time: 37:06) the latest starting time (Refer Time: 37:07) if we do all these we find that the critical path is A D G and this what we have already done. That is activities A D and G lie on the critical path, and these are the floats which are associated with activities B C E and F.

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So, this becomes our final network that A D and G are on the critical path and activities B E C and F have some float. That is their earliest starting dates are not that critical. There can be a small gap or there can be a resource reallocation such that they can be

moved around a little bit. With this we come to an end of our discussion today. And there are so many reference books. So many texts that can help you understand this concept of bar charts the critical path method and calculating the critical path. And I am sure you can do that. What we will do next is to handle a little bit of the uncertainty that is associated with the completion times of each of these activities.

In this discussion today I have also pointed out that we have assumed at time for each activity is known to us. Now what happens? If that time has an distribution in terms of the probability of completion we may have an activity which can be completed in 5 days it may also take 10 days. So, once that uncertainty is brought in how do we handle this uncertainty in our approach is what is the subject for the next discussion. I look forward to see you at that time.

Thank you.