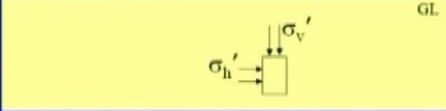


Application of Soil Mechanics
Prof. N. R. Patra
Department of Civil Engineering
Lecture – 38

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Earth Pressure at Rest

In a homogeneous natural soil deposit,



The ratio σ_h'/σ_v' is K
coefficient of earth pressure at rest (K_0)

To arrive K_0 state, there are no lateral strains during loading (1-D consolidation)

Last class we have started an earth pressure in the retaining wall and different earth pressure theory and this is the basic introduction. ((Refer Time: 00:29)) this earth pressure has been used design of reinforced earth soil mass. So we have covered with this earth pressure at rest, if there is a soil in a homogenous natural soil deposit. For example, here it is shown. The ratio of sigma h prime to sigma v prime is called K, that means that is your earth pressure called K, that means k is your coefficient of earth pressure at rest, it is called coefficient of earth pressure at rest which termed as a this thing has been term K 0. So to arrive this K 0 state that means coefficient of earth pressure at rest means, there are no lateral strains during loading that means 1 D consolidation should be there.

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Estimating K_0

For normally consolidated soils,

$$K_0 \approx 1 - \sin \phi' \quad (7.3)$$

For overconsolidated soils,

$$K_0 = (1 - \sin \phi') \text{OCR}^{\sin \phi} \quad (7.4)$$

From elastic analysis,

$$K_0 = \frac{\nu}{1 - \nu}$$

Poisson's ratio

Now estimating K_0 . For normally consolidated soils, K_0 is equal to generally we take 1 minus $\sin \phi$. For over consolidated soils, K_0 is equal to 1 minus $\sin \phi$ prime O C R - over consolidation ratio to the power $\sin \phi$. For elastic analysis, that means for linear or elastic analysis, K_0 is equal to μ by one minus μ . μ is your Poisson's ratio.

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Active/Passive Earth Pressures

smooth wall

Let's look at the soil elements A and P during the wall movement.

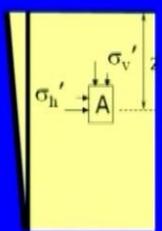
We have started these also last class active and passive state or active and passive earth pressure if you look at here, there is a smooth retaining wall. Now this wall is moving away from the

field. It has been shown moving away from the field because of your backfill material, so backfill soil then this is called in active state. Active state means wall moves away from the soil. If you look at here, because here is your soil mass. It try to push this wall that means in this case particularly in this domain, wall is moving away from the soil. But in this domain, wall is pushing this soil that means wall moves towards the soil in this region.

In this region, wall moves away from the soil that means wall moves away from the soil, it is called active state; and wall moving towards the soil, it is called passive state. If you look at here, soil mass in the A, it is called in the active state. Soil mass in the P, you can say that it is a passive state or may be wall movement because of the wall movement is happened.

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Active Earth Pressure



$\sigma_v' = \gamma z$

Initially, there is no lateral movement.

$\therefore \sigma_h' = K_0 \sigma_v' = K_0 \gamma z$

As the wall moves away from the soil,

σ_v' remains the same; and

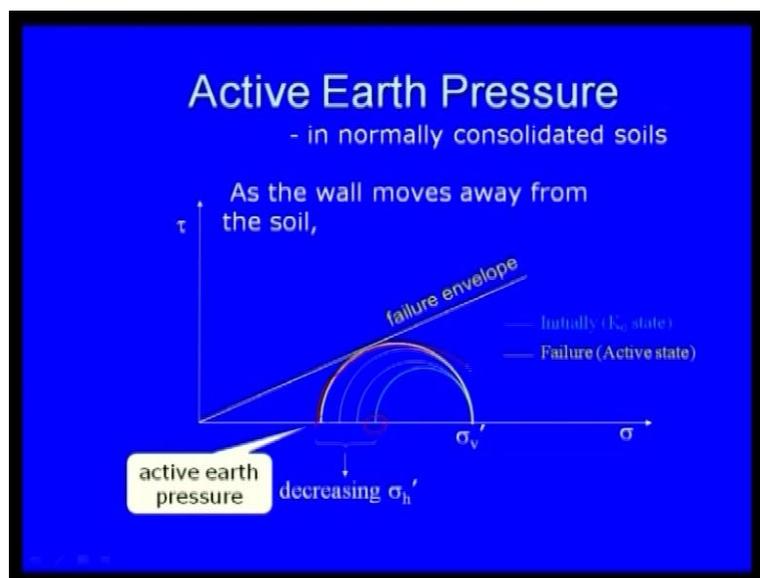
σ_h' decreases till failure occurs.

Active state

Next part, active earth pressure; if you look at here, sigma v is your overburden that is generally calculated as gamma z. So what happened when soil has means coefficient of earth pressure at rest that means soil at rest, there is no movement. In this case, this is wall and soil is there, there is no movement that means soil at rest, so that means there is no lateral movement. So sigma h prime, sigma v prime is your overburden that is your gamma into z. Gamma is your unit weight of soil, z is equal to 1 depth of the soil and sigma h prime is K 0 times sigma v. K 0 is equal to sigma h prime by sigma v prime. So which is equal to K 0 times gamma z.

As the wall moves away from the soil, initially what happens wall is the constant, there is no movement. Now wall is moving away from the soil, so what will happen σ_v remains the same that means overburden pressure remains the same. At the same, σ_h decreases till failure occurs that means σ_h that is your lateral pressure decreases till failure occurs. So this is called your active state that means if I define in the active state is soil mass before the active state, if there is a soil mass, in the soil mass, there are if I take into consideration of plane strain conditions in two dimensional. So in this case, there are two states; one is vertical that is your σ_v and σ_h is your lateral. So once this soil is trying to push this wall that means wall is moving away from the soil; so in this case, what will happen, σ_v that means your overburden pressure, overburden stress will remain constant, it is not going to change. However, σ_h , it decreases, decrease in the σ_h is decreasing; if this is a σ_h , it is decreasing decreasing, so till the failure occurs.

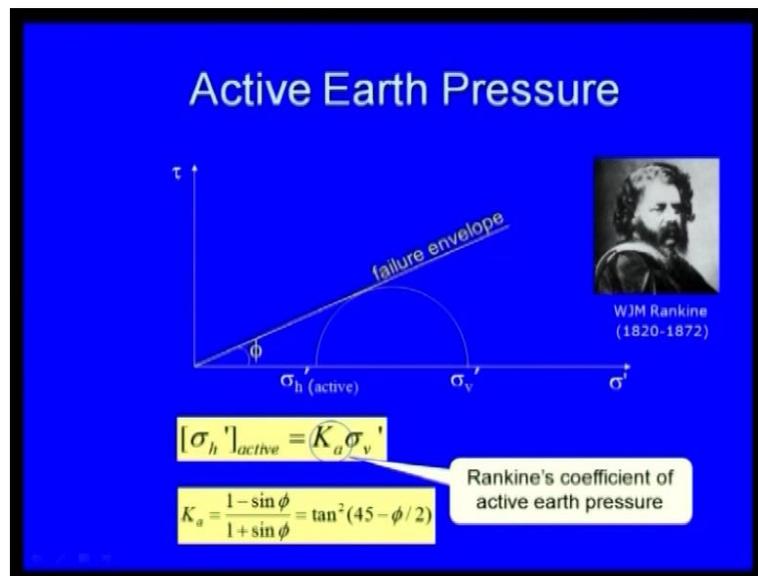
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If I take for normally consolidated soil draw the Mohr circle for particularly active earth pressure conditions. So in this case, this is your initial state of your stress. If I draw the Mohr circle, so this is your σ_v prime, this is σ_o prime and this this comes out your Mohr circle. So initial this is your K_0 condition, that means earth pressure at rest conditions. So once this soil movement starts what will happen σ_h decreases. If you look at here, this is your σ_v prime, this is your σ_h prime, now what happen initially σ_h prime is you're here. Now

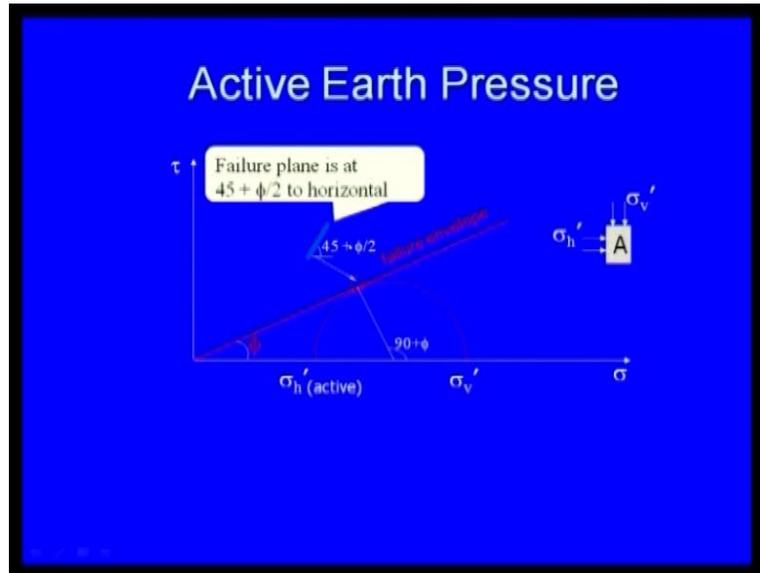
as the wall moves, σ_h prime is decreases that means it is decreasing till failure occurs. How do know that till failure occurs that means the Mohr circle envelope, Mohr circle will touch your failure envelope. So this case is called, this Mohr circle is for your active earth pressure for your active earth pressure.

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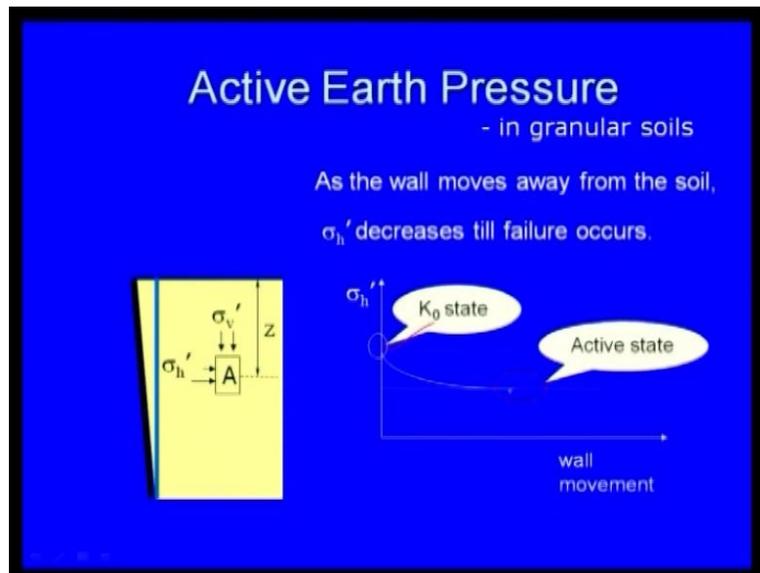
Now active earth pressures, it has been given by Rankine's 1820 to 1872 W J M Rankine, so if σ_h if I write it σ_h prime active, because σ_h is decreasing that means it is coming towards the left, that means it is your K_a into σ_v prime K_a into σ_v prime. So K_a equal to $1 - \sin \phi$ by $1 + \sin \phi$, which is equal to $\tan^2 45$ degree minus ϕ by 2. This is your Rankine's coefficient of active earth pressure – K_a .

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Now failure plane once once you draw the Mohr circle sigma v prime and sigma h prime active state, the failure plane is at 45 degree plus phi by 2 is your horizontal. And this is your failure plane at 45 degree plus phi by 2 this horizontal, to this horizontal.

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Then in granular soil, if you draw the active earth pressure, how it /// how this earth pressure distribution become, if there is a as wall moves away from the soil; if you look at here, this is your wall and this is your soil. As wall moves away from the soil mass, so what happen, initially

it is in K 0 state. K 0 state means earth pressure at rest condition, initially it is K 0 state. So what will happen, it will decrease decrease and it will remain at failure, beyond thus there will not be anymore further movement. So it will be active state, this is your wall movement beyond thus there is no further movement.

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Active Earth Pressure

- in cohesive soils

Follow the same steps as for granular soils.
Only difference is that $c \neq 0$.
Everything else the same as for granular soils.

$$\sigma_{h' (active)} = K_a \sigma_v' - 2c\sqrt{K_a}$$

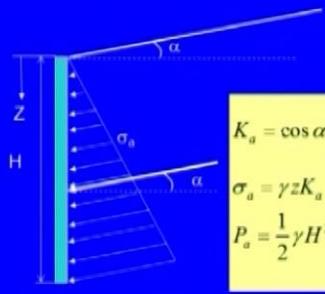
$$\sigma_a' = \sigma_o' K_a - 2c\sqrt{K_a}$$

Eq. 7.8 P. 330

Now for calculating this active earth pressure in cohesive soils, if I divide into 2 parts of these soil. One is your cohesion less soil, other is your cohesive soil. Follow the same steps as for granular soils. Only the difference is that c is not equal to 0. In case of cohesive soil, c is not equal to 0. C is your unit cohesion. Everything else as same as for your granular soil, so sigma h prime for active is equal to K a sigma v prime minus 2 c root over of K a. So sigma a prime is equal to sigma 0 prime into K a minus 2 c root over of K a, this is the derivation for active earth pressure calculations in cohesive soils.

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Rankine Theory with Slope Backfill



Eq. 7.19 P. 336

$$K_a = \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$$

$$\sigma_a = \gamma z K_a$$

$$P_a = \frac{1}{2} \gamma H^2 K_a$$

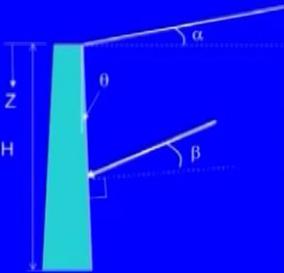
The Rankine active force is parallel to the slope of the backfill

Now Rankine's has given because this active state has been given by your Rankine's, based on that this theory whatever he has given based on his name, this is called Rankine's theory. So Rankine's theory initially what we have discussed, this is a plane ground surface; earlier we have discussed for planar ground surface or may be plane ground surface, there is no variation. Now this ground surface has been inclined at an angle alpha that means with a slope backfill in that case how this resultant of earth pressure acted, if it is a state, if it is a plane then what will happen, it will act like this and the distribution will become like this. So it will act at a at a distance of h by three, once this ground surface is slope that means slope backfill, obviously this will make an angle, this is your resultant pressure, it will make an angle with your horizontal at an angle alpha, which is equal to your slope angle of your alpha. So K a has been derived as cos alpha into cos alpha minus root over of cos square alpha minus cos square phi divided by cos alpha plus cos square alpha minus cos square phi root over, where sigma a is equal to gamma z K a; and P a – this is more important. P a is your resultant earth pressure, which is equal to it is simple geometry or area of the triangle.

If I take it, it is your half gamma H square into K a. So Rankines' active force is parallel to your slope of the backfill that means this Rankine active earth pressure or active earth force, you can say active force is parallel to your parallel to your slope of the backfill. This is your slope of the backfill, parallel to your slope of this backfill.

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Generalized Case for Rankine Active Pressure in granular soils



Eq. 7.17 P. 335

$$\psi_a = \sin^{-1} \left(\frac{\sin \alpha}{\sin \phi} \right) - \alpha + 2\theta$$

$$\beta = \tan^{-1} \left(\frac{\sin \phi \sin \psi_a}{1 - \sin \phi \cos \psi_a} \right)$$

$$K_a = \frac{\cos(\alpha - \theta) \sqrt{1 + \sin^2 \phi - 2 \sin \phi \cos \psi_a}}{\cos^2 \theta (\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi})}$$

$$P_a = \frac{1}{2} \gamma H^2 K_a$$

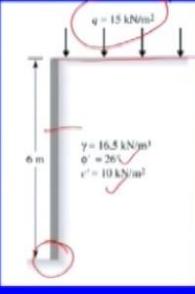
The Rankine active force is **no longer** parallel to the slope of the backfill

Now generalized case for Rankine active pressure in granular soil, the Rankine active force is no longer parallel to the slope of the backfill, in which cases one is your parallel to your slope of the backfill; other cases, it is some cases also parallel not to the backfill. In which cases, when this retaining wall, if you look at here, when this retaining wall is not purely vertical. It makes an angle, this is called it makes an angle theta that means retaining wall itself it is battered there is an angle. So this is your theta, this angle is your theta and this angle is your alpha. Alpha is your slope angle, this ground surface is sloping that means this is your this is your ground surface, it makes an slope with an alpha. Now in this case, this will be your beta, the resultant earth pressure will make an angle beta and beta is equal to beta is equal to tan inverse sin phi, sin psi a one minus sin phi cos psi a. Now psi a has been defined as a sin inverse sin alpha by sin phi minus alpha plus two theta. So then based on our psi a and beta, we can calculated if you know psi a and alpha K a, you can calculated, so P a is equal to half gamma H square into K a.

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Example

Determine the distribution of active pressure



$q = 15 \text{ kN/m}^2$

$\gamma = 16.5 \text{ kN/m}^3$
 $\phi' = 26^\circ$
 $c' = 10 \text{ kN/m}^2$

$[\sigma'_h]_{\text{active}} = K_a \sigma'_v - 2c' \sqrt{K_a}$

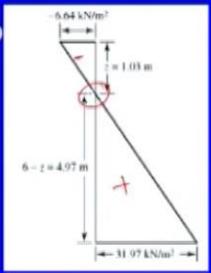
$K_a = \tan^2(45^\circ - 26^\circ/2) = 0.39$

$Z = 0$

$\sigma'_h = (0.39)(15) - 2(10)\sqrt{0.39}$
 $= -6.64 \text{ kN/m}^2$

$Z = 6$

$\sigma'_h = (0.39)(15 + 6 \cdot 16.5) - 2(10)\sqrt{0.39}$
 $= 31.97 \text{ kN/m}^2$



-6.64 kN/m^2

$z = 1.03 \text{ m}$

$6 - z = 4.97 \text{ m}$

31.97 kN/m^2

Now there are certain examples, we can find it out also solved examples, we can how this earth pressure calculation, how there is a example. Suppose this is a retaining wall, for example, this is a retaining wall of height 6 meter and gamma is equal to say this back fill has to be made, its unit weight of this soil is equal to 16.5 kilo Newton per meter cube, phi prime is equal to 26 degree, c prime is equal to 10 kilo Newton per meter square. And q surcharge is equal to 15 kilo Newton per meter square. Now the question is find it out or determine the distribution of active earth pressure that means in this case in this case there is a surcharge, q is equal to 15 kilo Newton per meter square, q is equal to this is called your surcharge. Q is equal to 15 kilo Newton per meter square, this is you can say that this is surcharge. And if you look at this ground surface is plane, it is not slope that means your alpha is equal to 0. It is a very straightforward very simple problem where starting.

Now if you come back to here, sigma h active is equal to because in this case, if you look at here this is not purely cohesion less soil. Rather it is c phi soil that means c is there, also phi is there. So in this case, the soil is c and phi, both c and phi parameter is there. So if you go back to this whatever they have derived for c phi soil, so sigma h active is equal to K a sigma v prime minus 2 c root over of K a. So K a is equal to we can find it out K a active earth pressure, you can coefficient you can find it out K a is equal to active earth pressure coefficient, tan square 45 degree minus phi by 2. So phi is given 26 degree, so it is your 26 degree by 2, it is coming about

to be 0.39. Now at z is equal to at the initially how do we start this earth pressure distribution diagram. If you put, if you look at this equation, what are the parameter, one is your K_a , other is your σ_v' . σ_v' is equal to γz that means you will start the variation, γ is fixed, that means the variation will be your z . So you start with the z , you start with at the surface, so let us say z is equal to 0 that means σ_h' is equal to K_a , K_a is equal to $0.39 \times \sigma_v'$, this comes to be 0.39. So at z is equal to 0, so this is your σ_v' then $2c$ into root over this, so this is coming about to be γ into z , this is to be 0. γ is equal to 16.5, this to be 0, it comes out be minus 6.64.

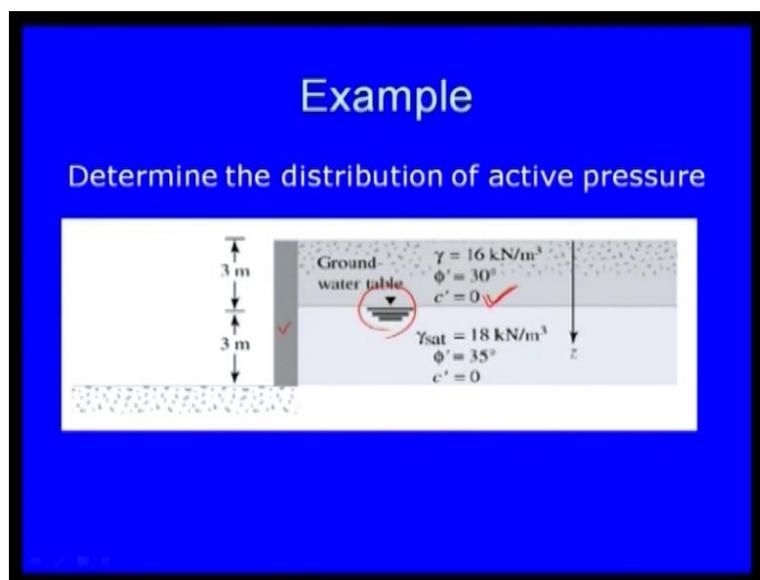
So if I take it, if I take it this is wrong, sorry this is your K_a into σ_v' . If you look at here, σ_v' , why it is not 0 σ_v' ? In this case, σ_v' is equal to if you look at here, in this case, σ_v' is equal to γz ; γz is your overburden plus your surcharge. The surcharge is already there 15 kilo Newton per meter square. The surcharge has to be surcharge effect has to be taken into consideration, so that's why here z is equal to 0 γ into z which is equal to 0. Now σ_v' is equal to q ; q is equal to 15, so it will be 0.39×15 minus minus which is your $2c$ and root over of 0.39. If you look at here, minus $2c$ is equal to ten, c is equal to ten. And root over of your K_a is equal to root over of this, this is your 0.39. So σ_h' is equal to minus 6.64 kilo Newton per meter square.

Now once you have started at z is equal to this, z is equal to 0, you can take also at intermediate point z is equal to two, z is equal to three, z is equal to four also. You can find it out, if you are a beginner. Then you can start with this z is equal to 6, that means at the base of the wall, so z is equal to six, σ_h' is equal to this, because 15 plus γ into z γ into z . So γ is equal to your 16.5 into z is equal to 6, this comes out to be 31.97 kilo Newton per meter square. What is it mean? If you look at this example, that means at the surface at the surface when z is equal to 0 that means there is a negative force, there is a negative pressure that means σ_h' minus 6.64, it is a negative pressure.

So if you come back, now this, if I draw the pressure distribution diagram or earth pressure distribution diagram, if you look at this earth pressure distribution diagram, here it is coming about to be at the top, it is your minus 6.60, 64 kilo Newton per meter square. At bottom, we are getting or bottom we are getting 31.97, here 31.97 kilo Newton per meter square. Now if I draw this is the pressure distribution, earth pressure distribution diagram, this part will be minus, this

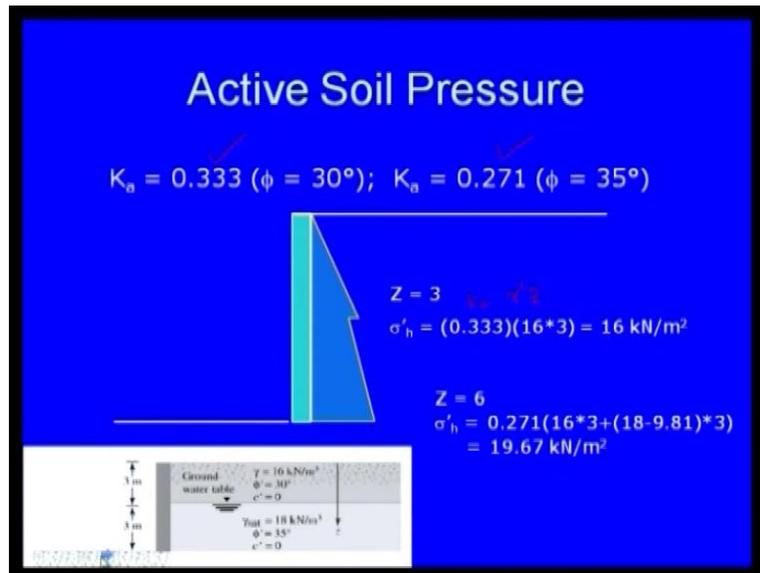
is your positive or plus. So now next step if you look at you will have to find it out at what distance, because this negative will go up to what distance you take it earth pressure as if in this case what will happen, in this case your earth pressure will be 0. So you will have to find it out the point where is your earth pressure is equal to 0. If you take this 0, then you will have to point it out what is the z distance other thing you know c, k and sigma v in terms of sigma v prime, the z term is coming. It comes out to be z is equal to 1.303 meter; where from negative, it will start, it will become 0, then it will go towards your positive.

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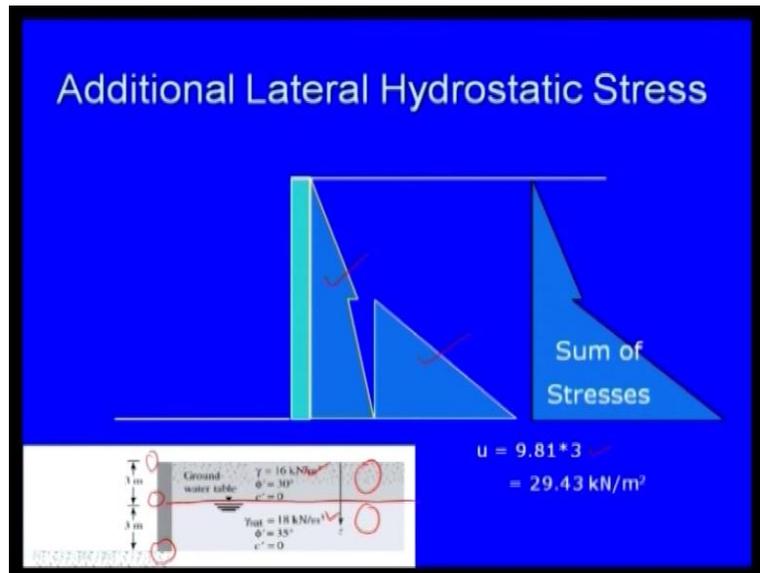
Now second example determine the distribution of active earth pressure, if this is your wall, there is a retaining wall or this is your retaining wall. You consider this grey color this is your retaining wall, and there are two soils, one is your one case the phi is equal to zero, that means it purely cohesion less soil, sorry it is c is equal to 0 so phi is equal to thirty degree that means this is your purely cohesion less soil. And second case c prime is equal to 0, phi prime is equal to 35 degrees, second layer also it is also cohesion less soil that means both the cases cohesion less soil of gamma different and there is one case additional is there ground water table is lying at a distance of three meter below the top. So this is the condition, so find our determine the distribution of active earth pressure.

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Now as usual you find it out what is the value of K_a , one minus sin phi by 1 plus sin phi. Here phi for layer 1, here phi is equal to 30 degree. For layer 2, your phi is equal to 35 degree. For layer 1, what is the value of K_a 0.333; for layer 2, what is the value of K_a 0.271. So then you find it out at z is equal to 3 that means at z is equal to 3 this is that the boundary that means two soil surface boundary and water table is there, so σ'_h is equal to γz into K_a that means this is your $K_a \gamma z$. So for top layer it is 16 into 3 into K_a 16 kilo Newton per meter square. Then again z is equal to 6, at 6 the value is coming about to be 19.67 kilo Newton per meter square.

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If you look at here how this pressure distribution diagram comes into picture. If you look at here, if I am taking let me taking into two parts, for these z is equal to three meter, where is your pressure will start, it will start from 0, it will end here. For z is equal to three meter, this soil also comes into picture, you can calculate also. You can (()), this is your resultant, this is your pressure distribution diagram. Now with this, we have not taken into consideration of water table. So additionally lateral hydrostatic stress also taken into consideration. If you take it to, u is equal to what is your hydrostatic pressure, u is equal to 9.81 into 3 that is your 29.43 kilo Newton per meter square.

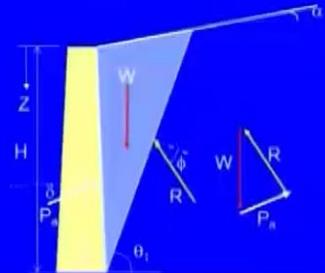
In this case, this is your additional hydrostatic pressure will come into picture, because water table is lying at the interface of both the soil layers. So in this case, if you go if you add your additional layer of hydrostatic stress then what will become with this, this is because of soil, this is your earth pressure and this is because of your water, if you add it how these earth pressure distribution diagram it looks. This is your earth pressure distribution diagram.

Now if I summarize it, now if there are 2 soils of different properties. In this case, one soil γ is equal to 16 , γ is equal to 16 kilo Newton per meter cube. Other case, γ saturated is equal to 18 kilo Newton per meter cube; ϕ' is equal to 30 degree and here ϕ' is equal to 35 degree. In this case, if you look at this case, first you find it out at z is equal to 0 , then find it out earth pressure at z is equal to 3 . So once you comes at the transition, this

is your phase where this soil as well as this soil is there. You calculate for this soil z is equal to three, how much is your pressure. For these soil also you calculate how much is your pressure, then you calculate at z is equal to six what is your pressure, then you because of soil, you find ah what is your net means earth pressure distribution. Then after that once there is a water table is there, additional pressure because of your hydrostatic stress that you add it then this becomes your sum of your stresses. These are the two typical examples I have solved, explained.

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Coulomb's Active Pressure



Coulomb's theory was developed in 1776.

The failure surface is assumed planar and the friction angle between soil and wall is δ .

The active force (P_a) is calculated based on equilibrium.

Different failure surfaces are attempted until the largest P_a is obtained.

$$K_a = \frac{\sin^2(\beta + \phi)}{\sin^2 \beta \sin(\beta - \delta) \left[1 + \frac{\sin(\phi + \delta) \sin(\phi - \alpha)}{\sin(\beta - \delta) \sin(\alpha + \beta)} \right]}$$

$$P_a = \frac{1}{2} \gamma H^2 K_a \quad (7.26) \& (7.27)$$

The next page is your Coulomb's active earth pressure distribution diagram. This Coulomb's earth pressure ah maybe we can discuss in the next class.

Thanks a lot.