Applied Seismology for Engineers

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Week – 08 Lecture - 03

Lecture – 19

Hello everyone, welcome to lecture 19 of the course Applied Seismology for Engineers. This particular lecture is a continuation of the topic Local Side Effect and Ground Response Analysis. Prior to this particular topic, we have covered two more lectures on this particular topic. So, initially, we discuss different kinds of waves which will come into the picture whenever there is an earthquake and how the vibration is getting transferred from the epicenter of the earthquake to distant locations. Because of the interaction of these waves when they are propagating through different materials, the material will experience compression, rarefaction, or it will experience shearing, as a result of which there will be different kinds of stresses being mobilized in the material. When I say material, primarily I am focusing on propagation material or the material which is existing between the propagation path, between the source and the site.

So, we discussed in earlier lectures that primary waves will be there, which will cause compression and rarefaction. As a result, whenever these are passing through a particular medium, there will be compression and rarefaction, and finally, the material will come back to its original volume and then it will continue. Secondly, there will be shear waves or secondary waves. As the name suggests, these are the waves which reach a recording station after the primary wave. As the name also suggests, these are called shear waves. So, whenever these waves are passing through a particular medium, they induce shear stresses in that particular medium.

Local site effect, as the name suggests, is primarily because of the presence of soil at a particular site, how the vibrations, which are transferred through the bedrock medium, will be amplified, de-amplified, and how there will be a change in the frequency content, duration, and frequency content of the motion between the bedrock and the different layers that are available between the bedrock and the ground surface at your site of interest. Since the process is mainly governed by the soil which is locally available at the site of interest, the process collectively is known as local site effect. In order to quantify local site effect, we discussed that there are different methods: empirical methods, numerical methods, and semi-empirical methods. So, we are discussing numerical methods, which further we have discussed as ground response analysis. That means how the ground is going to respond. In order to understand that, whatever analysis we are going to do is called ground response analysis. A couple of terms which you came across when we were discussing this particular topic. Firstly, was where the motion has been recorded, whether the motion is rock outcrop motion, soil outcrop motion, or at bedrock medium. That will define actually what the characteristics of the motion are and where the motion has been recorded. It is already highlighted in earlier lectures that it is quite important to understand before using a particular motion, primarily in site-specific studies, that one should know at what site condition the motion has been recorded.

That means if the motion is recorded as outcrop motion, then the same has to be transferred to bedrock characteristics before using the same motion for ground response analysis. And there are other cases which we have discussed in previous lectures, that is, lecture 17 and lecture 18. In addition, we also discussed when there is a soil layer existing between the ground surface where primarily a building will be located and the bedrock, through which the vibration has been transferred from your focus to the site of interest. So, between the ground surface and the bedrock medium, there will be more than one layer depending upon the characteristics of these layers and the shear strain which will be induced by the propagation of shear waves. We also discussed SH waves propagating vertically upward. These are primarily responsible for changes in the frequency content, duration, and amplitude of waves, which are mostly affected by different layers of the soil.

Then, we discussed that whenever we are talking about the soil, it will not be only the soil layer available at the ground surface, but there will be an n number of layers of soil which will be available. So, depending upon whatever the unknown condition, compatibility equation, displacement compatibility, strain compatibility, we have to take that into account. Thirdly, we discussed that the resistance soil is going to offer to any external loading will be dominated by two parameters. We are referring to Kelvin-Voigt solids where both the damper as well as the spring are attached in series. Then, in such a case, the response of the soil will be approximated by KV solids or Kelvin-Voigt solids. So, we discussed in earlier class that when we go for ground response analysis, though the soil is non-linear in nature, we can approximate based on three methods. One is linear ground response analysis, where we will consider that based on the initial assumption, the response of the soil will be governed by one set of dynamics or properties. We do not check whether these are the properties which are actually mobilized in a particular soil; that is called a linear approach. We consider, initially assume some value of damping ratio, some value of shear modulus, and start solving the equation.

When we say about solving the equation, that means solving the one-dimensional equation of motion. We take into account the damping ratio of the soil, take the damping ratio of the bedrock medium, take shear modulus of the soil, take the shear modulus of the bedrock medium, and using compatibility and displacement and strain compatibility, we will try to figure out what the potential solution is. So, whenever we say about the solution of the one-dimensional wave equation, we have the general solution, but depending upon the soil property and the rock properties, we try to solve further this equation so that we will be able to correlate the motion at the top of a particular soil layer with respect to the motion which has been induced either at the bedrock level or any particular soil layer as you are moving from bedrock to the surface. Generally, whenever the motion is available from an earthquake at the bedrock level, we will consider and try to understand how the bottom-most layer, which is located just above the bedrock, is going to respond during a particular earthquake loading.

So, we will take the input motion at the bedrock level, which has been transferred from the focus, and the soil properties which are available in the bottom-most layer, try to transfer the motion from the bottom-most layer to the top of that particular soil layer using compatibility equations and the solution of the one-dimensional wave equation which we discussed in lecture 17. In lecture 18, we discussed that the soil layer, which we are interested in to find out how much it is going to modify the motion, was basically undamped. Undamped means that there is no damping or there is no resistance which has been offered by the damping. So, solely the soil response is governed by how much shear modulus the soil has; that will govern or control how the displacement values are going to change as the wave is propagating through the particular soil layer. Continuing with that particular example, we also discussed that the bedrock medium which we have considered corresponds to a rigid half space. That means all the incident energy which is actually propagated to the site of interest is actually mobilized towards the surface rather than any portion of that particular energy being contained in the rigid medium or the medium below it. So, the entire seismic energy which the wave was carrying at the interface between the bedrock and the surface is propagated towards the surface or towards the next layer.

So, in today's case, that is lecture 19, as I mentioned, it is a continuation of the previous two lectures; that is why it is called part 3 of ground response analysis. Here, we will be discussing about how to find out a solution to one more field example. That means there is slight modification in terms of soil properties, keeping the rock properties the same. When you go for ground response analysis, we will be interested to know what the parameter is governing the response of the soil, and what is the parameter governing the response of the rock medium.

We have the general equations which have been obtained as solutions of the one-dimensional wave equation. We will use those equations, we will modify those equations because when we were discussing the solution of the one-dimensional wave equation, we primarily focused on the idea that the soil is mobilizing the resistance only by means of shear modulus. But whenever we will be discussing soil which has both damping and shear modulus properties, there will be an additional component which we will take into account and accordingly, corresponding to that, we will try to find out what the solution of the one-dimensional wave equation is, apply the compatibility equation, and then try to find the amplification factor. So, let us discuss about it further. As I discussed in the previous case, that is in lecture 18, we discussed about undamped soil which is located above rigid bedrock. Then we brought one-dimensional wave equation into account, such that when we derive that particular equation, the shear stress is mobilized in the particular soil but only dependent on the shear modulus of the soil. So, if you take one-dimensional equation of motion into account, which we have used in the previous example or in Lecture 18, the displacement value or the shear stress value was only a function of internal properties of the soil, which was shear modulus.

In today's case, we will be discussing about damped soil, which is a more practical example because, in actual site conditions, you will find soil is also offering resistance in terms of damping, which is directly a function of the rate of loading. If you are applying the loading at a very fast rate, the same soil may offer very high resistance. If you are applying loading at a very slow rate, the soil may offer very less resistance. So, that means that is dynamic soil properties, which is, though, offering resistance to external loading, but it is a function of the rate at which the external loading is applied. This we have already discussed in Lecture 17 and Lecture 18 when we were discussing about KV solids as well as undamped soil. So, here is an example:

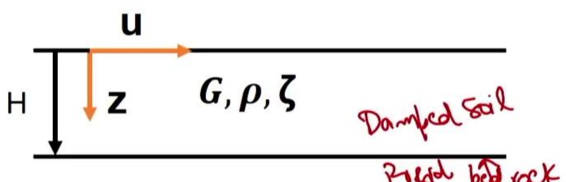

Figure. 1 Soil deposit of thickness H

we are having a soil medium, which is damped soil, located above bedrock. So, this is rigid bedrock, and rigid bedrock means where the incident waves have actually reached from the focus. So, whenever we say we have recorded ground motion at the bedrock, this is the level at which ground motion is available. Bedrock motion—remember, this is not outcrop motion. So, outcrop motion, again, when this will be exposed onto the ground surface, you call it outcrop motion. So, this is the level where bedrock is available, and this is your damped soil. Whenever I am saying "damped soil," that means the resistance soil is offering is having two components: one is shear modulus, which is given over here also (G value), and the other one is damping ratio—how much damping the soil is having, which is actual damping to the critical damping. And the next part is the mass density of the medium, that is, rho. So, these are the three properties of a particular soil layer. In addition, H is the total thickness of the particular soil layer, which is actually contributing to modification in the bedrock properties incident at that particular level.

So, I will be interested in this particular case. When the soil is undamped, we had solved this equation. In this particular case, the soil is damped. We are interested to find out when incident motion is there, how motion will be changing its properties, its frequency content, primarily because this is linear. So, we will be only taking into account the transfer function values—how this is going to change the properties of the motion from bedrock to the top of that particular soil layer, having thickness capital H given over here. H at any moment is measured with respect to the top of the soil layer, measuring downward. U is the displacement values, which we are targeting to find out even at any particular intermediate level. So, you start at Z equals zero; that means you are talking about at the surface, providing there is no layer above this particular soil layer. Z equals H means you are talking about the soil-bedrock interface, and any value of Z between zero to H means you are talking about any intermediate layer located between the top of the soil layer and the bottom of that particular soil layer.

You have the governing equation. Use that particular equation where you are able to correlate the value of U with respect to the inherent properties of the soil as well as the motion characteristics, the omega value, the circular frequency. So, this is the case, and the bedrock is also rigid. So, there is no elastic half-space. In other ways, we are not taking into account the damping characteristics of the medium, the bedrock medium. So, we will be only having the damping characteristics in the soil medium, no damping characteristics in the rock medium. So, another case, similarly, can be like you are having damped soil located above elastic half-space. So, we started with undamped soil over a rigid half-space. That means the soil is offering resistance; the bedrock is not containing any seismic energy. It is simply transferring the energy to the above layers, and the soil is only offering resistance to shear modulus. There is no damping—that was case one.

The second one is you move one step ahead in one time. So, the second one is the damped case. Now, the soil, in addition to damping ratio, in addition to shear modulus, is also having damping ratio. But the rock is still rigid; there is no elastic space. And then, next time, it can be damped soil and elastic half-space. Similarly, the approach will remain the same. The only thing is, what are the inherent properties of the soil or bedrock? Those components will keep on adding in the governing equation, and accordingly, you will try to find out how much is the transfer function. Using that transfer function, being a linear approach, we will try to use the transfer function, multiply that transfer function with respect to Fourier amplitude of the bedrock medium, try finding out the Fourier amplitude at the surface. Then, using that Fourier amplitude, we will transfer it to the time domain and try getting the acceleration time history.

The only difference between this particular case and the previous case is the value of the transfer function. The functional form of the transfer function, in the previous case, was only the value of omega thickness and shear modulus or shear velocity of the medium. In this particular case, because the medium is also offering resistance from damping ratio, there will be some additional components representing the damping ratio in the medium.

The one-dimensional wave equation we have already discussed: $\rho\, \partial^2 u/\partial t^2 = G\, \partial^2 u/\partial z^2$

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} \qquad 9$$

We have already discussed initially we determine this particular equation in terms of angular displacement. Then, we also modified that particular equation with respect to the value of u. Now, this particular equation, if you see, the value of displacement with respect to time and with respect to space is solely dependent on the value of shear modulus. So, this is primarily the equation for undamped soil. When we talk about damped soil, we have already also discussed about KV solids.

$$\tau(z,t) = G\gamma + \zeta\dot{\gamma} \qquad 10$$

That means the total stress developed in the medium is a summation of stress, which is by the spring, which is giving you resistance linearly with respect to loading, and the other one is with respect to the dashpot system, which is directly proportional to the rate of loading. So, where the stress value is not only the component of G times gamma but is also a function of damping ratio multiplied by the rate of loading, $\partial\gamma/\partial t$. So, again, with this particular part, we will continue. So, this is the basic equation based on which the shear stress will be induced. Now, this particular component will try to incorporate in equation number 9, and then we will get the solution of the one-dimensional equation of motion. First, we will get this one-dimensional equation of motion. So, initially, when we determined the one-dimensional equation of motion for undamped soil, this was the equation where the resistance is directly proportional to the shear modulus. Considering now about damped soil, which is also having resistance because of shear modulus, you are having two components. In such a case, this equation will slightly be modified, and this will be the revised equation of motion.

$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} + \zeta \frac{\partial^3 u}{\partial z^2 \partial t} \qquad 11$$

This is a one-dimensional equation of motion for KV solids, where you can see the displacement value with respect to time is having some component with respect to space coming from shear modulus and some component of displacement coming from damping ratio. Now, this equation we will try to use and try solving the equations again. If you remember, the solution for the undamped case: A exponential iota theta plus B exponential, those components were there—that will come in the next slide. Similarly, to this particular part also, you will have the corresponding solution of the one-dimensional equation of motion. Just remember, this was the equation of motion for undamped soil; this is the equation of motion for damped soil. So, the solution—now you can see—initially, we had the solution.

$$u(z,t) = Ae^{i(\omega t+k^{*}z)} + Be^{i(\omega t-k^{*}z)} \quad 12$$

What we are interested to find out is how the displacement is changing within the space Z with respect to time t: A exponential iota omega t, where omega is the circular frequency of external loading condition, and then you have again the wave number. But there is one star over here because, initially, if you remember, in the case of undamped soil, the value of wave number was ωH/Vs. Omega is circular frequency because of external loading, and Z is the thickness, and Vs is the shear velocity. In this particular case, there is also some component which is coming from damping ratio. As a result of that additional component, the wave number has become k star. Similarly, in the second case, you are also having an additional component. So, if you can recollect, some component is propagating upward, and some component of wave is propagating downward. There, you are having the one-dimensional equation of motion, the solution of the one-dimensional equation of motion, which is given over here. So, in the undamped case, what we did was start solving this particular equation, considering that at the topmost layer, that means at the top of this, provided there are no layers further, that means this is a stress-free condition. So, you will take that stress-free condition, apply it to this particular solution, and try finding out how the displacement value at the top and the displacement value at the bottom of these particular layers are correlated with respect to each other, which was called the amplification factor or, in general, we have used the term transfer function. So, k star in the previous equation u Z comma t equals to a exponential omega t plus k star x plus b exponential iota omega t minus k star x or k star z where k star is complex wave number.

$$k^{*} = \frac{\omega}{v_s^{*}}, v_s^{*} = \sqrt{\left(\frac{G^{*}}{\rho}\right)} \quad 13$$

Let's see what is the functional form of complex wave number you see over here we have used the wave number h was not there earlier omega over Vs was the complex wave number which was representing the wave number for undamped soil in this particular case the wave number is omega over Vs star. Omega is circular frequency because of external loading Vs star means there is some additional component which is coming in case of damped soil. Vs star is square root of G over rho G is the shear modulus and rho is the mass density of the medium but it is not G it is G star. G star is correlated with respect to G so this is shear modulus and you may call it as complex shear modulus or some component which is having shear modulus and some component of damping ratio also.

$$\check{G}^{*} = G(1+2i\zeta) \quad 14$$

So, G is shear modulus multiplied by one plus two times iota chi which is representing the damping ratio remember this is damping ratio of the medium. So now the basic equation the functional form of the equation remain the same but only thing the k becomes k star in order to bring damping ratio into account how it is bringing omega over Vs star Vs star is complex shear velocity which is correlated with respect to square root of G star over rho G is again complex shear modulus which is related to shear modulus times one plus two twice iota chi is damping ratio and so you are having you are incorporating the value of damping ratio by means of this complex form wherever iota is coming even we will see later on also that is representing additional component coming from just because of the medium is offering resistance from damping ratio or additional parameter which is offering resistance to external loading is damping ratio.

$$v_s^* = \sqrt{\left(\frac{G(1 + 2i\zeta)}{\rho}\right)} \quad 15$$

Again so Vs star you can correlate simply represent replacing G star by means of G one plus twice iota chi the equation remain the same so if you are having the damping ratio of the medium if you are having the shear modulus of the medium you can find out how much is the complex shear velocity of the propagation medium remember if the medium is not offering resistance this value will become zero if it is not offering resistance due to damping the second component value will become zero and you will come back to its original form of undamped case that is G over rho considering the damping ratio value is generally very less so there will not be significant variation if you calculate shear modulus or shear velocity and complex shear modulus and complex shear velocity but still there will be a difference because of this particular damping ratio. Again depending upon whether significant difference will be there or not that will be controlled by how much shear strain is getting mobilized in a particular soil layer during a particular earthquake motion because these shear modulus as well as damping ratio these are dynamic properties of the soil so being dynamic that means depending upon how much shear strain is getting mobilized by means of external loading condition will define how much will be the shear modulus how much will be the damping ratio and collectively based on the combination of shear modulus and damping ratio will determine how much is the resistance soil is offering or is going to offer due to external loading condition. So when we will discuss further about this particular part we will also see how a particular soil layer is going to respond differently during different earthquakes that means if some earthquake is mobilizing very less value of shear strain in the soil the same soil may behave differently in comparison to another earthquake which is inducing different value of shear modulus or shear strain with respect to the previous earthquake certainly the local site effect will be significantly different even at the same site for that reason many a time you will see even there is an earthquake which has happened at distant location but because of these two properties collectively the resistance soil has offered is significantly different leading to many times amplification in the bedrock motion and subsequently this amplified ground motion will be subjected to buildings these will undergo failure ground if possible it will undergo leak affection even at larger distances. So that is primarily because of dynamic properties or the properties which are not constant for a particular soil layer but are dynamically changing depending upon what external loading condition what shear strain is being mobilized in the particular soil layer.

$$v_s^* = \sqrt{(\frac{G}{\rho}(1 + 2i\zeta))} \quad 16$$

Okay, considering the component which we have just discussed about Vs star so you can see over here Vs star is again G by rho 1 plus iota so this term will be there 2 will also be there which was given in the previous case.

$$\mathbf{k}^* = \frac{\omega}{v_s^*} = \frac{\omega}{v_s(1+i\zeta)} = \frac{\omega}{v_s}(1 - i\zeta) = k(1 - i\zeta) \qquad 17$$

In this particular case we will discuss about omega over so k star becomes omega over Vs star omega over Vs (Vs is correlated with respect to Vs star as Vs 1 plus iota chi) and then subsequently you can find out the complex wave number equals to k which was the wave number for the case of undamped case multiplied by 1 minus iota chi so that is going to give you the complex wave number and this is the wave number corresponding to undamped case.

$$F(\omega) = \frac{1}{\cos(k^* \times H)} = \frac{1}{\cos(\omega * \frac{H}{Vs} * (1 + i\zeta))} \qquad 18$$

The transfer function which was going to give you the ratio of displacement between the top of the soil layer and the bottom of this particular soil layer because we are considering the thickness of the soil layer as the entire thickness value equals to z or h then transfer function you can find out the value of q corresponding to z equals to 0 corresponding to z equals to h take those two ratio into account as discussed for undamped case we will get the transfer function remember for the case of undamped case the value of transfer function was 1 over cosine of k h that was the value of transfer function for the case of undamped case or undamped soil in case of damped soil you are having the value of complex wave number that is k star times h 1 over cosine of omega times h over vs multiplied by 1 plus iota chi so that is going to give you the complex wave number complex shear velocity and subsequently the transfer function value how it is going to be modified with respect to an undamped case so that is how the procedure remains the same but you are bringing into account the value of damping ratio at every step whether you are talking about complex shear velocity when you are talking about complex wave number when you are talking about complex shear modulus similarly when you are talking about transfer function if you can recall the solution which we have discussed in previous case we have taken the transfer function bring which was a function further of shear modulus or shear velocity alone multiplied the transfer function with respect to the Fourier amplitude of bedrock motion in this particular case.

$$F(\omega) = \frac{1}{\sqrt{\cos^2(kH) + \sinh^2(\zeta kH)}} \qquad 19$$

So, the transfer function value again if you see the terms inside you can find out cosine square of k h plus sine hyperbolic square chi k h that is the term you will get in the denominator from the term which is shown in the figure 18. So, this is your transfer function for the case of the damped case. Earlier, this was the component alone for the damped case, undamped case. Now, this additional component is coming because of damping present in the system. So, because of damping, you are getting an additional component as a function of the transfer function: 1 over square root of cosine square k times h plus sine hyperbolic square chi k h. That is the value of the transfer function or the functional form of the transfer function you are getting for the case of damped soil located over rigid half-space. Remember, we are still on rigid half-space. Now, this is the transfer function value. If you see, it is a function of the thickness, it is a function of the wave number, which is further a function of shear modulus or shear velocity, and the damping ratio. So, if I am interested to find out the value of transfer function variation, there are three components based on which the transfer function is varying. One is the value of omega; that is, how much frequency content is available for the soil to respond. Secondly, how much is the shear modulus or shear velocity of a particular soil medium? Thirdly, how much is the damping ratio of a particular soil medium? So, these three parameters are going to control now f omega value. If I am interested to see the behavior of each of these components, we can subsequently change the variation of each of these parameters and see, as shown in the next slide.

$$\mathbf{k_1} = \frac{\omega}{\mathbf{v_s}} \qquad k_2 = -k \times \zeta = -\frac{\omega}{v_s}\zeta$$

So, remember, k is having two components. One is coming from the real part; that is, omega over vs, and the other one is coming from the imaginary part; that is, this part was there, which is given over here as k2 or omega chi over vs. So, where k star is a complex wave number with the real part represented by k1, the functional form of k1 is given over here, and the imaginary part is given by k2, which is given over here: minus k times chi.

$sinh^2 y \approx y^2$ **for very small y**

$$\mathbf{F}(\omega) = \frac{1}{\sqrt{\cos^2(\mathbf{kH}) + (\zeta\mathbf{kH})^2}} \qquad (20)$$

Remember, if sine hyperbolic y sine hyperbolic square y will be equal to y if the value of y is very small, keeping that into account and remembering the functional form of the transfer function which we have just discussed, the value of transfer function can be approximated as 1 over square root of cosine square k h plus chi k h square. We can compare this with respect to the equation which was given in this particular case. So, this was the equation which is given over here. So, this is the equation which is given over here. This was initially sine hyperbolic square of this particular term, which, as per this approximation, will be equal to chi k h square. Now, here we can see it is directly a function of the damping ratio, shear wave velocity, and the operating frequency of the motion, which will come into picture in omega form. So, when we go for the solution, we will be interested to find out now. Again, we can relook into the equation simply by turning the functional form of wave number over here as omega over vs in both the terms.

$$F(\omega) = \frac{1}{\sqrt{cos^2\left(\frac{\omega H}{v_s}\right) + \left(\zeta\frac{\omega H}{v_s}\right)^2}} \qquad (21)$$

Now, here we can see the value of transfer function or the mode of transfer function. When we will go for finding out the mode, that will make sure that once you are going to the surface, getting the Fourier spectra, you are getting real parts. So, you see over here, you are having the value of omega, which is a representation of how many frequency contents are available in your external loading condition, what is the shear modulus, or what is the shear velocity, or how much resistance because of shear modulus of a particular medium is available, and the last one is damping ratio, which is directly the function of the rate of loading, how much it is available. These three components are going to confirm how much is the value of transfer function, or depending upon this value of transfer function, how the motion from the base of the particular soil layer will be transferred to the top of that particular soil layer, keeping only the soil layer is damped; there is no damping coming from the bedrock medium. So, the modulus value, the mode of transfer function, that gives you the amplification factor. Using that value of the amplification factor multiplied with respect to Fourier amplitude spectra of the base of the soil column or the base of the soil layer will give you how much is the Fourier amplitude at the top of that particular soil layer. So, the modulus is the amplification factor. So, again, you see if you take the mode of this particular part f omega, you will get the value of the amplification factor, which is directly going to give you how much amplification is going to happen between the base of the particular soil layer and the top of that particular soil layer at each frequency content. Because you will be multiplying the transfer function at each frequency content or corresponding to each frequency content what is the Fourier amplitude? Multiply with respect to the mode of transfer function or the modulus of the transfer function.

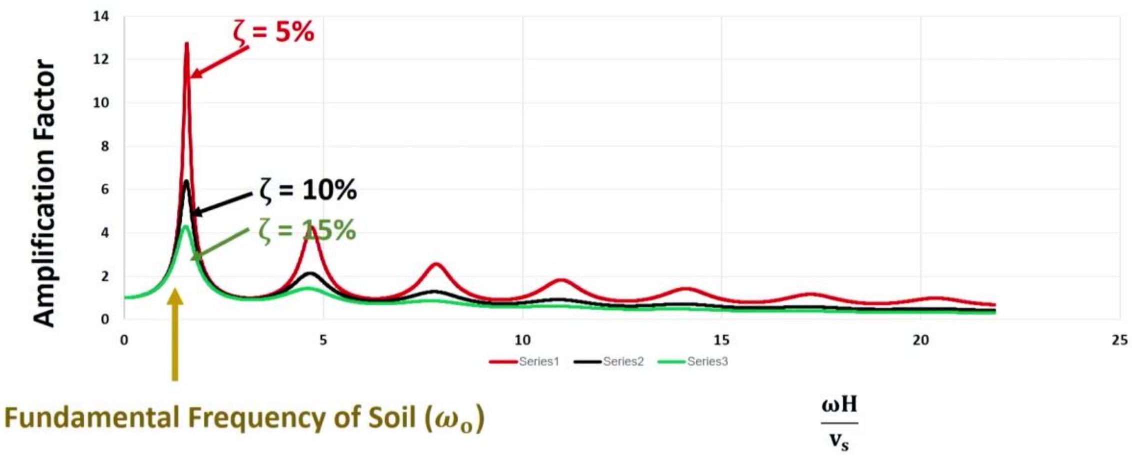

Figure 2: Influence of frequency on steady-state response of damped, linear elastic layer

Now, here we can see the amplification factor or the modulus of transfer function is a function of two parameters added over here. Similarly, if you change the value of shear wave velocity, again, subsequently, we can see the effect with respect to the frequency content. So, you can see over here, this is the frequency content of the motion. So, you bring any frequency content into account corresponding to omega value. You put in the equation, which was shown in the previous slide, shear modulus or shear velocity we are considering the same because it is already user-defined. Then you can see as you are starting from lower frequency content, it is reaching a peak value. You can keep on changing the frequency content for keeping one value of damping ratio. So, lower is the value of damping ratio, the system will respond more, or there will be large fluctuation in the response of the system, which is given and indicated by the plot over here. As you increase the damping ratio, initially it was for 5%, then you see 10%, there is significant reduction in the response of the soil layer or reduction in the amplification factor. Further increase to 15%, you can see the amplification factor is further reduced, and this significant change in the amplification factor is happening at lower frequency content because generally, this is the range in which the natural period of the soil column will also exist. That is why if you replace this with respect to the undamped case, you will find out the responses at separated by pi by 2; you will find the responses are separated, and then subsequently you are observing resonance condition. Because this is a damped case, there is actually no infinite amplification factor, but depending upon the value of damping ratio, the amplification factor will have a very small value, as shown for 15% to very high value of close to 13 for a damping ratio of 5%. Again, if you move from low to high frequency content, there is not significant change in the amplification factor, whatever is the value of damping ratio. So, depending upon the peak first peak you are getting, you can find out that is directly an indication of approximately the natural period of the soil column also exists close to the peak. Sometimes, you can also have subsequently second peak, third peaks are also like that; that is an indication of second or third natural frequency of particular multiple layer system.

So, now, based on this particular graph, we are able to find out how much is the amplification factor, which is a representation of how much change in the displacement between the bottom and the top of a particular soil layer for a damped soil located over a rigid half-space. The frequency content that corresponds to local maxima, as we discussed, is an indication of the natural frequency of the soil. Subsequently, you can have second, third peaks as indications of second and third natural frequencies, omega 1, omega 2, omega 3 values for a multiple layer system. In addition, we have also seen the response or the effect of damping ratio is more prominent for lower frequency content, primarily because the natural period of the soil column will also be in that particular range, 1, 2, 3, up to 5 Hertz. That is the range in which, if we refer to site classification, that is generally the range in which you can find out for different site classes the natural period of the soil column exists. So, the same thing we can see over here also, and as you move to higher frequency content, whether the damping ratio is 5%, 15%, or 20%, you will not see significant change in the amplification factor values. And this observation, that is, no significant change at higher frequency contents, significant change with respect to damping ratio at lower frequency content, that will finally determine how much amplification or de-amplification will happen at lower frequency content. When we are going for this particular linear analysis.

$$\omega_o = \frac{\pi v_s}{2H} = \frac{2\pi}{T_s}$$

Now omega value corresponding to the first peak, you can find out the value of omega. If you consider the local maximum into account and differentiate the transfer function, you can correlate the value of omega equals to pi over pi by 2 VS over H, which can be also written as 2 pi over TS, where TS is the natural period of the soil column.

$$T_s = \frac{4H}{v_s}$$

And that's how you can also find out the natural period of the soil column is 4 H over VS, or if you are interested to find out the natural frequency of the soil column, FS, you can also write it as Vs over 4H. That means if a soil column is given to you, having shear velocity of VS and the thickness of the soil layer is capital H, using this particular equation, you can find out how much is the natural period of this particular soil column. Taking what is generally the stiffness of the soil medium, starting from very loose to stiff to very hard soil, you will be able to find out that the natural frequency of this particular soil column in general will be ranging from 1, 3, 5 Hertz. That's the general trend in which the natural frequency of the soil columns will exist. This is the result where the amplification factor at low frequency content was reaching significantly high value in comparison to high frequency content.

Qn.02. For a site having an average shear wave velocity of 340m/s and damping ratio of 5%. Compute the time history of acceleration at the surface of the linear elastic soil deposit of 4m thick overlying a rigid bedrock considering the deposit for undamped condition.

Okay, let's see an example so that it will be clear. This was a site having shear velocity of 340 meters per second and considering it is damped soil, so you are having a damping ratio of 5%. It is asked to find out the acceleration time history at the top of that particular soil layer, which is having shear velocity of 340 meters per second and damping ratio of 5%. Now, it will not be undamped case because damping value is already given over here, so I am just striking it out. The value of damping ratio is given as already 5%. So, what will be our approach?

NOTE:

- **The problem is processed using Chile Earthquake, 1985, having PGA = 0.12g**
- **Any ground motion can be used for the procedure.**
- **To perform Fourier transformations, ensure that Microsoft Excel has activated the "Data Analysis" tool pack.**
- **The Fourier analysis in MS Excel is restricted to a maximum of 4096 points. Ensure the number of points considered in the acceleration time history is 4096.**

Remember this equation, similar to lecture number 18, is also solved corresponding to input motion from the 1985 Chile earthquake. The PGA value of, or the peak ground acceleration value of this particular earthquake, was 0.12 g. So, if you take the earthquake record corresponding to the recording station, which has been considered over here, and plot it such that time value is there on the x-axis and acceleration values are on the y-axis, corresponding to the peak value of this particular plot is 0.12 g. That's how you can interpret the graph. So, any ground motion you can use because we are interested to find out the motion. So, we have to be very specific in the beginning itself. Okay, let's see, I am interested to determine the motion because of the response of a particular motion, and when I am taking the motion, I am considering the earthquake, or in this particular case, the 1985 Chile earthquake having PGA value of 0.12 G. Because this is going to give you how much is the frequency content, what is the frequency content at the recording station where this particular earthquake was recorded, which will be treated as bedrock motion for this particular example. So, to perform, we generally start because we will be discussing in terms of Fourier spectra, so we will be using Fast Fourier Transformation in Excel. You can go to the data analysis tool pack and then activate Fast Fourier Transformation in your Excel. Similarly, in other tools, also you can utilize, and you can solve this particular example. As we mentioned, the Fourier amplitude, the

Fourier analysis is restricted to a maximum of 4096 points, so one has to ensure that 4096 points in acceleration time history should be there to do the Fourier analysis.

The procedure for computing GRA is as follows:

1.Generation of Fourier Series of Bedrock motion, FA: Conversion of data from the Time domain to the Frequency domain is performed through Fourier Analysis

- **Data → Data Analysis → Fourier Analysis → feed the input and output range (refer to Figure. 3.1 for the input and output ranges used) → OK**
- **The result corresponding to the initial 10 data of acceleration is provided in Fig. 3.2**

So, firstly, we will be determining the bedrock motion, which is already given to us in terms of Fourier spectra or Fourier series. So, the generation of Fourier series of bedrock motion or Fourier analysis of bedrock motion of the 1985 Chile earthquake, such that whatever motion record is there in time domain will convert to frequency content. Again, if you remember, last time also we discussed, depending upon the frequency at which the record is available, that will also determine how much is the highest frequency which is available in your record, so that we have discussed already. So, we have to take into account how much is the range of frequency content which is available in your existing record, which has to be used while converting this from time domain to frequency domain. So, results corresponding to 10 initial points are already given in figure 3.2. This conversion from acceleration time history to Fourier amplitude spectra is similar to whatever has been discussed in lecture 18, so I will skip some of the parts.

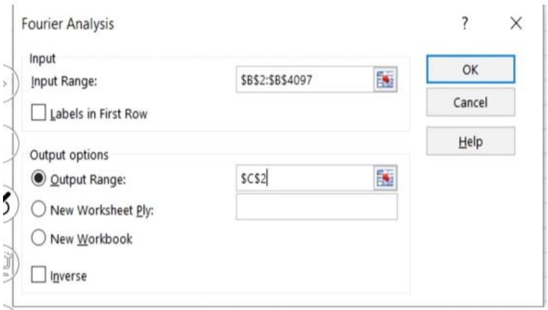

Fig.3.1 – Input and output ranges for computation

Time(s)	Acc(g)	Fourier Amplitude
0	-0.00045	1.23514217851749
0.005	-0.00034	0.509959898345385+8.29304769247872E-002i
0.01	-0.00015	0.505800021257452+0.190868940909305i
0.015	8.85E-05	0.412057826504709+0.445870885262561i
0.02	0.00033	0.340700518550535+0.946706698469754i
0.025	0.000543	-0.233045415360519-0.710943091104957i
0.03	0.000697	1.02621793770719+0.718114502849389i
0.035	0.000767	1.25176744404983-0.326332601786579i
0.04	0.000739	0.180365680132803+0.672497849516099i
0.045	0.00061	1.0282212141-0.479995344323163i

Fig.3.2 – Fourier Amplitude corresponding to initial 10 data points

You can see over here, the record is available at 200 Hertz or 0.005 seconds interval. One record of acceleration has been sensed by the sensor, which is available for the 1985 Chile earthquake mentioned over here. That means maximum up to 200 Hertz, and correspondingly Nyquist frequency, you can find out that is going to give you how much the highest frequency content available in your record that is given as 100 Hertz over here. So, this is again acceleration time history, and then when you are converting it to Fourier spectra where you have to give what is the range of frequency content, one can find out, which you can determine based on the rate at which the data was recorded during a particular earthquake. You can find out the Fourier amplitude on the left-hand side of the box, which appears when you go for Fourier analysis.

You have to basically tell what is the input motion range and what is the frequency content range, and then it will convert to Fourier analysis.

2. Generation of Frequency Content :

- **The Nyquist frequency is the highest frequency in the Fourier series = 1/ Δt = 1/ 0.005 = 200Hz.**

- **The minimum frequency, f1 = 1/(0.005*4096)**

- **The successive frequency , f2 = f1 + (1/(0.005*4096))**

- **As the Fourier amplitude of the second half of points would be the complex conjugate of the first half, we limit the frequency calculation corresponding to half of the sample points for the sophistication of data handling(i.e., Up to 100Hz). The same is also evident from the highlighted values of Fourier amplitude. (Fig.3.3)**

So, the generation of frequency content has been discussed earlier also. The highest frequency content you can find out corresponding to 1 over 0.005 Hertz, 0.005, that is going to give 200 Hertz. The minimum frequency content you can find out based on the time interval taken and 4096 maximum or upper limit of the points, and that's how you can find out more number of points, and then the highest frequency content we have already discussed that will be like the entire frequency range is corresponding to 200 Hertz. That means initial 100 Hertz, there will be some frequency content, and then after 100 Hertz, 100 to 200 will be similar to whatever has been observed from 0 to 100. So, it will be like 0 to 100, and then 100 to 0 in that particular order. The cycle or the amplitude of your Fourier amplitude of the series will be repeated because it will be conjugate complex function.

1	Time(s)	Acc(g)	Fourier Amplitude	Freq(Hz)
2045	10.215	0.259481	-3.48134496086477E-002+1.15999688430846E-004i	99.8046875
2046	10.22	0.26737	-3.48134234236079E-002+9.27995812676774E-005i	99.85351563
2047	10.225	0.262088	-3.48134030576072E-002+6.95995870821919E-005i	99.90234375
2048	10.23	0.242977	-3.48133885105274E-002+4.63996776531306E-005i	99.95117188
2049	10.235	0.21045	-3.48133797822999E-002+2.31998246953344E-005i	100
2050	10.24	0.166052	-3.48133768729195E-002	
2051	10.245	0.112403	-3.48133797823003E-002-2.31998246961046E-005i	
2052	10.25	0.053022	-3.48133885105281E-002-4.63996776515208E-005i	
2053	10.255	-0.00796	-3.48134030576072E-002-6.95995870820254E-005i	
2054	10.26	-0.06618	-3.48134234236093E-002-9.27995812669558E-005i	
2055	10.265	-0.11747	-3.48134496086462E-002-1.15999688431401E-004i	

Fig.3.3. Frequency content and complex conjugate nature of Fourier amplitude (highlights)

So, this is already discussed, and that's how you can find out the Fourier amplitude. You can see over here also the Fourier amplitude corresponding to different times, and then you can see it has started corresponding to 4096. So, this one you see up till 100 Hertz, you are getting different values, and after 100 Hertz, the values are getting started repeating. Okay, so the same exercise will be repeated to find out the Fourier amplitude content of the input motion.

- **Transfer function, F(ω) = $\frac{1}{\cos(\omega * \frac{H}{Vs} * (1 + i\zeta))}$**

  where ω (rad/s) = frequency content of the ground motion

  = 2 * pi()*Frequency

  H = Thickness of the strata = 4m

  Vs = Shear wave velocity of the layer = 340 m/s

  ζ = 5% =0.05

- **The number of points to be considered is the same as that of the frequency content.**

Then, calculation of the transfer function, we knew that is the transfer function which we have discussed, cosine of omega H over Vs, 1 plus iota chi, that was the transfer function which was given. Then, you will take the modulus of that. Then, in the bottom, you will have cosine square of some terms and then sine hyperbolic square terms, which was given in the previous slide, referring to that particular part. Remember omega is the frequency content of the input motion, which we have just discussed in the previous slide, which we have also obtained based on Fourier series analysis. So, 2 pi times frequency content, that is the total length, H is the thickness of the soil strata, which is given as 4 meters, shear velocity, which is given as 340 meters per second, damping ratio is given as 5 percent or 0.05. The total number of points to be considered for transfer function will also be equal to how many number of points are available in your Fourier series because when you transfer this bedrock motion to the surface, there will be a product of transfer function and Fourier amplitude. So, if you are having 4096 points corresponding or corresponding up to 100 Hertz, then the same value of transfer function also will be calculated over here. The amplification factor is the absolute value of the transfer function, that we have already discussed.

$$F(\omega) = \frac{1}{\cos(\omega * \frac{H}{Vs} * (1 + i\zeta))}$$

Calculate the Denominator ,

- **D = IMCOS(COMPLEX((ω *H/Vs),(ω *H*ζ/Vs)))**

So, calculate the denominator, whatever has been given in the previous slide, that cosine omega H over Vs, 1 plus iota chi, that we have to firstly find out. This is a functional form of transfer function, so we'll try finding out the complex part using IMCOS of complex, and then you give whatever is the input motion. So, H value, Vs value are constant, and omega value based on the frequency content, you can find out the omega value and use these terms along with the function Excel function IMCOS. Then, within bracket complex, and using this function, you can find out how much is the value of the denominator, which is given over here, which is the transfer function value. Remember, this is not amplification factor; this is a transfer function value. It's the value of the denominator. So D is basically indicating whatever is given in the denominator in this particular equation. So, this part is actually D. Once you determine the value of D, you can find out how much is the value of. So above equation comprises all the calculation of cosine of complex terminology using IMCOS. IMCOS is going to give you cosine of complex terms which are inside the cosine bracket, which is given in that particular equation.

Transfer function, F(ω) = IMDIV (1,D)

Now, the transfer function is 1 over D, so you have to actually find out inverse, but because this is a complex function, so again, you can use IMDIV. What is in the numerator, what is in the denominator, that you can find out using this particular functional form. So, F omega will be equals to IMDIV within bracket 1, comma D. So D is whatever system is in there in the denominator, which has been calculated already using IMCOS complex. So, this D is whatever has been calculated using this particular equation. When that part is calculated, you can find out F omega value. Remember, F omega is the transfer function, and we have to find out the amplitude of the transfer function; that will be the amplification factor.

Freq(Hz)	Denominator, D	Transfer function	FA x Transfer function
0.048828125	0.999993502527893-6.51374937357727E-07i	1.0000064975139+6.51383402020922E-07i	1.23515020387096+8.04551114222256E-07i
0.09765625	0.999974010195157-2.60548282027051E-06i	1.00002599047354+2.60561825751282E-06i	0.509972936359465+0.0829339610879751i
0.146484375	0.99994152325255-5.86227286157881E-06i	1.00005848013281+5.86295853481986E-06i	0.505828481453184+0.19088306843487i
0.1953125	0.999896042117993-0.0000104216604171194i	1.00010396858173+0.00001042382758141i	0.41209601989129+0.445921537045873i
0.244140625	0.999837567376571-0.0000162835269873647i	1.00016245874679+0.0000162888182242291i	0.340740447596486+0.946866048862381i

Table 3.4. Transfer function F(ω) values

So firstly, let's see the value of D, which is given over here. Corresponding to value of F, corresponding to this value of F, you can find out omega value as well is known to us, Vas well is known to us, then we can find out how much is the transfer function. Damping value is also known to us, so you can use those functional form and the function call IMCOS. That is going to give you the value of D. IMDIV of 1, comma D will give you the transfer function amplitude of the transfer function. You have to find out using the absolute value of the transfer function and then multiply with respect to the Fourier amplitude. So right now, we are only considering the transfer function. Multiply with the Fourier amplitude. Fourier amplitude we have already calculated in earlier slide by using input motion and transferring it from time domain to frequency content. So, this also in frequency domain. The transfer function is also we are estimating with respect to the frequency content. So, both the things, Fourier amplitude as well as transfer function, both are in frequency domain. That has to be ensured that equal number of points you estimate for Fourier amplitude as well as for the transfer function.

4. Fourier Series of the ground surface:

(a) Fourier series up to 100Hz of frequency content –

- **Fourier series of the ground surface = FA * Transfer function**
- **In Microsoft Excel, any operation involving complex quantities must be performed using compatible inbuilt functions.**
- **For the product of FA and Transfer function, use IMPRODUCT function.**

(b) Fourier Series from 100 – 200Hz of frequency content –

Then Fourier amplitude at the ground surface will be a product of Fourier amplitude at the base multiplied by the transfer function. You will get the Fourier amplitude at the surface, which is given over here. So, you can use any function which is giving you the complex product of transfer function and the Fourier amplitude spectra. That you can use using IMPRODUCT, because every time there is a complex function because of damping ratio involved in your input parameter. So, you have to use some inbuilt function which are there in Excel, because I have solved this particular example using Excel. So, in case you do not have any other tool, you can at least use Excel and try solving this equation by yourself. So, IMPRODUCT is going to give you the product of Fourier amplitude, which is only having real part, and the transfer function, which has real as well as imaginary part. The product of these two, you can find out using IMPRODUCT. So, this we have already discussed. The 100 to 200 will be a repetition of 0 to 100 hertz.

- **Add new sheet (Sheet2) → Number the first column of the sheet in ascending order for the number of values of FA corresponding to 100Hz**
- **→ Copy- Paste the values of Fourier Amplitude in the next column → Use IMCONJUGATE of FA to get the complex conjugate values → click the first column → Data→ sort Z to A**
- **Copy the resulting column and paste at the end of the FA values in Sheet 1. figure3.5.**

Freq(Hz)	Denominator, D	Transfer function	FA x Transfer function
99.8046875	0.490172751371332-0.335281864518777i	1.38983845876933+0.950660004115719i	-0.0484953474128077-0.032934533320053i
99.85351563	0.486772471227371-0.336075957080468i	1.39119788501267+0.960506578186433i	-0.0485214956452344-0.0333094196263754i
99.90234375	0.48336542767939-0.336866316496619i	1.39249816477979+0.970457754394484i	-0.0485451433264437-0.0336880196568342i
99.95117188	0.479951663972091-0.337652928058031i	1.39373720518661+0.980514255081404i	-0.0485662103511082-0.0340703547452054i
100	0.476531223442373-0.33843577709545i	1.39491286351511+0.990676781961732i	-0.0485846148084382-0.0344564453180417i
			-0.0485846148084382+0.0344564453180417i
			-0.0485662103511082+0.0340703547452054i

Figure 3.5. Fourier Amplitude at the ground surface of points above and beyond 100Hz.

Okay, this thing we have already discussed. So, after 100 hertz to 200 hertz, you have to just use IM conjugate, and then it will give you the value of whatever was there from 0 to 100 will be from 101 to 200 in reverse order. So, you can see over here D value, transfer function, and the product using IMPRODUCT, you are getting the value of FA times of Fourier amplitude times transfer function. That is going to be the value of this product. If you remember the solution, this product is going to give you the Fourier amplitude at the surface, but still, it is having complex part also.

- **Conversion of data from Frequency to Time domain is done by Inverse Fast Fourier Transform**
- **Data→ Data Analysis→ Fourier Analysis → Provide the input (FA at ground surface) and output ranges → Choose Inverse → OK (Refer to Figure 3.6 for the output)**

Transfer function	FA x Transfer function	Inverse Fast Fourier Transform
1.0000064975139+6.51383402020922E-07i	1.23515020387096+8.04551114222256E-07i	-4.09433419021654E-003
1.00002599047354+2.60561825751282E-06i	0.509972936359465+0.0829339610879751i	3.59829002242016E-003-2.7598544230234E-006i
1.00005848013281+5.86295853481986E-06i	0.505828481453184+0.19088306843487i	-2.2687802846892E-003+3.48026809843249E-006i
1.00010396858173+0.00001042382758141i	0.41209601989129+0.445921537045873i	2.83500699014847E-003-6.52328089703613E-006i
1.00016245874679+0.0000162888182242291i	0.340740447596486+0.946866048862381i	-9.23752372831849E-004+2.83404567705259E-006i

Figure 3.6. IFFT of acceleration data

So, we have to see now acceleration time history So, whatever you are having real and imaginary part, we have to remove the imaginary part and keeping real part into account. We will try to find out acceleration time history. So again, for that, you go to data Fourier analysis and provide input that is Fourier amplitude at the surface. Fourier amplitude at the ground surface is equal to Fourier amplitude at bedrock multiplied by the transfer function. And then you do the inverse, choose inverse, which is already there in the Fourier analysis box. Just check on that and then press OK. That is going to give you how much is your now. You see over here also Fourier amplitude time transfer function; it is going to give you the Fourier amplitude at the surface. Do inverse fast Fourier transformation by checking the inverse checkbox. You will get again these values in from frequency content to time domain. Again, you are having imaginary part as well as real part, so you have to actually remove the imaginary part, and rest of the part you will get. It is acceleration time history. So IFFT of Fourier series will give you acceleration time history. This is for Fourier spectra at ground surface. It is going to give you the Fourier amplitude at the ground surface, and then you inverse fast Fourier transformation, you will get acceleration time history at the surface.

6. Plotting

- **Plot 1** - **Bedrock Acceleration vs Time - Figure. 3.7**
- **Plot 2** - **Fourier Amplitude of the bedrock motion vs Frequency content (use IMABS to get the absolute value)**
  - Figure 3.8
- **Plot 3** **-Amplification function vs Frequency content**
  (Amplification function = IMABS of Transfer function)
  - Figure 3.9
- **Plot 4** **-Fourier Amplitude of the ground surface vs Frequency – (Use IMABS for the absolute value) Figure 3.10**
- **Plot 5** **-Ground surface Acceleration vs Time - Figure 3.11**

=IMABS(Amplitude)*IMREAL(Amplitude)/ABS(IMREAL(Amplitude))
Abs of the first amplitude of acceleration in cell H2
= IMABS(H2)*IMREAL(H2)/ABS(IMREAL(H2))

Now, remember acceleration time history, which I just showed you, it is having real as well as imaginary part. So, you have to use IMABSOLUTE. It will only give you the real part of FA times transfer function, which has been converted from Fourier series to time domain. So, you can have bedrock motion, then you can have Fourier amplitude of bedrock motion, or Fourier spectra of bedrock motion. Then you can have amplification factor, which is the magnitude of transfer function. Then amplification factor multiplied by Fourier amplitude will give you Fourier spectra at the surface. Then do the inverse Fourier transformation. You will get acceleration time history at the ground surface. So, the procedure will remain the same. Only thing, the governing equation of motion, primarily two differences are there with respect to undamped cases. Firstly, the governing equation of motion rho times dou square u over dou t square equals to G times dou square u over dou z square has additional component because of damping. Subsequently, when you go for the solution, you will have, rather than K, you will have K star, which is complex wave number, which is having some component of damping into it. Using that solution, you try to find out transfer function. Then bedrock motion transferred to frequency domain. Multiply with respect to transfer function. Get the Fourier series at the top. Inverse fast Fourier transformation, you will get acceleration time history at the surface. So, this approach will remain the same. Only thing, if you go for elastic half space, there will be component of damping from the rock also, which so far has not come into picture. So, these are the typical outputs which you can get from the.

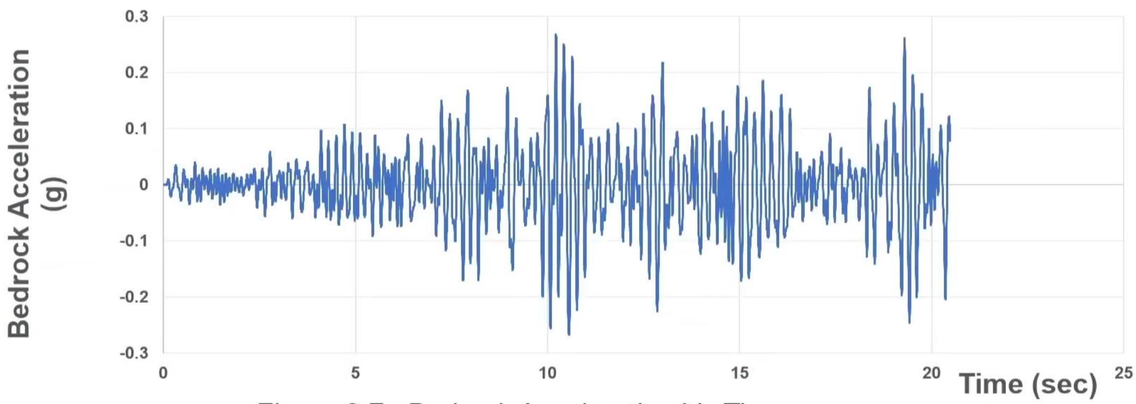

Figure 3.7– Bedrock Acceleration Vs Time

So, this is bedrock motion.

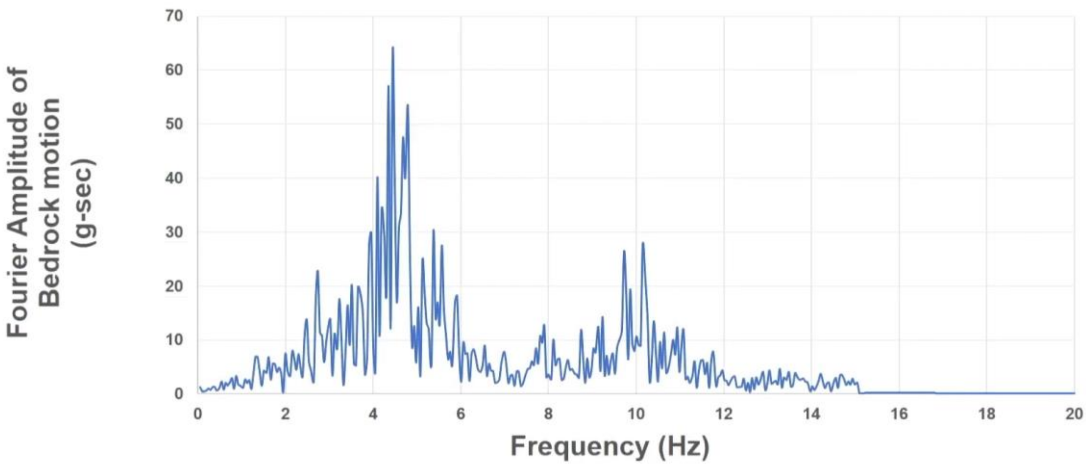

Figure 3.8–Fourier Amplitude of Bedrock motion Vs Frequency

Corresponding to bedrock motion this is a Fourier spectra.

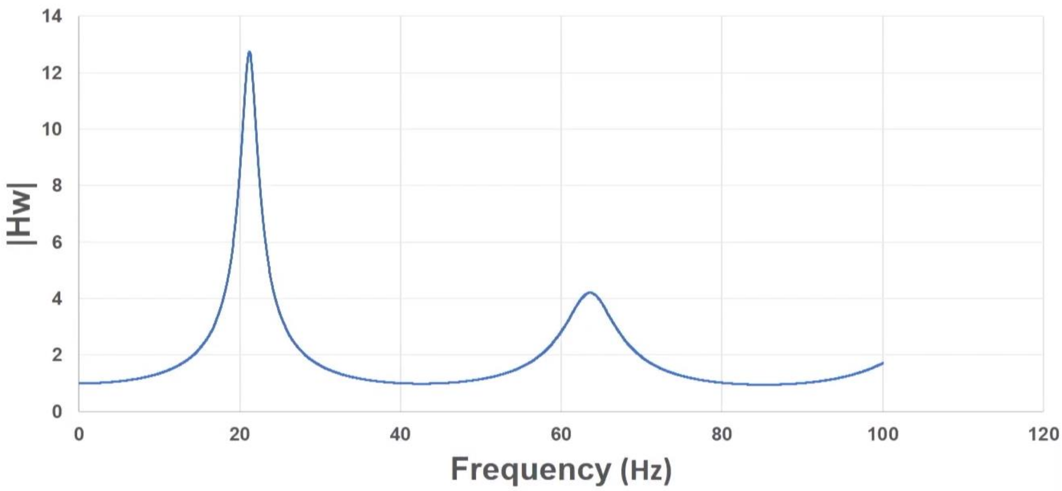

Figure 3.9.–Amplification due to soil properties Vs Frequency

Then transfer function magnitude or amplification factor values with respect to different frequency content.

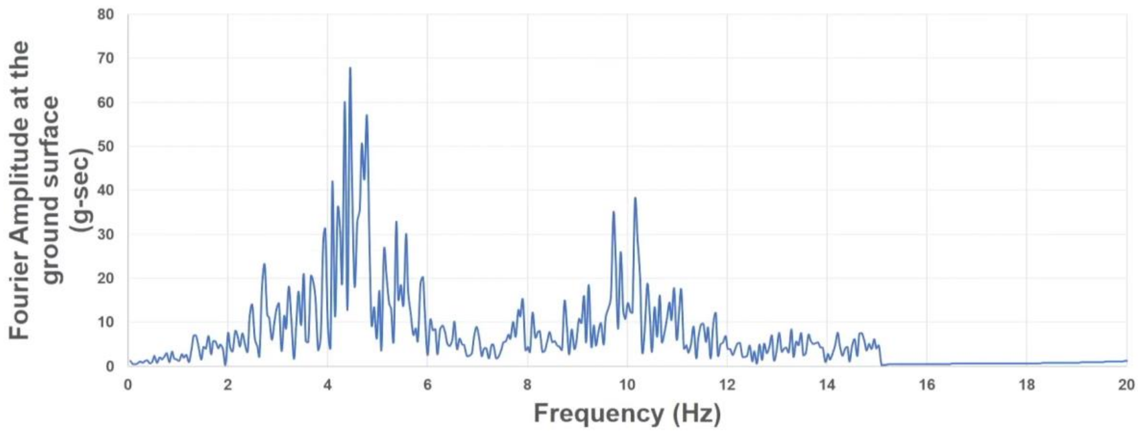

Figure 3.10–Fourier Amplitude at the ground surface Vs Frequency

Then this is Fourier amplitude at the ground surface.

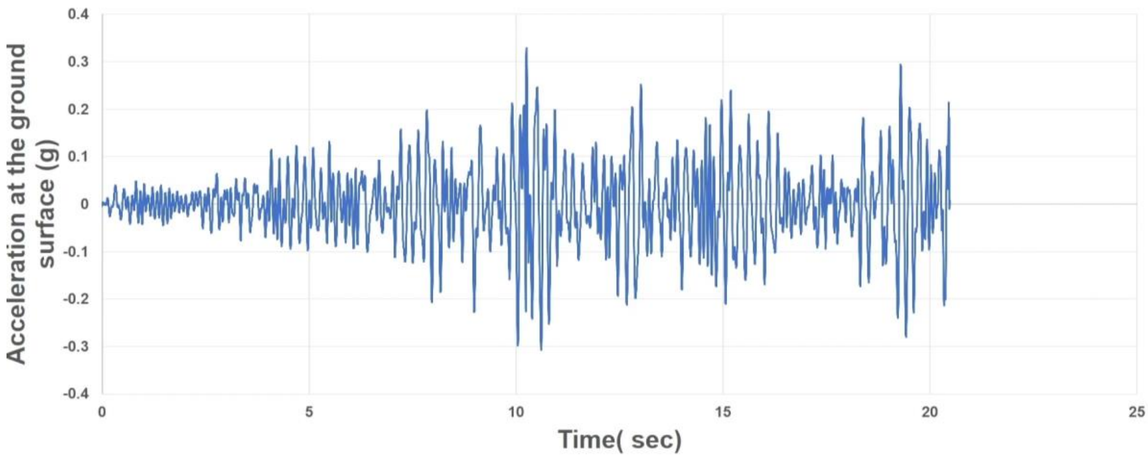

Figure 3.11– Acceleration of the Ground motion Vs Time

Conversion of Fourier amplitude to acceleration time history. So, this completes the objective. The objective was to find out if bedrock motion is given, acceleration time is yet bedrock is given, how much will the acceleration time is at the top of 4-meter soil layer thickness having shear velocity of 340 meters per second and damping ratio of 5%. Then here is the answer.

So, thank you, everyone, and practice this particular numerical by yourself. That's why everything has been solved in Excel. So, if you are able to manage Excel, you can solve this yourself and learn how this particular example of linear ground response analysis one can attempt to solve. Thank you, everyone.