Advanced Soil Mechanic
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Lecture – 05
Stress action on a plane example

In the last lecture, we have discussed about stress acting on a plane, we will see a simple example, to demonstrate how to find out stress acting on a plane by knowing the stress tensor acting at a point.

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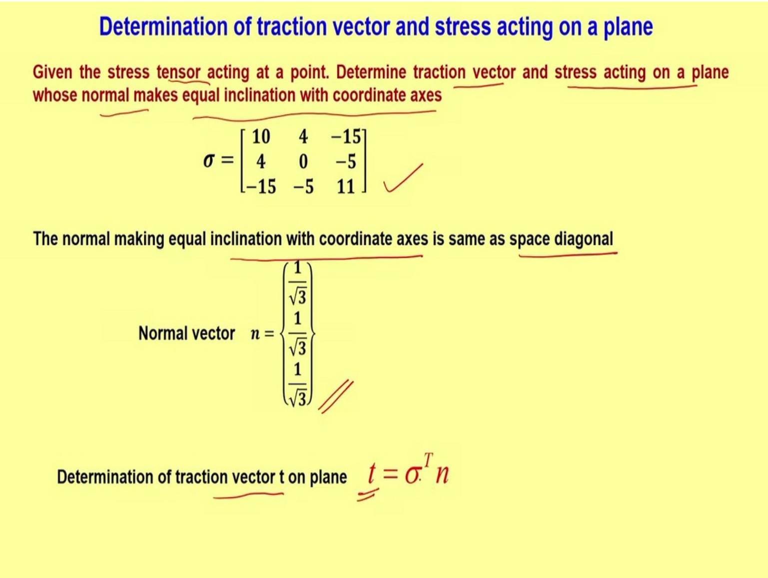

So, how do we do this? So, we will refer to 1 example problem given the stress tensor acting at a point. we are asked to find out what is the traction vector because, without knowing traction vector, it is difficult to find the stress acting on a plane. So, first thing is to find out traction vector and then stress acting on a plane whose, normal makes equal inclination with coordinate axis.

So, now, this particular sentence we need to understand carefully, we are asked to find out stress acting on a plane now, what is stress acting on a plane stress acting on a plane is about the normal and shear component of the traction vector. So, that is what we have to find out, but for finding out we need to know the normal vector. Now, what is the normal vector. So, it is told that it is a plane whose normal makes equal inclination with coordinate axes we have learned this in the lecture.

So, what is meant by normal making equal inclination. It is similar to that of a space diagonal, spaced diagonal makes equal inclination with the axes. So, here we have been given with a stress tensor with its components. So, σ is given. So, normal making equal inclination with coordinate axis it is same as the space diagonal. Now, what is that it is equal inclination. So, we know that the normal vector n is equal to $1/\sqrt{3}$, $1/\sqrt{3}$, $1/\sqrt{3}$. This also we have discussed in the lecture. So, how do you find the traction vector? So, traction vector t is equal to σ transpose into n. So, that will give you traction vector.

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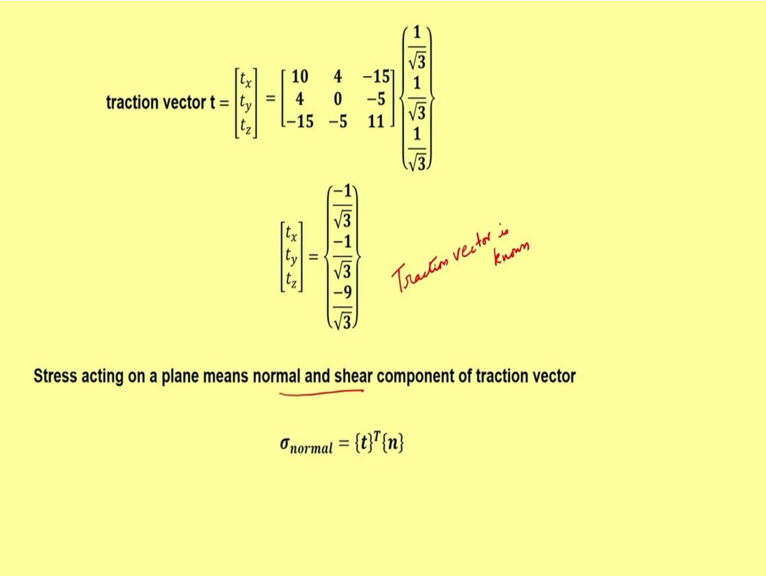

So, traction vector t is equal to t_x, t_y, t_z which is equal to the stress tensor into the normal vector. So, do the matrix multiplication one will get t_x, t_y, t_z is that is equal to $-1/\sqrt{3}$, $-1/\sqrt{3}$, $-9/\sqrt{3}$. So, the traction vector is known. Now, we need to find out what is the normal stress. So, how do we get that?

The stress acting on a plane means normal and shear components of traction vector. So, first we will see how to get σ_{normal}, that is nothing but the transpose of traction vector multiplied by the normal vector.

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$$\sigma_{normal} = \begin{Bmatrix} \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{-9}{\sqrt{3}} \end{Bmatrix} \begin{Bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{Bmatrix}$$

$$\sigma_{normal} = \frac{-11}{3}$$

$$\tau_s = \sqrt{t^2 - \sigma_{normal}{}^2}$$

$$\tau_s = \sqrt{\frac{83}{3} - \frac{121}{9}}$$

$$= \sqrt{\frac{128}{9}}$$

So, that is what is done here, the traction vector t and the normal vector n. So, you do this simple matrix multiplication, we will get -11/3. So, the normal stress or the normal component of traction vector is now known. Now, what is left with we are left with the shear component. So, how do we do that? We know now, what is the traction vector? We know what is the magnitude of σ_{normal}.

So, one can always find out the magnitude of shear component which is given by

$$\tau_s = \sqrt{t^2 - \sigma_n^2}$$

where t^2 is the magnitude of the traction vector and t^2 this term. So, you will get τ_s equal to

$$\sqrt{83/3 - \sigma_n^2}$$

that σ_n^2 is 121 / 9, well that will give you $\sqrt{128/9}$. So, that is about the normal stress and the shear stress component, which has been asked. So, what we have done? We have done the determination of traction vector, knowing traction vector one has obtained the stresses acting on the plane.