

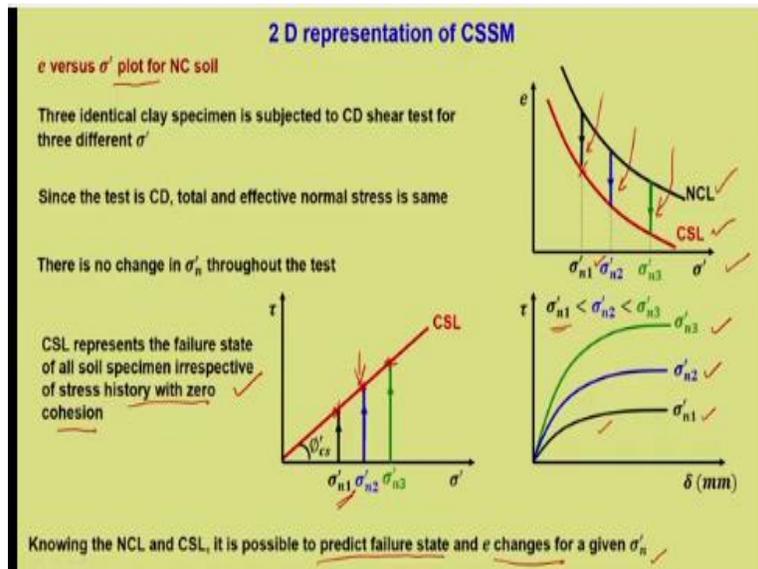
**Advanced Soil Mechanics**  
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**Lecture-46**  
**CSSM-2D Representation**

Welcome to all the participants, in the last lecture we have introduced critical state soil mechanics, we have seen different elements of critical state. The various parameters and those parameters considered to be the fundamental behaviour or properties of soil. Now in today's lecture we will see the two dimensional representation of critical state soil mechanics, what we initially introduced is that we are integrating the aspect of deformation or maybe volume change characteristics along with stresses.

And hence it becomes a three dimensional representation. So, before moving on to integrating into a 3D space, let us first understand the 2D space properly and what are the important factors that come into play? What are those elements which we know but we have not projected in this manner? That is what we will be seeing in today's lecture.

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Basically the 2D representation of CSSM critical state soil mechanics. Now as I told in the beginning for some lectures we will parallelly understand both representation of critical state.

One is  $e$  versus  $\log \sigma'$  or  $e$  versus  $\tau$  versus  $\sigma'$ ,  $e$  void ratio,  $\tau$  shear stress and  $\sigma'$  is the normal stress, effective normal stress. But we are interested in the representation of in terms of stress invariance.

So, we have  $v$  which is the specific volume  $q$  and  $p'$ , so we will be discussing both. So, in today's 2D representation also we will start with  $e$  versus  $\sigma'$  plot for NC soil. Now when we do this discussion, we have to keep in mind we have to understand CSSM discussion from the perspective of stress history and from the perspective of drainage condition.

So, then we say, then we have to appreciate the point that this initial conditions in terms of stress history and the drainage conditions is not going to affect finally the critical state parameter or the critical state framework, so we need to discuss from those angles. So, first let us see  $e$  versus  $\sigma'$  plot for NC soil, normally consolidated soil. We start with this particular curve  $e$  versus  $\sigma'$ , please note it is not  $\log \sigma'$ .

Let us consider three identical clay specimen, which is subjected to consolidated drained shear test. So, now understand that we are first considering the drained test subjected to CD shear test for three different  $\sigma'$ . This is what we generally do in triaxial testing also, we subject the soil to three different confinements or maybe in the case of direct shear test, we subject the soil to three different confining normal stress and then we find out the shear stress.

So, similarly we are considering now three identical clay specimen which is subjected to CD test. And since it is normally consolidated, you can see the stress strain the shear stress versus shear displacement it is not in terms of strain, it is in terms of displacement for three different normal stresses where  $\sigma'_{n1}$  is less than  $\sigma'_{n2}$  less than  $\sigma'_{n3}$ . Now for  $\sigma'_{n1}$ , we know that  $e$  is going to change.

So, these the information that we have known in our previous lectures I was every time I was hinting this point when we were discussing these points in module 2 as well as in module 3. Now we need to keep those points in mind while discussing this module as well. So, what is the first thing that we understand for normally consolidated soil, it is compression, now if it is compression we know that the void ratio is going to decrease?

So, this is one information and that is what is shown here,  $e$  is decreasing. Now why it should decrease in a vertical manner? Downwards, so downwards we have got the answer why vertical. That means that  $\sigma$  normal stress, normal effective stress is not changing during the course of shearing. And this is true, because this is a consolidated drained test, so shearing is under drained condition.

And when it is under drained condition there is no development of excess pore water pressure. So, whatever is the initial condition  $\sigma'_n$  which is the confinement, please note this is an important point here. Like what changes during shearing? The shear stress changes during shearing and it is done at a constant confining pressure. And confining pressure here is  $\sigma'$ , so that is the effective confining pressure.

So, that is not changing throughout the test because it is drained test. So, that is why corresponding to  $\sigma'_{n1}$ , we find that void ratio reduces and it touches the critical state line at this point where it fails. Now let us say  $\sigma'_{n2}$ , the same thing happens and  $\sigma'_{n3}$  also the same thing happens. And wherever the path here it is not actually stress path, when the path of  $e$  variation touches the critical state line, then the failure occurs.

Since the test is CD, total and effective normal stress is same, so that is why it is not moving on left or right rather it is vertically down. There is no change in  $\sigma'_n$  throughout the test. Now if you plot this in terms of  $\tau$  and  $\sigma'$ , how will it look like? So,  $\sigma'_{n1}$  is vertically upwards here, why because this is not changing. And  $\phi'_{cs}$  and this is the critical state line,  $\sigma'_{n2}$  and  $\sigma'_{n3}$ .

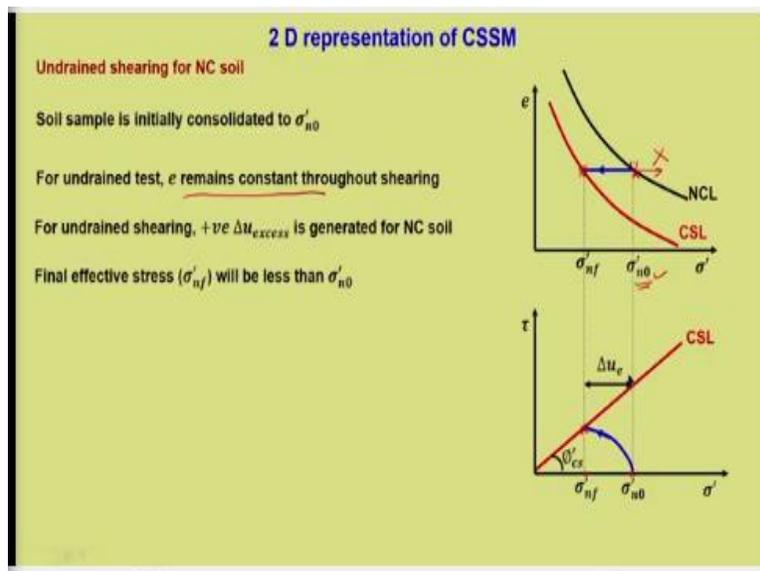
So, we have these points lying on the critical state line. Now critical state line represents the failure state of all soil specimen irrespective of stress history with zero cohesion. You will hear this sentence quite a number of times throughout this lecture. Because it is very important that it gets into our head properly, why? Because that is the concept based on which the critical state is portrayed, like at critical state irrespective of any condition.

So, we will have quite a few examples hinting the same point, and this statement will be in a way repeating throughout this module. So, what it states is that whatever be the initial state, now we are considering only the normally consolidated state, you will see towards the end of this lecture this particular sentence will be even more clear to you rather it all falls on this critical state line, this one.

And with zero cohesion, and this particular aspect we have discussed in the previous lecture, what did we discuss? At critical state, the strain which the soil undergoes is significant, now at that level of strain in general there cannot be any sort of cementation or cohesion. So, that is the principle based on which it is told that critical state line always passes through origin. Now knowing the NCL and the critical state line, it is possible to predict failure state and  $e$  changes for a given  $\sigma'_n$ .

So, given these conditions it is quite easy once we define the normally consolidated and critical state line, it is very easy for us to predict this the failure and the change in which how the  $e$  changes during the test but with some prior information.

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Now let us see undrained shearing of normally consolidated soil. Let me again remind you that the concepts that we are discussing it is not new rather you know this and you have studied this in one or the other form. If you have not studied during your undergraduate at least you have

studied this in the previous modules. So, we are just placing those information at the appropriate place, so please follow the previous lectures very carefully.

So, that this explanation does not need much time for you to grasp. Again we start with  $e$   $\sigma'$ . Soil sample is initially consolidated to  $\sigma'_{n0}$ , so this is the point to which it is initially consolidated, so it is the starting point. Now this is reminding you it is an undrained shearing, so it is kind of consolidated undrained test where the soil is initially consolidated to  $\sigma'_{n0}$ .

For undrained test, so this again becomes a quite a handy information. So, if you do not appreciate that point that void ratio remains same during undrained test, you will not be able to understand this better. So, for undrained test  $e$  remains constant throughout shearing. So, we know that this is the starting point; it is very easy to predict, what will be the path towards the critical state line at constant shearing?

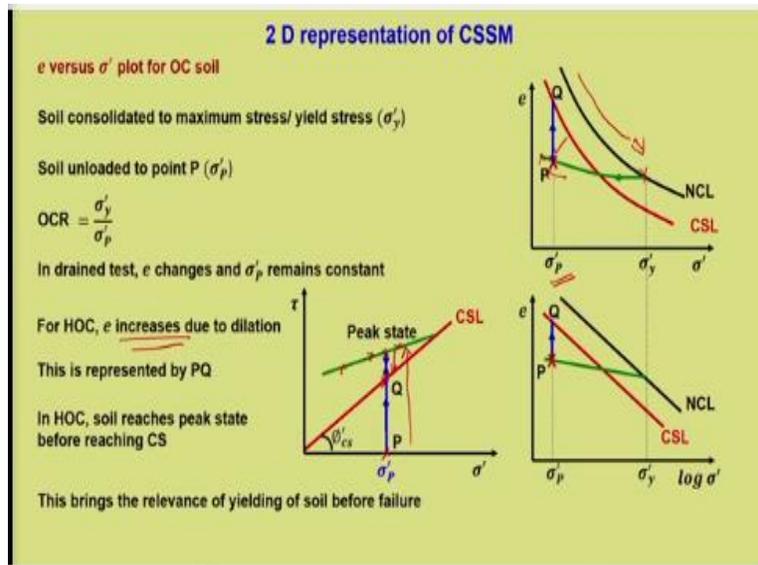
Definitely the soil the state cannot move in this direction, this is impossible state, we have seen that, NCL becomes the right boundary. Now during shearing  $e$  remains constant, so how it should move? It has to move in this direction, so this is the only possible way by which the state of the soil would change. That is this is the starting point and during shearing for NC at constant  $e$ , this is the path till it reaches the critical state line.

Now here you can see in comparison to the previous slide the starting point and the end point is different. In the earlier case this was constant, that is  $\sigma'_{n0} = \sigma_{n0}$  whatever is the starting point that remains same, because  $e$  was changing. But in this case  $e$  is constant because it is undrained and due to the development of excess pore water pressure the initial confining stress keeps reducing, the effective confining stress keeps reducing.

For undrained shearing positive  $u$  excess is generated for NC soil. Now this again becomes an important information from our previous modules, what? During undrained shearing or during drained shearing the tendency of normally consolidated soil is to compress. So, corresponding to that in an undrained test there will be positive pore water pressure, so that is what is written here. Final effective stress  $\sigma'_{nf}$  will be less than  $\sigma'_{n0}$ , so this is the point of  $\sigma'_{nf}$ .

So, let us see this on  $\tau$   $\sigma'$  plot, so this is the point where it starts, now the stress path will be. So, the  $\sigma'_{nf}$  will be this, now this is the point from where the  $\sigma'_{nf}$  comes down and this is the stress. Now this is the starting point, it has to end on the critical state line, so this is the point. So, this is the effective stress path in  $\tau$   $\sigma'$  plot and this we have already seen it moves leftwards and this is the pore water pressure.

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So, that is about the undrained response of normally consolidated soil. Now  $e$  versus  $\sigma'$  plot for OC soil, now let us see what will be the implication of stress history. We will start with the same discussion  $e$  versus  $\sigma'$ , now what is happening? For over consolidated state what has to happen? The soil has been consolidated to this particular stress level and then it is further unloaded, so this is the unloading path.

So, this is unloading path and hence the final point reaches here. Now what is the over consolidation? From here to here is the over consolidation. So, soil consolidated to maximum stress or yield stress what we call  $\sigma'_y$ , this is the maximum stress to which it is subjected to in the past. Soil unloaded it to point P, so let us say this is the point P, where the stress is  $\sigma'_p$ , so  $\sigma'_p$ , so OCR is  $\sigma'_y / \sigma'_p$ .

Now this is represented on  $e \log \sigma'$  plot. All these representations can be considered for all sort of representation, you should not get confused why  $e$  versus  $\log \sigma'$  is discussed only for OC not for NC, it is applicable for all. It is like I am using different framework for representation all of them are equally valid for all the cases. So,  $\sigma'_y$ , the point P  $\sigma'_p$  marked same.

In drained test  $e$  changes  $\sigma'_p$  remains constant that is what we have already seen. For HOC  $e$  increases due to dilation, now this is again another important fact which we should be knowing. Otherwise from here how the stress state would change, we will not be able to understand better. So, for OC we know that it is dilation, now for dilation what happens?  $e$  has to increase, so how the stress state will move this is represented by PQ.

You can see that during shearing it is moving upwards, and why it is moving vertically upwards? Because  $\sigma'_p$  remains constant because it is a drained test with excess pore water pressure 0. So,  $\sigma'_p$  is not going to change, initial and final states are the same, now  $e$  instead of moving downwards in it is moving upwards, now what is this? This is due to dilation which the OC soil undergoes during shearing; same is represented in  $e \log \sigma'$  plot.

In  $\tau \sigma'$  how it will look like? So, it is again the same it is moving in the upward direction from  $\sigma'_p$  and it touches at this point where it fails. In HOC, now this is another important aspect we need to keep in mind. You know that in heavily over consolidated soil it would first exhibit peak and then come to ultimate state or the critical state. Now where is this shown here?

If we do like this it means that from this point the stress path moves and touches the critical state point. Now where is the question of peak coming into picture, in fact this representation is not fully correct, so what has to be done? We need to have understand that the stress path would first hit the peak line or the peak state and then return to critical state, and that too we have to remember it said  $\sigma'_p$  constant because it is drained test.

So, there is a peak state because this representation is also very clear to us in module 2, when we discuss the shear strength. Because corresponding to every critical state we have the peak state which is above the critical state. And there we have discussed effective peak angle includes both

the component that is a basic critical state component and the dilation component. So, that is why we have this particular state, and this is called peak state.

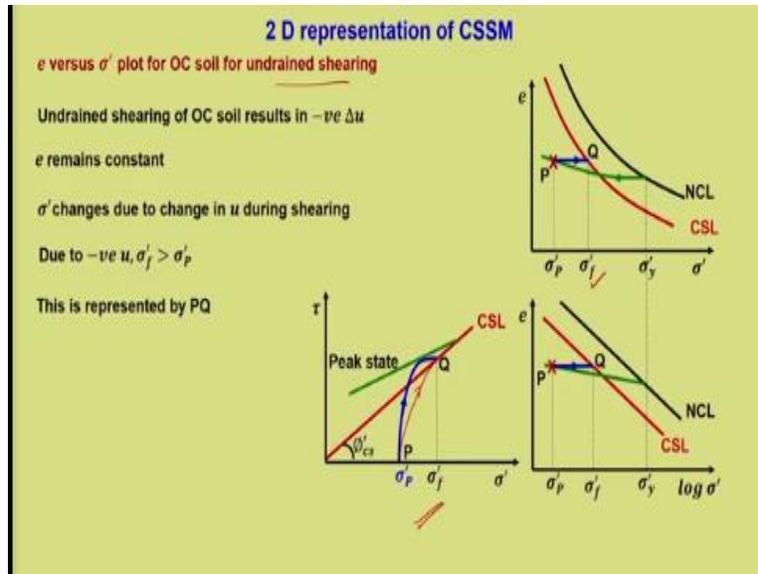
So, first the stress path has to reach this peak state. So, here it is vertically upwards, the stress path first moves, touches the peak and then returns to the critical state line, so this is the correct method of representation. HOC has to first, the state of HOC soil during drained shearing would first hit the peak state and then returns back to critical state which has been shown.

So, first it goes, touches there and then comes down, so that is what the representation is. Now this brings the relevance of yielding of soil before failure. Now you may ask why it is crossing critical state line, it would have failed by this time, it is not like that. For any soil to fail should yield first from elastic the point of yielding and failure, this is the sequence of events. Now in the case of over consolidated, it exhibits an elastic characteristics during shearing.

Now we know that at a very low strain itself it picks up and it reaches the peak. So, close to the peak it starts yielding, so once it start yielding strain softening happens and then the stress comes down, this much we know, this is very clear to all of us. So, initially the stress path goes towards the yield line sorry not yield line, the peak line yield there and after yielding it comes down to failure to critical state.

Anyway we will be repeating this in the following lectures as well where the same concept will be told again and it will be pretty clear. As of now we have not started the concept of yielding of the soil. Once we start discussing yielding this particular aspect will become very clear. So, as of now we need to understand that concept of yielding is important, only after yielding the failure can occur. And that is the reason why for a heavily over consolidated soil the stress path moves to peak line yield there and then returns back to critical state line.

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So, now *e* versus  $\sigma'$  plot for OC soil for undrained shearing. So, we have discussed about the drained characteristics, now what will happen in undrained shearing of OC? Now since OC has a tendency to dilate the corresponding nature in the undrained shearing will be the result will be negative pore water pressure, this we have already seen. We know that *e* remains constant,  $\sigma'$  changes due to change in *u* during shearing.

Now pore water pressure keeps changing during undrained, in HOC since the dilation is the tendency the corresponding pore water pressure will be negative. Now during negative due to this negative pore water pressure the initial effective normal stress would change. Due to negative *u*, now what is the implication of negative *u*?  $\sigma'_f$  which is the final or the failure normal stress will be greater than the initial normal stress, this we know.

Because it is getting added up, *u* is getting added up, now this is represented by PQ, now what will be PQ? It is very easy to understand undrained, because there is no change in *e*, and we know the relative position of critical state, so from here it is should be horizontally to Q, so that is  $\sigma'_f$ , so this is the final normal stress, same is marked in  $\log \sigma'$ . And this is the way in which the stress path moves, and this we have seen.

You remember in the case of stress path representation like we have marked like this that is moving towards right. Now at that particular point of time we did not have this complication of

touching the peak and then coming down to critical state after yielding. We did not intentionally discuss those points there but then actual manner of stress path is this; this is the correct way of representation.

Like the stress path moves above the critical state line, it is not that it is moving from critical state to peak, do not consider ever like that. It is going towards peak yielding and then coming down to the critical state and that is very clear from the strain softening response of OC. And here this stress path moves and it moves towards right and it is touching at this point Q, so this is  $\Delta u$  which is negative.

So, because of this negative your normal stress is greater than the initial normal stress. In undrained test there is a point P that is  $\sigma'_p$  for which  $\Delta u$  is 0. Now we need to understand that, in fact I have not shown the total stress path here. Total stress path, now whatever is the effective stress path if it is towards the left of total stress path it will be positive a pore water pressure will be positive.

If the effective stress path is towards the right of total stress path then the effective pore water pressure will be negative. This also, please refer back to stress path plot for understanding what I just told. Now in this case you can see that from P it moves and then turns right wards, now depending upon the level of over consolidation or depending upon the over consolidation ratio whether it is a lightly over consolidated or whether it is a heavily over consolidated.

There can be a situation where this particular envelope will result in zero pore water pressure, excess pore water pressure at the failure state. So, in undrained there is a point P for which  $\Delta u$  developed will be zero, because it is moving from positive to negative. So, there can be a situation where positive goes to zero, so the final excess pore water pressure is close to zero. So, that is what it means, so there can be one particular point P.

Now this is a heavily over consolidated maybe towards a lightly over consolidated state we can expect such a situation. The point is the intersection of unloading line with critical over consolidation ratio line. Now critical over consolidation ratio line we have discussed which is the



Now 2D analysis of normally consolidated soil subjected to CU and CD in  $v, q, p'$  plot. As I told we are discussing in both the framework. Now whatever we have discussed for  $e, \tau, \sigma'$  plot we will just repeat it for  $q, p'$ . Because ultimately for three dimensional integration we need  $v, q, p'$  plot. Soil is initially isotropically normally consolidated to  $p'_c$ , it simply means that ICL, that is a isotropic consolidation line is same as normally consolidation line.

And it is consolidated to point  $A'$  where the pressure is  $p'_c$ ; the effective mean stress is  $p'_c$ . So, this is the point A, drained shearing is represented by stress path  $A' B'$ . Now we know that  $A' B'$  from the module 3, we know it is at an inclination of 3. So,  $ESP = TSP$  because it is drained. So, this is  $ESP = TSP$  for CD that is consolidated drained with a slope of 3 in  $q-p, p'$  plot.

So, this is what is represented, this is  $A' B'$  is the effective stress path or total stress path for CD. So, here because of CD it is same and it is at an inclination of 3, how will it look like in  $v \ln p'$ ? So, this is the representation which we want, so  $A'$  will be on ICL or NCL. Now from here, we know that  $v$  changes or  $v$  reduces due to compression. So,  $B'$  is the failure point which is there on the critical state line, so this is the point where it fails.

And that is denoted as  $A''$  and  $B''$ , the homologous points of  $A'$  and  $B'$  on this plot is  $A''$  and  $B''$ . So, this is the path or this is the change in the state of the soil towards failure, so for  $A''$  to  $B''$ . Now consider initial  $u$  is zero, there is no initial pore water pressure. So, initial condition of CU is same as CD, so there is no change in the point  $A'$ , so  $A = A'$ .

Slope of ESP of CU depends on  $A_f$ , in the previous plot we did not have much of discussion because it is direct reduction in  $\sigma'$  when you have pore water pressure. But in this case we need to determine the slope of effective stress path for a undrained test. And that also we know  $3 / 1 - 3 A_f$  is the slope. So, we know that it depends on pore water pressure parameter  $A_f$  the slope.

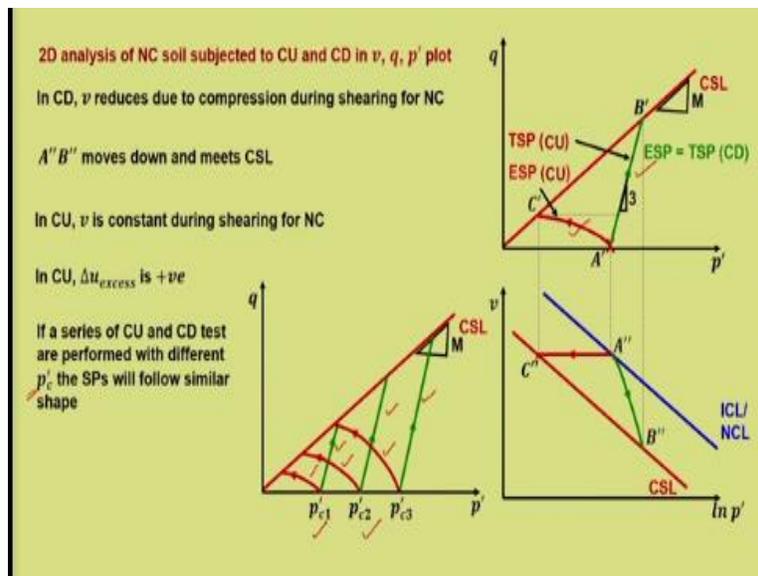
And in the case of normally consolidated, we know that mostly it moves towards the left and why? Because  $A$  is close to it is a positive value. So, ESP of CU will be towards left for normally consolidated, this is the way in which it moves and this is  $C'$ . You might be thinking why we

have to draw with this lower slope, why not here, why not here? In fact both these points should match.

Now we know since this is a undrained test, there will be no change in  $v$ , so we have to first draw the curve here where this will be horizontal. So, this point and this point has to match, and that is why it is moving in this direction. So, it  $A' C'$  is the effective stress path for undrained shearing. And this is the point where it touches for consolidated undrained test; our total stress path is valid only up to this particular point.

So, this effective stress path for CU and this is the total stress path for CU, so the same line, because there is no initial pore water pressure. So, this  $C'$  comes and this is horizontal and  $v$  remains constant, that is the  $C''$  point.

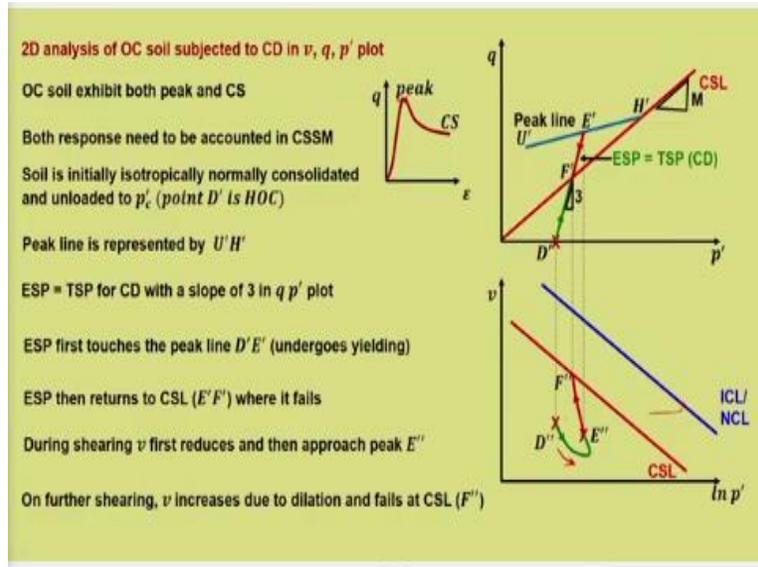
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In CD,  $v$  reduces due to compression during shearing for NC, so this is the reduction.  $A' B'$  moves down and meets critical state line, so this is the one which we have already explained. In CU,  $v$  is constant during shearing for normally consolidated, so this is not changing, so this is the point where it fails. In CU,  $\Delta u$  excess is positive which we have already seen. Now if a series of such CU and CD test are performed with different  $p'_c$ , that is the initial effective mean stress.

Then the stress path will follow similar pattern, so this is the pattern or the trend in which the TSP and ESP would move. So, this is the ESP for drained and this is the ESP for undrained, so this you can see that for different combination of  $p'_{c1}$ ,  $p'_{c2}$  and  $p'_{c3}$ . We can see that there are different the same kind of response and this is the same kind of effective stress path for drain. So, you will have a series of such stress paths.

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Now let us see 2D analysis of OC soil subjected to CD in  $v, q, p'$  plot. The OC soil exhibit both peak and critical state, both response need to be accounted in CSSM. Soil is initially isotropically normally consolidated, and then unload it to  $p'_c$ , that is the point  $D'$  is heavily over consolidated, let us this is the point  $D'$ . So, what does it mean, it means that it is loaded, so this is the peak line, so what it means is that soil is initially isotropically normally consolidated.

So, it is normally consolidated then it is unloaded to point  $p'_c$  that is the initial effective volumetric stress. So, point  $D'$  is the heavily over consolidated, this is the point  $D'$  and the peak line is represented by  $U' H'$ . So, we know that we need this particular point over here, why? Because it has to reach there and then come back to critical state line.  $ESP = TSP$  for CD with a slope of 3 in  $q p'$  plot, now that is not going to change.

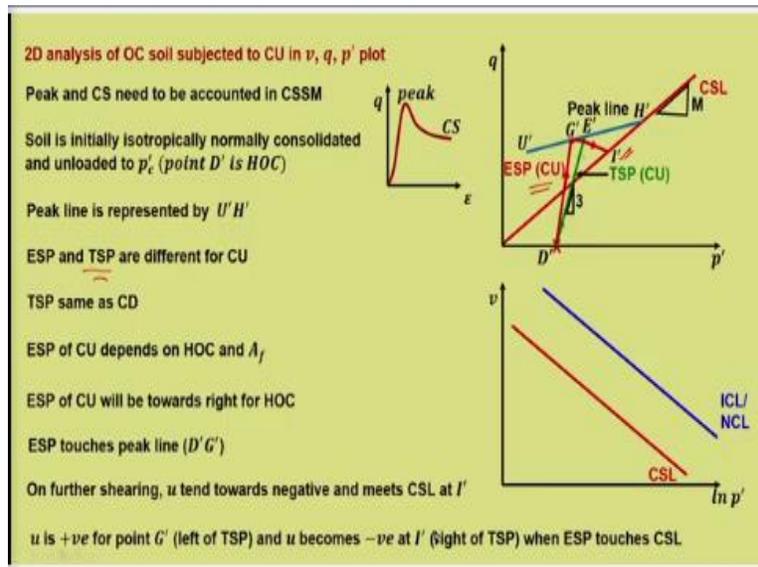
ESP first touches the peak line  $D' E'$  undergoes yielding, so this we have already discussed, so I am not going to spend more time here. It reaches  $E'$  and then ESP returns to critical state line

where it fails and that is at the same slope of 3. So, it comes back and fails at critical state. So, this is ESP which is same as TSP for consolidated drained test, now this is the point D, D''.

During shearing  $v$  first reduces, now I will tell you like from here it is isotropically consolidated, it reaches somewhere here and then it is unloaded. So, there will be an unloading line on which the point D'' will fall. So, now we know that when it is sheared the  $v$  first reduces because there will be some initial compression which is shown by this response. And once the initial compression is over then in a drained test there will be increase in this specific volume.

So, this is the point E' somewhere here, so it has already started yielding. So, then once it is started dilating then it will be in the upward direction,  $v$  will start increasing. So, on further shearing,  $v$  increases due to dilation and fails at CSL the point F''. So, that F'' is the point towards which it fails, so  $v$  initially reduces then increases. So, whatever we have learned in the module 2 it is all mapped onto a critical state framework.

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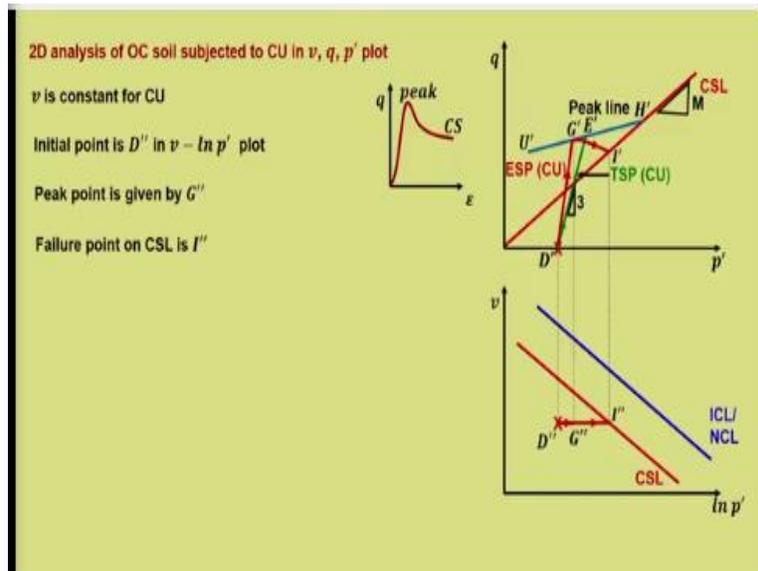
2D analysis of OC soil subjected to CU, now we will see the undrained response. Again peak and CS need to be accounted, first it is normally consolidated, unloaded the same starting point. Peak line  $U' H'$ , ESP and TSP are different for CU, it is no longer same, so this we need to keep in mind. TSP same as consolidated drained, so this is will be the total stress path. And ESP of

CU depends on heavily over consolidated soil and the value of  $A_f$ , and mostly  $A_f$  will be negative, so we know it will move towards right for heavily over consolidated soil.

ESP touches peak line, that is  $D'G'$ , and then further shearing  $u$  tends to negative and meets critical state line at  $I'$ , so this is the way in which it will move. So, this much detailing we have not done in our earlier lecture. So, in the critical state framework we understand this clearly how the shearing happens. So, this is the effective stress path for consolidated undrained, and this is the total stress path.

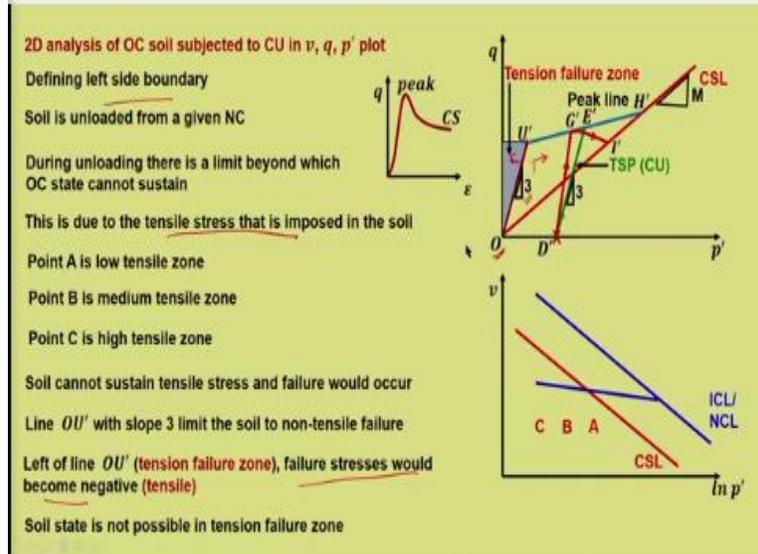
So,  $u$  is positive for point  $G'$ , now this is the point which I was trying to make, now this is the total stress path, this point  $G'$  is towards the left. Now relative position of this keeps changing depending upon the state of the soil and the soil type. And towards the end it comes towards the right of the total stress, so this is because the pore water pressure is negative for  $I'$ .

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Now again  $v$  is constant for CU, so  $D'$  we know, now it is very easy to plot in what direction the state of the soil would change, so it has to be horizontal. Peak point is given by  $G'$ , it has to be in the same line, there is no because  $v$  remains constant in CU. Failure point on CSL is  $I'$ , so it is a horizontal line and this is denoted by  $I''$ .

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Now 2D analysis of OC soil, it is again now we need to discuss about an important aspect. The soil sample is loaded to a particular point on the NC and then we unload it, now how long you can unload because the soil is subjected to a kind of swelling. So, there will be a limit towards the left side of the critical state framework. The right side boundary we know, it is normally consolidated line or isotropic consolidation line.

And there should be a boundary towards left otherwise what does it mean, towards left means it is more tensile because it is minus. So, you are minusing the initial effective stress, so it keeps reducing. And hence that reduction will be curtailed by some boundary, so that is what we are going to learn now that is defining the left side boundary towards this area. So, soil is unloaded from a given NC, so this is the unloading line.

During unloading there is a limit beyond which OC state cannot sustain, this is due to the tensile stress that is imposed in the soil. Point A let us say it is a low tensile zone; here the possibility of tensile stress is less because it is close towards the normally consolidated line. Point B is a medium tensile zone and point C is a high tensile zone, this is a possibility of high tensile zone in point C. Now soil cannot sustain tensile stress and failure would occur.

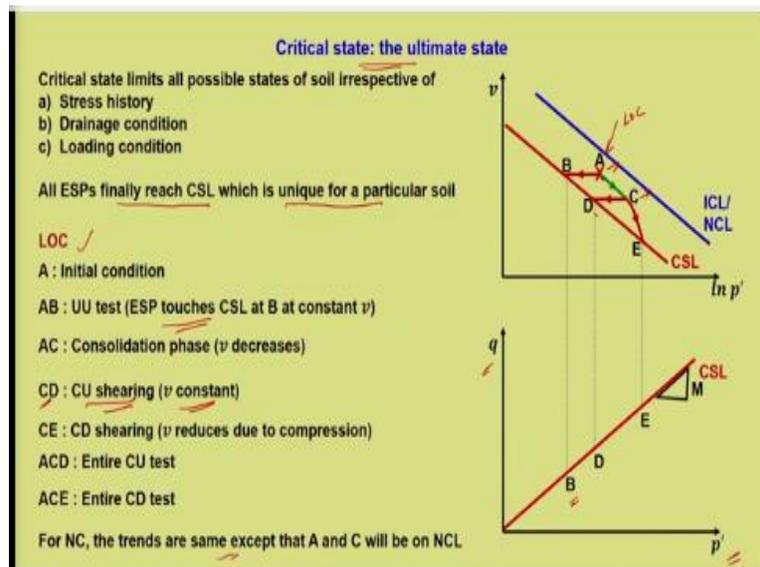
This we know soil is very, very weak in tension, because these are granular. Line  $OU'$  with slope 3, limit the soil to non tensile failure. Now this is the point  $OU'$ , it is slope 3 similar to this total

stress path, so it is a slope 3 and this is the left most boundary. And that towards this direction this will be a non tensile failure zone, and this particular zone will be a tensile failure zone. Left of line OU' that is towards this is the tension failure zone.

So, whatever has been marked here is tension failure zone, and the failure stresses would become negative. Now please keep in mind this particular sentence, we will know this better in our subsequent lectures where we will show that beyond this slope. Now this is at a slope of 3 in  $q$   $p'$  representation, this slope considered as 3 beyond which it will be a tension failure zone. Why it becomes a tension failure zone?

Because the failure stresses tends to become negative beyond this slope of 3, as it increase the slope beyond 3 the failure stresses will be negative. We will be showing this in our subsequent lecture. So, this marks the boundary or the tension cut off or that is the region of left boundary below which soil cannot exist. So, soil state is not possible in tension failure zone.

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Now let us summarize this by saying critical state: the ultimate state. Critical state limits all possible states of soil irrespective of stress history, drainage condition and loading condition, this is what we have seen, let us put it all together in this slide. All ESPs finally reached critical state line which is unique for a particular soil. So, let us start with a lightly over consolidated state of the soil, till now we were discussing mostly about NC or HOC.

For this example let us consider lightly over consolidated state. Let us say A is the initial condition let us say this is A, now you can see that it is towards the left of NCL and hence this state is LOC. AB which represents UU test, that means ESP touches critical state at B at constant  $v$ . Now let us say A is the starting point and we know we have conducted UU test, in UU test there is no consolidation and it is an undrained shearing directly.

Now when you do undrained shearing we know that  $v$  is not going to change, it remains constant. And hence it is very easy to draw the state of soil how it changes. So, this is the way in which it changes, it is horizontally towards critical state line that is the point B. And the same is mapped on to the critical state line in  $q$   $p'$  plot; this plot is needed only at failure. So, B is a failure point that is mapped onto  $q$   $p'$  plot.

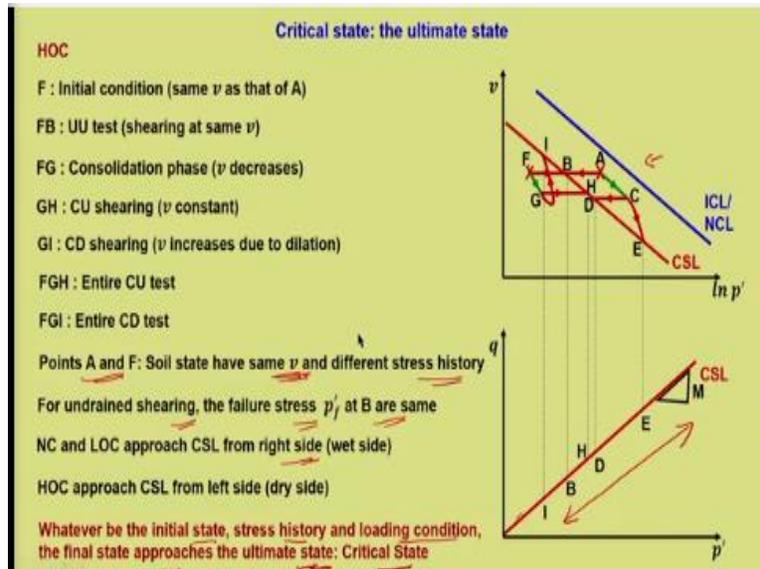
AC: let us say we are consolidating it first before shearing, so that consolidation phase means  $v$  reduces and this is represented by AC. Now we do not have to map this here because C is not a failure point. CD: indicates consolidated undrained shearing and  $v$  is constant, now from point C, it is very easy, it is horizontal towards the critical state line and that is the point D. This CD is not consolidated drained, it is the point CD, so it represents consolidated undrained shearing.

So, that is lying on the critical state line, so it is mapped to critical state line. CE: is consolidated drained shearing,  $v$  reduces due to compression. Now from here C, it is sheared in a drained manner, consolidated drain shearing but it is consolidated to same point C, so how it will move? It will further reduce in  $v$  and it moves down and that is the point CE, where it indicates the consolidated drained test. And this is on the critical state line, so it has to be mapped onto critical state line.

ACD, represents that is A, C, D represents the entire CU test, so A, C and D represents the entire cycle of the consolidated undrained test. And similarly ACE is the test for consolidated drained one, so entire consolidated drain test. For normally consolidated, the trends are same except that A and C will be on NCL. Now we have discussed for lightly over consolidated state, now if I say

that it is normally consolidated only thing is this will shift towards this point, and all other trends remaining the same.

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Now let us see for heavily over consolidated state, F is the initial condition same  $v$  as that of A. Now we have the same specific volume but the point is the stress state or the stress history is different, it is heavily over consolidated. From here UU test, you can see that this is the way and it fails exactly at the point B because the initial specific volume is same. Now these are some of the facts which we have not seen in our previous lectures, and which is very legitimately clear in this particular lecture.

FG is the consolidation phase, so it is coming down, then this only consolidation. GH is consolidated undrained shearing, so  $v$  is constant. So, that represents GH and H has to be mapped down. Then GI is consolidated drained shearing, now in the case of HOC understand  $v$  will be increasing because of dilation. And that is the point I where it meets the critical state, it is mapped down I. So, FGH, FGH is the entire CU test and FGI is the entire CD test.

Now you can compare points A and F, soil state have same  $v$  and different stress history. So, just because of different stress history means it does not mean that it will have different failure point because it has same  $v$ . So, what is that initial state of  $v$  is playing a role, so for undrained

shearing failure stress  $p'_f$  at B are same for both the stress history A and F. Normally consolidated and LOC approach CSL from right side, so that is the wet side.

So, whatever be the NC and LOC will approach critical state from right towards left, that is from the wet side, and HOC from left side that is the dry side. Now the summary is whatever be the initial state stress history and loading condition, the final state approaches the ultimate state which is the critical state, you can see it here. Whatever be the final state can all be mapped on this critical state line in  $q$   $p'$ , where the failure occurs.

We have discussed about various initial conditions for both OC and NC but then all can be mapped onto critical states, so that is what it means critical state the ultimate state. So, let us try to summarize today's lecture.

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**Summary**

- The 2D frame work of CSSM in terms of  $e$ ,  $\tau$ ,  $\sigma'$  and  $v$ ,  $q$ ,  $p'$  is discussed
- NC and LOC approach CSL from right side (wet side)
- HOC approach CSL from left side (dry side)
- In drained test,  $e$  changes and  $\sigma'_p$  remains constant
- In HOC, soil reaches peak state before reaching CS
- Concept of yielding becomes important
- In undrained test, intersection of unloading line with COCRL correspond to  $\Delta u = 0$
- $\Delta u$  is  $+ve$  for initial state right side of COCRL due to compression during shearing
- $\Delta u$  is  $-ve$  for initial state left side of COCRL due to dilation during shearing
- Similar to NCL, tension failure zone forms the left side boundary for soil state to exist
- Irrespective of the initial state, stress history and loading condition, the final state approaches the ultimate state: Critical State

The 2D frame work of critical stage soil mechanics in terms of  $e$   $\tau$   $\sigma'$  and  $v$ ,  $q$ ,  $p'$  is discussed. Normally consolidated and lightly over consolidated soil approach critical state line from right side which is the wet side. HOC approach critical state line from left side which is the dry side. In drain test  $e$  changes and  $\sigma'_p$  remains constant, in heavily over consolidated soil reaches peak state before reaching critical state, that is yielding has to happen before failure.

The concept of yielding becomes important, and that is the reason why we will be discussing yielding of the soil in our subsequent lectures. In undrained test intersection of unloading line with critical over consolidation ratio line correspond to  $\Delta u = 0$ . Now this in some cases it is considered as critical state line as well.  $\Delta u$  is positive for initial state right side of critical or over consolidation ratio line due to compression during shearing, we have discussed this, whatever towards the right of COCRL will have positive view during shearing.

The same is negative when it is towards the left of COCRL due to the tendency to dilate. Please do not read this as due to dilation, because dilation represents drain behaviour. So, whichever soil which has got the tendency to dilate will have negative pore water pressure. Similar to normal consolidation line, tension failure zone forms the left side boundary of the soil state to excess. NCL becomes the right side boundary similarly we have a tension failure zone which defines the left side boundary.

Irrespective of the initial state, stress history, loading condition, the final state approaches the ultimate state that is the critical state. So, that is all about two dimensional representation of critical state. Now we will discuss a few more aspects of critical state in the subsequent lectures followed by soil yielding, that is all for now, thank you.