Advanced Soil Mechanics
Prof. Sreedeep Sekharan
Department of Civil Engineering
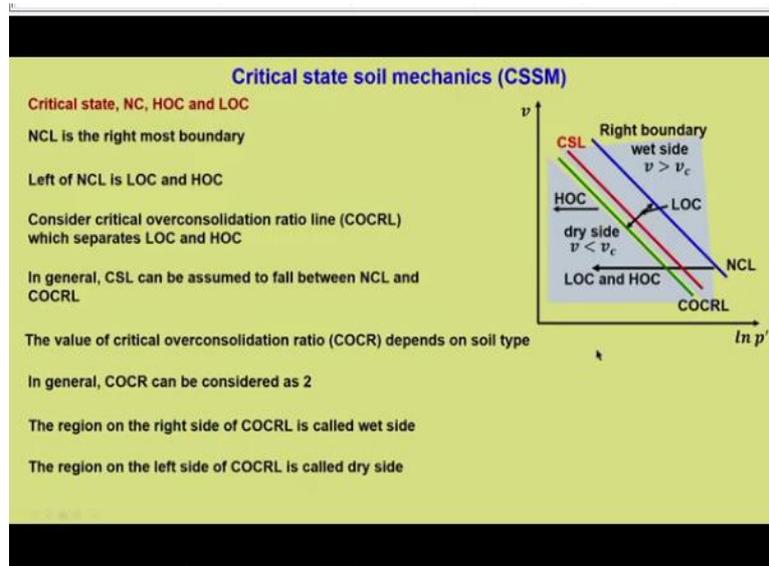**Indian Institute of Technology-Guwahati**

Lecture-45
Introduction Critical State Soil Mechanics

Welcome back all of you. In the last lecture, we have started fourth module on critical state soil mechanics, we were dealing with the introduction of critical state soil mechanics, we will continue with that lecture in today's lecture as well. In the last lecture, we have understood a few aspects of critical state which we already know initially, but we are trying to understand in the context of a framework.

So, we have not actually discussed the entire aspect of critical state line. So, we will focus more on that in today's lecture and see where it position itself with respect to the normal consolidation line or the swelling reloading line. So, today's lecture will focus on critical state, normal consolidation, heavily over consolidated state and likely over consolidated state, why this is important is we need to understand what is the starting point of a given soil or rather I would say state of the soil with respect to critical state.

According to critical state framework, we know that the failure would happen when the soil state reaches a critical state that is how we define it. Now, where the soil position is initially with respect to critical state will help us appreciate or understand better in what manner or in what amount of what is the factor of safety that is available before failure.

(Refer Slide Time: 02:19)

Critical state soil mechanics (CSSM)

Critical state, NC, HOC and LOC

NCL is the right most boundary

Left of NCL is LOC and HOC

Consider critical overconsolidation ratio line (COCRL) which separates LOC and HOC

In general, CSL can be assumed to fall between NCL and COCRL

The value of critical overconsolidation ratio (COCR) depends on soil type

In general, COCR can be considered as 2

The region on the right side of COCRL is called wet side

The region on the left side of COCRL is called dry side

We have already said that normally consolidation line is the right most boundary, it can be normally consolidated or you can say it as isotopic consolidated, whatever be it remains the same only thing is for tracks shell we are specifically interested in isotropic consolidation. So, for all practical purpose our normal consolidation line we can treat same as that of isotropic consolidation line.

So, in v ln p', we can draw NCL, we have already seen that and that is the right most boundary and this NCL has a slope of λ. Now, the space which is towards the left is always the likely over consolidated or heavily over consolidated depending upon what amount of unloading the soil has undergone with respect to its past maximum pressure, the past maximum pressure lies on normal consolidation line or isotropic consolidation line from where it is unloaded which creates lightly over consolidated state or heavily over consolidated state depending upon how far the state of the soil is with respect to NCL.

So, this is the left most part is LOC and HOC. Even though we know all these facts, it is very important that we visualize this in the critical state framework. Consider critical overconsolidation ratio line which separates lightly over consolidated and heavily over consolidated. Now, we have already seen in module 2, that the behaviour of heavily over consolidated is fairly different from that of the normally consolidated response.

Understanding this response is very important to understand critical state framework as well. So, first we need to understand what separates heavily over consolidated from normally consolidated. Now there is another category which is likely over consolidated, which

resembles its behaviour more towards the normal consolidation, means it may not exhibit peak. So, where should we draw a line, so that it separates the normally consolidated or maybe lightly over consolidated from that of heavily over consolidated.

And that is given by critical overconsolidation ratio line. So, overconsolidation ratio helps us to understand how much or what is the degree of over consolidation the soil is subjected to. Now, we let us understand it to be a line that is critical overconsolidation ratio COCRL to be a line which is parallel to NCL, because then the ratio will be the same. Now, this line also has an inclination of or the slope of λ.

Now, the portion now, what it means is this particular line is the line between lightly over consolidated state and heavily over consolidated state, this line separates it. So, any state which is on this particular line on the left of COCRL will be HOC and towards the right of COCRL will be lightly over consolidated and on this line it is NCL that is marked here. In general now, where does the critical state line fall?

So, in general, CSL can be assumed to fall between normal consolidation and COCRL. So, the critical state line falls somewhere in between NCL and COCRL, we need to understand this that these are all certain concepts. Now, according to that, it is somewhere here. So, this portion is lightly over consolidated, towards the side also likely over consolidated, towards this side also lightly over consolidated. Now whether a lightly over consolidated soil will exhibit peak or not these are all dependent on that specific type of soil, but definitely towards the left of COCRL it is going to exhibit a heavily over consolidated behaviour.

The value of critical overconsolidation ratio depends on the soil type. So, what is the overconsolidation ratio below which it is a LOC and above which it is HOC, it is dependent mostly on it on soil type, but in general COCR that is critical overconsolidation ratio can be considered as to in most of the cases for practical purpose we can consider the overconsolidation ratio as to which is termed as critical overconsolidation ratio.

Now, below 2 if the OCR value is less than 2, we consider it to be LOC and above 2 can be considered as HOC, these are all some bare ideas like it is not a very strict philosophy, it is kind of a rule of thumb below which it is considered to be LOC, the region on the right side

of COCRL. Now, this COCRL is the reference towards the right side is called wet side. Now, this particular definition of wet side, dry side we have seen in compassion curve.

What is towards the right of OMC we call it has wet side of optimum and towards left it is called dry side of optimum. Similar to that it is defined that towards the right side of COCRL it is wet side and towards left it is dry side. Now, wet side, dry side I am introducing because the basic philosophy of critical state soil mechanics put forth by Schofield and Roth, they also use these terminologies.

So, it is important, also it is good that we understand these terminologies better. So, that is why I am introducing and as such wet side, dry side does not have much significance if you understand it is towards right or left. So, the region on the right side is wet side and towards left off COCRL is called the dry side. Now you need to keep in mind the difference between COCRL and CSL is not much.

So, sometimes you can find certain discussions wherein the wet side and dry side is also talked in terms of critical state line as well. So, we can see that dry side of critical state line or towards wet side of critical state line. In fact, the overconsolidation ratio separating LOC, HOC it is marginal. So, one can always say that towards dry side of critical state line, it exhibits HOC behaviour.

So, wet side, dry side here, we are specifically discussing with respect to critical overconsolidation ratio line, but keep in mind one can also refer to in terms of critical state line as well. Now, here it is marked as wet side where specific volume that is v is greater than v critical, v critical is the critical void ratio with reference to COCRL and if it is less than COCRL then it is dry side.

Keep in mind we can also refer to v_c as the specific volume along the critical state line as well. So, please keep in mind this difference in general the definition is like this, but can also refer v_c as the specific volume on critical state line.

(Refer Slide Time: 11:10)

Undrained strength

The relationship $\tau_f = \tau_{cs} = \sigma'_{cs}\,tan\emptyset'_{cs}$ is applicable for effective normal stress

Undrained strength need to be represented differently

Strength decreases with increase in e $Gw = S_r e$

Strength is related to w

Undrained strength refers to the strength when the soil is sheared without change in w

$\tau_f = \tau_{cs} = S_u$ $\tau_{cs} = \sigma'_{cs}\,tan\emptyset'_{cs}$

$S_u = \sigma'_{cs}\,tan\emptyset'_{cs}$ $\sigma'_{cs} = \dfrac{S_u}{tan\emptyset'_{cs}}$

$e_{cs} = e_\Gamma - C_c log\sigma'_{cs}$ $e_{cs} = e_\Gamma - C_c log(\frac{S_u}{tan\emptyset'_{cs}})$

$log\left(\dfrac{S_u}{tan\emptyset'_{cs}}\right) = \dfrac{e_\Gamma - e_{cs}}{C_c}$

S_u changes with change in e or w

S_u is not a fundamental soil parameter

S_u is a state dependent parameter

Now, let us see some important aspects related to undrained strength, we have discussed this S_u is the undrained shear strength of soil which we discussed in the module 2. So, we will see in terms of critical state framework how does it look like. Now, we know the relationship τ_f, which is the failure state and for critical state framework, the failure state is same as the critical state τ_{cs} which is equal to σ'_{cs} tan φ'_{cs} is applicable for effective normal stress.

Now, this relationship it is familiar to us and we talk this in terms of critical state or the effective normal stress. Now, undrained strength we need to represent this differently, which is not similar to that of this particular relationship, this particular relationship we know it is more like a Mohr coulomb failure envelope with inclination of φ'_{cs}. So, τ versus e how does it look like?

So, this is shear stress up versus void ratio and this is the critical state line. Now, here this in the case of undrained shear strength, we have τ_{cs} that is the critical state strength is equal to undrained shear strength and the void ratio corresponding to that is e_{cs}. Now, we can see here that strength decreases with increase in e as can be seen here. Now, there can be S_u here, there can be S_u here.

So, as e increases the value of S_u comes down. Now, we know that the relationship $Gw = S_r$ *e. Now, this is a saturated state, G is a constant. So, as e increases w increases that means, w and e is related. So, it also means that with increase in e S_u decreases, which also mean which is very correct also as the water content increases, the S_u value comes down.

So, strength is there by related to w as well, we are talking about undrained shear strength very specifically and undrained shear strength is highly sensitive to its initial condition in terms of e or w. Now, we also know S_u refers to the strength when soil is sheared without change in w. So, it is at constant w condition, we are doing the shear test. So, there is no change in w why because there is no change in drainage condition.

And hence the e also remains constant. So, we can write $\tau_f = \tau_{cs} = S_u$ which is the undrained shear strength, we also have the relationship $\tau_{cs} = \sigma'_{cs} \tan \varphi'_{cs}$. Now, in this case, we can write $\tau_{cs} = S_u$. So substitute for $\tau_{cs} = S_u$. Now, whether this equation holds good or not. So, based on our previous understanding, please refer back to module 2.

What we have discussed when we discussed about UU test like we have only one effective stress circle because of which we are not in a position to define the effective stress Mohr coulomb envelope. But while discussing that, we also have seen if φ'_{cs} is known, one can always draw the Mohr coulomb failure envelope for the very one effective stress circle that you obtain from the UU test which means there is a point of intersection between the effective Mohr circle envelope with the tresca envelope which is given by S_u.

So, there is a point where it intersects. So, this particular equation holds good and that can be explained in this particular manner. So, this is τ versus σ' and this is the Mohr coulomb failure envelope. Now, this is the tresca failure criterion wherein S_u is the undrained shear strength. So, this point is what we are discussing about. So, this point this particular equation is very much valid.

So, $S_u = \sigma'_{cs} \tan \varphi'_{cs}$. So, we can write $\sigma'_{cs} = S_u / \tan \varphi'_{cs}$, we also have the relationship $e_{cs} = e_\gamma - C_c \log \sigma'_{cs}$, this comes from our previous lecture. So, please refer back. So, we can write $e_{cs} = e_\gamma - C_c \log$, this σ'_{cs} we can substitute it here. So, we have S_u upon $\tan \varphi'_{cs}$. So, we can write this goes on the left side, we have $C_c \log S_u / \tan \varphi'_{cs} = e_\gamma - e_{cs}$ upon C_c.

So, here you are relating S_u as a function of e_{cs} and other parameters. So, here you can see S_u changes with change in e or w which we have already explained, since it is changing with the state it is not a fundamental soil parameter and that is where we have started off with C' and φ'. So, for undrained shear strength, it still holds good that it is not a fundamental soil parameter.

S_u is a state dependent parameter. So, what is the state of the soil for undrained shear strength it is more dependent on state. State means, how and where the soil initial point is positioned with respect to the critical state line. So, that is defined by e or w. So, it is a state is defined by void ratio in the critical state framework we define it using void ratio. So, initial void ratio gives where it is with respect to the final state which is on the critical state. So, that is why S_u is known as the state dependent parameter. So, we will come to what is meant by state in the subsequent slide.

(Refer Slide Time: 18:25)
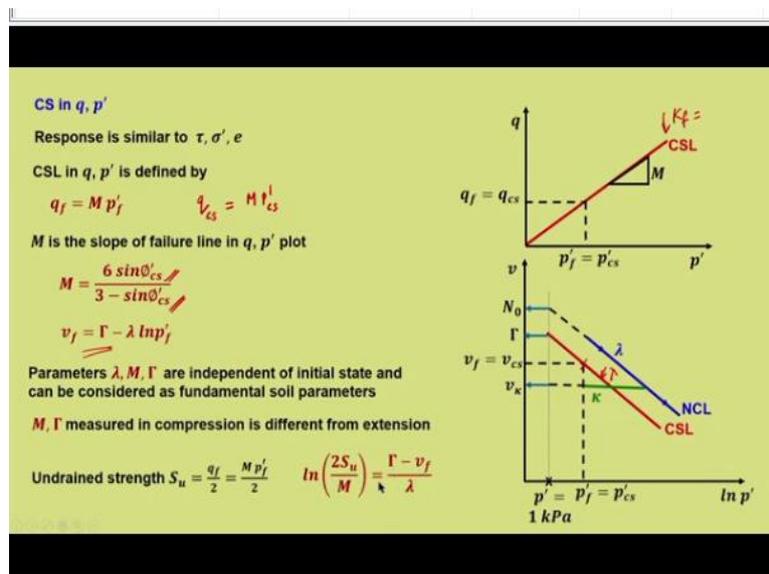
So, critical state now, let us see in q, p' format, we are explaining in 2 ways; critical state directly we know it comes from τ σ' response, whatever we know from module 2 and all those concepts remains same in q-p' framework, because you will see later the critical state is mostly defined in terms of stress invariants which is q- p'. So, we will be discussing both.

First we will discuss in τ σ' format and then we will come to q- p' till we introduce all the concept, further on we will deal only with q- p' $_v$ space or you can also write it in terms of q, p' $_e$. Because e and v is there is only a difference of 1. So, 1 + e gives me. So, both are valid, but while understanding critical state framework, let us discuss both. So, whatever we have discussed now, in terms of τ σ' e we will see that in q, p' and v. So, responses similar to τ σ' e. Now, when we discussed q, p' v in the previous lecture, we have not discussed the failure state that is what is how q $_f$ and p' $_f$ is correlated?

So, we are yet to discuss that, so that we will discuss now. Now, this particular response we have not discussed in the previous lecture. So, it is q, p' and critical state line is given by this; this is nothing but the K_f line that we have seen before. Now, how it becomes critical state line because, here the φ' considered as $φ'_{cs}$. So, that is why K_f automatically becomes a critical state line.

Let us assume $p'_f = p'_{cs}$. Now, critical state, the failure state is same. So, $p'_f = p'_{cs}$ and for which $q_f = q_{cs}$. Now, this is related. Now, the slope of the critical state line is given by capital M. So, critical state line in q, p' is defined by $q_f = M$ into p'_f very important expression which you need to keep in mind. So, critical state in q, p' v space is defined as q_f is equal to M into p'_f.

It is also good to write $q_{cs} = M$ into p'_{cs}. So, both are valid. What is M? M is the slope of failure line in q- p' plot and we already have defined or we have derived this expression for the slope in q- p' again please refer back we have already done it and for compression case it is 6 sin $φ_{cs}$ / 3 - sin $φ_{cs}$. Now, you can see here we are denoting it by $φ'_{cs}$.

So, that is why here the failure line K_f is equal to critical state line. Now, let us see this in terms of v ln p'. Now, this is the NCL with a slope of λ, the critical state line is position like this, because we know the right most boundary is NCL and there is no state beyond NCL. Now, this is the unloading reloading line with a slope of κ. Again we have already discussed this.

Now, let us see here $p'_f = p'_{cs}$ which is lying on the critical state line where $v_f = v_{cs}$. Now, all the important parameters are defined that is p'_{cs} q_{cs} and v_{cs}. Now, we already have a reference line or the reference point on every line which corresponds to p' equal to 1 kPa. So, here on NCL, it is N_0, for the unloading reloading line is $v_κ$ and for critical state line is capital γ, which can be represented.

So, all the initial states of the line has been defined based on which the equation for this line NCL and critical state line can be written. So, we can write the equation for critical state line in v ln p' as γ, that is this point - λ is the slope because this is also λ - λ ln p'_f. Now, p'_f means p'_{cs}. So, every point on the critical state line can be defined using this expression v_f is

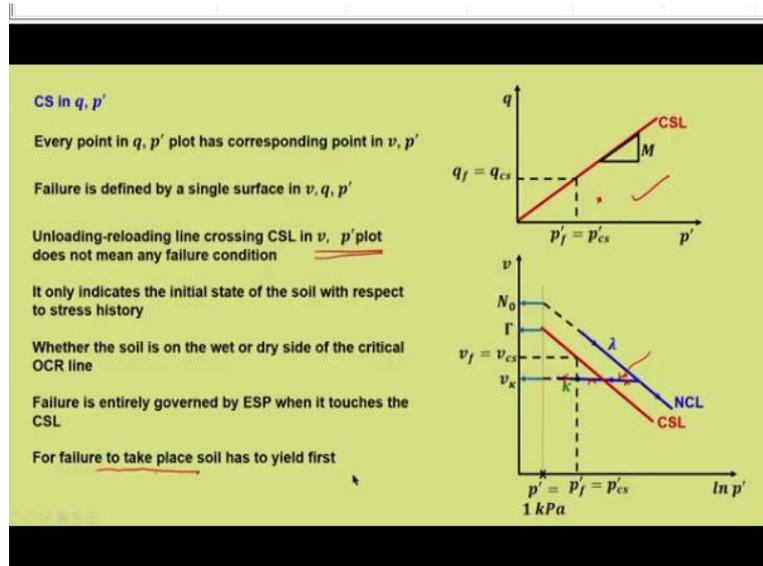equal to γ which is corresponding to 1 kPa - and then it depends upon where actually the point is.

So, this is the p'_f. Here the parameters λ , M and γ are independent of initial state why, because these are the parameters which represents the critical state line and can be considered as fundamental soil parameters. Earlier we have understood in τ σ' space, the τ σ' e space that C_c which corresponds to λ and the other parameters it can be used for defining the fundamental parameters of soil.

So, similarly in q v p' space you can write λ M and γ these are the fundamental parameters of soil and it can be considered in the case of critical state framework. M and γ that is the slope M and γ measured in compression is different from extension that we have already seen. The slope of K_f line in extension is different for q- p' plot.

So, similarly, M is also marginally different in the case of extension case. So, now, we are basically discussing with reference to competition line. So, we will not bother about that right now. Also, we know that in the previous slide we have discussed about undrained shear strength. Now, here also we can very well define undrained shear strength q $S_u = q_f / 2$. Now, q_f is $\sigma'_1 - \sigma'_3 / 2$ is the radius of the circle which gives you tresca criterion.

So, S_u is very much equal to $q_f / 2$ and $q_f = M$ into $p'_f / 2$. So, we can also write ln 2 S_u / M = $\gamma - v_f / \lambda$. The way we have written previously in terms of log. So, how this is possible? This is possible because you are writing the expression for ln p'_f, which is equal to $\gamma - v_f$ upon λ which is equal to ln p'_f and p'_f is nothing but to S_u / M which from here. So, that gives the expression for S_u in terms of critical state parameters.

(Refer Slide Time: 26:28)

CS in q, p'

Every point in q, p' plot has corresponding point in v, p'

Failure is defined by a single surface in v, q, p'

Unloading-reloading line crossing CSL in v, p' plot does not mean any failure condition

It only indicates the initial state of the soil with respect to stress history

Whether the soil is on the wet or dry side of the critical OCR line

Failure is entirely governed by ESP when it touches the CSL

For failure to take place soil has to yield first

So, now, let us further try to explain every point in q- p' plot has the corresponding point in v, p'. So, that is true whatever we are having here wherever we are taking we are taking a point here can be correspondingly marked in the v ln p' plot as well. Failure is defined by a single surface in v, q, p' which is the critical state of that particular soil. Now, unloading reloading line crossing critical state line in v- p' plot does not mean any failure condition.

Now, this is something which I am explaining, because when we discuss this in the normal classroom, we have found that some students always have this doubt that you can see here critical state line is placed here. Now, from here the unloading line it goes and it crosses the critical state line. Now, sometimes there is a misconception that it is crossing the critical state lines. So, the failure has already occurred, it is not true.

Critical state line is only a relative position of the failure condition, the unloading reloading line it only determines the stress history of the soil. So, crossing this critical state line in please underline in v, p' or v ln p' plot is not a failure condition. Obviously, when it crosses or when it touches the critical state line in q-p' plot it is the failure condition not in v ln p' plot.

So, please do not confuse yourself this with respect to v ln p' it is only the relative position or the state of the soil when I say that it is on unloading reloading line or on NCL, it is only the initial state of the soil and during shearing it moves towards the critical state line, it is not that merely this crossing does not mean that the failure has occurred. It only indicates the initial state of the soil with respect to stress history; you know that different points on the unloading

line with respect to the given NCL gives different OCR, whether the soil is on wet or dry side of the critical OCR line.

So, that is the only purpose of these points. Failure is entirely governed by effective stress path when it touches the critical state line which comes from this particular figure. So, when the effective stress path when it starts from a particular point and when it reaches the critical state line, then that explains whether the failure has occurred or not. For failure to take place soil has to yield first, so for this also we know for failure to take place, the soil has to yield first.

That means we need to introduce another concept, which we will see shortly that we need to define what is soil yielding? Now, when we discussed in the case of module 2, we never discussed this point explicitly, but it is there, but we are not discussing that explicitly, but in this module, we need to clearly state when the soil is going to yield. So, without a yielding failure cannot take place.

So, I am not going into the details right now, you can underline this fact that for failure to take place soil has to yield first. This particular understanding is extremely important when you try to understand the behaviour of OC. So, let us hold on for the time being about this yielding, but make sure that we understand that for failure to take place soil has to yield first. So, that is all the preliminary discussion about critical state in both the format's that is τ as well as q, v, p' format.

(Refer Slide Time: 31:07)
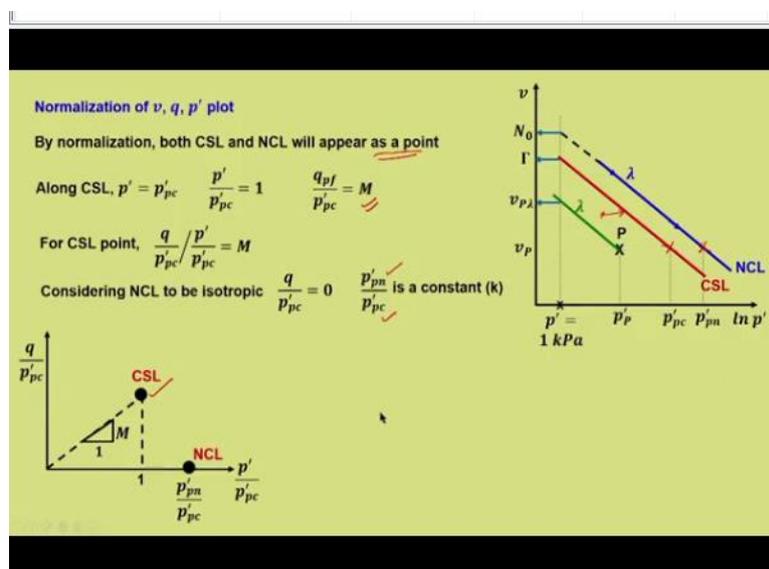
Now, we will discuss a bit about normalization of v, q, p' plot with respect to some initial condition or some specific parameters, so, that the comparison becomes easy. Consider a point P. The state of P is defined by so, this is the point P. Now, state of P can be defined by what is you can assume a line which is parallel to NCL with slope λ passing through P. So, the state of this particular point P can be defined by p'_p, v_p as well as q, p in q, p' plot.

Now, corresponding to this particular v_p, if we draw a horizontal line, we can also find p' corresponding to that on the critical state line and that is p'_{pc}. Now, the value of specific volume corresponding to 1 kPa on the line through P is given by $v_{p\lambda}$. The effective stress corresponding to v_p on NCL line is given by p p', p' p M. Now, p' represents it is effective p, mean stress.

This p represents that it is with reference to point p and n represents that it is on NCL similarly, here c represents it is on critical state line. So, there are certain reference points now, v_p is the specific volume for point P. Now p' corresponding to v_p has been marked and that the normalizing parameter is p'_{pc} and $v_{p\lambda}$. So, these 2 parameters now, for every point in this space, you can find a corresponding $v_{p\lambda}$ and p'_{pc}.

Now, you may be wondering why the normalizing parameter is not p'_p, because this p'_p is dependent on the position of the point p, whereas, p'_{pc}, it is a more stable point, because for every p you are considering the point or the location of this point with respect to critical state line. So, that is why p'_{pc} can be a good normalizing parameter.

Obviously, this 1 kPa reference is also a very good normalizing parameter. P'_{pn} is also used as normalizing parameter. So, what is the distance of point P from NCL that is p'_{pn}. So, this can also be used as a normalizing parameter, but rather than normally consolidated line critical state line is of more importance, why that distance actually represents the factor of safety.

So, for all practical purposes the normalizing parameter can be considered as $v_{p\lambda}$ and p'_{pc}, we can write v_p that is specific volume along this line is equal to $v_{p\lambda}$ - $\lambda \ln p'_p$. Now, when we substitute $\ln p'_p$ that is this particular point we are discussing v_p of point P. So, v_p of point P = $v_{p\lambda}$ - $\lambda \ln p'_p$. Now, when you know the initial point p'_p is known, v_p is known, $v_{p\lambda}$ can be determined.

So, that becomes a reference parameter. So, $v_{p\lambda} = v_p + \lambda \ln p'_p$. So, $v_p = \gamma - \lambda \ln p'_{pc}$. Now, v_p is this particular point. So, you have this v_p on critical state line as well. We can very well write $v_p = \gamma - \lambda$, λ is the slope of this, $\lambda \ln p'_{pc}$ because this is the reference point. This equation we have already discussed previously for critical state line.

So, v_p is known now, v_p depends on the position of p, γ is the critical state parameter which is known, λ is known, p'_{pc} can be determined. So, $\ln p'_{pc} = \gamma - v_p / \lambda$. So, both the normalizing parameters can be determined. Now, when you normalize the whole of the v, q, p' plot how does it look like? So, let us normalize with respect to p / p'_{pc} and p' / p'_{pc}.

Now, how does it look? By normalization both critical state line and NCL will appear as a point. Now, both instead of a line it becomes a point how? So, let us say that this is the point which is the representation of that particular soil in q-p, p' q / p'_{pc} p' / p'_{pc} space, this is the normalized plot. So, this is the point corresponding to CSL. Let us see how? Now, along critical state line this line $p' = p'_{pc}$.

So, any point we take, let us say take this point again this is p'_{pc}. So, $p' = p'_{pc}$. So, p' by p'_{pc} = 1. So, this is not going to change. So, this 1 is represented by this particular point. So, you have a line along $p' / p'_{pc} = 1$. So, that is fixed for a critical state line. Now, $q_{pf} / p'_{pc} = M$, how that is q_p / p'_f, that is $q_f / p'_f = M$. Now, p'_f is nothing but p'_{pc} because it is only critical state line.

So, we know that that particular ratio is M. Now, for critical state point, we know that q / p'_{pc} - p' / p'_{pc} is equal to M why? Because here $p' / p'_{pc} = 1$ in this particular case and q / p'_{pc} or this particular one is already equal to M. So, this becomes M. So, where you find a particular point with a slope M which intersects this particular line is the point of critical state line.

So, that is how we can say that critical state line is a point in this normalized space. Now, for considering NCL to be isotropic let us say that the initial point is isotropic. So, in that case, $\sigma'_1 - \sigma'_3 = 0$ that is q = 0. Hence, $q / p'_{pc} = 0$. So, this q is 0. So, NCL will be represented by a point anywhere on this particular line where it is 0. Also, p'_{pn} / p'_{pc} is a constant.

You can see that p'$_{pn}$ is this and p'$_{pc}$ is this; this remains always a constant wherever you take, so, that constant is not going to change. So, always it can be represented by this particular point. So, NCL is represented by a point. Now, depending upon the different type of soils the relative position will change that is a long this it will change or along this it will change. But these CSL and NCL can be represented by a particular point.

(Refer Slide Time: 39:50)
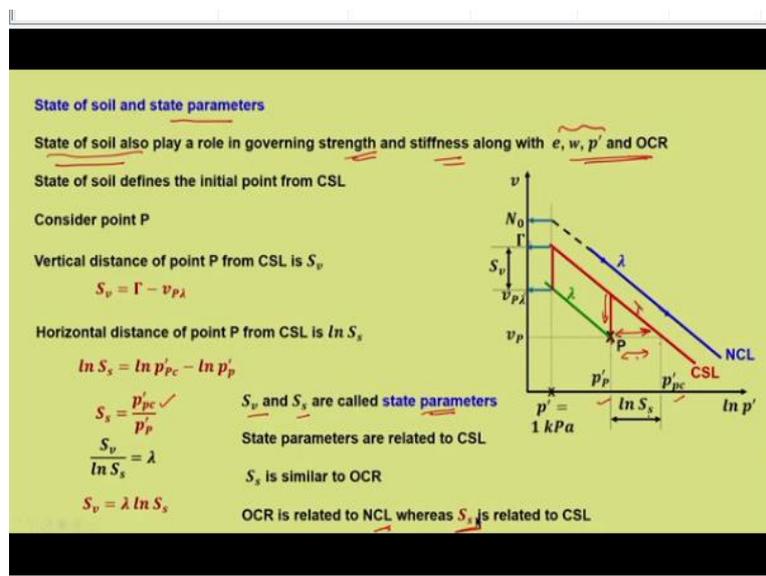
Now, we have discussed about state of the soil. So, we will discuss state of the soil in detail and the state parameters. And NCL can be represented by a particular point. Now we have discussed about state of the soil. So, we will discuss state of the soil in detail, and the state parameters. Now, state of the soil is relevant for a given point. Now, here the reference point is s peak P. State of soil also play a role in governing strength and stiffness, along with e, w, p' and OCR.

So, whatever be these are parameters that is going to define what is the strength and stiffness of the soil? So similarly, it is important to understand how state of the soil also play a governing role in defining the strength and stiffness. It is one of the same because once you know this, you are defining the state, but the parameters in v ln p', in terms of state parameters, we will define.

State of the soil defines the initial point from CSL; you can see here, this point P, how distant, it is from CSL is the kind of factor of safety. Consider point P. So, this we have already done, vertical distance of point P from critical state line is S_p. So, when we define state of soil we are defining what is the state of this particular soil initially with respect to

critical state line? So, that is why we are defining the vertical distance of point P from critical state line, you can see this red line.
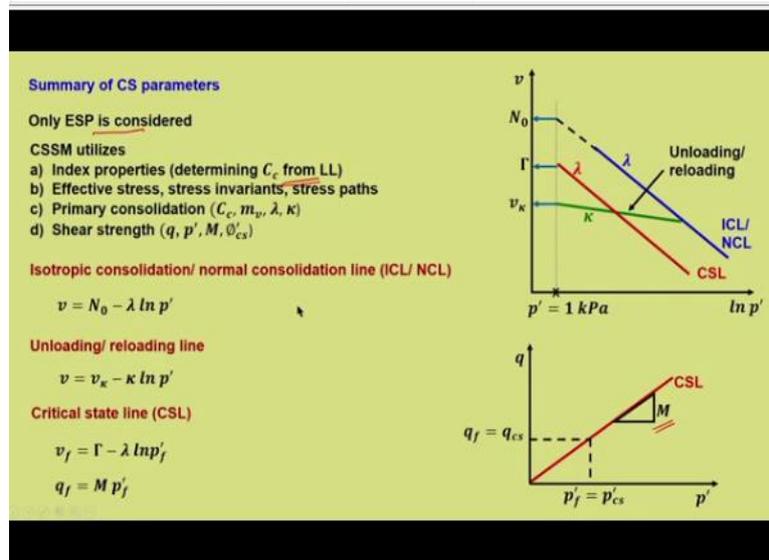
Now since these are parallel. This is also the same as this distance and this particular distance is represented by S_v, which is the state parameter. So, that is what we are going to define. This state parameter is what we are going to define. So, the vertical distance is S_v. Now, S_v can be written as $\gamma - v_{P\lambda}$, what is γ? This distance is γ and this distance is $v_{P\lambda}$.

So, S_v from the geometry you can very well write $\gamma - v_{P\lambda}$. Horizontal distance of point P from CSL is given by this one ln scale. So, ln S_s. So, this is the horizontal distance. Now ln S_s can be written as ln $p'_{pc} - p'_p$. So, S_s can be written as p'_{pc} / p'_p, this p'_{pc} / p'_p, you can see that this is very much similar to the definition of OCR.

OCR is with reference instead of p'_{pc}, it is with reference to the stress state on the NCL, which is called as yield stress. So, here $S_v / \ln S_s = \lambda$ here, if you consider this particular line and this. So, this is ln S_s. This is ln S_s and this is S_v, so $S_v / \ln S_s$ is equal to the slope which is λ. So, that is given as λ, so we can always write $S_v = \lambda * \ln S_s$. So, this is the relationship between the state parameter.

So S_v and S_s are called state parameters of that particular point or what the soil state is with respect to critical state is given by the state parameters and the vertical and horizontal or S_v and S_s. State parameters are related to critical state line, S_s is very similar to OCR, this definition of S_s is very similar to OCR. In OCR is related to normally consolidated line, whereas S_s is related to critical state line.

(Refer Slide Time: 43:48)

Summary of CS parameters

Only ESP is considered

CSSM utilizes
a) Index properties (determining C_c from LL)
b) Effective stress, stress invariants, stress paths
c) Primary consolidation (C_c, m_v, λ, κ)
d) Shear strength (q, p', M, $\emptyset'_{cs}$)

Isotropic consolidation/ normal consolidation line (ICL/ NCL)

$$v = N_0 - \lambda \ln p'$$

Unloading/ reloading line

$$v = v_\kappa - \kappa \ln p'$$

Critical state line (CSL)

$$v_f = \Gamma - \lambda \ln p'_f$$

$$q_f = M p'_f$$

So summary of critical state parameters, now what we have done is in the name of introducing critical state, we have discussed quite a lot of things in these 2 lectures, but what is very essential as we move forward when we interpret critical state soil mechanics. So, that summary, we are providing here. So, whatever we have learned finally, these facts, what we're going to discuss now is very important.

So, only effective stress path is considered, you have ICL or NCL. ICL means isotropic consolidation line or NCL with slope λ, CSL with slope λ, unloading reloading line with slope κ, the initial point reference p' = 1 kPa, $N_{0\gamma}$ and v $_\kappa$. And we need to see only effective stress path is considered for defining failure. So, then we have q p' plot where q $_f$ upon p' $_f$ is given by slope M.

So, M λ κ they are important. CSM utilizes the index properties, determining C $_c$ from liquid limit, because C $_c$ is the one which we get from the consolidation plot. So, this C $_c$ is very important and we know that C $_c$ can be determined from liquid limit. So, CSSM utilizes index properties in one way or the other. Then effective stress that is σ'$_n$ that is the normal stress or the consolidation stress σ' which is always in terms of effective.

Stress invariants, because q and p' we are considering and stress path, whether it is drained or untrained how it moves. So, CSSM utilizes these information as well. The primary consolidation data that the C $_c$,m$_v$, λ and κ. Shear strength q, p', M and φ' $_{cs}$. So, all these we have discussed and critical state framework utilizes these properties.

Isotopic consolidation are normally consolidation line, which is given as this, where it is defined as $N_0 - \lambda \ln p'$. Unloading reloading line, which is defined as $v = v_\kappa - \kappa \ln p'$. All this we have discussed, I am just summarizing it and critical straight line CSL is defined as $v_f = \gamma - \lambda \ln p'_f$. So, you can see that with reference to this here, you are adding the subscript f in for critical state line.

And the additional one what is needed is $q_f = M$ into p'_f. So, these are the essential relationships, which we have to keep in mind for dealing with critical state framework.

(Refer Slide Time: 46:57)
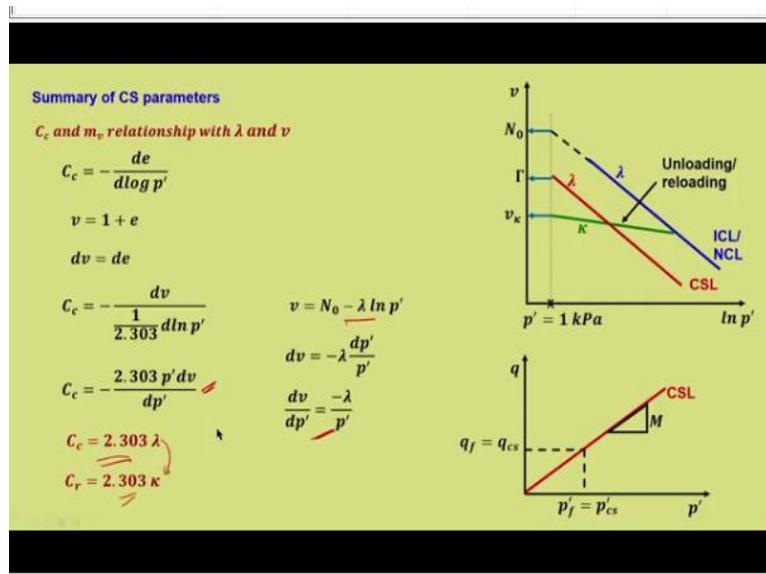
We also need to understand the relationship of C_c and m_v with λ and v, because we need to cross compute these as and when required. We know that $C_c = - de / d \log p'$, from our initial definition of C_c and that corresponds to normally consolidated portion, $v = 1 + e$, $dv = de$, substitute for in the previous expression $C_c = - de$ is replaced by dv and $d \log p'$ can be written as $1 / 2.303 \, d \ln p'$.

Now $d \ln p'$ is $1 / p'$ into dp', so that will give you $C_c = - 2.303 \, p'$, that goes into a numerator dv / dp'. Now we have the expression for normal consolidate line as $v = N_0 - \lambda \ln p'$, dv, if you differentiate this, you will get $- \lambda \, dp' / p'$. So, we can write dv upon $dp' = - \lambda / p'$. Now you can see, this expression and this dv / dp' can be substituted.,

C_c can be written as 2.303λ. So, knowing C_c λ can be determined. So, C_r that is for unloading reloading line, the same equation holds only is instead of λ, it becomes κ.

(Refer Slide Time: 48:37)
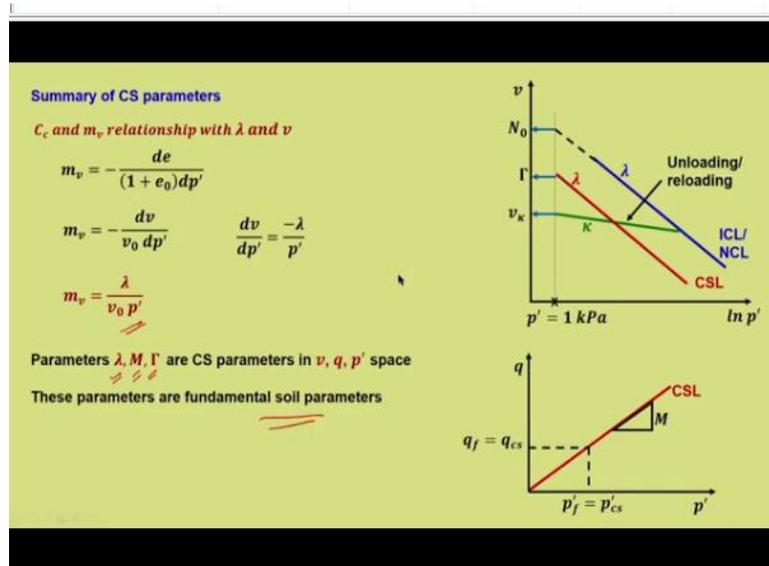
C_c and m_v relationship with λ and v will continue, we need to find out the relationship for m_v. So, m_v again from the basic definition, we have - de upon $1 + e_0$, e_0 is the initial void ratio upon dp'. So, m_v is equal to again substituting we have dv $1 + e_0$ is initial specific volume v_0 dp', dv / dp' we know that is - λ / p'.

So, m_v = λ / v_0 p'. So, parameters λ, M γ are critical state parameters in v, q, p' space. Please keep these relationships what we discussed as summary in your minds, because these are very important as we move further. These parameters can be considered as fundamental soil parameters.

(Refer Slide Time: 49:43)

Some additional note on critical state before we wrap up the introduction part, the strain required to reach critical state depends on the initial soil state, whether it is on wet or dry

state of critical. Soil sample strain without change in shear stress, effective stress and volume at critical state. So, there is a shear straining happening at constant condition. Shear strength tests are normally terminated at strain, less than 20%.

It depends upon the codal provision, what is the termination criteria? This strain may not be sufficient to achieve critical state for soil samples on wet side of critical. Again I told that these are all concept put in place. Now in reality what actually is happening to the soil we really do not know? So, we have some termination criteria at a high strain and high strain we consider it to be maybe less than 20%.

So, we presume that the state what we define as critical state has been achieved by the soil during shearing, but we are not sure. Now, we also need to keep in mind that even at 20% strain there are possibility that the soil has not reached its critical state. So φ' that you define based on this 20% strain actually may not be φ'_c as it may be even less, so what, actually it is we do not know, but we presume it.

Now, for wet side of critical state it may so happen that it will not reach the required critical state at 20% strain. Now wet side of critical means what it is mostly referring to normally consolidated or lightly over consolidated state. Now for this there is a progressive densification that is happening upon yielding. After yielding it is strain hardening. So, it can build up more and more.

So, that is the possibility why we say that it may not reach to its critical state, whereas in the case of OC state what happens with sharing, there is dilation, so the progressive softening that is happening that soil particles are disintegrated or separated from each other. So, here the critical state is fairly easy. The strain may be sufficient for whatever 20% what we discuss this strain may be sufficient for soil samples on dry state, dry state means nothing but the heavily over consolidated state.

It is ideal to conduct shear tests on lightly over consolidated samples for determining critical state. Since the initial state is close to critical state. So, if you are planning to obtain the critical state parameter one suggestion is that if the soil is lightly over consolidated then fairly, it is easy for the soil state to reach critical state if you see that in terms of v ln p' plot you can see that LOC is more close to critical state than HOC and NC.

So, if we start the sharing it likely over consolidate state it is fairly, we can say that by 20% strain yes there are all possibilities that the soil would reach its critical state. Critical state line passes through origin in q, p' plot, no strength at zero confinement. Now if there is cohesion, definitely that is going to be there at zero confinement, that is what we know. Now in q, p', if we are representing the K_f line in terms of critical state line, it passes through origin.

Now what is the logic here? If there is cohesion, it does not mean that it is not considered let us see what actually does it imply? So, there is no strength is zero, we are not considering any strength for the soil at zero confinement. That is we are discussing with respect to critical state line that we have to keep in mind. In case of true cohesion, we are not discussing about apparent coercion.

But if there is true cohesion soil will exhibit some sort of cementation characteristics and if there is cementation characteristi$_{cs}$, obviously, there will be some strength at zero confinement. There will be strength at zero confinement, but in the case of critical state line we do not consider this. Why it is because the strain required to reach a critical state is enough to break the true cohesion or cementation.

So when the particle undergoes shearing, it is undergoing volume change and we know that under critical state there is the movement of soil particles more like a fluid frictional fluid we call it. So, there is a turbulent motion of particles. Now when we define such a situation for critical state it is very implicit, that the particles bonding need not be considered at critical state. That is a logic which states that there is zero strength at no confinement condition.

So, critical state line passes through the origin in q, p' plot. It is hard to exactly achieve critical state during shear test for the reasons we have already stated, it is often assumed that critical state has reached for the strain level considered in the test. Now the concept of critical state is also a hypothesis and in the testing in the actual practice for its implementation we presume that the termination criteria that we adopted if the soil has not failed.

That particular strain level corresponds to critical states. So, whenever we are using this termination criteria we need to keep in mind that it is based on the assumption that we are following this.

(Refer Slide Time: 56:09)

So, let us summarize the last 2 lectures, where we have discussed or we have introduced the critical state soil mechanics. Critical state soil mechanics is a concept that integrates consolidation and shear strength behaviour for defining failure state which has not been done in Mohr coulomb failure envelope. The 2D representation of void ratio versus effective stress and shear stress versus effective stress is integrated to obtain 3D representation of critical state we have not done this yet that will be towards the end of this module that we will see how to integrate the concept.

According to CSSM all soils will fail on a unique failure surface defined in q, p' e or q, p' p space. Critical state, in general is attained at shear strain greater than 10% why instead of 20% I have used 10% is in some codal provision it is 10%, in some it is 15%. So, in general, less than 20% is the understanding. Critical state parameters can be considered as a fundamental soil property.

Accordingly φ'_{cs} because we discussed it in terms of τ, σ' e plot. So, φ'_{cs} e $_\gamma$ C_c can be considered as fundamental soil parameters. Normal consolidation line NCL, unloading-reloading line and critical state line are important for defining CSSM, we have seen that. In q, p' v space, slope of NCL and CSL is λ. And the slope of unloading-reloading line is κ. NCL forms the right side boundary for the soil state to exist; we will be discussing this again in the subsequent lectures. Left of NCL is LOC and HOC.

(Refer Slide Time: 58:21)

Summary

- Importance of OCR and yield stress ratio discussed

- Soil bulk stiffness is dependent on λ and κ depending on loading/ unloading

- Critical overconsolidation ratio line (COCRL) is the boundary between LOC and HOC

- The region on the right side of COCRL is called wet side (LOC and NC) and left of COCRL is dry side (HOC)

- Undrained strength expressed in terms of CS parameters

- Normalization of v, q, p' plot discussed

- State of soil and state parameters defined

- Parameters λ, M, Γ are CS parameters in v, q, p' space and are fundamental soil parameters

- CS is a concept

- It is practically difficult to assess whether the soil has reached CS during shearing

- In most cases, it is assumed that the soil reached CS during shearing (at high strain condition > 10%)

Importance of OCR and yield stress ratio discussed. Soil bulk stiffness is dependent on λ and κ, depending on whether it is loading or unloading and hence it is not a fundamental parameter. Critical overconsolidation ratio line is the boundary between LOC and HOC. The region on the right side of COCRL is called wet side and left is dry side, which is heavily over consolidated.

Undrained strength expressed in terms of $_{CS}$ parameters both cases for τ space as well as q, p' space has been discussed. How to normalize v, q, p' plot has been discussed. State of soil and state parameters has been defined. Parameters λ, M and γ are critical state parameters in v, q, p' space and are fundamental soil parameters. Finally, we need to keep in mind that critical state is a concept. And there can be departure of this concept from actual reality. It is practically difficult to assess whether the soil has reached critical state during sharing.

In most cases, it is assumed that the soil reached critical state during sharing that is at high string condition, which is practically greater than 10%. So, that is all for today's lecture. Now, we have introduced critical state framework. In the next lecture we will see more about 2D representation stage by stage we will progress. And then finally, we will discuss about 3D formulation, how to integrate the concept. So, that is all for now. Thank you.