

Unsaturated Soil Mechanics
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Week - 09
Lecture - 26
Extended M-C Criterion – I

Hello everyone. So, far we have gone through the Bishop's Effective Stress Principle, which is modified after Terzaghi's Effective Stress Principle for the Unsaturated Soils. Bishop introduced one more strength parameter called effective stress parameter ξ which is function of matrix suction. The estimation of ξ function from the suction control direct shear test and suction control tri axial test are seen.

So, today will understand the development there were some more developments that took place later on because the ξ parameter estimation is a very difficult especially for fine grained soils where the estimation of ξ over a wide suction range is very difficult. So, independently there were some more test that were conducted by other researchers, further to evaluate whether the independent stress state variables that were used by Bishop are valid or not. So, such test are called null type test, so which will discuss today.

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SHEAR STRENGTH

○ Null tests (Fredlund and Morgenstern, 1977):

Test number	Initial stresses (kPa)			Changes in Stresses (kPa)		
	Total stress, σ	Pore-air pressure, u_a	Pore-water pressure, u_w	$\Delta\sigma$	Δu_w	Δu_a
23	420.7 ✓	278.7 ✓	109.6 ✓	+71.4	+70.3	+70.7
25	495.3	406.8	143.5	+68.6	+68.3	+66.9
27	234.2	138.3	100.3	+68.8	+68.5	+80.8
29	274.6	202.2	22.4	+68.5	+68.3	+68.8
31	411.4	338.3	160.2	+68.1	+68.0	+67.3
33	549.0	476.4	297.2	+69.0	+68.0	+68.4
35	410.9	338.5	208.3	+69.5	+69.3	+69.7
37	547.5	473.7	343.9	+67.9	+67.5	+67.4
39	549.4	477.1	347.6	-70.2	-69.5	-69.8
41	412.6	340.7	211.4	-140.5	-140.3	-139.8



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Fredlund, D. G., & Morgenstern, N. R. (1977). Stress state variables for unsaturated soils. *Journal of Geotechnical and Geoenvironmental Engineering*, 103 (ASCE 12919).

So, the null type test these tests are to understand the validity of different the stress state variables. So, it is based on manipulation of different combination of stress state variables. Changes to whether total volume are degree of saturation is observed are used as the measure for understanding whether they are independence stress state variables or not. So, null tests are conducted by Fredlund Morgenstern in 1977, this is very popular work and the concept are used even till date. So, this is published in A.C. geotechnical and geo environmental engineering in 1977.

So, in this work they have conducted more than 19 null test on compacted cerulean soil, cerulean clay by controlling different stress parameters. So, as in tri axial cell one can control the deviatoric stress, the all-round pressure, the pore air pressure and pore water pressure, of course, we have coarse for (Refer Time: 03:20) here and then you have higher entry porous disk located at this location. So, therefore, the u_a within the soil mass and a u_w within the soil mass can be controlled independently. So, therefore, they have changed these parameters such a way that whether all the parameters are increased to the same extent or decrease to the same extent.

So, here if you observe the test number 23 from their work, so, this is a initial conditions of the soil, where the soil was at equilibrium, total stress was 420.7 and pore air pressure is 278.7 and pore water pressure is 109.6. You can assume some units a kilopascal. Now, the $\Delta \sigma$ is increase to plus 71.4 by changing the water pressure to plus 70.3, and changing the air pressure to plus 70.7, the want values of these parameters to be the same.

So, while doing this experiments slight deviation was there; however, nearly changes in the values are constant. So, due to these changes, the sample dimension the sample volume did not change, or the water content did not change during the testing. Similarly, there is a another test 25 number, this initial condition and so the changes and stresses is same in $\Delta \sigma$ Δu_w and Δu_a . Similarly, several other test data are shown here. Either you increase this you increase all of them plus or you decrease all of them.

So, in the same direction they have been applied, and when these changes are made the sample volume did not change; that means, the shear stresses are not initiated within the soil sample. So, this is because the $\sigma - u_a$, that is a effective stress the that net normal stress we use; which is equals to $\sigma +$, there is a small increment Δ

sigma in the test number 23, minus u a. Here, u a is increased the small value delta u a test number 23.

So, as this delta sigma and delta u a are maintained to be constant, so they get cancelled. So, essentially you get sigma minus u a. Similarly, if you take u a minus u w the increase u a plus delta u a and u w plus delta u w. This delta u a and delta u w both are one and the same. This remains u a minus u w. Due to increase in the stresses as shown here. So, therefore, these 2 variables can be used as independent state variables for defining the shear in the soils. Because earlier Terzaghi's had showed that it is effective stress that defines the state of the soil for shearing.

And here Bishop has propose that sigma minus u a, the net normal stress and u a minus u w these 2 should be used as stress state variables and they are independent to each other. And that is proven by these null test by Fredlund and Morgenstern. So, sigma minus u a and u a minus u w when they are kept constant. There is no shear induced in the soil sample.

So, therefore, there is no change in the volume, or degree of saturation. So, these are confirmatory test for Bishop's effective stress equation for unsaturated soils.

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SHEAR STRENGTH



○ Null tests (Fredlund and Morgenstern, 1977):

$$(\sigma - u_w) \text{ \& \ } (u_a - u_w); \quad \sigma' = (\sigma - u_w) + \alpha(u_a - u_w)$$

$$= (\sigma - u_w) + \alpha(u_a - u_w)$$

$$= (\sigma - u_w) + \alpha(\sigma - u_w)$$

#	Group - 1		Group - 2		Group - 3	
	$(\sigma_3 - u_w)$ kPa	$(u_a - u_w)$ kPa	$(\sigma_3 - u_w)$ kPa	$(u_a - u_w)$ kPa	$(\sigma_3 - u_w)$ kPa	$(u_a - u_w)$ kPa
1	44.8-31.0 = 13.8	31.0-(-27.6) = 58.6	72.4	58.6	13.8	72.4
2	77.2-63.4 = 13.8	63.4-(+4.8) = 58.6	72.4	58.6	13.8	72.4
3	13.8-0.0 = 13.8	0-(-58.6) = 58.6	72.4	58.6	13.8	72.4
4	110.3-96.5 = 13.8	96.5-(+37.9) = 58.6	72.4	58.6	13.8	72.4

Table. Independent stress state variables controlled in the tests (Source: Bishop and Donald, 1961)



Fredlund, D. G., & Morgenstern, N. R. (1977). Stress state variables for unsaturated soils. *Journal of Geotechnical and Geoenvironmental Engineering*, 103 (ASCE 12919).

Further their observation states that, it is not just one combination of sigma minus u a and u a minus u w to use as 2 independent state variables, stress state variables. They

further say that even if we use combination of this is one set $\sigma - u_a$ and $u_a - u_w$ minus u_w . Another combination $\sigma - u_w$ and $u_a - u_w$, and a other third combination is $\sigma - u_a$ and $\sigma - u_w$. Any of these 3 combinations can be used for defining the stress state in soil.

So, here the $\sigma - u_a$ or $\sigma - u_w$, and $u_a - u_w$ are maintained to be 13.8 and 58.6 changing from the initial value like this 44.8, so 13.8 and 58.6 are maintained. And in the second test same values are maintained, but different values of σ and u_a are given. And similarly different values of u_a and u_w are given, but the $u_a - u_w$ remains constant.

So, in this all 4 test the $\sigma - u_a$ and $u_a - u_w$ are kept constant, then the volume of the sample did not change. So, group one that is $\sigma - u_a$ net normal stress, and $u_a - u_w$ this combination qualifies as independent stress state variables. Similarly, the group 2 is analyzed there is $\sigma - u_w$ and $u_a - u_w$.

So, here the same combination is maintained for all different test by varying $\sigma - u_w$ and $u_a - u_w$ independently. So, then the $\sigma - u_w$ and $u_a - u_w$ these 2 combination kept constant for all these 4 test and we will now the volume remains constant. Similarly, in group 3 the $\sigma - u_a$ and $\sigma - u_w$ are maintained to be constant for all these 4 test then the volume remains constant.

So, therefore, the effective stress equation that is given by Bishop, that is $\sigma' = \sigma - u_a + \xi(u_a - u_w)$ equals to $\sigma - u_a + \xi(u_a - u_w)$. This is what we used as effective stress principle, modified effective stress equation from Bishop's proposal. And here the effective stress also could be used as $\sigma - u_w + \xi(u_a - u_w)$. Or this could be a $\sigma - u_a + \xi(\sigma - u_w)$.

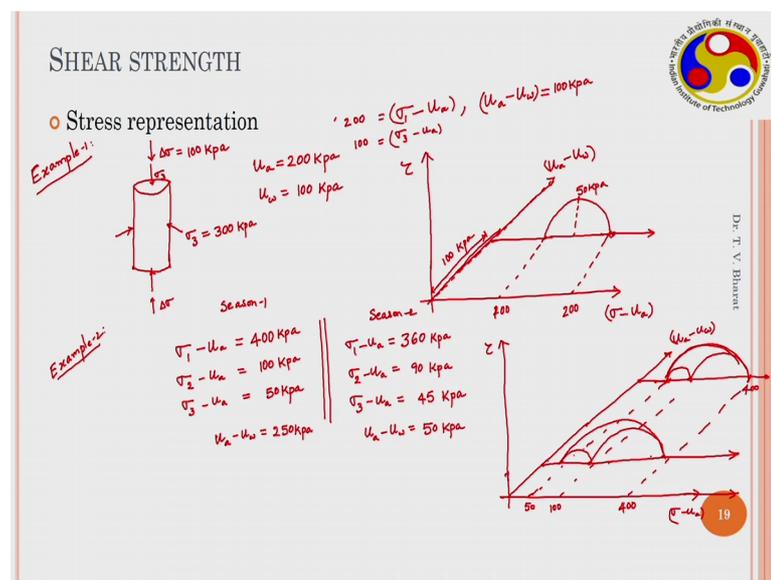
So, any of these 3 combinations can be used to formulate the effective stress principle, and it could be used to define the shear strength of the soil at any given stress state. So, therefore, this test reveal that Bishop's independence stress state variables indeed valid for the unsaturated soils. And moreover any of these 3 combinations are group one group 2 group 3 could be used as independent stress state variables.

Fredlund and Morgenstern after their stress state variables definitions and stress state variable confirmatory test by in null type test in suction control tri axial and suction

control consolidation cells, they are come up with new form of modified column failure criterion for unsaturated soils. Before going into that, let us understand how the stress can be represented graphically so that the Fredlunds extended more coulomb theory can be very well understood.

So, here the stress representation for unsaturated soils can be understood by solving some problems. Here there is one particular problem there is taken; that is some example.

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So, one shear testing is conducted on clay soil using tri axial setup. So, soil sample which is at equilibrium with an all-round pressure σ_3 is equal to 300 kilopascal. So, this is a all-around pressure, and a deviatoric stress of 100 kilopascal is acting. So, this is σ_3 and that is $\Delta\sigma$, and there is a pore air pressure on the sample is 200 kilopascal. And pore water pressure is 100 kilopascal. This soil state can be represented graphically as we know the stress state variables for unsaturated soils we can choose net normal stress there is $\sigma - u_a$ and $u_a - u_w$.

So, they as this is tri axial test you will have σ_1 and σ_3 . Therefore, we can draw the Mohr's circle. So, this can be plotted as τ on y axis and $\sigma - u_a$ on x axis. But as we have suction also, matrix suction also. So, that can be plotted on third axis.

So, this is third axis, so z axis for u a minus u w. So, if that is represented on y z axis. Now the stressed of the soil can be represented on u a minus u w is 100 kilopascal. So, this is located at 100 kilopascal here. So, this is 100 kilopascal. The more circles stress circle can be drawn on this axis, so which is at 100 kilopascal on the z axis. So, here the σ_3 minus u a and $N \sigma_1$ minus u a can be obtained. σ_1 minus u a that is a net normal stress in the y direction on the soil sample is 300 plus 100, that is 400 minus u a is to 200.

So, therefore, this is 200 kilopascal σ_1 is 400, minus u a is 200. So, therefore, this is 200 kilopascal, and σ_3 is 300 and u a is 200 therefore, this is 100 kilopascal. So, therefore, there is 100 kilopascal here, and there may be 200 kilopascal somewhere here. So, this is this is 100 kilopascal and this is 200 kilopascal, σ_1 minus u a is 200 kilopascal. So, maximum stress is at the centre, that is 50 kilopascal and maximum stress exist at 45 degrees from the major principle stress and this is a maximum values 50 kilopascal.

So, this is how the state of the stress can be represented graphically on a 3 dimensional plot. Similarly, let us take other example. So, there is a soil mass a which is located at on a slope. So, therefore, it has the 3 stresses acting major principle stress, and the net normal stress along with major principles is σ_1 minus u a is 400 kilopascal. And intermediate stress is 100 kilopascal, and minor principle stress is 50 kilopascal.

So, if this stresses vary during the monsoon state. The stress state changed from one season to another season. This is season one and in the season 2. The stress state changed to σ_1 minus u a is 360 kilopascal, and σ_2 minus u a is 90 kilopascal. And σ_3 minus u a is 45 kilopascal. And in the season one the matrix suction in the soil is 250 kilopascal.

So, this is relatively dry season. And in the season 2 the matrix suction decrease to 50 kilopascal. So, how to represent the change in the stress state of soil? This can be again drawn on the 3 axis plot, σ minus u a on x axis, and τ on y axis, and the third axis for u a minus u w.

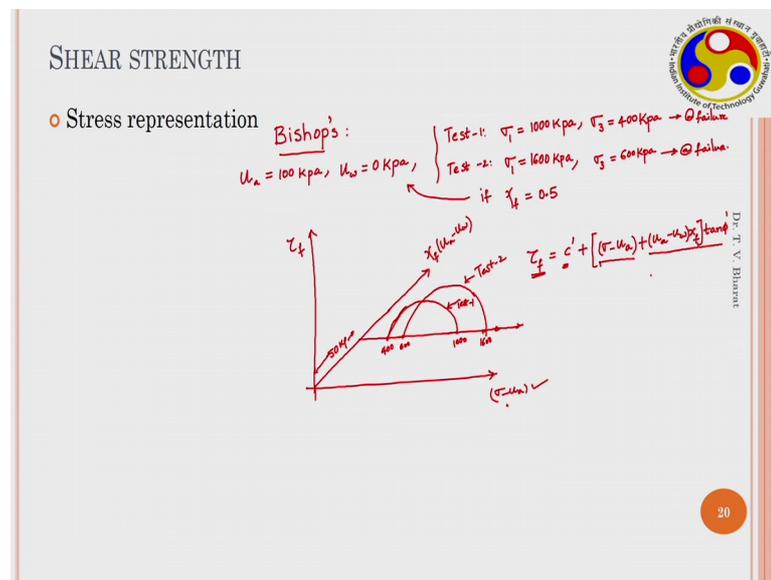
So, here the on the suction axis at 250 kilopascal somewhere here, you draw horizontal line. So, this is σ minus u a parallel to the σ minus u a axis. So, here on this the

sigma 1 minus u a is 400, say 400 and sigma 2 minus u a is 100, and sigma 3 minus u a is 50. So, you have one stress so this is how it is plotted.

So, this is so this value is 50, and this value is this goes parallel to this axis. So, this is 100 and this is 400. Similarly, in other season 2, in the matrix suction decrease to 50. So, assume that this is 50 and horizontal if you draw and here this became 360. So, slightly decreased this value and here it is 90 and here it is 45.

And this is how it can be plotted stress state of the soil.

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So, in case of Bishop's effective stress principle we have a one test data. So, that is u a is 100 kilopascal and u w is 0 kilopascal. So, in axis translation let us understand how the Bishop's modified more coulomb equations can be represented graphically. So, let us assume one problem where u a is 100 kilopascal and u w is kept at atmospheric, so this is 0 kilopascal.

And in one test in test one sigma 1 is 1,000 kilopascal and sigma 3 is 400 kilopascal at failure; these are observed data. And in test 2 when the sigma 3 is maintained 600 kilopascal, the soil field at sigma 1 of 1600 kilopascal at failure. If xi f is known is equal to 0.5, because it depends on matrix suction. So, matrix suction is constant in these 2 test. So, xi f will be constant for these 2 test is xi f is known there is 0.5, then how to

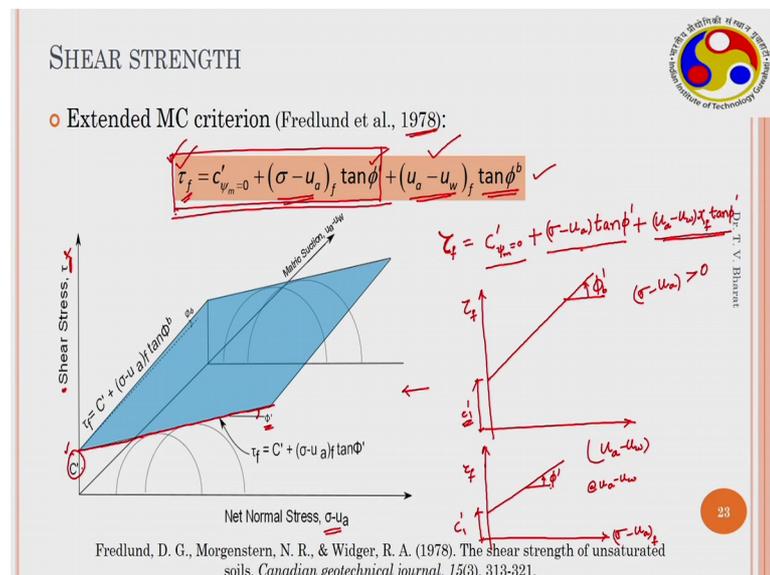
represent this? Here in the same way you can represent on wax is sets the tau an x axis is sigma minus u a.

So now this is third axis, now in this case because the equation is tau f is equals to C dash plus sigma minus u a plus u a minus u w time xi f times tan phi. So, this is intercept plus this whole thing, the one axis is sigma minus u a another axis is tau f, and other axis could be xi f times u a minus u w. So, this xi f u a minus u w. Then at one particular xi f into u a minus u w, u a minus u w is suction is 100 kilopascal, xi f is 0.5.

So, this 50 kilopascal, xi f times u a minus u w that is 50 kilopascal. Then in one test the soil failed at sigma 3 400. And sigma 1 is 1,000 kilopascal. So, this the test one 400 and 1,000. In the test 2 when the sigma 3 is 600 kilopascal, it failed at 1600 kilopascal somewhere here.

So, this is test 2 and this is test 1. This is how it can be represented because we have when we have all of 3 axis defined one is tau f another one is sigma minus u a. And another one is u a minus u w times xi f as third axis then the stress state of the soil can be very well defined using graphically, by graphically.

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Now, we can understand how graphically this stress state of the soil can be represented.

So, that Fredlund Fredlund Morgenstern and Widger in 1978 proposed extended more coulomb criterion according to this criterion. The tau f is equals to C dash that is a

cohesion intercept at suction is equals to 0. That is saturated state, plus $\sigma - u_a$ at failure times $\tan \phi'$ plus $u_a - u_w$, this is matrix suction at failure times $\tan \phi_b$. So, here as they came up with Fredlund and Morgenstern in 1977 came up with 2 stressed parameters that can be defined independently. And they confirm Bishop's a modified mc criterion.

So, similar same stress state variables are used in this extended mc criterion area. So, in this extended mc criterion which is nearly similar to the Bishop's modified more coulomb criterion; which I can write it for you here, τ_f is equals to C' . This is same at $x_i m$ is equals to 0, plus $\sigma - u_a \tan \phi'$ plus $u_a - u_w$ times $x_i f \tan \phi'$; because it is at any $\sigma_1 - \sigma - u_a$ plus $x_i f u_a - u_w$ that is effective stress times $\tan \phi'$.

So, when we write as 2 terms by multiplying $\tan \phi'$, then you get this. First term is same, second term is same, third term is there is slight difference. The $x_i f \tan \phi'$ is modified it to be $\tan \phi_b$. Even though it appears to be a small difference between these 2 equations, there is a significant advantage in writing this Bishop's expression into this particular equation; that is, when you draw shear stress on x y axis, that is τ_f or τ and net normal stress on x axis, then the extended mc criterion says that the shear stress for unsaturated soils can be defined with the surface or plane.

So, this plane represent this particular equation. So, if it is τ_f versus $\sigma - u_a$. So, then these 2 terms on the right hand side this particular expression is valid. So, as C' at x at $\psi_i m$ is equal to 0, so here that the C' dash this particular z axis on $u_a - u_w$ this is 0. So, then this particular thing is not there then you plot $\sigma - u_a$ and shear stress. So, when you plot, so the linear relationship is this. This angle of internal friction ϕ' and this $\sigma - u_a$ axis and this is τ_f axis. This is a cohesion intercept.

So, this is cohesion intercept, and this is angle of internal friction, so this equation is valid. And if you happen to plot τ versus $u_a - u_w$, that is when you look at this 3 dimensional plot from this direction, then net normal stress will not appear for you, and then this can be plotted as this. So, that is τ_f versus $u_a - u_w$, this versus $u_a - u_w$. Then you have $\sum C'$ and a this angle is ϕ_v dash τ_f versus $u_a - u_w$ this is how it is plotted. Because on this τ_f versus $u_a - u_w$, the σ

minus u_a is 0, because on this 2 dimensional plot or 2 axis plot this $\sigma - u_a$ is 0. Therefore, this is simply τ_f versus $u_a - u_w$ can be plotted in this manner. You have an intercept and you have a ϕ^b .

So, if this is plotted at one particular $\sigma - u_a$; which is more than 0, then the intercept becomes some intercept C_1 . Similarly, if τ_f and $\sigma - u_a$ are plotted, at one particular $u_a - u_w$, then you have some intercept. So, this is not equal to C_1 , this equal to C_1 or C_2 or something. So, this angle is ϕ^b . So, the graphical representation of stress state of the unsaturated soils can be very well defined using this particular form of more coulomb criterion, given by Fredlund at all.

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SHEAR STRENGTH

Extended MC criterion (Fredlund et al., 1978):

Test-1: $u_a - u_w = 10 \text{ Kpa}$, $(\sigma - u_w)_f = 100 \text{ Kpa}$, $\tau_f = 55 \text{ Kpa}$

Test-2: $u_a - u_w = 10 \text{ Kpa}$, $(\sigma - u_w)_f = 300 \text{ Kpa}$, $\tau_f = 150 \text{ Kpa}$

Test-3: $u_a - u_w = 300 \text{ Kpa}$, $(\sigma - u_w)_f = 170 \text{ Kpa}$, $\tau_f = 240 \text{ Kpa}$

Test-4: $u_a - u_w = 300 \text{ Kpa}$, $(\sigma - u_w)_f = 295 \text{ Kpa}$, $\tau_f = 300 \text{ Kpa}$

$\tau_f = c' + (\sigma - u_w)_f \tan \phi'$

$150 = c' + 300 \tan \phi'$

$55 = c' + 100 \tan \phi'$

$c' = 7.55 \text{ Kpa}$

$\phi' = \tan^{-1} \left(\frac{60}{125} \right) = 25.4^\circ$

$300 = c'_2 + 295 \tan \phi'$

$240 = c'_2 + 170 \tan \phi'$

$\phi' = \tan^{-1} \left(\frac{60}{125} \right) = 25.4^\circ$

$c'_2 = 158.4 \text{ Kpa}$

$\phi' = \frac{25.4 + 25.4}{2} = 25.5^\circ$

$c' = 2.35 \text{ Kpa}$

$\phi' = 25.52^\circ$

$\phi^b = 27.48^\circ$

Let us the understand how to estimate the shear strength parameters. We have C dash and ϕ dash as the shear strength parameters for saturated soils. In case of unsaturated soils, you have C dash ϕ dash and ξ f C dash ϕ dash and ξ f of say ψ matrix suction using Bishop's more coulomb criterion.

So, this is called a modified more coulomb criterion given by Bishop. In case of Fredlund at all 1978, this is C dash ϕ dash and ϕ^b dash, this is single value is not a functional form, but however ξ f is a functional form. So, because this is only one particular value ϕ^b dash just like ϕ dash, so estimation is easier. So, let us understand

how to estimate the shear strength parameters? c and ϕ from the suction controlled direct shear test data.

We have 4 test conducted. In test one the $u_a - u_w$ in test from the $u_a - u_w$ is maintain to be 10 kilopascal, and σ'_a at failure is equal to 100 kilopascal maintained. And τ_f is shear stress at failure is observed to be 55 kilopascal. This is a direct shear test data. And test two $u_a - u_w$ is 10 kilopascal, and σ'_a at failure is equals to 300 kilopascal, and τ_f is 150 kilopascal.

In test 3, $u_a - u_w$ is 300 kilopascal, σ'_a is 170 kilopascal, and τ_f is 240 kilopascal. And test 4 $u_a - u_w$ equals to 300 kilopascal, and σ'_a at failure is 295 kilopascal, and τ_f is equal to 300 kilopascal.

So, this is how the direct shear test by controlling the suction is conducted. So, here in 2 test the matrix suction is constant which is maintained to be 10 kilopascal, then the net normal stress is changed from the first test 100 kilopascal, it is changed to 300 kilopascal in the second test. And 2 more test are conducted, by maintaining different value of matrix suction in these 2 tests, that is 300 kilopascal and 300 kilopascal, but the net normal stress is now changed from 170 in the third test to 290 kilopascal in the 4th test.

So, we have this observation, τ_f is value there is a shear stress at failure for all these 4 test. So, using this simple data, if you conduct for different test, then you will get all these strength parameters estimated. But in case of Bishop's more coulomb criterion to obtain the Bishop's strength parameters, we need to conduct series of tests for getting a smooth variation of τ_f with respect to matrix action.

So, if you have wide range of matrix suction values that may vary in your soil, at all these different matrix suction data points, at all these different matrix suction values the τ_f need to be estimated by conducting so many number of test and then τ_f can be estimated. Because here the ϕ is a single value and this is not a function form. So, therefore, just simply 4 different test if you conduct you get all those 3 Fredlund strength parameters estimated.

So, using the first 2 tests using Fredlunds equation when $u_a - u_w$ is constant, so, under particular $u_a - u_w$, so then the plot is between if you recollect. So, the plot is between τ_f and σ'_a . So, this is at one particular $u_a - u_w$, this u_a

minus u_w is not 0. So, it is at one particular matrix suction so then the angle is ϕ' at one particular $u_a - u_w$, the angle is ϕ' and the intercept is C_1' some values because if the $u_a - u_w$ is 0 then intercept is C' . But this C' some other intercept, because we are plotting at one particular $u_a - u_w$.

So, therefore, this equation is simply τ_f is equal to $C_1' + \sigma' \tan \phi'$ at failure and $\tan \phi'$. So, if we substitute from this from this 2 test, this is 150 is equal to $C_1' + \sigma' \tan \phi'$ and here this is 55 is equal to $C_1' + 100 \tan \phi'$.

So, if you solve these 2, that is if you take difference of this, then this is $\tan \phi'$ is equal to 95, 95 by 200. So, ϕ' is 25.4, ϕ' is 25.4 degrees. So, already got one strength parameter. So, from this if you substitute this value here, you can get C_1' as 7.55 kilo Pascal.

Similarly, if you solve this 2, so here at a 300 kilopascal this is at $u_a - u_w$ is 10 kilopascal. Here this is at $u_a - u_w$ of 300 kilopascal. So now, τ_f is 300 is equal to this is C_2' , because this a different matrix suction value. So, the intercept will be different. So, therefore, C_2' is different from C_1' , plus $\sigma' \tan \phi'$ is 295 $\tan \phi'$ this is 240 C_2' plus 170 $\tan \phi'$. If we solve these 2, by subtracting from one to another, we get ϕ' is equal to $\tan^{-1} 60$ by 125, which is equal to 25.64 degrees. And if you substitute this value, and you get C_2' as 158.4 kilopascals.

So, as ϕ' we generally assume that it is constant it does not vary even with change in the matrix suction. So, ϕ' we can average root. So, ϕ' is equal to average of 25.4 plus 25.64 divide by 2. So, you can consider this to be 25.52. So, this average value we can use. So, coming to the first intercept, 7.55 kilopascal and second intercept is 158.4 kilopascal. From the equation τ_f is equal to $C' + \sigma' \tan \phi'$, plus $u_a - u_w$ at failure times $\tan \phi'$. So far we have utilized only this part. So now, when this is plotted as τ_f versus $u_a - u_w$, you get some intercept here and here this is ϕ' .

So, this intercept changes. So, this intercept is C_1' and where $u_a - u_w$ is equal to 0 this becomes C' . And at different $u_a - u_w$ say, initially $u_a - u_w$ is 10 kilopascal, 10 kilo Pascal this is C_1' , and at $u_a - u_w$ of 300 kilo

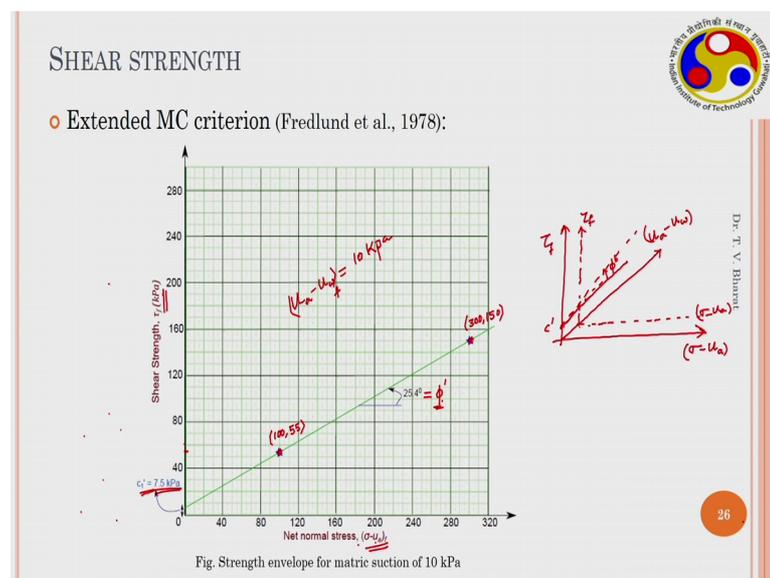
Pascal somewhere here. This is C 2 dash so this is C 1 dash, this is C 1 dash and C 2 dash.

So, therefore, now C 2 dash which is equal to 158.4 kilopascal, the expression is C dash plus u a minus u w that is 10 kilopascal times tan phi b. Sorry, 300 and the other expression the C 1 dash is 7.55 is equal to C dash plus 10 kilopascal is a suction times tan phi b dash phi b. If we solve this, we get phi b is equals to tan inverse of 150.86 divided by 290. This is 27.48 degrees. So, if you substitute this value here, then you get C dash is equals to 2.35 kilopascal.

So, the strength parameters are C dash equals to 2.3 kilo Pascal, and phi dash is equals to 25.52 degrees and phi b is equal to 25.48 degrees. So, these are the Fredlund all strength parameters from the extended mc criterion. So, this is how the direct shear test data can be analyzed for understanding the shear strength parameters from the extended mc criteria.

So, here same expression can be used only 4 test data are required to analyze the whole data. And then if this is available this set of data is available. So, entire surface can be plotted and at any given point. At any given matrix suction value at any given net normal stress the shear stress at failure can be obtained; a very easy to do because you already analyze graphical technique of representation of stress earlier.

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So now this will be easy to do it graphically. So, once you take a graph sheet and then give proper dimensions for the x axis and y axis. So, here the x axis dimensions are varied upto 320 kilopascal, because net normal stress in our case where is up to nearly 300 kilopascal. So, similarly the shear strength, τ_f is also varied in this particular manner. And here the initially the net normal stress σ_{u_a} at failure and τ_f are plotted. So, because the matrix suction is not 0, you do not have any test conducted fully saturated strait.

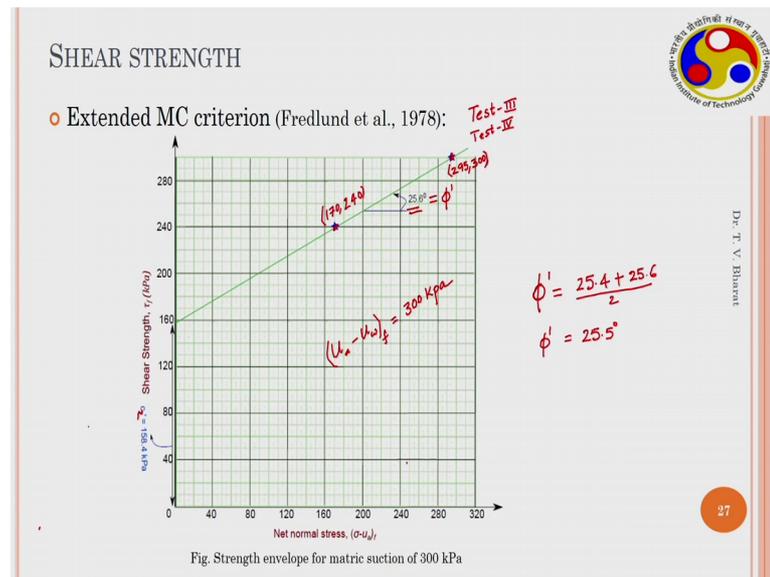
So, the matrix suction varied for 2 different test one is at 10 kilopascal and another one is at 300 kilopascal. Therefore, you have an intercept which is a C_1 dash not C dash. So, how to obtain the C_1 dash. Now one particular test data there is at $u_a - u_w$ is equals to 10 kilo Pascal, you have 2 test from this 2 test, the σ_{u_a} that is a net normal stress is 100 kilopascal, this is 100 corresponding to the shear strength of soil is 55 kilopascal, so 55 is here.

So, this is 60 and this is 55, so this is the point. And second test at the same matrix suction is conducted. So, because τ_f σ_{u_a} on x axis and $u_a - u_w$ on y axis z axis, then had one particular $u_a - u_w$. So, when you plot at one particular $u_a - u_w$ that is 10 kilo Pascal, this is what we are plotting. So, here if the $u_a - u_w = 0$, then intercept is C dash. But here this $u_a - u_w$ is not 0 therefore, the intercept is slightly higher than that, because in this angle is slightly different. So, if C dash same then this is a thing and if it is slightly C dash is increasing with increase in the suction. So, this angle is ϕ_b , that is what we got.

So, therefore, here one particular net normal stress at $u_a - u_w$ corresponding τ_f is given here. And another point at the same matrix suction value; that is at 295 kilopascal, the τ_f is 150 kilopascal. This x axis is x axis is 300, and y axis is 150. And here x axis is 100 and y axis is 55. So now, if we join this line with a straight line, the angle is ϕ dash and intercept is C_1 dash.

So, this is intercept, this is same as what we got by solving those 2 equations.

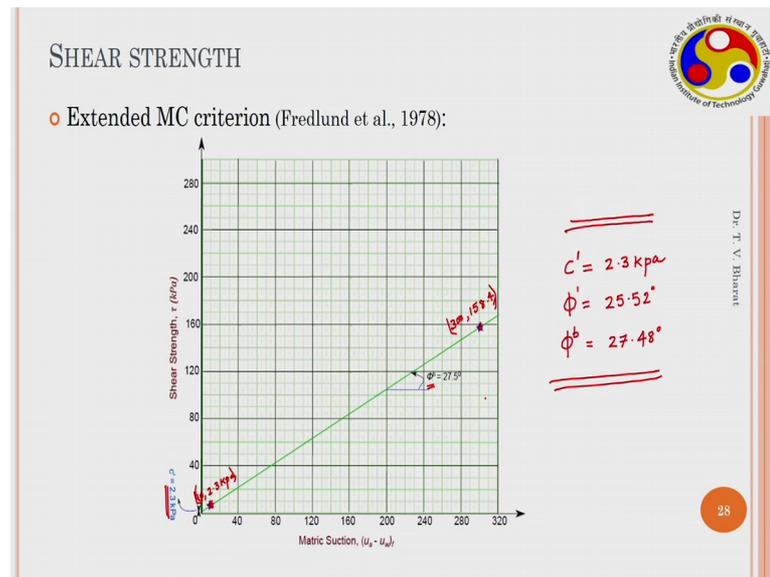
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Similarly, other 2 test data that is test 3 and test 4 are taken. So, another graph sheet is taken and the axis are noted down on x axis and y axis. X axis an again this is sigma minus u a at failure, and y axis is tau f. Now when this is plotted, one particular value of a sigma minus u a that is 170 and tau f at failure is 240 is plotted and another point. Sigma minus u a from the 4th test is 295 and tau f is 300 kilo Pascal on y axis.

So, when these 2 are connected with a straight line, you get C 2 dash, you get C 2 dash that is intercept at u a minus u w of this is at u a minus u w of 300 kilopascal. So, the intercept is 158.4 kilopascal and the angle this is again phi dash is 25.6 degrees. So, in the earlier, earlier this is u a minus u w is 10 kilopascal. And we got one C 1 dash and phi dash. And when at u a minus u w at failure equal to 300 kilopascal, and you get 158.4 kilo Pascal intercept. And phi dash is 25.6, so when we take a average, we get the angle of internal friction, average value that is from earlier 25.4 plus 25.6 here. If you take average you get 25.5, so this is phi dash.

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Now, we have the other axis the ratio shear stress, tau on y axis and matrix suction u a minus u_w on x axis then the first intercept there is C' is 2.3 kilopascal. And other intercept that is 158.4 kilopascal. This is y axis for the matrix suction of 300. And here 10 is matrix suction therefore, x is this one and y is this.

So, from this if you join these 2 points you get ϕ^b angle and the intercept which is 2.3 kilopascal. So, the C' that is cohesion intercept is 2.3 kilopascal, and a ϕ' which is already an average data which is which we used from the 2 4 test, from the first 2 test we got one value and the other 2 test we got another value. So, from this average we got 25.52 degrees and ϕ^b which we got from these to intercept values at 2 different suction.

So, that is 27.48 degrees. This data is same as what we got analytically by solving those equations. So, this particular problem can be solved using with a graphically, or by simply solving those equations by substituting test data from 4 independent test, so.

Thank you.