Unsaturated Soil Mechanics
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Week – 06
Lecture - 18
Pedo-Transfer Functions (PTF)

Hello everyone, we are discussing several simple theoretical models that exists for the determination of soil water characteristic curve and one of the such models is Arya Paris model which is called Pedo Transfer Functions.

So, these Pedo transfer function utilize grey say distribution data to predict the soil water characteristic curve and then utilize bulk density and mass fraction data to predict the soil water characteristic curve.

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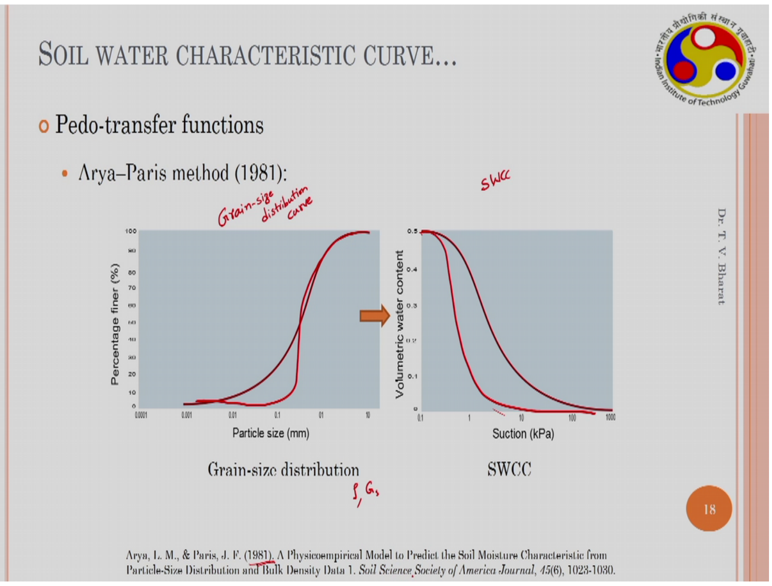

And we discussed that Arya Paris method, which utilizes the grain size distribution data and also which considers the density of the soil specific gravity etcetera to determine the hydraulic conductivity functions.

As you see this is grain size distribution curve and this is soil water characteristic curve. If you see the grain size distribution curve which is also S-shaped one for well graded soil and soil water characteristic curve is also S shaped. There is a similarity between these two as there is a uniformity if the soil is uniform soil.

So, you will have curve similar to like this then the SWCC also correspondingly changes to somewhat like this because as you have uniform soil all the particles are nearly of same size. So, then pores are also may be equally sized then the air enter value gets reduced and once air starts entering into the largest pore of the soil and these pores are nearly uniform then water content suddenly drops and reaches a very small value or it reaches the residual water.

So, we could see that there is a similarity between grain size distribution and the SWCC there is procedure laid down by Arya and Paris in 1981. So, these the reference for that, which is published in soil science society of America journal in 1981.

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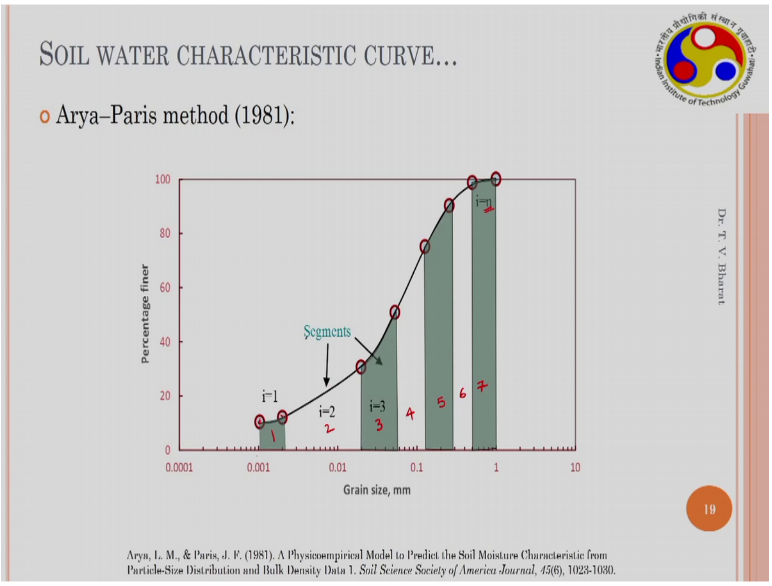

The procedure consist of first initially taking the grain size distribution curve and dividing rain size distribution curve into number of segments. This is segment 1, this is segment 2, 3, 4, 5, 6 and so, on.

So, 1 to n number of segments it is divided into how do we divide into number of segments. Generally based on number of data points you have based on that you can select one segment.

For example, these two data points form one segment and next two data points can be used to form the next segment. Similarly you can make number of segments. So, if you have n plus 1 number of data points then you can form n segments. So, if you have ore number of data points on this curve. So, it is more likely that you get more number of segments and data is more accurate the obtained SWCC is more accurate.

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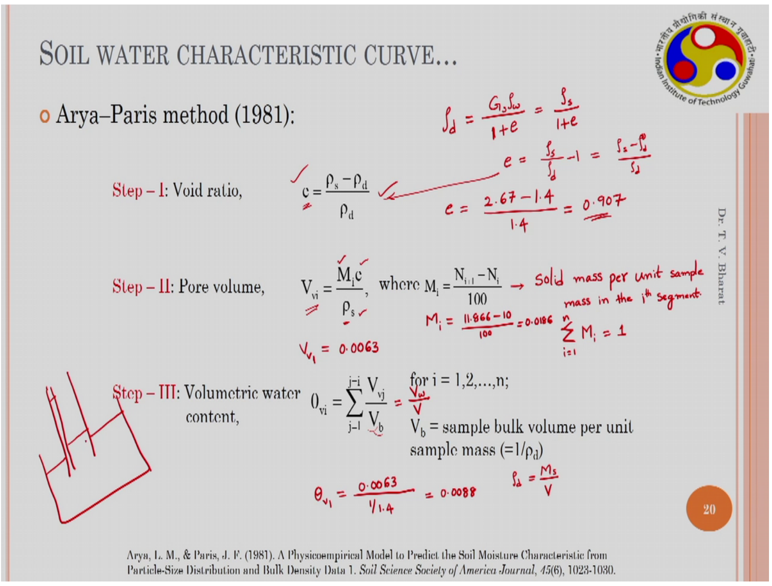

So, in Arya Paris method first the void ratio is obtained void ratio is obtained by this formula. So, which is based on the estimation from rho d the dry density because we know the dry density is equals to G s rho w by 1 plus e.

So, this is nothing, but G s rho w is rho s 1 plus e. So, you can write this expression for void ratio as rho s minus rho d r, then you can write the expression for e rho s by rho d minus 1 which is nothing, but rho s minus rho d by rho d. So, this is what we got here.

So, this void ratio is determined if you know the dry density of the soil. And one assumption is made here that the void ratio does not change as the saturation takes place. Initially soil is a dry state as and when you saturate the soil sample or the water content increases the void ratio remains same. So, void ratio should not change so, that is the assumption that is made here.

So, generally this is applicable for coarse grain soils like sands loam etcetera. So, then you have if you determine the pore volume. So, then we determine the pore volume. So, the pore volume is equals to the mass fraction times void ratio divided by rho s; rho s is density of soil solids. So, here the mass fraction is the solid mass per unit sample mass this is solid mass per unit sample mass in the i th segment.

So, in the i th segment. So, the sum of these M i, i equals to 1 2 number of segments this is equals to 1. So, sum of the all mass fraction at different segments together is 1. So, which is determined by difference between the percentage i plus one th data point and i th data point divided by 100.

So, this is how the mass fraction is determined. So, then once the mass fraction is known the void ratio is determined from the previous step and rho s is already known because G s should be given the specific gravity should be known, then rho s can be substituted and you get pore volumes.

So, here the void ratio is volume of voids by volume of solids and the rho s is mass of solids by volume of solids. So, this volume of solids get cancelled and here you have the mass fraction. So, when this one this one this two are mass you know the units are gram and you have volume of voids here.

So, that is the pore volume that is what we are determining here. So, once the pore volume is known the volumetric water content can be determined from this formula. So, here the V b sample bulk volume therefore, this is pore volume divided by total bulk volume.

So, sample bulk volume per unit sample mass is 1 over the dry density of the soil as the dry density is mass of solids per total volume. So, 1 over rho d gives the value of bulk volume per unit soil mass.

So, this is pore volume divided by the bulk volume should give the theta because this is volume of water divided by total volume. So, that is what you are getting here. So, the volume of water divided by bulk volume or total volume you are getting we are getting.

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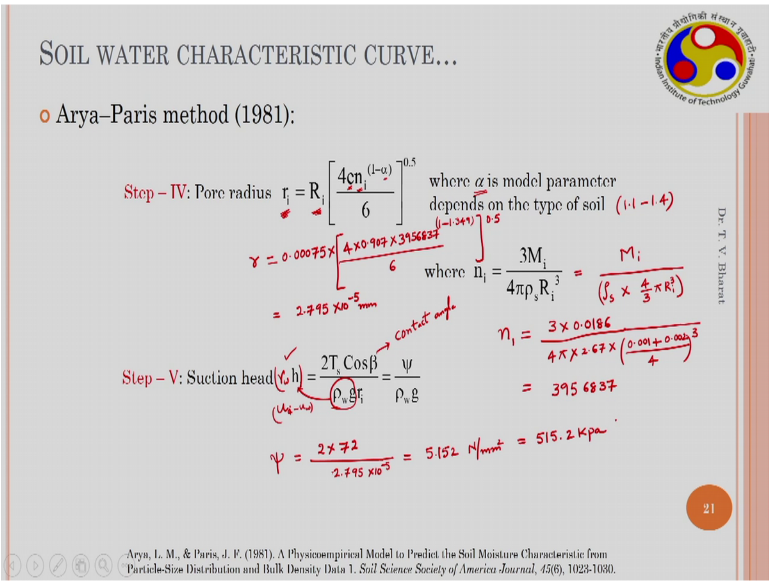

So, once that is determined once theta is determined one of the straight variables is determined then you have the pore radius. So, pore radius Arya Paris assumes a relationship between pore radius and particle radius are they assume that the pore radius and particle radius are related by this formula, where r i that is the pore radius at i th segment is equals to the r i capital R i that is the radius of the i segment particle times four times wide ratio into n i; n i is the number of n i is the number of spherical particles having mean radius of R i.

So, we assume all the particles to be spherical. So, then n i is the number of particles spherical particles having mean radius R i power 1 minus alpha. Alpha is one model parameter divide by 6 and whole power 0.5. So, this alpha is the model parameter which depends on the type of soil which varies between 1.1 and 1.4. So, this is kind of fitting parameter once we obtain the data we can vary this value to fit close to the data. So, here the number of particles spherical particles can be obtained by writing mass fraction divided by density of solids times volume.

So, volume is volume of the spherical particle is four third by R q. So, there is how we obtain number of particles that exists here. So, number of particles we obtain like this. So, once we obtain number of particles we can substitute here void ratio is already determined alpha you can assume and capital R i is known mean particle radius the in that particular segment is known, here if the segment size is smaller that is more number of data points on the grain size distribution curve exists then this will be a good approximation otherwise you will have a poor value of R because when the R varies in the significant manner in the given segment, then we take an average value and may not be good approximation.

So, then once you get the pore radius we can utilize this particular formula the h i equals to 2 T s by 2 T s cos beta by r i this if it is written in this rho w g is nothing, but gamma w if it goes here then this is pressure. So, there is pressure at air water interface the pressure drop is equal to 2 T s cos beta by r.

So, beta is conduct angle. So, here conduct angle is assumed to be 0 if we know some conduct angle we can also use. So, then from this we get the suction head. So, we do one calculation for one particular data set.

So, for the first segment. So, this void ratio is equals to g s is 2. 67 minus void ratio is equals to 2.67 minus the dry density is 1.4 divided by 1.4 this gives the void ratio of 0.907. So, the pore volume is pore volume can be determined.

So, the pore volume can be determined using this formula. So, here M i is the percentage finer we obtain is n 1 is 10 percent and n 2 is 11.866percent. So, this divided by 100 which will give you a value of 0.0186 and void ratio is already known there is 0.907 and rho s is 2.67. So, when you are substitute we get V v i for the first segment V v 1 is equals to 0.0063.

The volumetric water content is theta v 1 is equals to 0.0063 divided by 1 over 1.4. So, this is equals to 0.008. So, this is the volumetric water content. So, this is fines here the assumption is that soil pores will be filled one after the other in the first segment first the water enters that is fine particles will get filled first. And then only the larger particles gets filled.

So, naturally when you immerse number of number of capillary tubes into water one large tube and one small tube what we observe is. So, the pinch you gets more water and larger tube gets lower water, but then this sticks. Simultaneously, but in this case is assumed that water enters into the smallest pore first and once the smallest pore gets completely filled, then water enters into the other pores other larger pores.

So, other larger pores will be kept empty until all the smaller pores get completely filled with water that is the assumption we make here. So, this particular volume of water will get filled first. So, therefore, for the segment two this theta v 1 plus theta v 2 should be added.

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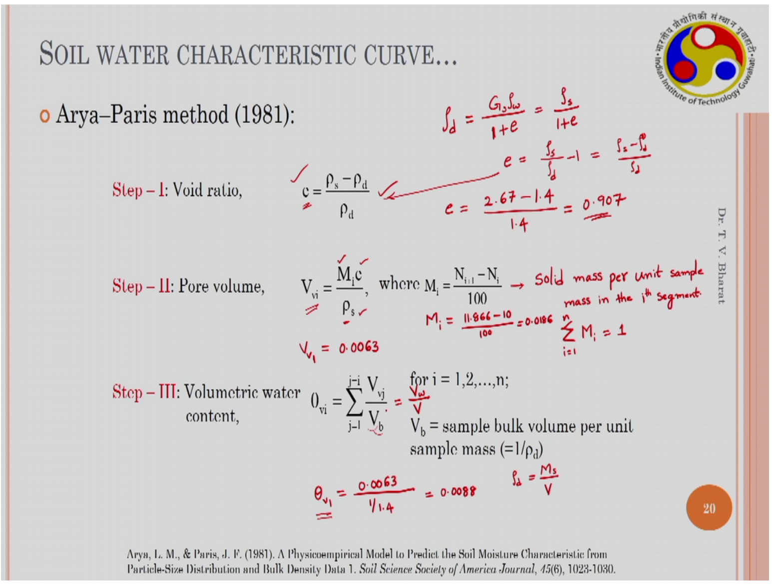

So, therefore, here for i equals to 1 it should be simply this much then pore radius is here number of particles n 1 in the first segment is equals to 3 times 0.0186 divided by 4 phi rho s is 2. 67 times r i is a mean value that is 0.001 that is the first data point corresponding to the diameter divide by 2.

So, the diameter is 0.001 mm plus second data point is at 0.002 mm this is diameter therefore, divide by 4 m am using this divide by two should be radius this divide by two should be radius and for average value divide by 2. So, I am using 4 here. So, this whole cube. So, the number of particles are 3956837 these many number of particles that are present in that particular segment.

So, from that if I estimate the radius the radius is equals to 0.00057 there is the average obtained from this one times 4 times void ratio 0.907 times number of particles 3958737 whole power 1 minus alpha here alpha assumed to be in Arya Paris assumed alpha value to be 1.349 problem after best fit divide by 6 whole power 0.5.

So, this value is equals to 2.735 into 10 power minus 5 mm. So, therefore, suction head which can be estimated which is equals to by substituting T s is 72 milli Newton per meter and beta is equals to 0 contact angle is 0 cos 0 is 1 and this is this r 1 is simply 2.795 into 10 power minus 5 and here we can write the pressure. So, the pressure drop pressure drop or matrix suction psi is 5.152 Newton per mm square are this is nothing, but 5 15.2 kilo Pascal.

So, for a given theta.

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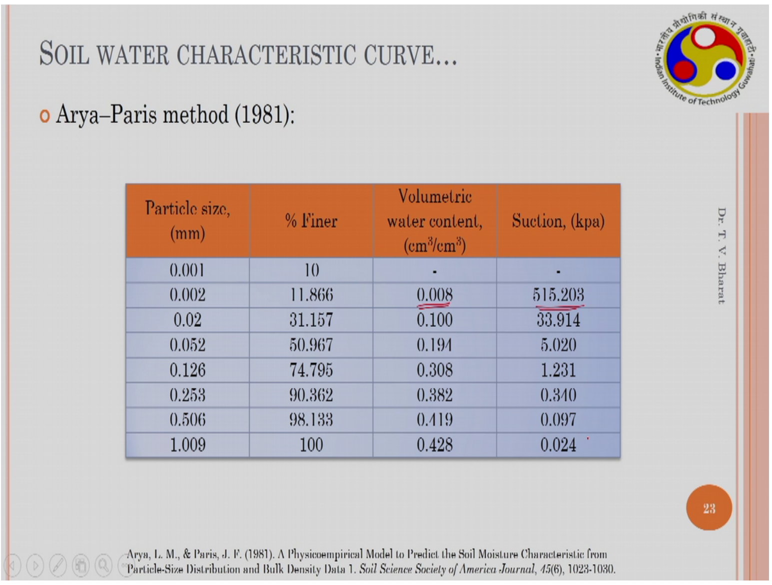

So, this is the suction. So, all the values are tabulated here the particle size are here and percentage finer is here. So, the volumetric water content we determine for the first segment is 0.008 and corresponding suction is 515.203 similarly for all the other data points which is listed here.

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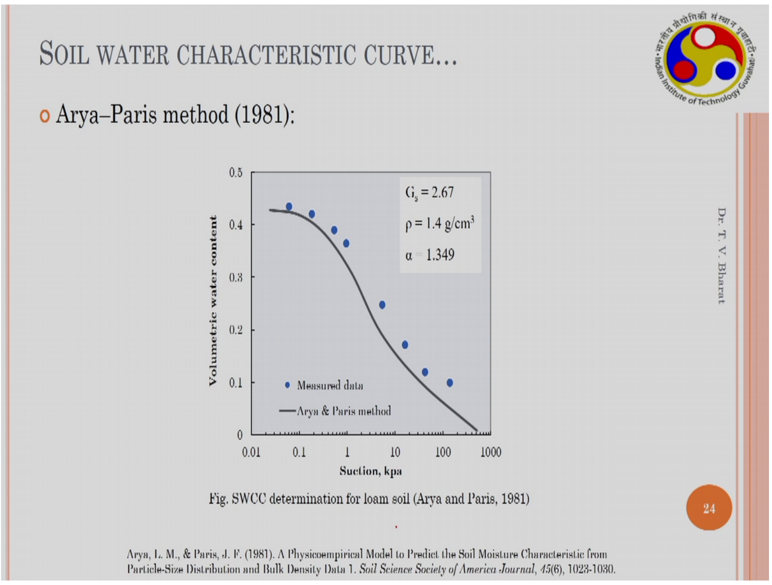

once the data which is this is measured data obtained from a pressure plate operators or axis translation technique by Arya Paris which is show here and this is theoretical data obtained from a Arya Paris method which I have shown here how it is obtained.

So, this is the this good compare this si the good match between theory and experiments considering all different assumptions we made that particles are spherical and particles gets filled based on their pore size only one after the other and considering all the assumptions the fit is very good and Arya Paris method is widely used for coarse grind soils and there are other models which are often used for even fine grain soils also there are models developed by Fredlund and other researchers which are spread in commercial software like soil vision etcetera.

So, we will see some more example we have already seen how the Arya Paris method estimates the soil water characteristics curve on well graded soil. Let us now see when we take uniformly graded soil coarse grain soil fine grain soils grab graded soils what is the estimated soil water characteristic curves based on Arya Paris method let us take a coarse grain soil, which is uniformly graded. So, here we can assume.

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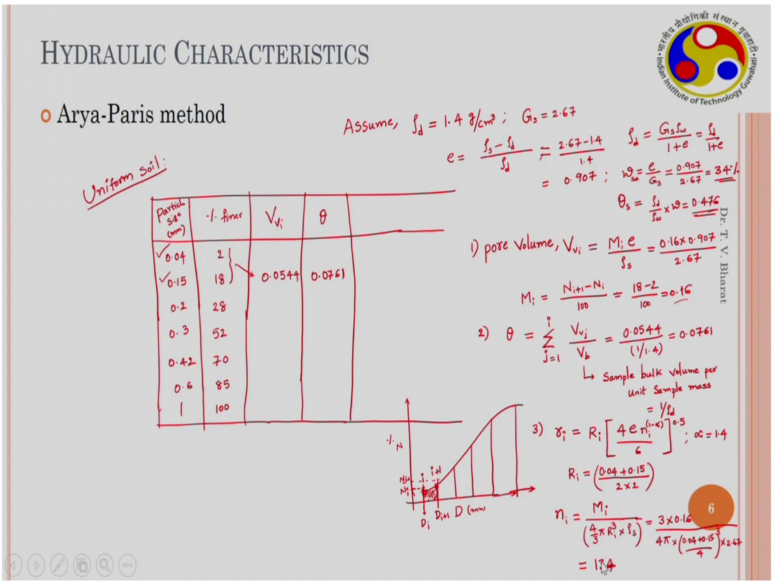

For all these soils you can assume the dry density is one point four gram per centimeter cube and specific gravity is 2.67 then we can estimate the void ratio also. We can estimate the void ratio which is rho s minus rho d by rho d this is from the equation for rho d rho d is equal to G s rho w by 1 plus e.

So, from this e is estimated this is nothing, but rho d by 1 plus e. So, from this we can estimate the void ratio.

So, this void ratio value is 2.67 minus 1.4 by 1.4. So, this the value we estimated earlier also this is 0.907 this are void ratio. So, we can estimate when the soil is completely saturated. So, we can estimate what is the water content.

So, the water content would be e equal to w G s by S r therefore, e by G s when degree of such ratio is equals to one that is fully saturated system then void ratio is 0.907 divided by G s is 2.67.

So, which gives the value of 34 percent. So, this is at fully saturated. So, therefore, corresponding theta s is equals to rho d by rho w times double u. So, this is equals to 0.467.

So, this is 0.476 this is ferocity of the soil. Now let us take one uniform soil and which is coarse grain. So, the particle sizes in mm and percent finer is given.

So, values are 0.04 0.15 and 0.2 0.3 0.42 0.6 and 1. So, these are the particle sizes and corresponding percentage finer is 2 18 28 52 70 85 and 100.

So, soil is finer than 1 mm 100 percent finer than 1 mm size and 2 percent only finer than 0.04 mm. So, we can estimate we can calculate the other values we can estimate the volumetric water content and matrix suction using Arya Paris method.

So, before that we require the estimation of several other things like pore volume. So, pore volume is represented with V v i which is equals to a mass fraction m i times void ratio divided by rho s. So, this mass fraction is nothing, but n i plus 1 minus n i. So, this the variation in percentage finer between two different particle sizes as we have described earlier that when we have a particle size distribution percentage finer present and particle size in mm.

If this is the grain size distribution curve then it should be divided into several segments for each segment we consider we calculate the pore volume average pore volume and average mass fraction value.

So, the mass fraction value for this particular two ranges of the segment are estimated. So, here if it is N i and this is N i plus 1 this is N i if this is i th one and this is i plus 1 this is N i and this is N i plus 1. So, the mass fraction is calculated using this N i plus 1 minus N i divided by 100. So, which is the for the first case if I write 4 volumes here v i for the first case using this segment this value is 18 minus 2 by 100 that is 0.16 times void ratio is 0.907 divided by rho s is 2.67.

So, this value comes out to be 0.0544 for this particular segment. Similarly once we know the pore volumes we can calculate the volumetric water content theta. So, theta is equals to summation of all different pores starting from first pore to y th pore V v j divided by sample bulk volume per unit sample mass that is V b.

So, this is the sample bulk volume per unit sample mass which is nothing, but 1 over rho d. So, this is equals to this value is 0.0544 divided by 1 by 1.4. This comes out to be 0.0761. So, volumetric water content is 0.0761 for the first segment then the third step we can estimate the pore radius pore radius is assumed to be related to radius of the particle in this manner radius of the particle times four times void ratio times N i power 1 minus alpha divided by 6 whole to the power of 0.5.

So, here alpha is the model parameter here I am assuming alpha to be 1.4 and N i is number of spherical particles and e is the void ratio and R i is the average radius of the particles in this particular size range in the first segment.

So, R i we can consider to be as the diameter is known. So, when the diameter of this particular type the size and i plus 1th size is known. So, the diameter divided by the 2 and average volume when we take this is nothing, but 0.04 and 0.15 this is the size range 0.04 and 0.15.

So, divided by 2 gives the average diameter and divide by 2 gives average radius and void ratio is known and number of spherical particles can be calculated by calculating the volume of each individuals spherical particle and in the denominator the total mass of soil solids in that segment.

So, that is the mass fraction divided by. So, four third pi r cube by rho s. So, the mass fraction divide by rho s should give total mass of soil solids and this is the volume of individuals spherical particle when we assume that the particles are spherical in nature then the number of particles we get. This is when we simplify we get 3 m i divide by 4 pi r cube into rho s. So, this is 3 times mass fraction is 0.16 divide by 4 pi r i cube is 0.04 plus 0.15 by 4 cube and rho s is 2.67.

This gives value of 134. So, there are 134 number of particles each particle is assumed to be spherical then 134 spherical particle exists when that particular size range.

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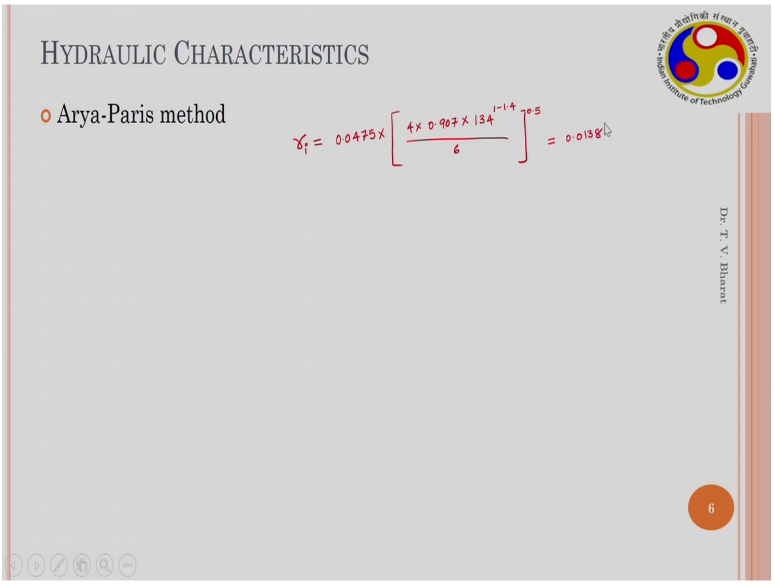

So, we can calculate the pore radius average radius of the particle is 0.0476 times 4 times the void ratio is 0.907 times number of particles is 134 power 1 minus alpha is assumed to be 1.4 then divided by 6 whole power 0.5. So, this gives the value of 0.0138.

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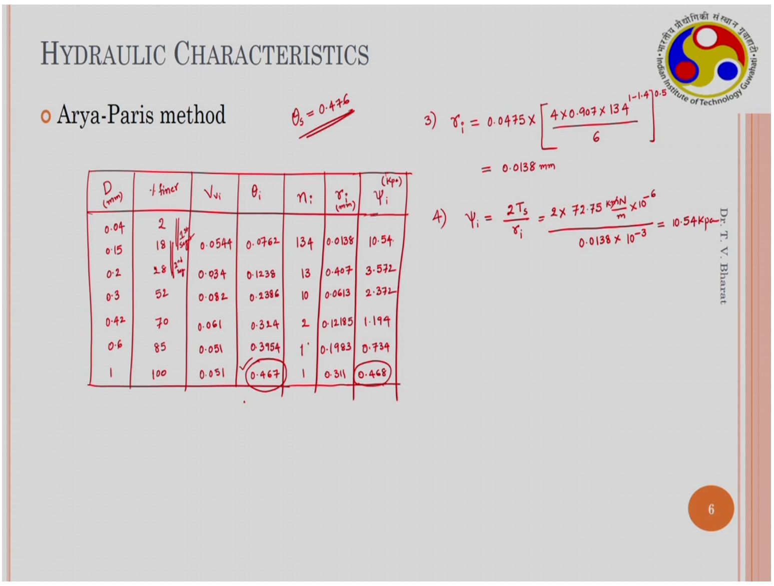

So, now let us calculate the pore radius and the values are brought here into this page. Now the pore radius is r i is equal to the pore radius r i is equals to the average radius of the particles in the first segment is 0.0475 times 4 times the void ratio 0.907 times number of particles 134 power 1 minus 1.4 this alpha that we assume to be point 1.4

divide by 6 whole to the power of 0.5. So, the pore radius comes out to be 0.0138 mm. So, we can also substitute the values of number of particles in each segment and pore radius and the suction value the matrix suction.

So, the number of particles are 134 in the first segment and pore radius is 0.0138. This is an mm and here it is in kilo Pascal which is reported. So, now, the suction is estimated the suction value is 2 t s by r i 2 times the surface tension is 72.75 milli Newton per meter milli Newton per meter. So, if it is represented in kilo Newton then this is kilo then 10 power minus 6 kilo Newton per meter divided by pore radius 0.0138. In mm if it is represented in meter then it is 10 power minus 3 meter. So, which are comes out to be 10.54 kilo Pascal.

So, the suction corresponding to volumetric water content of 0.0762 is 10.54 kilo pascal. So, when if we calculate the values of V v i theta n i r i and psi i for all the other data from this second segment this is second segment corresponding the values are 0.034 this 0.1238 this is 30 number of particles 0.407 this 3.572 and from the third segment 0.082 0.2386 here number of particles are only 10 when these are truncated.

0.0613 and 2.372 and for the percentage finer corresponding to 0.42 mm diameter where 70. So, the values corresponding to this segment 4 is 0.0610 0.0324 two number of particles 0.12185 1.194. So, here this is 0.051 0.03954 one number of particle and 0.1983 0.734. Here the real number also can be used we do no t need to truncate and then use integer alone we can actually use a real number here.

So, here even though it does not mean anything, but real value can be used here this is 0.051 and this is 0.05467 this is 10.311 0.468. If we use real numbers here the values of this psi changes slightly. So, the suction here is changed from 10.54 to nearly 0.468 when the volumetric water content is changed from 0.0762 to 0.467. If I bring your attention to the theta here this is nearly saturated volumetric water content because the corresponding suction is 0.0468.

So, if you bring the suction value close to 0 then we will have theta value close to theta s the estimated theta S is 0.476 which is very close to the estimated value by Arya Paris method here. So, therefore, the estimated values are quite good.

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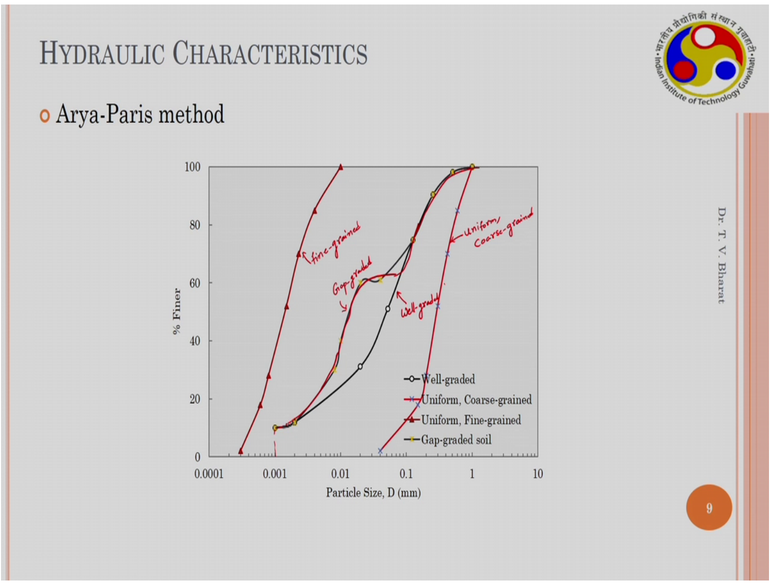

And similarly we can also estimate this values for fine grain soils and also for gap graded soils. So, here this is the well graded soil. So, this is given by Arya Paris methods that data is shown here the range of particles vary from nearly 1 micron to close to 1 mm.

So, large variation in the particle sizes exists for this a particular soil. And this is considered to well graded. Here and this is compared to this is a uniform soil and which is a coarse grain soil because most of the particles ranges are here. So, they are larger than 7 5 micron size most of the particles. So, this is considered to be uniform soil of coarse grain and this is considered to be fine grain soil and this is coarse grain soil and this is gap graded.

So, this is a gap exists. So, this variation in particle size is like this. So, this is gap graded. So, if the particle size distribution varies in this manner the soil water characteristic curve estimated by Arya Paris method varies in this manner.

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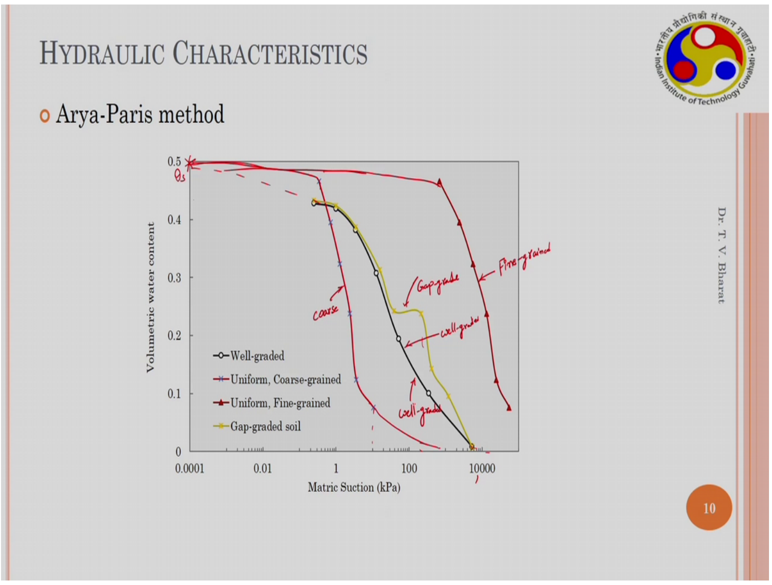

So, this is well graded. And this is of fine grained soils which shifted towards right because it has very high suction values suction values even varied beyond 10 MPa this for fine grain soils and this is coarse grain soil.

And this is well grade and this is gap graded. If you see the air enter value nearly varied. So, this is how soil water characteristic curve for this particular fine grain soils and this is entire SWCC curve because less than this value we would not get and if know the initial value of theta that is theta s we can connect these two points then this is how this would vary. So, this is nearly air enter value.

Air enter value is very high as high as 200 kilopascal in this range and the beyond that the degree of such ration are volumetric water decrease in this manner for fine grain soils that we can study in this particular example. And for coarse grain soils the maximum suction value is hardly 10 kilo Pascal and beyond that it decreases quickly and this how the SWCC curve for coarse grain soils and for well graded soils initial volumetric water content. If this is same this would go like this and this reaches value is also well distributed the suction range is wider for well graded soil.

Suction range is not wider for uniform soils for gap graded even you have a bump here in SWCC as there is a bump there and bump is also found here. So, this Arya Paris method exactly simulates or replicates the way the particle size distribution curve varies on the SWCC the Arya Paris method exactly maps, the way the particle size distribution curve varies in the same manner the soil water characteristic curve would vary however, as final particles increase in the soil. So, other than the capillary force come into play then Arya Paris method which is purely based on capillary phenomena would not hold good and the values are expected to be varying significantly.

So, for coarse grain soils Arya Paris method is very good method for the prediction of SWCC from grains distribution curve. Later on there are several researchers like (Refer Time: 41:05) they came up with several equations for predicting the entire soil water characteristic curve. Using the grain size distribution curve here, they fit the grain size distribution curve using a one equation which is similar to soil water characteristic curve. So, by estimating the fitting parameters directly the SWCC can be predicted. So, such models are available in soil vision software.

Thank you.