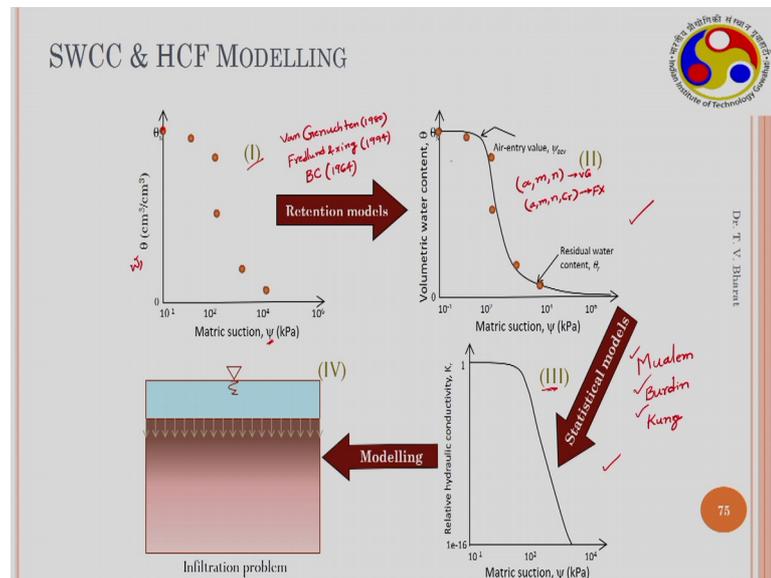


Unsaturated Soil Mechanics
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Week – 06
Lecture - 17
Fitting of SWCC & HCF modelling

Hello everyone, we were looking at different soil water characteristic of models and hydraulic conductivity functions, for the estimation of flow through Unsaturated Soils. So, we are preparing ourselves to take up the task of understanding the flow through unsaturated soils. Before we can get into the flow, we need to understand the SWCC models that are available for obtaining smooth soil water characteristic curves and hydraulic conductivity function, well established HCFs from the measured water retention data from the laboratory or from the field and either by measuring the hydraulic conductivity function data as well or prediction from the SWCC model.

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So, this whole process is done in this particular manner in the first stage we were obtaining the soil water tension data as of which is relationship between volumetric water content versus matrix suction. Often this data also we may not get directly, because we may need to use a different suction controlling or suction measurement techniques in the laboratory for the estimation of theta versus psi over a whole range of suctions. So,

often we get in terms of may be (Refer Time: 01:53) water content double u . So, then using the relationship between double u and θ by using the density of soil, how it varies with either drawing or wetting knowing this data we can convert to θ . Once we convert to θ we can plot θ versus ψ .

So, we get a discrete data like this, the θ equals to θ_s corresponds to very small suction or suction is close to 0. So, that is saturated volumetric water content which is porastive soil and as a suction increases the θ value decreases. So, this is the discrete data we obtain; however, this discrete data may not be directly useful and we require a smooth well defined function between θ and ψ , we utilize several existing retention models such as van genuchten model Van Genuchten model of 1980 or Fredlund and Xing 1984 model or any other model. You can use a Brooks Corey also BC model which is may be 1964, but; however, with BC model we have measure issue that up to the air entry value, we have a θ equals to θ_s and beyond that it decreases non-linearly.

However, here the function is not smooth at air entry value. Therefore, we may not get a smooth function as we get with other models. So, when we use these models these are the retention models, we can obtain a smooth function like this using some optimization techniques that will be discussed now after this slide. So, using some optimization techniques we can obtain a smooth function like this. So, then once we obtain a smooth function, using different statistical models such as a Mualem model are Burdin such statistical models one can use are even Fredlund Xing use Kunz model. So, suppose models could be used to obtain hydraulic conductivity function.

In the last class in the previous class we discussed how to obtain the hydraulic conductivity functions by knowing the SWCC model parameters. Once a m and n parameters in Van Genuchten are a m n C r parameters in Fredlund Xing model. Once these parameters are known using different statistical models like Mualem Burdin etcetera are Kunz. So, we have analytical solutions for few cases and for few cases we have to solve numerically and we can obtain hydraulic conductivity function like this.

So, once we have SWCC model in the second stage and hydraulic conductivity function in the third stage using these two function functional forms, we can use to model the flow through unsaturated soils either it could be infiltration problem through soils, as it infiltrates the hydraulic conductivity starts increasing. So, if we have wetting data, then

we can directly utilize and similarly the suction value decreases because of the suction head that exists in the soil mass because of which there is a flow that is taking place. So, therefore, we can utilize this equation as well provided these are wetting data, then we can directly utilize to understand how the flow takes place through a initial dry soil.

Similarly, if you have a drying data drying SWCC and drying hydraulic conductivity function, then we can understand the evaporation behavior or drainage behavior through initially saturated soil.

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SWCC & HCF MODELLING

One-dimensional flow through a variably saturated soil:

$$\frac{\partial \theta(\psi_m)}{\partial t} = \frac{\partial}{\partial z} \left[K(\psi_m) \left(\frac{\partial \psi_m}{\partial z} + 1 \right) \right]$$

$$\frac{\partial \theta}{\partial \psi_m} \frac{\partial \psi_m(\theta)}{\partial t} = \frac{\partial}{\partial z} \left[K(\psi_m) \left(\frac{\partial \psi_m(\theta)}{\partial z} + 1 \right) \right] \rightarrow \theta(z,t)$$

Specific moisture capacity: $C(\psi_m)$

Hydraulic conductivity function: $K(\psi_m)$

Soil-water retention curve: $\psi_m(\theta)$

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As I discussed in the previous classes that the one dimensional flow through partly saturated soils can be understood using this kind of equation, which is similar to the consolidation equation. So, here you have the different variables theta, which is again a function of suction. So, $\frac{\partial \theta}{\partial t}$ is equal to $\frac{\partial}{\partial z}$ of K hydraulic conductivity which is function of suction therefore, is hydraulic conductivity function times $\frac{\partial \psi_m}{\partial z}$ plus 1. So, will see why we get plus 1 if you have a horizontal flow that is taking place then you will not have this plus 1. So, if you have a vertical, then plus one is added that we will see in when we discuss about the flows.

Say if you simplify this or to modify this equation to get the same dependent variables on either side of the equation, the you can convert to suction throughout then you will have a slope $\frac{\partial \theta}{\partial \psi_m}$ times $\frac{\partial \psi_m}{\partial t}$. So, this is the slope of the soil water characteristic curve which is called the specific moisture capability, which is equal to

do by do z of hydraulic conductive function and this is the same as the earlier form. Now the K of psi m is hydraulic conductivity function and psi of theta is the soil water characteristic curve. Here we require smooth functions to solve this equation

Another important point here is the slope of the soil water characteristic curve should be defined at every point; that means, the slope are derivative of the soil water characteristic curve, should be well defined. In case of Brooks Corey method the dou theta by dou psi is not valid at air enter value because at that in fluxion point which is the very sharp inflexion point at air enter value. So, therefore, this value is not valid. So, therefore, that is the reason why we require very smooth functions like hydraulic conductive function and SWCCs to solve this particular equation to get theta volumetric water content as the space and time. So, this is the data we are looking for. So, when we give this as an input we get this theta.

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SWCC & HCF MODELLING

o Gradient-based optimization strategies for (I) --> (II):

Objective fn. $O(x)$

$O(x) = \frac{\sum_{i=1}^N (\theta_i^{(1)} - \theta_i^{(2)})^2}{N}$

$\theta = \theta_r + (\theta_s - \theta_r) \left(\frac{1}{1 + |ah|^n} \right)^m = \theta_r + (\theta_s - \theta_r) \left(\frac{1}{1 + (\frac{h}{a})^n} \right)^m$

$\frac{dO}{dx} = 0; x = x_{opt}$

$\frac{dO}{dm} \text{ \& } \frac{dO}{dn} \rightarrow 0 \text{ (m,n)}$

$O(a, m, n) \rightarrow \text{minimized}$

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How to get soil water characteristic curve from the measured retention data? We have a several optimization strategies like gradient based optimization those are classical methods, to minimize the error between experimental data and theoretical data. So, here briefly discuss how this is generally done. From stage 1 that is the measured data measured water retention data to smooth soil water characteristic curve how we obtain. So, generally we build error functions are called objective functions, we build objective

function say which is function of say x , x is the variable which we want to determine or the model parameter we want to determine.

So, how these function, what is this function? Objective function objective function can be build using the measured data. So, for example, this can be determined as this. So, the measured data difference between measured data and the theoretical data. So, say you have N number of point's N number of measured data points. So, then volumetric water content theoretically determined for different suction values minus θ_m that is measured by you can obtain square divided by number of data points you have. So, this is root mean square error they call. So, essentially when you have SWCC data like this, θ on y axis and ψ on x axis. For different values of ψ you have say N number of data points for corresponding N number of data points you can assume you assume one particular VG model.

So, which is may be $\theta = \theta_r + \theta_s - \theta_r \left(\frac{1}{1 + \alpha \psi^n} \right)^m$. Here we assume one set of α , m and n here you can write in terms of a also you can write $a \psi$ or ψ by a , this can be written in the form of $\theta = \theta_r + \theta_s - \theta_r \left(\frac{1}{1 + \psi/a} \right)^m$.

So, this is the general form which is used in geotechnical engineering. So, here when you use ψ the a has units of kilo Pascal because ψ has units of kilo Pascal here α has units of $1/\text{meter}$ because h is substituted in meters. So, therefore, either α , m and n simply a , m and n . So, you can assume one set of these model parameters, you can give a value to there is a this is related to air enter value if it is sandy soil you can give say 2 kilo Pascal e as the initial value. If it is a clay soil you can give a very high value say 1000 kilo Pascal or something and you can give some values to m and n may be n is equals to 1.5 or something and m value you can give two or any value.

If m and n are related are constrained, then you give the values accordingly. So, this is called initial guess. So, this the initial guess that is given. So, when you give this initial guess or you substitute these values, then you can obtain θ theoretical values. So, that may be somewhat like this. So, this may be theoretical and this may be experimental. This experimental or measured this is theoretical. So, then for any given suction value, so, there is an error, between measured and theoretical. So, this is what we are trying to

minimize. You build an objective function that objective function are error function contains what is the error between this data point and theoretical data point this data point and theoretical data point you can use some of squares or root mean square or mean square or any norm could be used to define an error function.

Once error function is defined with respect to any given model parameter, say here the error function are the model parameter here is x could be a m or n . If you have only one model parameter to determine, you may have an objective function like this. So, our objective is to minimize the objective function objective value or error function value and what is the corresponding x that is what we are trying to determine. So, this can be solved using simple optimization. So, you can determine in initial guess maybe this is the initial guess x_i and this is optimum; our objective is to get x optimum by assuming some x_i ; x_i could be here or here or anywhere on this curve.

So, we use different gradient techniques and using gradient techniques simple gradient technique is objective function with respect to x should be 0 at optimum value. At optimum value of x when x equals to x optimum this value should be 0 this objective function should be 0. See in case of the present thing which I discussing for example, if you have a dependent case where the objective function with respect to say a and another axis you have m and for example, if you have m and n restrain condition m and m and n are related.

So, in that particular case you have some functional form a three dimensional form. So, where you have one particular model parameter where one particular set of model parameter a and m where the objective function is lowest are minimum. So, therefore, $\frac{dO}{da}$ and $\frac{dO}{dm}$ should approach to 0 at optimum value. So, that set of m and n we determine. So, we build an objective function here O in terms of a , m and n if you have three parameters to determine are simply a and m only two parameters to determine then the objective function is minimized.

So, this is minimized there are several such optimization strategies using gradient, where sometimes the utilize double derivative also and they solve. These are called classical optimization algorithms or strategies they utilize the gradient of the objective function with respect to the model parameters and they determine the optimum parameters.

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SWCC & HCF MODELLING

- Modelling
 - RETIC (van Genuchten et al., 1991):

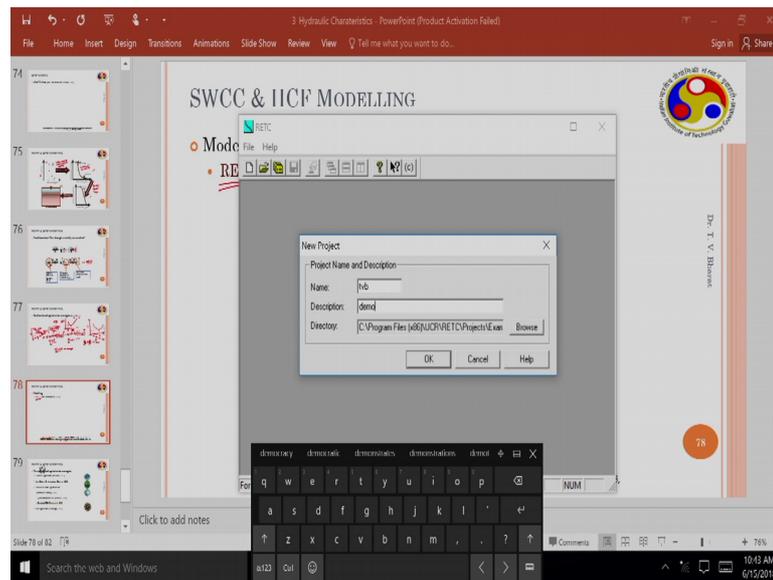
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van Genuchten, M. Th., F. J. Leij, and S. R. Yates. (1991). The RETIC Code for Quantifying the Hydraulic Functions of Unsaturated Soils, Version 1.0. EPA Report 600/2-91/063, U.S. Salinity Laboratory, USDA, ARS, Riverside, California.

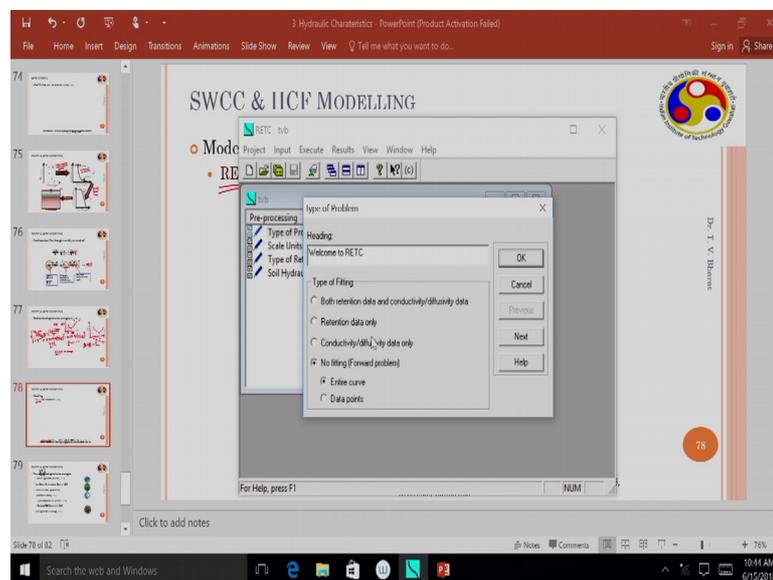
So, such strategy is used and software is built which is called RETIC; to estimate soil water characteristic curve the smooth SWCC function from the measured data, where we can utilize Van Genuchten model or Brooks Corey model. And in Van Genuchten again you can utilize different restrain conditions a how m and n are related are m and n are independent and several such cases could be utilized and we can obtain a smooth function. And now I will demonstrate this is the free software free b, which is developed by Van Genuchten and Nieche aids etcetera and in 1991. There are several improved versions are available, one can simply download h there is a website called PC progress. If you give the keywords as pc progress at c in Google you will be guided to the webpage and you can download the RETIC software which is a very simple software, I will demonstrate it now.

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So, this is the RETC software. So, initially when you select a file, a new project here you can give some name and you can give some descriptions at demo.

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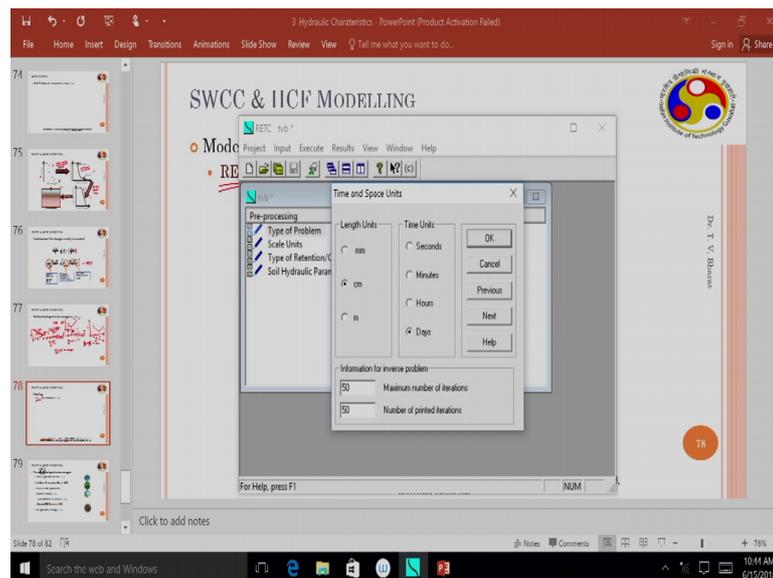


Here it will ask you what are different problems type of problems, scale, units type of retention or conductivity model soil hydraulic parameters. So, when you choose that type of problem, a new window opens up here it says welcome to RETC and type of fitting you want to do both retention data and conductivity or diffusion data. So, retention data is soil water characteristic data and conductivity or diffusivity data. Generally we do not

have conductivity data I have some retention data for loom soil from Aria of Paris paper 1981. So, I will use that data to demonstrate how it fits using a gradient based optimization algorithm called 11 Breg mark wet algorithm.

So, I will select only retention data because I have only retention data and when I select next.

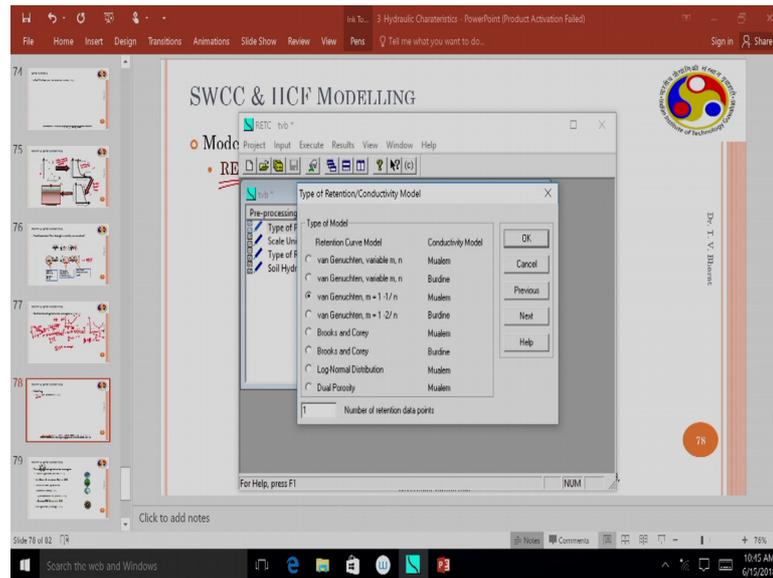
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I can choose different units, here the input for the suction should be done soil water potential, that is suction head should be substituted therefore, we select meters, I can substitute suction head in meters or centimeter or mm. So, generally we have suction values very high suction values such as 500 kilo Pascal, 1000 kilo Pascal etcetera generally in for fine grained soil, which we deal in soil mechanics. So, therefore, it is better to select in meters and it could be anything because if we are not dealing with time units now, because we are not estimating hydraulic conductivity now.

So, here for the inverse problem if you have then maximum number of iterations number of printed iterations you can chose I will leave everything default.

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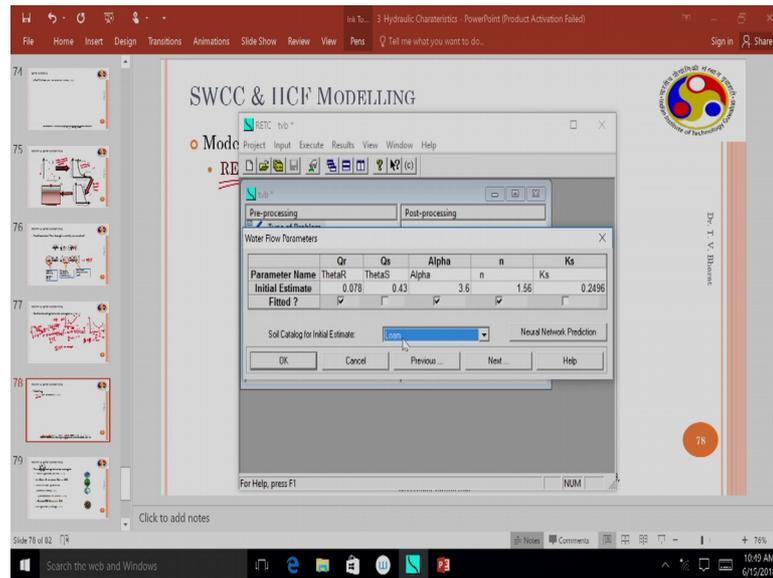


So, here you have several models Van Genuchten variable m and n model. So, this I have discussed where you do not have any restriction between m and n , only the restriction is that n is select n is chosen to be more than 1, n less than or equal to one results into nonphysical data at suction value close to 0. So, you have corresponding conductivity models also you can chose, you can chose Mualem or Burdin. So, therefore, accordingly m and n parameters are estimated by optimization, the hydraulic conductivity functions are also determined immediately.

So, you have Van Genuchten model, m equals to one minus 1 by n this is the particular solution. So, I have discussed this when we made k equals to 0 you have particular solution for that this is an analytical solution available. So, corresponding analytical solution you can chose again Mualem model similarly in combination with Burdin they derived another solution k equals to 0, for which the hydraulic conductivity function has the particular solution, so, for which m equals to 1 minus 2 by n . Similarly, you can chose Brooks Corey model or Brooks Corey along with Mualem and Burdin are log normal distribution or dual porosity models these models I did not discuss, but other models I have discussed in the previous classes.

So, you can chose any model we can chose Van Genuchten m equals to m minus 1 by n model and correspondingly the Mualem can be selected for hydraulic conductivity function estimation.

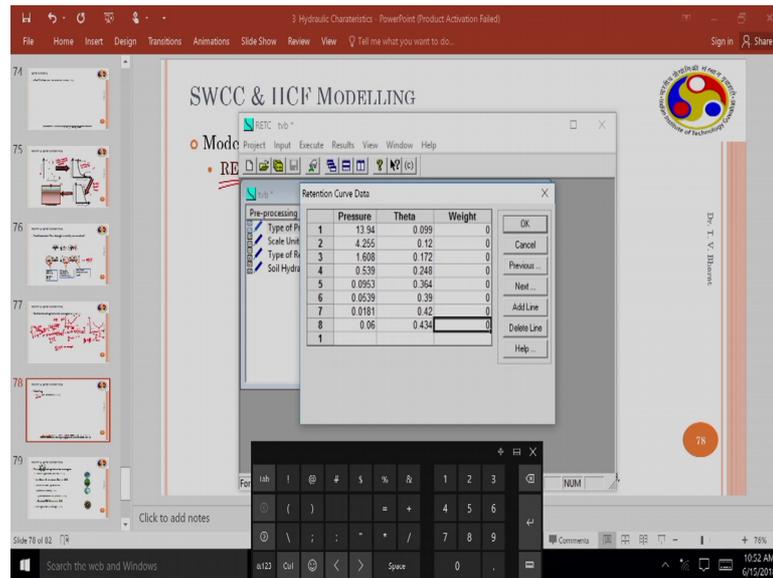
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So, then if you click next; it will ask you it will show you different parameters that are available that is theta r this is theta r the residual water content do we know this data are know if we do not have we can fit it theta s is the data is known, then you do not need to fit it. You do not need to fit it you can select this data. So, as of now leave it as it is and alpha value I will fit, n value I fit and Ks if I do not have if I do not need to fit, then I can select some value here I am leaving a default value like this.

So, now you can chose from this menu different soils possible soils you are fitting for. So, actually I am fitting for loam, this information is also not required for the optimization, but here this information is used. So, that the initial guess will be generated based on this data. You can recollect what I said for the gradient based optimization techniques we require an initial guess. Based on initial guess using the gradient we move to the next step and reach the optimum value. So, how to chose initial guess is the big issue? See here the initial guess is chosen based on the type of soil. So, if you select loam I have the loam data. So, I select the loam. So, then initial guess will be taken accordingly then if I select next.

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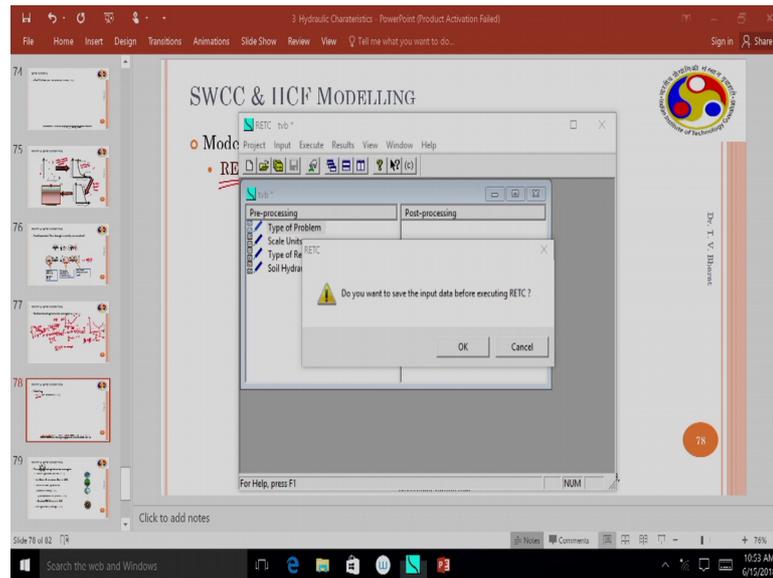


So, here I can input the pressure versus theta here pressure should be inputted in meters. So, I can input the data that is 13.94 meters pressure corresponding to theta value of 0.099 and pressure value of 4.255 and corresponding theta as pressure is decreasing the theta value should increase.

So, this is 0.12 pressure is 1.608. So, the theta data is 0.172 and this is 0.539, this value is 0.248 and this 0.0953 and this data is 0.364 and this 0.0539 and this value is 0.39 and this is 0.0181 and this value is 0.42 and 0.06, this value is 0.434. So, this is the data we have. So, here you can give weightage based on whether the measured data is accurate or not, there may be slight chances of when you do the test several times. So, this the measured data therefore, there may be some small error that could be there. So, you can give weightage whether this data is reliable or this particular data set is data point is reliable or not.

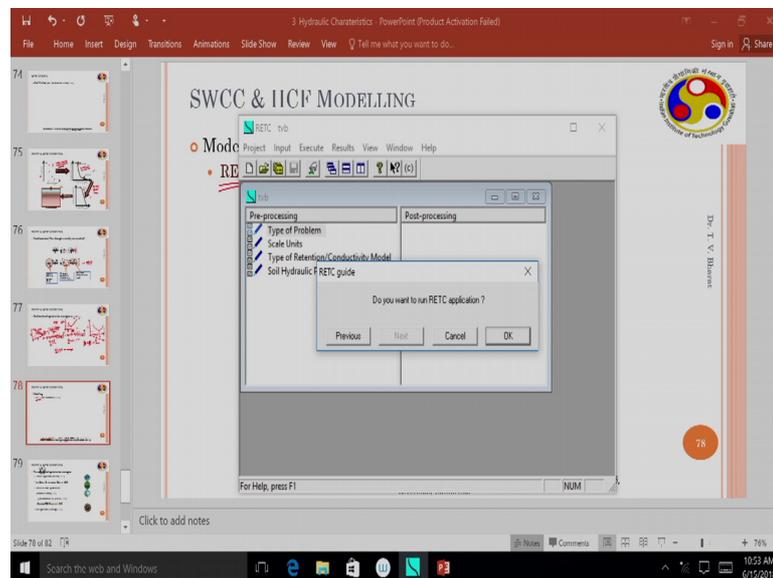
So, here I am not giving any weight or anything I am just leaving the default values.

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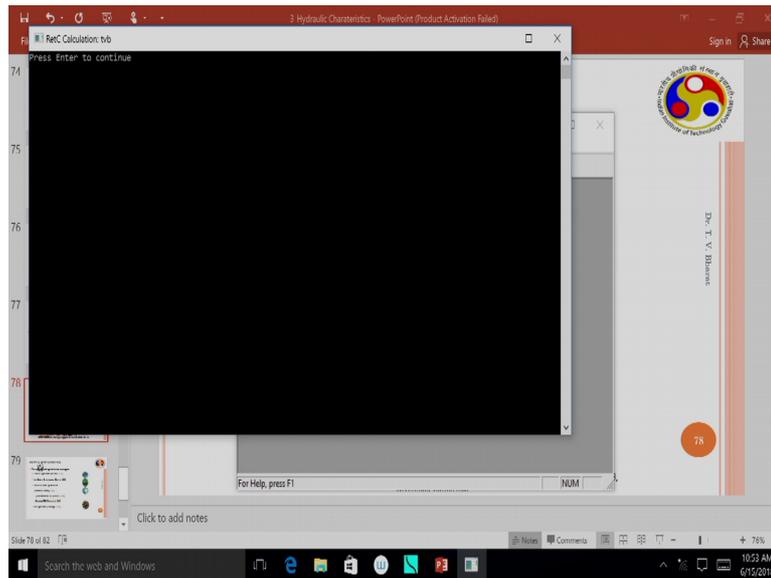
So, then when I go for next step, do you want to save the input data before executing a RETC I select.

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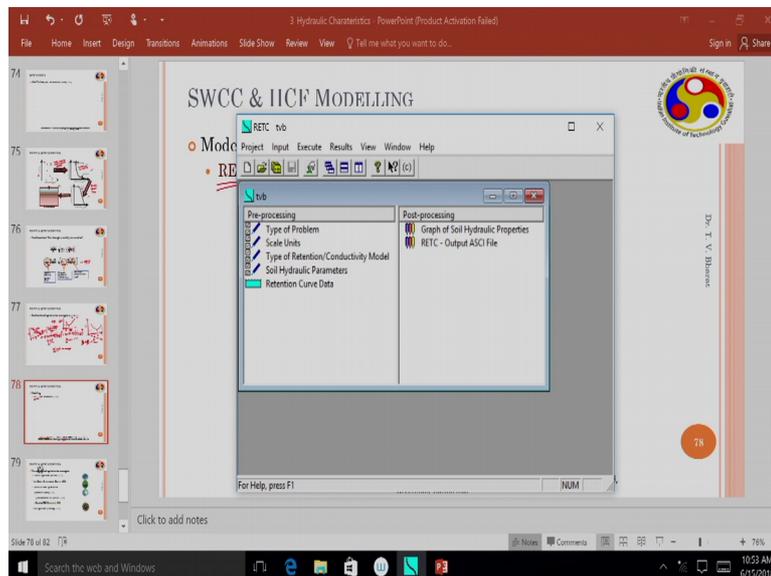
Then it ask you to do you want to run the application yes.

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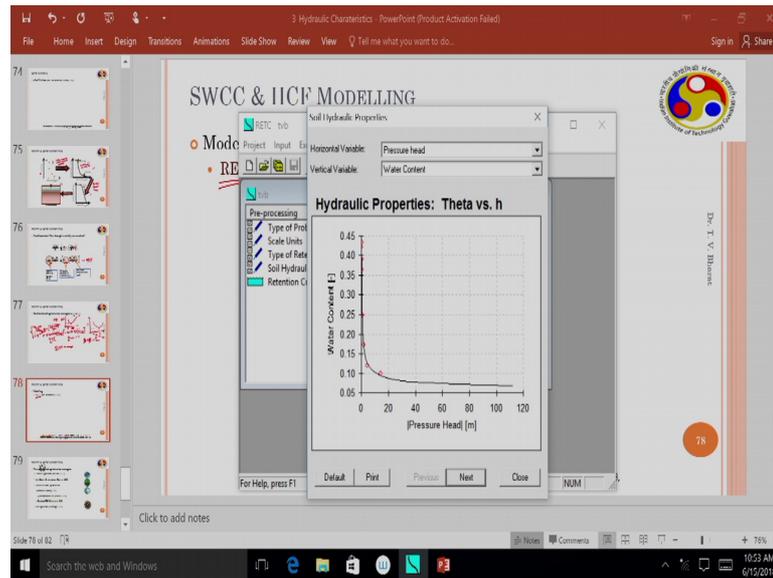
So, then it show you this one and you need to give enter. So, once you enter.

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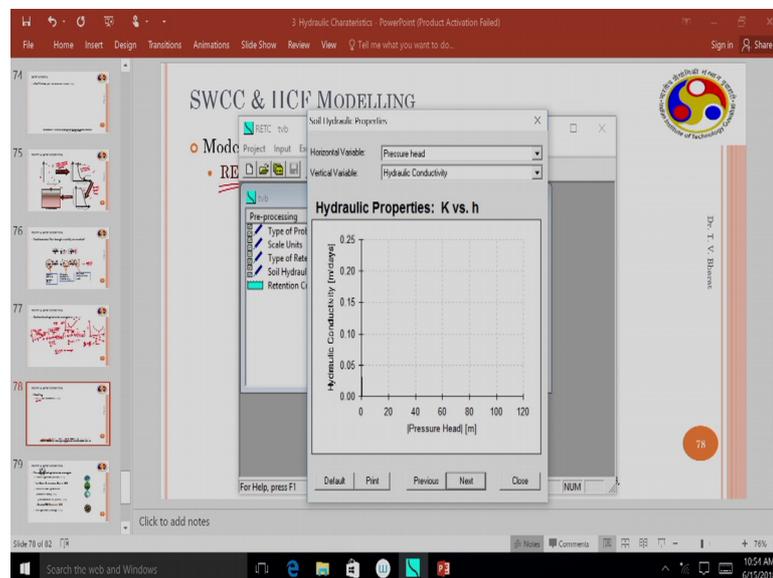
So, you have the output file.

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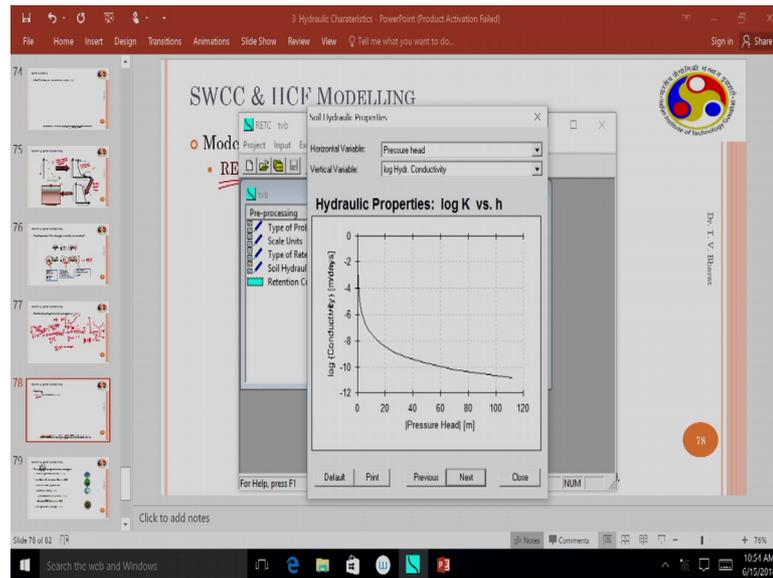
This is the fitting that has been done. So, here the water content is drawn on y axis and pressure head. So, that is in meters is plotted on a x axis. So, these are the data points. So, these red marks are the data points that we have inputted.

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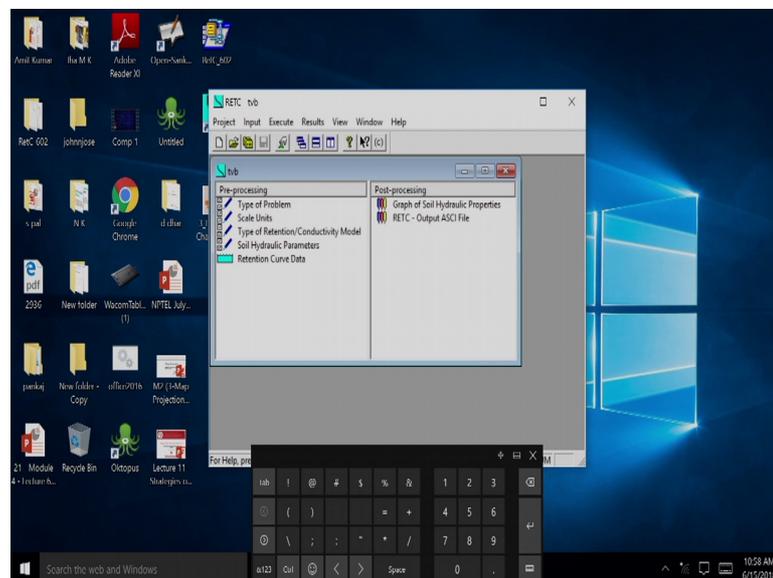
And this line is fitted line and this is hydraulic conductivity versus pressure head.

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So, which we cannot fit here in log scale if you see log conductivity versus pressure head, this is how the hydraulic conductivity varies with suction head; as a pressure head increases the hydraulic conductivity decreases.

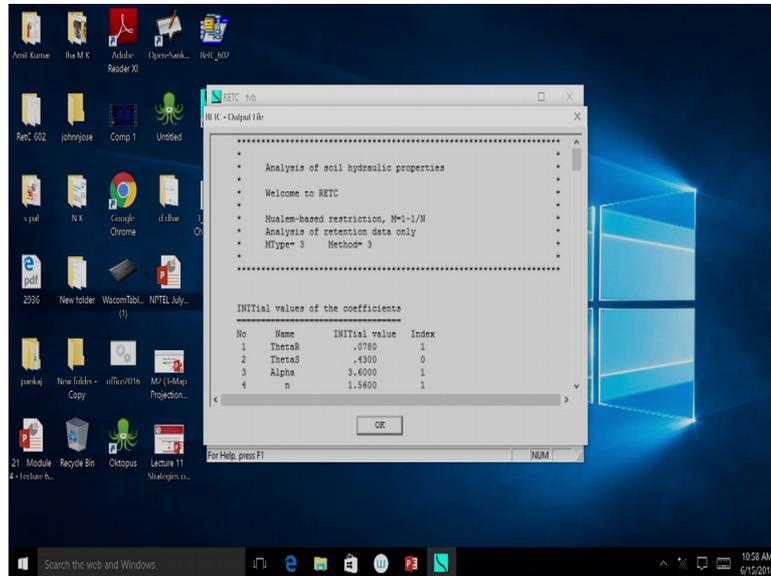
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So, this is the output file and if you see the graph, this is the water content versus pressure head which is very well fit very well fitted. So, this red marks are the data points inputted and this is the this black line is the fitted curve. So, when you plot the

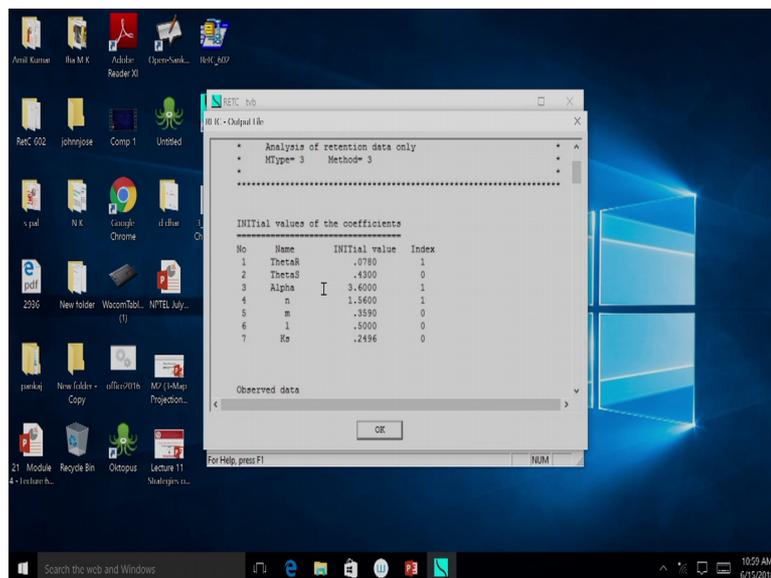
conductivity in meter per days and log scale and pressure head in meters if you plot as a pressure head increases the conductivity decreases. So, this is the plot you have.

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And we can see the RETC output file, which contains the information of what we used. We used Mualem based restriction that is m equals to $1 - 1/N$ used.

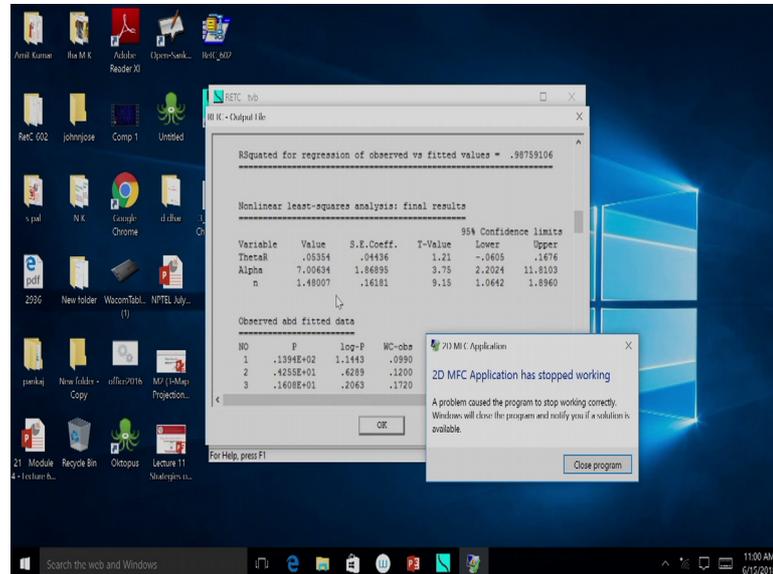
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And this is the initial values of the coefficients theta R, we gave 0.078 and theta S 0.443. So, this initial value is not fitted, but this is selected as an initial guess. But 0.43 this

value we have inputted and alpha n m are taken as a initial guess and case we have given as 0.2496.

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So, then after the analysis the r square we got is 0.987 very good r square value. So, non-linear least square analysis is done and the finally this is the output. The theta R is estimated to be 0.0535 and alpha is 7 and n is 1.48 as n and m restricted case. So, once we get the n we can determine the m as well. So, this is the data we get. So, once we get the m and m are n and alpha values and theta R. So, these three parameters unknown parameters are known and we have the other known parameters like theta s and other data we can use and plot it in excel on our own and that is how the SWCC smooth function is obtained by RETC software.

So, this is how RETC free software could be used for the estimation of smooth soil water characteristic curve. So, we can also obtain the hydraulic conductivity function. However, if you are interested in using Fredlund and Xing model to the RETC software, does not allow you to use Fredlung Xing. So, then you need to use any other software like GeoStudio, but these are commercial software's, where you can generate or you can write simple code in MATLAB and you can optimize.

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SWCC & HCF MODELLING

- Nature-inspired optimization strategies:
 - Genetic Algorithms (Holland, 1975)
 - Ant Colony Optimization (Dorigo, 1992)
 - Particle Swarm Optimization (Eberhart & Kennedy, 1995)
 - Quantum-behaved PSO (Sun et al., 2004)
 - Perturbed PSO (Bharat et al., 2008)
 - Bee algorithms (Karaboga, 2005)

The slide features a graph of a non-smooth objective function $f(x)$ versus x . Handwritten red annotations include a point (a, m) , a vector \vec{O}_1 , and a vector \vec{O}_2 . A vertical stack of icons on the right represents various nature-inspired algorithms: a fish (Genetic Algorithms), an ant (Ant Colony Optimization), a bird (Particle Swarm Optimization), a bee (Bee algorithms), and a colony of bees (Bee algorithms). The slide is attributed to Dr. T. V. Bharat and includes a page number 79.

So, the gradient based algorithms have some issues such as the initial guess needs to be thoroughly examined, because generally the objective functions may not be as smooth as I have shown

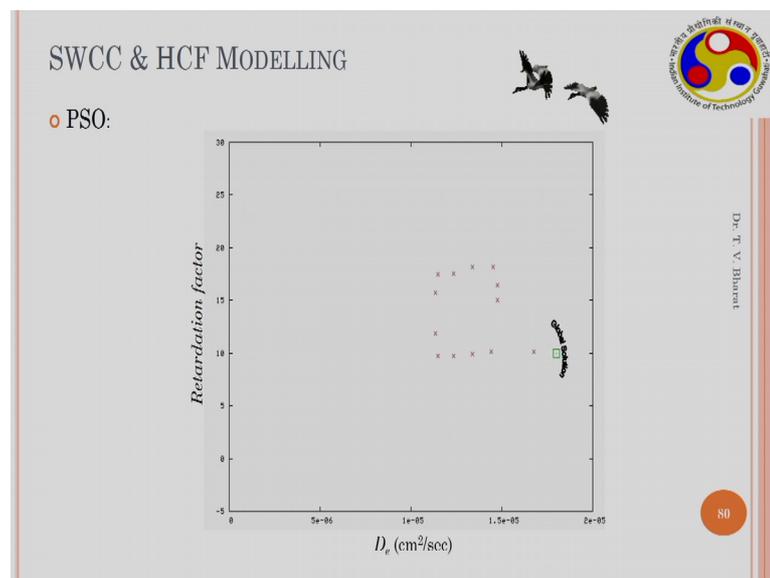
So, the objective function could be like this, but because the objective function versus say x , but because we are using some experimental data measured data there may be some error. So, this may not be as smooth as this you may have some roughness and you may have a data like this. So, in that particular case, so, this maybe your global minimum, but your function may get stuck here itself at local minima itself. So, if you chose an initial guess somewhere here, this may be your initial guess and it will traverse and it will get stuck here itself it may not reach here or it may be 0 here itself. So, the objective function get zero at local minimum value.

So, therefore, the estimated parameters may be erroneous. So, therefore, there are some global optimization algorithms. So, those are based on nature inspiration. So, such algorithms are genetic algorithm, ant colony optimization, particle swarm optimization in that there are any variations of course, in genetic algorithm also there are many variations and ant colony also there may be few variations and particles swarm optimization there are many variations like quantum behaved PSO, perturbed PSO which is our work from our research lab and bee colony algorithms etcetera.

So, these algorithms very limited algorithms have listed here, but there may be enormous number of algorithms based on the observations in the nature. So, here for example, if I describe ant colony the way the ants move from their colony to the food sources by optimizing the path between the colony and the food source, that is one of the inspiration for building an optimization algorithm, there is an optimization involved in doing that in the social behavior of ants, so, based on that the algorithms were built. Similarly, the particles formed to optimization is based on the behavior that is exhibited by the flocky birds and fish colonies etcetera. So, birds they try to fly in a very formed shaped and there will be a leader and followers and they usually form this v shape and they fly to minimize that drag force.

So, there are several rules to maintain such v shape and then while flying. So, such optimization behavior is brought into the algorithm and which is used for solving several engineering problems.

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So, for example, the particles and optimization, which has a simple rules for example, you have two parameters x_1 and x_2 you can chose initially some number of solutions on the shear space this is the shear space. The x_1 in our case could be a and x_2 ij our case could be m or n. So, two parameter optimization problem we have this is sets space this is sets space we know the limit limits of the sets space as well because a can vary only between some particular range say 0 1 kilo Pascal to minimum 1 kilo Pascal to very

large value like 5000 kilo Pascal 10000 kilo Pascal because this is relate to air enter value.

And m also m and n can be restricted in a given range. Now, possible number of solutions are chosen on the search space and each solution indicate a set of a set of a 1 and m 1 and you chose another one this has a 2 and m 2 . These are the two possible solutions for our problem and similarly you have chosen n number of such solutions on a search space i equals to 1 and this is the i equals to 2 and similarly you have n number of solutions. Now, i equals to 1 corresponding to this you can obtain the objective function O 1 and corresponding to i 2 you can obtain the objective function O 2 . So, now, one you have once you have the objective function O 2 and O 1 and O 2 you can compare these two. And if objective function is lower, this value is lower than O 2 that means, the global minima could be close to this particular value close to.

So, what you could do is you can make this particle to move in this direction in the direction of 1 . So, one possible way of doing that is for example, you have one particular particle here which is at distance say 0 and 2 and another particle are we call particles these number of solutions are called particles it could be birds also. So, that means, here I have another particle which is located at say 4 and say 5 .

How to move this particle to this value? So, directly to move this particle to here, what you could do is the difference could be added to this value and when it will jump into there. So, for example, the difference is of 4 and the difference is 3 if you add this point to this and this point to this it can directly jump here but we do not want to do that directly it should not jump.

Because this may not be the global solution, it should only move in this direction. So, for that what you could do? You can multiply with some factor. So, see theta or alpha or something. So, this is less than 1 . So, then it can jump to a new solution in that particular direction. So, similarly you compute the objective functions of all different particles and which ever has a lowest objective function, all different particles will move towards that particle for example, this particle has a lowest out of all the different particles, then all different particles will start moving towards him.

So, one iteration is complete, in the next iteration they have new positions. So, again you can compare all the objective functions and you can obtain what is the global best and

also because they have their own experience to move from here to here, they can compare within their history who is best this position is best or this position is best based on the local best and global best the particle will start moving towards the optimal solution and finally, at the end of the iteration once the search is complete all the particles will converge to one particular point that is a local minimum. So, I will demonstrate this. So, this is one particular problem for diffusion problem, where I have two parameters to optimize that is diffusivity diffusion coefficient and retardation factor, this of containment flow through saturated soils for that particular problem.

So, initially I have chosen number of particles on the such space, say these are the number of particles and they started searching and moving towards the global. So, this is initially the best solution. So, everybody started following this fellow and then they started coming close to and this also started searching and found that there is a better solution in and around. So, it also started perturbing and finally, all the particles at the end of the search process they converge to a global solution.

So, that is how we search the global solutions using swarm intelligence or nature inspired algorithms. So, there are many such algorithms. So, one can use to determine.

(Refer Slide Time: 37:31)

The slide is titled "SWCC & HCF MODELLING". It contains the following text and notes:

- Stage (I) ^{SWCC} --> (II) ^{K-fn}:
- $\sqrt{G}, m = 1 - h \rightarrow K_r$ ✓
- $m = 1 - h \rightarrow K_r$ ✓
- $m, n, \text{Variable} \rightarrow K_r$ ✓

The slide also features a logo of the Indian Institute of Technology Gandhinagar in the top right corner and the name "Dr. T. V. Bharath" on the right side. The number "81" is in a red circle at the bottom right.

So, once you get the smooth soil water characteristic curve using optimization techniques, we can determine the hydraulic conductivity function that is stage 2 is SWCC, obtaining smooth SWCC stage 3 is obtaining K function. So, once SWCC is

obtained the k function can be determined using either analytical solutions if there available or numerical solutions. If Van Genuchten model with some restrictions are chosen m equals to one minus one by n or something then you are using Mualem model then corresponding hydraulic conductivity function is known.

The solution is known as we discussed earlier and you can obtain, single Van Genuchten model using Burdin equation is known then we can utilize particular solution which was derived and we can get the hydraulic conductivity function. Similarly, if you are using m n variable case, so, then corresponding K_r in terms of incomplete beta function could be used to obtain the hydraulic conductivity function.

Similarly several number of hydraulic conductivity functions can be determined based on what type of SWCC modeling is utilized. This is how we obtain hydraulic conductivity function. So, once we have SWCC and hydraulic conductivity function, once we have these two functions well defined functions, we can utilize them for the flow problems. So, once we enter into the flow we can understand how these functions can be utilized and solved.

Thank you.