

Unsaturated Soil Mechanics
Dr. T.V.Bharat
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Week – 04

Lecture - 10

Hydraulic Conductivity Functions and Determination of State Variables

Hello everyone. We have seen an important constitutive relationship; it is soil water characteristic curve, which is a relation between water content and suction or negative pore water pressure, within the water.

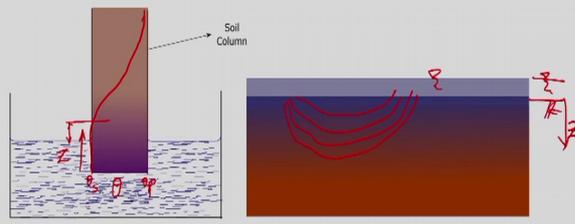
We have seen how theoretically this can be deduced for idealized cases, and after that we going introduce, another constitutive relationship called hydraulic conductivity function. Because, whenever there is a flow that takes place through partly saturated soils or unsaturated soils, what hydraulic conductivity we should use to simulate the flow through soils. Because, as the water content in the soil increases the hydraulic conductivity or the flow would be higher, and when the water content decreases, flow rate would be lower.

So, with what hydraulic conductivity we should simulate the flow conditions, is a question.

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UNSATURATED CONSTITUTIVE RELATIONS

o Soil Hydraulic conductivity function (HCF)





Dr. T. V. Bharat

Richards' equation: $\frac{\partial}{\partial z} \left[k(h_m) \left(\frac{\partial h_m}{\partial z} + 1 \right) \right] = \frac{\partial \theta}{\partial t}$

$\frac{\partial h_m}{\partial z} + 1 \rightarrow \frac{\partial \theta}{\partial z}$
 $k(h_m) \rightarrow \text{Hydraulic cond. fn. (HCF)}$
 $h_m(\theta) \rightarrow \text{SWCC}$

$\text{matric suction head} = \frac{(u_w - u_a)}{\gamma_w}$
 $\frac{\partial \theta}{\partial t} \rightarrow \frac{\partial \theta}{\partial t}$

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So, I have considered two simpler cases, where you have a soil column, immersed in water reservoir. Then, there is a flow that is taking place into the soil mass. So, the water content of the soil is also increasing, as a flow takes place into the soil, and there is a moisture content variation as we have seen earlier. So, there may be some change in the moisture content θ , this is θ_s , and this may be θ_r or θ_0 , and this is height or z . θ_s , it would be up to the air entry value, and then beyond that the volumetric water content decreases. And it reaches to the initial value; initial water content within the soil sample, maybe at air dried state.

So, now flow takes place due to what? There is a flow that is taking place against the gravity here in this case. So, it is not the elevation that dictates; there is something else that dictates that is a chemical potential of the pore fluid; that chemical potential within the soil system that dictates the flow. So, therefore, there is an upward flow that is taking place. So now, when the flow takes place, with what rate the water uptake takes place within the soil, is a question. So for that, we need to understand whether the Darcy's law is valid or not.

And similarly, there is another case where you have soil mass; this is the ground-level and this is a water table; there is a ponding that has taken place due to rainfall. So, then, there is a flow of water that takes place into the soil mass; there is infiltration that takes place. That is, in the direction of gravity only. In the direction of gravity, there is a flow that takes place. So, how to simulate these things? We have one flow equation. We will discuss this little later, when we discuss about the flow through unsaturated soils; which is called Richards' equation which is commonly used to simulate the flow through unsaturated soils, like the cases I have discussed here.

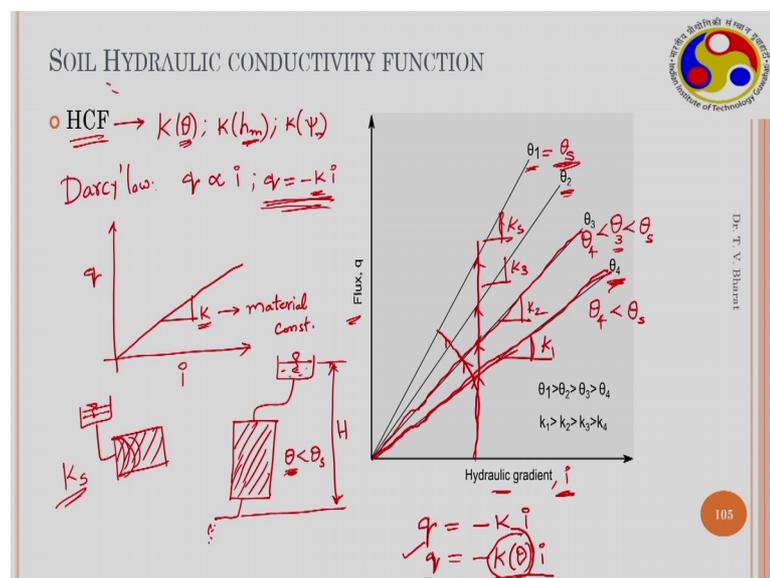
So, if you observe the flow equation; in partly saturated soils there is a θ by θ ; z ; z and t , these are independent variables; and you have something called k_h term. Hydraulic conductivity, k is the hydraulic conductivity which is a functional form which depends on matric suction head. This is a matric suction head, which is a matric suction head; which is nothing but, u_s or u_w , divided by γ_w . So, there is a matric suction head. And, z again, an independent variable h_m is the same suction head, and θ here, is a volumetric water content; volume of water per total volume.

So here, essentially, we require, a relationship, between h versus θ , because, you have dependent variable here in as a h ; here you have θ . So, this could be expressed in terms of θ completely, if you have a relation between h versus θ . This is your soil water characteristic curve. This we have defined earlier. How the matric suction varies with volumetric water content is given by soil water characteristic curve.

Here, there is another relationship, another constitutive relationship that is required to solve this particular equation; which is called hydraulic conductivity function; conductivity function. So, here this is what I have written, soil hydraulic conductivity function, HCF. So, this is HCF and this is SWCC. These two constitutive relationships are fundamental in unsaturated soil flow. When the flow takes place through unsaturated soils, these two constitutive relationships are fundamentally important for understanding the flow behavior.

So now we will, let us try to understand the HCF, Hydraulic Conductivity Function.

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So, when there is a flow takes place, we commonly use the Darcy's law, which states the flux, or the Darcy velocity; small q is proportional to i , hydraulic gradient that is Darcy's law. If there is a equation constant, we put k that is a hydraulic conductivity. Here, we put negative sign, because the flow takes place from higher head to lower head. So, this is the Darcy's law.

Now, the question is, whether the Darcy's law is valid or not. Because when I plot, a relation between q that is a flux; and i , hydraulic gradient, I should expect a unique relationship between these two. So, that means, the slope of this one is hydraulic conductivity. So, this is constant. This should not depend, or this should not vary with hydraulic gradient, or it should not vary with anything else. It only depends on the pore structure, pore geometry etcetera. However, this is an independent. This is a material constant.

Let us see what happens in case of unsaturated soils. In unsaturated soils if I plot hydraulic gradient on x axis that is i , and a flux, that is q , on a y axis. If I maintain, if I conduct a test, hydraulic conductivity test in a soil column by maintaining one particular head, one particular head I have maintained; and this is the soil column hashed area. Here, I have I happened to maintain particular water content. So, this is possible. Generally, what we do in a hydraulic conductivity test is that, the soil is completely saturated and after that only we conduct the test and obtain the case hydraulic conductivity; saturated hydraulic conductivity.

But in this case, I want to maintain certain θ , which is less than θ_s which is less than the saturated volumetric water content. That means, it is partly saturated condition. Even after maintaining certain head, I should be able to maintain certain volumetric water content. So, because in unsaturated state, you have two phase system in the pore space you have water and you may have other medium that could be air, or it could be other immiscible fluids, such as oil or something.

So, I can maintain certain water content by maintaining the other fluid content to be same. So, that is possible, right? So, if that could be maintained then, what relationship I obtain between flux and i ? Again, I get a unique relationship like this. This is where θ_4 is the volumetric water content I maintained. So, this is θ_4 is much less than θ_s , right? So, now, this is unique relationship again like Darcy's law, where flux is proportional to hydraulic gradient.

Now, I change my volumetric water content, or I slightly increase the volumetric water content by decreasing the content of other fluid. Could be oil and that content I reduced. So now, I maintained something called σ_3 . σ_3 is still less than θ_s , but σ_3 is so greater than θ_4 . So, this volumetric water content θ_3 is larger than

or higher than the θ_4 , but less than θ_s only. So, then what will happen? My flow rate will change; my flow rate will increase. And this is a flow rate I obtain. And again, this is another unique relationship. Similarly, I can keep on increasing my water content; θ_2 and θ_1 , or θ_1 may be equals to θ_s only. So, such a way condition also, I can bring in.

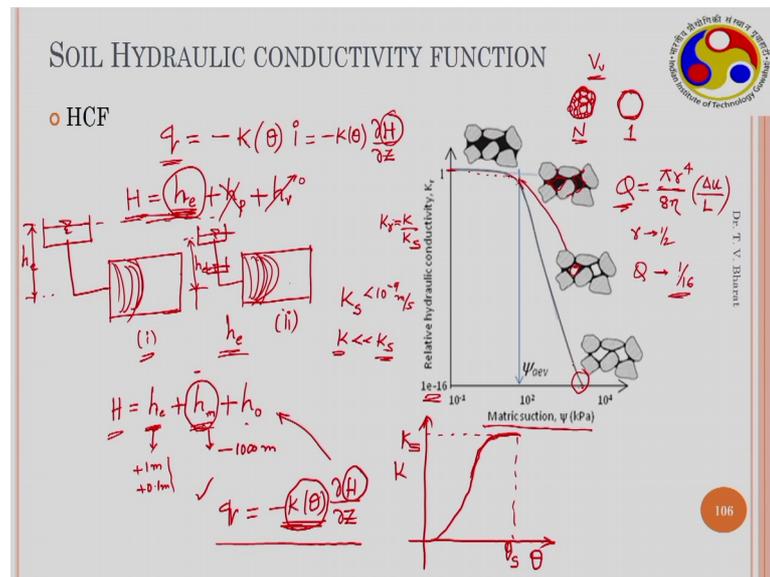
So, then my flow is changing. For the same hydraulic gradient, I may have different flow rates or different flux values. My flow rate is changed. Or, I have a soil sample. I just allowed the soil sample to; initially the soil sample is at air dried state. Then I maintained certain water level. So, then I allowed the flow to take place through the soil. Then what will happen? As and when the water passes through it, the hydraulic gradient also decreases because the level decreases.

So, slowly hydraulic gradient decreases, but my volumetric water content is increasing; and which causes the increase in the flux. So, I go in this path. So, either way it could be, it could be possible. By either maintaining constant hydraulic gradient I can achieve different flux, or different flow rates with different volumetric water contents. Or my flux or flow rate is, should, can change with time due to increase in the moisture content.

So, therefore, whether the Darcy's law is valid now? This is valid. This could be used; Darcy's law still could be used. Darcy's law is, minus k_i . Is still could be used, provided k , this slope is k_1 . This slope is k_2 . This slope is k_3 . This slope is say k_s . Because, this is a θ_s . Corresponding to θ_s , the hydraulic conductivity is θ_s . So, k itself is changing with change in the moisture content. So, when the θ volumetric water content is very small, then k is smaller. As volumetric water content increases to the saturated water content, then it is it became k_s ; so unless until you assume a functional form for k , it is not possible to use Darcy's law.

So, Darcy's law could be used, by defining k as, k as functional form. K is function of θ . So, this functional form is called hydraulic conductivity function. Hydraulic conductivity function is k as a function of θ , or it could be k as a function of h_m matric suction head, or k as a function of suction, matric suction, or simply suction. Because, θ is again dependent on suction; so we can define k of θ , or k of h_m .

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So, we just said that, q should be written as minus k of θ , functional form. And i , i is what? Hydraulic gradient, so that is equals to, minus k theta. So, there is a dou. You know one dimensional case. This is simply, dou H by dou Z . dou z or dou x . So, this is a special variable.

Now, what is H ? This H , this is total head. Generally, in saturated soils, the total head is defined as, the head due to elevation, or elevation head, plus pressure head, plus the kinetic head or velocity head. Usually velocity heads, velocity heads we ignore; because, they are negligible compared to other heads. Velocity of water through soil pores is negligible. So, we generally we ignore.

So, now, when you construct the elevation head and pressure head; I will take one simple example, where you have a soil, which is connected to a reservoir. I maintained certain water level. So, this is the water level. This is the elevation head. Now, flow is taking place through this. There is a event. So, flow is taking place. This is one experiment I have conducted. I can conduct another experiment, by maintaining different elevation head. So, the height has reduced. Now, in this particular case, elevation head is reduced. Compared to this, the elevation head has reduced.

So, if this happens to be clay soil; compacted clay soil, such as, bentonites or something, you would observe that, elevation head does not make any difference in the flow rate through soils. So, the wetting front, if you observe within the soil mass you would see

that it moves at the same rate as it moved in this particular case; first case. That is because, first of all in saturated system itself, hydraulic conductivities of saturated soils in bentonites is very less which is less than may be 10^{-9} meter per second; which is very very, small. So, in that particular case, the hydraulic gradients are not important. So, diffusion dominates.

So, in this particular case also, because this is unsaturated soil, the k values are much less than k_s . The hydraulic conductivities are far less than saturated hydraulic conductivities. So, head would not make any difference. So, even if you have a very small head like this the flow rate would be same. So, then what is it that governs? Elevation head remains same. Pressure head is not there. Pressure head, is not anything is maintained, so then total head remains same. So here, the elevation head is changed. And, pressure head is not changing, or which is not there.

So, when the total head is changing, but the flow rate remain same, what could be the reason? When you are changing the elevation head, when you maintain at say 10 meters also, the flow rate will be same. So, this is not increasing as high as this. So, then what is it that governs the flow here? The flow here governs, other than the elevation head etcetera. There are other things, like the chemical potential.

So, the total head now should consist of in unsaturated soils, the elevation head plus if you have pressure head, positive pressure head. But then generally, this is a negative pressure in unsaturated soils. So, this pressure head is a negative pressure head within the soil mass. Now, this one is a chemical potential head or matric suction head that can be represented as h_m . Matric suction head plus if you have salts then, it could be osmotic suction head. So, this is what that governs the flow behavior.

So, as a matric suction head in this 2 particular case is significantly larger compared to the elevation head you are talking about, this a negative value. This is a negative quantity, which is very large compared to the elevation head you are talking about. The matric suction head, as I have mentioned in earlier cases that it could be as high as several 1000 meters. So, if this is a several 1000 meters, negative 1000 meters, and this you are maintaining 1 meter, plus 1 meter plus 0.1 meter, if you are varying in this manner. So, it does not make big difference on the flow rates. So, that is a reason why

your flow rate remains constant, even though you are varying the positive head; the elevation head.

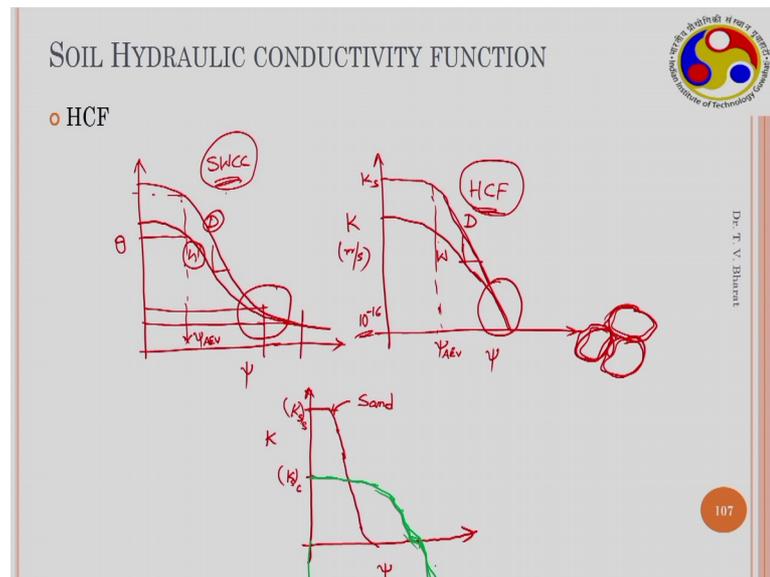
So now, the Darcy's law, which is q equals to minus k , θ times θ times, $\frac{dH}{dz}$, this H should contain total head. The total head should contain elevation head, plus matric suction head, plus osmotic suction head all the heads should be there. Then, this modified form of Darcy's law could be used for flows. So now, we are not going into the solving the, flow equations and then understanding the flow through unsaturated soil. We will do it little later. Now our interest is on hydraulic conductivity function. So, what is this hydraulic conductivity function?

If I plot, the hydraulic conductivity versus volumetric water content on x axis, I would expect that, the hydraulic conductivity increases as a volumetric water content increases; and it reaches a nearly constant value at θ_s . θ is equals to θ_s . θ_s , is saturated volumetric water content or this is (Refer Time: 20:07). So, at that particular point this is, this will become k_s that is the saturated hydraulic conductivity. Or, this could be written, this could be plotted, in terms of matric suction. So, here its written relative hydraulic conductivity that is nothing but your k_r is k/k_s .

So, when the soil is completely saturated, the hydraulic conductivity is k_s . So therefore, k_r is k_r is 1. As the water content within the soil pores decrease; that means, only the contact angle is changing; the meniscus curvature is changing; small change in the water content. But definitely, the flow takes place through these larger pore space. So, therefore, hydraulic conductivity significantly does not change.

So, until there is a, air entry that takes place. This is a, air entry point; so at this particular point, where air has entered into the soil. So, a sudden drop that occurs in the hydraulic conductivity, and with decrease in water content further, it significantly decreases. And it becomes close to 0 at residual state. At residual state, the water content exists as a thin film around the particles. So, in that particular case, the flow channels are not existing. Therefore, the flow does not take place. Vapor flow makes this.

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So, now we have seen that the hydraulic conductivity function when we plotted in terms of k in terms of suction we have seen that the hydraulic conductivity initially changes; hydraulic conductivity initially changes slightly, and then after that it decreases significantly. So, after reaching the air entry, we have seen that, there is air enters into the larger pores or largest pores of the soil system. When the larger pores are getting emptied, air enters. The macro flow channels do not exist anymore; and flow has to take place through the micro channels; small thin channels.

So, compared to the macro channels, the micro channels offer more resistance for the water movement. Therefore, the flow rate or the hydraulic conductivity decreases drastically. So, if you consider thin pipes; considering a thin, so many numbers of pipes, and considering one single pipe and the volume of voids in both these cases remain same. We have considered so many numbers of pores here. N number of pores and only single pore here single larger pore and a small pores of N number volume of voids in both the cases remain same.

So, in that particular situation, the flow through this larger pore will be highest compared to this particular case. Because, larger pores first of all, if you, from the Poiseuille's equation if you consider, the volumetric flow rate, q is equals to πr^4 , divided by 8μ into change in the pressure by L . So, apart from the viscosity, and the length of the pipe etcetera, and change in the pressure; and this is a constant. Now, the flow rate is

directly proportional to the r power 4. And the r is reduced to half, then Q will reduce to, 1 by 16th. So, flow rate will decrease one-sixteenth value.

So, similarly here, because you have, so much of, here, it will have more resistance for the flow to take place, because, you have so much of surface area so much of surface area. There is a huge resistance that is offered from the wall surface. So therefore, here as soon as the larger pore get emptied, due to the air entry. So, the hydraulic conductivity drastically decreases; and when the capillary action is taking place here, due to the reduction in the water in the larger capillaries, the hydraulic conductivity decreases drastically.

When compared to volumetric water content versus suction, that is SWCC; that relationship may be somewhat like this. The hydraulic conductivity function k versus ψ changes in this particular manner. So, this is air entry. This is air entry. Beyond that, there is a change in the volume, volume of water. The water content decreases. So, because, decrease in the water content especially this slope is much steeper compared to this one. Because, here as the larger pores are getting emptied, the hydraulic conductivity decreases very sharp. So, probably I can adjust this, by plotting like this; much steeper one. This is much steeper.

So, once it reaches the residual portion, you would see that the hydraulic conductivity goes to nearly 0. Because, here, when you apply a huge suction the volumetric water content decreases in significant amount. So but then, that is a mechanism here. But here, the mechanism is that, when the water exists as a thin film around the particles, as we have seen earlier this is a particle, and this is another particle. So, water exists only around as a thin film around the particles.

So, in that particular case, the pores are emptied, but flow cannot take place through these channels. There is no channel exists. Actually, channel broke down, because, the larger pores, and small pores are also got emptied. Now, you have water only available as a film. Therefore, the hydraulic conductivity goes to nearly 0; 10^{-16} or something. These are theoretical values. But, experimentally we cannot measure 10^{-16} ; 10^{-10} meter per second values, right? So, that is the hydraulic conductivity function.

So, we require both these functions. SWCC and hydraulic conductivity function for understanding the flow behavior; so as SWCC is hysteresis, it has, this is a drying curve, and this is a wetting curve. Similarly, the hydraulic conductivity function also will follow hysteresis. So, this is the drying path and this is wetting path. So, drying hydraulic conductivity function lies above the wetting hydraulic conductivity function.

However, we will see that because at a given suction the water content in the wetting curve is smaller than the drying curve; therefore, when the water content decreases, so definitely the hydraulic conductivity also decreases. So, this follows the same logic as SWCC. As the HCF, hydraulic conductivity function replicates the behavior of soil water characteristic curve, often the hydraulic conductivity function is derived SWCC. We will see how to derive this hydraulic conductivity functions for a given soil from soil water characteristic curve, in the later lectures.

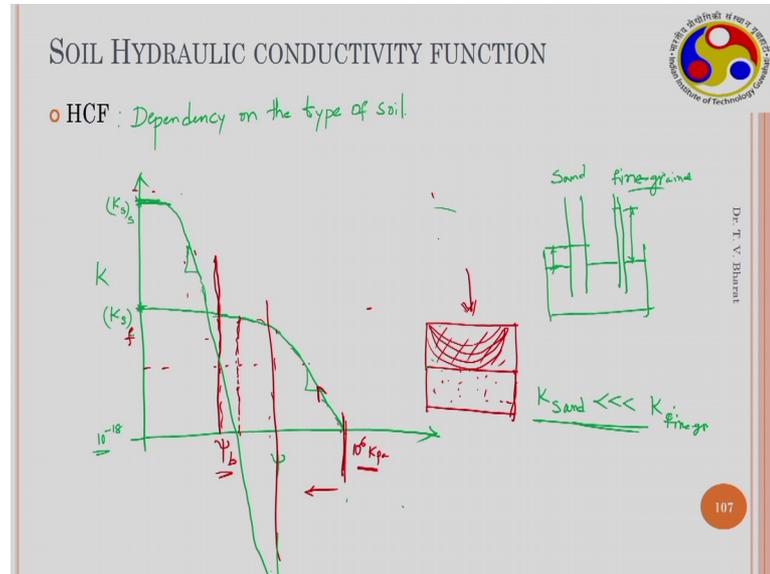
So other, there is a interesting concept that also we observe in hydraulic conductivity function is that, when you plot hydraulic conductivity versus suction for different soils. For hydraulic conductivity of sand and clay when you are plotting, definitely the hydraulic conductivity of sand is saturated hydraulic conductivity of sand is much higher, and saturated hydraulic conductivity of clay is much smaller to sand. This is sand. k_s of sand, and this k_s of clay. So, how do you plot? You have a small air entry; generally, sands will have very small air entry. Immediately, the air enters into the system. Because, when you take a small sand sample, saturated soil sample, with minimal effort, water comes out; under gravity itself.

So, maintaining saturated condition in sand is very difficult. Unless and until, you have a container, and pour the sand, and put the water. In normal condition, when you hold it in hand, the water drains out under the gravity. And then, you generally have a unsaturated soil; that means, under the gravity itself, the soil exists as unsaturated state only. That is the reason, why we usually make sand castles on beach side, because soil condition could be maintained unsaturated.

So, here hydraulic conductivity of sand is very huge; but, air entry is very small. So, immediately as and when the air enters into the system, the hydraulic conductivity drops and then it follows, it follows like this. So, this is for sand. So, in case of clays, the hydraulic conductivity is very small; and the air entry value is very high;, very very high.

Significantly higher than the sand and it follows, it follows this path and it goes in this manner. If I extend this curve, then it follows like this.

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Let us understand another important aspect in hydraulic conductivity function. That is, dependency on the type of soil. So, if I plot for 2 different soils, you would see that, for example for sand this is k and this is suction. For sand, the hydraulic conductivity is very high. So, the k_s of sand is very high. So, I expect that hydraulic conductivity slightly decreases with increase in the suction, up to the air entry. Air entry of sand is very small compared to; air entry value of sand is very small compared to the clays or other soils, finer, fine grained soils. Air entry of sand is smaller compared to the other fine-grained soils.

This is obvious, because when you hold a saturated sand in hand, water simply drains out under the gravity itself, because, sand does not have the ability to retain water against the gravity. So, because the capillary size is larger, it does not hold the water that much. So, that could be understood using simple capillary mechanism, when you have two capillaries; one is a larger capillary and another one is smaller capillary. The height of water in this capillary if you see the height is smaller in larger capillary like sand; this represents sand and this represents fine grained soil. So, fine grained soils will have larger air entry value. This is the air entry for sand, this is the air entry for fine grained soils.

So, therefore, generally it is difficult to maintain full saturation in sand, because, under the gravity itself the water drains out. So, that is the reason why, when we make a castle, sand castles; obviously, the water drains out and then it maintains only the unsaturated state; it has more strength compared to either fully saturated or fully dried state. So, here the air entry of the sand is very small and beyond that you expect that the hydraulic conductivity drops drastically. And then it goes to very small value maybe 10^{-16} or 10^{-18} . Or something meter per second very small value. Insignificant value, it becomes.

In case of clays, if you consider, clays or any other fine-grained soils, the hydraulic conductivity is much smaller compared to sand. So therefore, the saturated hydraulic conductivity of clays lies here. The first point lies here. So, then the air entry value if you see, the air entry value is very high compared to sand. So therefore, for very large values of suctions, it retains water and after that it decreases the hydraulic conductivity. Here, in this case, it is much steeper than this particular curve. Because, as a larger pore get emptied, most of it is like, there is no flow channels and immediately it goes to nearly 0, or very small value it becomes.

Fine grained soils, you have more number of smaller pores. So, more water is retained; and as and when they get emptied, slowly the hydraulic conductivity decreases. So, this is much steeper. In sand, this is much steeper. This is a very interesting observation because, interestingly beyond this particular suction, I name it as ψ_b beyond this particular suction clay or the fine-grained soil, this is fine grained soil. Fine grained soil will have hydraulic conductivity, this after, beyond this particular value. The fine-grained soils will have hydraulic conductivity. If you consider any suction beyond this ψ_b , at any given suction, the fine-grained soils will have higher hydraulic conductivity compared to the sands.

So, this is beyond ψ_b . This became vice versa, where fine grained soils will have a hydraulic conductivity much larger than the sands. So, sands will have lower hydraulic conductivity. So, this has important implications. Because, when you take a dry, fine grained soil, and when you take dry sand, this is sand. And this is fine grained soil, if you consider this setup. So, when the flow takes place through this; that means, initially they are dry. So, initially the suction is here [vocalized-noise], very high. The suction maybe

So, we have defined several state variables such as water content. It could be gravimetric water content or volumetric water content. And volume of soil, that is also required in many places, and suction; osmotic suction, matric suction. So, depending on that, whether you are estimating only the matric suction or total suction; total suction includes, matric suction plus, osmotic suction, and hydraulic conductivity or, hydraulic conductivity function.

Hydraulic conductivity is not a state variable. I just mentioned that all the state variables. Hydraulic conductivity is a material constant, but estimation of hydraulic conductivity at different states of soil. That is at, this is a functional form k of θ or k of ψ also need to be estimated. How to estimate or measure these values in the laboratory and field? That we will see now. There is a important relationship for SWCC, soil water characteristic curve. We require a relationship between water content versus suction, or volumetric water content versus suction. Water content determination, gravimetric water content determination is often, which is done in soil mechanics laboratory because, we often determine using oven drying technique.

But, in the field such scenario, where you have a hill slope, and there is a rainfall, because of which there is a water infiltration that takes place through the soil mass, we need to understand real time, how the water percolates into the soil, how θ affects t . Changes θ of x t or w of x t . w of x t or w of x y z t , changes. Water content changes with special distribution and time, and how this impacts suction, and how this impacts the further stability calculations. So for that usually, we put some sensors to monitor how the moisture content changes. These sensors, they use some physical laws; they use basic laws to estimate the moisture content, either by relating dielectric constant of the soils, or some other properties of the soils.

Therefore, they estimate directly with volumetric water content. Because, this is easy to estimate the volumetric water content because, how much in a given representative volume, how much volume of water is present in this total volume can be determined more accurately using these sensors. But, it is not possible to take the soil sample and bring it to the laboratory, and wait for 44 hours to oven it, oven dry it and then after that, getting the mass of solids and, total, sorry; mass of water and mass of solids, and getting the gravimetric water content is not feasible.

But when you do laboratory tests, often we put it in oven and then determine the volumetric gravimetric water content. So, often we require to estimate the volumetric water content from gravimetric water content or vice versa. Estimate the volumetric water content from the gravimetric water content. This can be achieved, theta volumetric water content is, volume of water, by total volume. Another hand, gravimetric water content w is mass of water by mass of solids. This can be achieved, if you know the density of soil. The dry density is, can be obtained from ρ_b by $1 + w$, w is in fraction, not in percentage.

So, then using this ρ_b by $1 + w$, we can obtain ρ_d . So, ρ_b can be obtained if you know the total mass and total volume of the soil. Then, you get ρ_d . Mass can be estimated, but how to estimate volume of the soil under different states. For example, you take a small soil sample. This, soil sample, initially is at completely saturated state. You probably mix the soil sample at 1.5 times of its liquid limit, you made a kind of slurry and then put it in a small container, and then allowed to dry.

When soil dries with time, how to estimate its volume, because, you want to estimate the complete distribution of water content. So, in that particular case, the estimation of volume is important. Another point is, often we estimate the SWCC's using different techniques in the laboratory, where we get water content versus suction gravimetric water content versus suction. This needs to be translated to theta volumetric water content versus suction. Or it could be represented, void ratio versus suction also. Void ratio versus suction also. But anyways, this is not very commonly used. Most commonly used one is theta versus psi. This is very useful to solve the flow equations.

But generally, in laboratory we get w versus psi. How to translate this one to this one? This may be somewhat like this. You will not get water content variation in especially clays is significantly high. Water content could be as high as maybe 10 percent. 10 percent here and it can become 10 to 20 percent even for bentonites. In Bentonites, it could be as high as, more than 100 percent. For powdered bentonites, slurries, it could be very high value. Then, w itself without percentage it could be as high as 10 maybe or 20. It could be as high as 10.

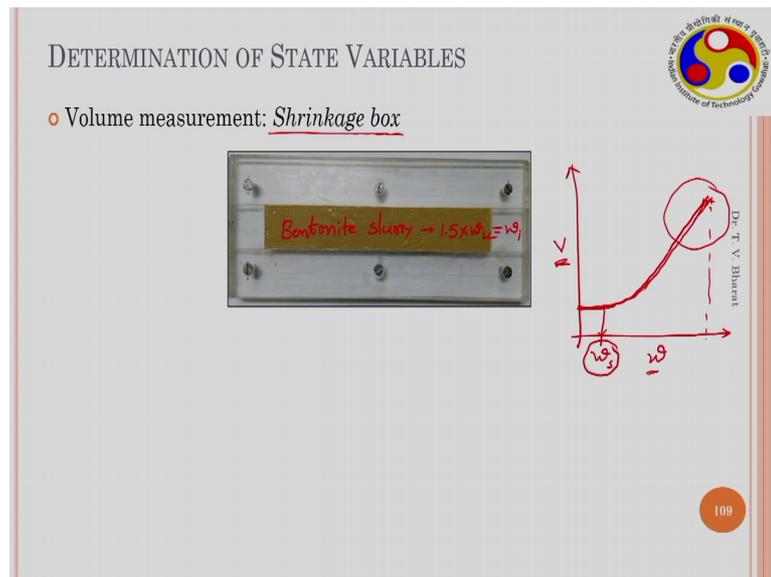
And from that, as the suction increases, this value decreases. And most of the time, we do not obtain the air entry from this particular representation. So, often we represent into

theta versus psi representation. So, theta versus psi representation will give you the air entry value. Or sometimes, it is also represented in terms of degree of saturation versus suction. This is more accurate way of estimating air entry value of the soil. So often, from the laboratory obtained SWCC in the form of w versus ψ will be represented in terms of θ versus ψ or s_r versus ψ . For that we need to understand how the volume of the soil changes during the drying process.

So, here from the initially saturated to complete dry state; how the water content is changing? For the corresponding water content, we require to understand how the volume is changing; so that we can estimate how θ can be, how θ can be estimated from this. So, then correspondingly, s_r can be obtained using, e equal to $W G$ by S_r . So for that we require a relationship between volume of the soil sample, versus water content, which is called volumetric shrinkage curve or volumetric shrinkage behavior.

So this is a relationship between another constitutive relationship; that is, a relation between, volume of the soil sample, versus the water content. Or, it could be void ratio versus water content. So, this constitutive relationship is important. Another two constitutive two relationships we already defined which is SWCC and hydraulic conductivity functions. So, to understand these 3 constitutive relationships, we need to estimate the water content measurement. We need to see and volumetric water content measurement volume measurement of the soil suction measurement and hydraulic conductivity measurement.

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So, we will start with our volumetric, volume measurement of the soils. So, essentially, we need to establish the entire volumetric water content. Where, total volume of the soil plotted on y axis, and water content is plotted on x axis; then, initially when you start with very high-water content, which is, maybe about, 1.5 times liquid limit of the water content. Then, as the water content decreases, the volume decreases. And beyond that, volume does not decrease significantly. And then, it reaches a constant volume at shrinkage limit point. So, this is shrinkage limit point. Generally, we estimate only this particular point. Using mercury replacement method, we estimate only this point. But, in this particular case we require entire relationship of volume versus water content. That is, volumetric shrinkage behavior.

So, we have several techniques. One is, direct measurement of volume. So, if you take a shrinkage box, where you have a groove to put soil mass. So, initially a slurry; this is a bentonite slurry, which is mixed with water at, 1.5 times. This is initial water content of the soil. So, where it is completely mixed and then, it is placed in this particular groove, and which is allowed to dry naturally, when it is allowed to dry, the volume of the sample decreases. Using a Vernier, we measure the volume. So, you measure the width, you measure the length, and you measure the depth of the soil sample. And this measurement may not be that accurate, because the sample may not maintain its uniformity. Then it shrinks.

So, this provides some information. And especially, initial state, it is much advantage and easier to measure. But, as when the soil gets dried so when the soil does not shrink uniformly anymore then it is very difficult to estimate the volume. So, with then for, you have alternative techniques. Other one is wax coating method which is very commonly used technique which is where soil samples when we obtain from in the field to maintain its water content. Generally, we put a wax coating around the samples, and then carry it to the laboratory. Same technique could be used for estimating the volume of the samples at different water contents.

So, in this particular technique, samples, initially, when the sample is mixed with water content, close to 1.5 times liquid limit and then allowed made to slurry and fill it in a shrinkage cup. And then once it comes to one particular consistency, after drying. Then, once you can handle a soil sample, then you can put a wax coating around it. And then you can directly immerse into the wax or you can directly immerse into water for measuring its volume.

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The slide, titled "DETERMINATION OF STATE VARIABLES", illustrates the wax coating method for volume measurement. It features a graph on the left showing Volume (V) on the vertical axis and Water Content (w) on the horizontal axis. A curve represents the soil's behavior, with a point marked by dashed lines extending to both axes. To the right, a diagram shows a soil sample (hatched rectangle) inside a shrinkage cup (dashed rectangle). Handwritten notes include the equation $V = V_{\text{Soil}} + V_w$, the maximum mass M_{max} , the wax density $\rho_{\text{wax}} = 0.9 \text{ g/cm}^3$, and the masses M_{Soil} and M , along with the wax volume V_{wax} . The slide also includes the IIT Bombay logo and the name Dr. T. V. Bhargava.

So, you have a soil sample. So, you have a wax coating around this. And now, this is immersed in water bath. Using Archimedes principle, the change in the volume of water or total volume can be obtained. Total volume of the sample can be obtained. This total volume consists of volume of soil plus volume of the wax. As you have wax around the soil sample water does not enter in to your soil sample. So, this can be obtained,

estimated using Archimedes principle. Now after that, you can peel of the wax around the soil and then weight it, you can weigh the wax mass. So, based on the density of wax, which is about 0.9 gram per centimeter cube, you can estimate what is the volume of wax that is coated around it. So, volume of wax is obtained. So, then you can obtain the volume of soil.

So, this technique could be used for individual soil sample. So, in the estimation of volume versus water content, for each data point on this particular curve, you need to conduct one sample. One test; in this particular manner and obtain the volume of the soil. And corresponding weight you can obtain. And then, you can oven dried, oven dry the soil sample and obtain the water content. So, water content and volume can be obtained in this particular technique by utilizing so many number of samples at different water content states, and a volume could be obtained in this particular way.

But this is a tedious technique because for each you need to prepare probably, tens of samples. And then each sample, once the time is allowed for drying, each sample need to be taken and then wax coating is applied. And after that it is immersed in water and volume is measured, and it is peeled off. Again, the weight is measured. And then it is kept in oven for the change in the water content. After 24 hours, you take the sample and estimate its water content. And, now water content versus volume is obtained. Similarly, for other soils, it should be followed. So, then you obtain this particular curve.

However, it is seen that and when wax molten wax which is a hot wax, which is applied, you need to wait for some time, to cool down the wax which is coated around it. So, during the cooling process, it exerts some pressure on the soil sample. So, initially when soil consistency is very loose, soft, then it causes a small shrinkage of the soil sample also. It does not maintain the same volume as it was earlier. So, during the drying process of the wax, when those wax from liquid to solid phase when it phase transformation takes place, it exerts some pressure on the soil sample. It changed the volume of the soil samples slightly. That is what we have observed recently.

And here, either the mass of the wax or directly mass of the soil sample; after peeling off can be obtained mass of the soil directly and before peeling off, the total mass also can be obtained. This difference also gives you the mass of wax. So, that way also mass of

wax can be obtained. So, this is how the wax coating method could be applied for estimating the volumetric shrinkage behavior of soils.

Thank you.