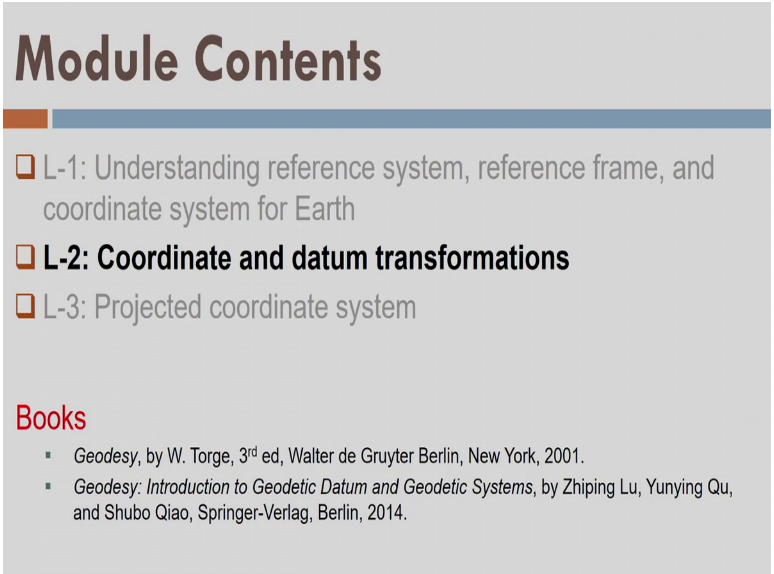
Higher Surveying
Dr. Ajay Dashora
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module - 2
Lecture - 03
Coordinate System and Reference Frame

Hello everyone. Welcome back on the course of Higher Surveying.

(Refer Slide Time: 00:37)



Well, in the last lecture of module 2, that was lecture 1, we defined what is the reference system, reference frame and coordinate system for the planet like earth, first we define what is the reference system. So, reference system we said it is a combination of 3 orthogonal axeses; which are centred at an origin. Then we said ok, this is the ideal definition of a reference system, what if I take measurements, then using some measurements on surface of earth, if I determine the exact location of the centre of the reference system and then also define where the coordinate axeses or reference axeses are passing, right.

So, that is called the realisation because we are doing some kind of mathematical calculation, some kind of observation. So, this kind of realisation is called reference frame; that means I have an ideal concept called reference system. If I realise it by some calculation mathematics and so on, the realised reference axeses are called reference frame. We have defined what is the inertial frame and non-inertial frame. Remember we said first that a point on the surface of earth is also rotating with the earth. So, we need to measure the time also, in other to define a point xyz correctly. Now, we also realize that if there are 2 points both are lying on the surface of earth and I want to locate them related to each other, I know that both points are rotating in the same speed and they are also revolving with the earth.

So, I need a reference frame or reference system that is, rotating with the earth, such a frame we called non inertial frame, or the body reference frame. Well, with this what is the advantage? The advantage is that I can skip the measure of meant of time, ok. Later on we also said for the earth if we have a non-inertial frame, we will call it terrestrial reference frame. International terrestrial reference frame is one of the reference frame. Well, after that we said that we need to attach a coordinate system with each and every reference frame to define a point P in 3 dimension.

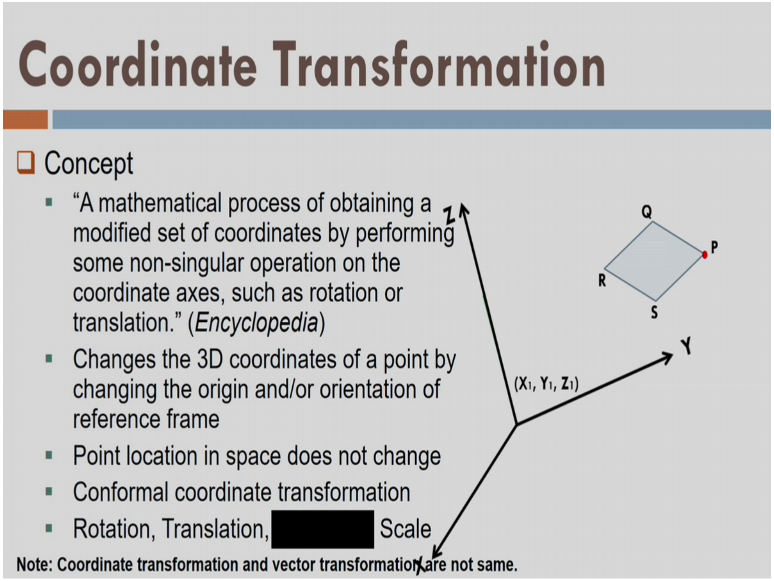
It can be defined with the help of physical coordinate system, that is distance elevation angle and Alzimer's angle. It can be defined with the help of Cartesian coordinates, like xyz. It can be defined with help of ellipsoidal or geodetic coordinate system, lambda phi h all 3 in small. Or it can be astronomic coordinate system, capital lambda capital phi capital H. In case of geodetic or astronomic coordinate system, we measure the height from a curvilinear surface. For geodetic system or ellipsoidal system, we measure the ellipsoidal height from ellipsoidal surface in the direction of perpendicular to the surface of ellipsoidal. So, we call it the direction is ellipsoidal normal.

If we do the measurement with respect to geoid or geoid model, then we call the height as orthometric height represented by capital H. Then we have defined 2 type of reference frame which are used for the whole earth and one is the geocentric; that means the origin axis is centered at the center of mass of earth. And there if it is not centered; that means, the reference axis the origin of reference axis is not centered at the center of mass of the earth, we call it geodetic reference frame.

Then we also defined different type of reference frame for the surface of earth. That is for the local reference frame. Well, with this definitions and concepts, in this lecture we are going to understand how to do the transformations between the coordinate system, and reference frames. So, that is why it is written coordinate and Datum transformation.

Remember the word Datum comes from reference frame. Reference frame is also called Datum. Datum could be flat plane like xy plane or it could be a curvilinear surface like ellipsoidal surface or geoid surface. Well, with this idea let us go ahead in this lecture.

(Refer Slide Time: 05:06)



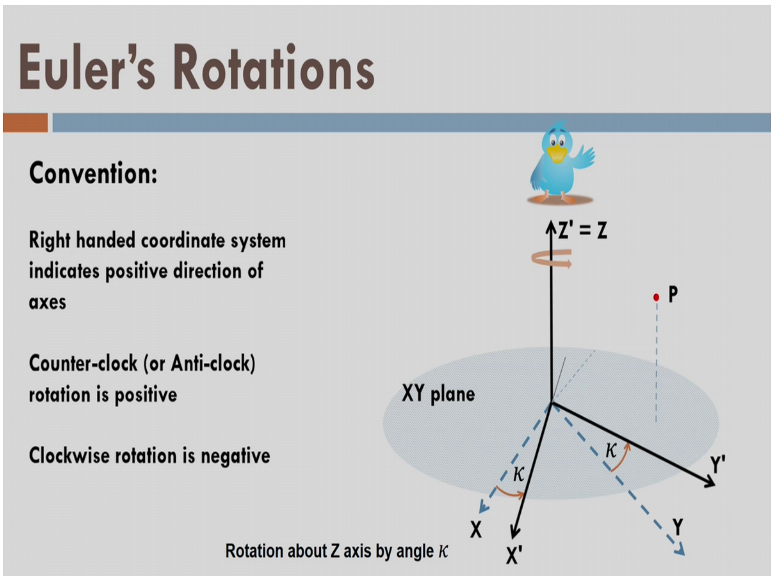
So, the coordinate transformation concept is I tried to find out it is definition on the encyclopedia and definition I am reading it word by word; a mathematical process of obtaining a modified set of coordinates by performing some nonsingular operation on the coordinate axis, such as rotation or translation where just note the highlighting points are, we are performing some nonsingular operation on the coordinate axeses. Meaning is I am not going to change the location of point P. My point P will on the surface of earth will remain intact rather, I am going to change the location of the coordinate axeses and it is origin.

So, that is the idea here, well, it is this concept is quite different from the vector transformation. In vector transformation, we change the location of point P. It generally happens in robotics. Well, suppose robot is moving it is arm. So, the point P which has located at the end of arm of the robot, it will move like this. So, remember in case of coordinate transformation, we are not going to change the location of point P. Rather we are just going to translate, rotate, scale the coordinate axeses or their reference system or reference frame. So, nonsingular operation means, it gives me it should be very trivial and it should give me a unique answer.

Suppose if I rotate my coordinate axeses by angle theta, I should get unique answer; that means, I should not have multiple answers there, fine. With this idea so, I can say that we should first understand, conformal coordinate transformation, ok. Conformal means if there are 2 lines and they are maintaining some angle like that, the this is the angle. So, this angle will remain same of transformation. Such a transformation is called conformal coordinate transformation. So, rotation translation scale, if they provide me this kind of transformation; that means, they are not going to change the angle between the 2 lines before and after transformation, they will become conformal.

So, let us see graphically what is the meaning here. Let us say this is my coordinate system X, Y, Z for point P. Now I am translating my reference frame, ok. I am rotating my reference frame. So, that is my new origin now after rotation and translation. X 1, Y 1, Z 1, ok. So, what are the new coordinates of point P? If I determine the new coordinates of point P, it is called coordinate transformation. So, I need to consider the rotation of the coordinate axeses and the translation of origin. So, if say there are P Q R S 4 points are there. And they are maintaining a rectangle; that means, if in case of a rectangle, the angle between the 2 lines which are at certain value that is 90 degree. So, after transformation, this angle should remain same, then and then I will call the transformation as conformal transformation.

(Refer Slide Time: 08:11)

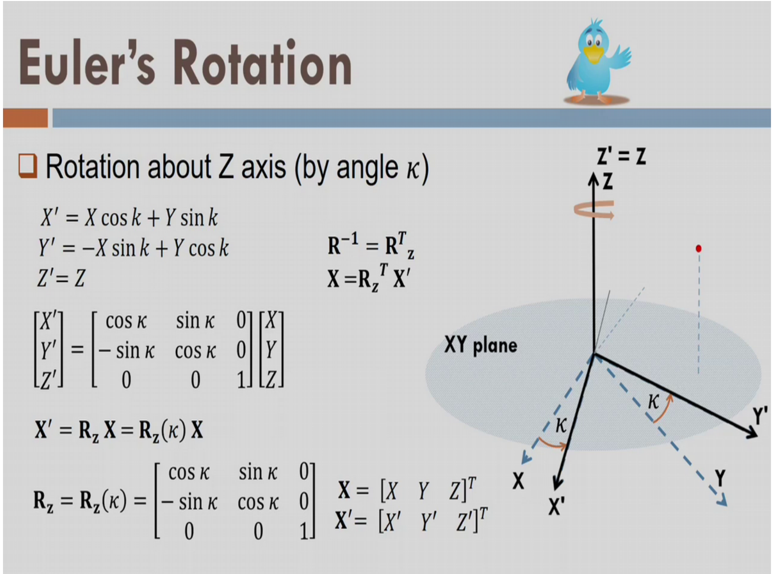


Now, let us look into the rotation which are conformal in nature. So, we have Euler's rotations. So, let us look the rotation about Z axis by angle kappa. What does it mean? Let us say there is reference frame X, Y, Z which is orthogonal, it is slightly appearing started, because I am trying to show in 3D. So, there is a point P, which I want to locate with respect to capital X, Y, Z reference frame. So, let us say this is my X Y plane and this is my look direction; that means being an observer I am looking from the top of Z axis or the arrow side of the Z axis.

Well, so, when I am looking from the top or from the arrow side, if I do the anticlockwise rotation, that is called positive rotation, ok. So, let us see this is my anticlockwise rotation. Let us say I call this angle kappa. So, the kappa rotation is now positive, ok. So, if you just look at thing, I have new coordinate axeses which are orthogonal in nature. Still, because they have rotated with the angle kappa and both axeses have rotated by angle kappa. So, I have new system X dash, Y dash, Z. Why because, nothing has changed with respect to Z because I have rotated the coordinate axis X Y with respect to Z. So, it is kind of this kind of rotation is happening, so, Z axis principle same shown by the thumb.

So, earlier I have these 3 coordinates if you see carefully. X, Y, Z, now I have these 2 coordinates, that is this X dash and Y dash shown by the black dotted lines. And blue line Z is not changed. So, I have not changed the line showing the Z coordinate of point P, well. So, this is my convention that right handed coordinate system indicates positive direction of axeses. Anticlockwise rotation is positive. So, clockwise is automatically negative rotation.

(Refer Slide Time: 10:20)



So, let us go ahead. So, starts the rotation about the Z axis by angle kappa. So, let us look what is the meaning here. So, as I already declared that there is no change of the Z axis, or the Z coordinate of point P. So, I have Z dash equal to Z.

So, let us say X Y plane and then again I am rotating by certain angle kappa here. So, you know this angle kappa. So, earlier axeses are X and Y. Now new axeses are X dash Y dash, fine. So, if I want to do or if I want to find out the coordinates which are earlier like that X, Y, Z now they are this one, ok. So, I can write X dash is equal to X cos kappa plus Y sin kappa, then Y dash equal to minus X sin kappa plus Y cos kappa. And Z dash equal to Z. You can prove it yourself also. It is very easy or even if you refer any book on coordinate transformation, or any literature on internet, you yourself can see this is very, very trivial work to do, you try yourself prove it.

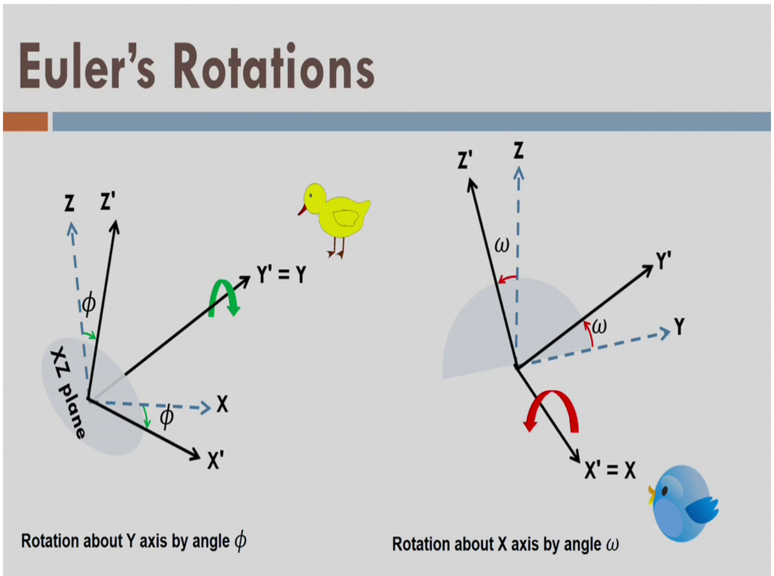
Now the more critical thing in this derivation is I can write the X dash, Y dash, Z dash coordinates in terms of X, Y, Z using a very elegant matrix form. And the elegant form of matrix showing the rotation angle kappa is shown in the next step in the same screen. So, that is cos kappa sin kappa 0, minus sin kappa cos kappa 0 0 0 1. So, if you multiply the 2 matrices on the right hand side of this equation you will get all 3 equations. Now is it not very nice that we can use the rotation matrices? And using the matrix notation it is very easy. So now, I am writing in the matrix notation capital X dash equal to R Z, that is the rotation about Z axis. So, it is nothing but R Z is my rotation matrix showing that it is representing the rotation about Z axis by angle kappa.

So, sometimes it is also written like R Z in bracket I write angle kappa. So, it is always required desired to write that way only. Why because R Z indicates some rotations happening about Z axis, and kappa indicates the amount of angle by which rotation is happening, well. So, capital X is indicating my input coordinate system or my original coordinate system, and X dash is representing the transformed coordinate system. So now, I can write the R Z kappa as my rotation matrix which is shown here, as well as X and X dash as my column vector matrices. So, again the look direction is from the top.

Now, more important property of this matrix R Z. R Z is a rotation matrix representing the rotation about Z axis. They are orthogonal in nature. And as a result what is the orthogonal matrix? If I take the inverse of the orthogonal matrix, I need to take only the transpose. So, I can write R inverse equal to R T; that means, if I want to convert back from X dash to X; that means, I want to find out X coordinate system given X dash, Y dash, Z dash. So, what will I do? I need to take an inverse of the matrix R Z. Since inverse is very easy, take the transpose then you are done.

So, you can now write X equal to R T Z X dash.

(Refer Slide Time: 13:54)

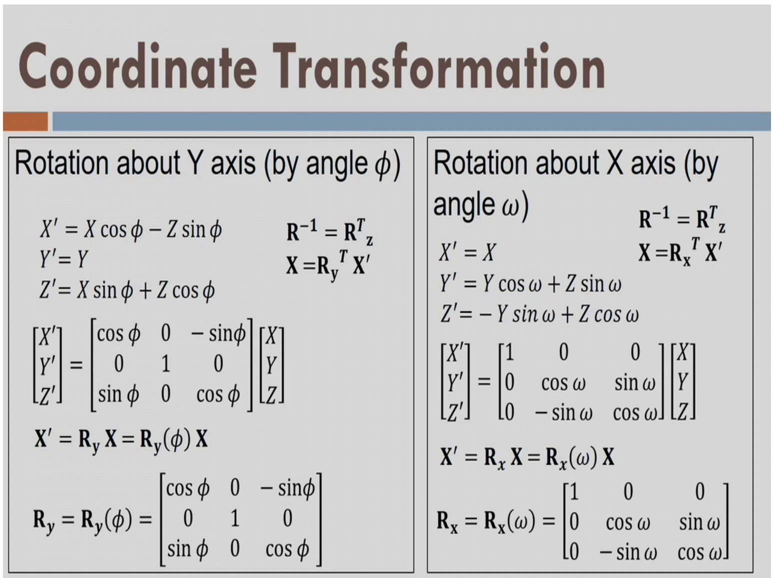


So, let us go ahead for the Euler's rotation about Y axis by angle phi and about X axis about angle omega. That is a representation here. So, let us look into the first about Y axis. So, let us see this is my look direction; that is, from the arrow side. If I look from the arrow side and anticlockwise rotation that will be positive rotation about Y axis. So, this is my anticlockwise rotation. Now I am rotating it this way. And this rotation is angle phi. Now my both axeses X and Z they are turned into X dash and Z dash. Let us look into the rotation about X axis by angle omega.

So, yes, this is my look direction shown by the fish here. So, fish is looking in the X direction from arrow side. And so, I have this positive rotation direction. Well, this is the rotation happening, and this rotation makes the Y and Z axis as Y dash Z dash. And the angle of rotation is omega. Now you can understand you can comprehend, you can easily appreciate it what do you mean by a rotation about certain axis, and what is a positive rotation, what is a negative rotation. And now you see X dash will remain equal to X. In the same fashion, if I rotate about Y axis. So, Y dash will be equal to Y there is no change in the Y coordinate of a point P, or any point P 1 P 2 and so on.

So, that is the idea one should be clear about the rotations. And they are called Euler's rotations omega phi kappa about 3 axis X, Y, Z respectively, ok. What about the rotation about Y axis?

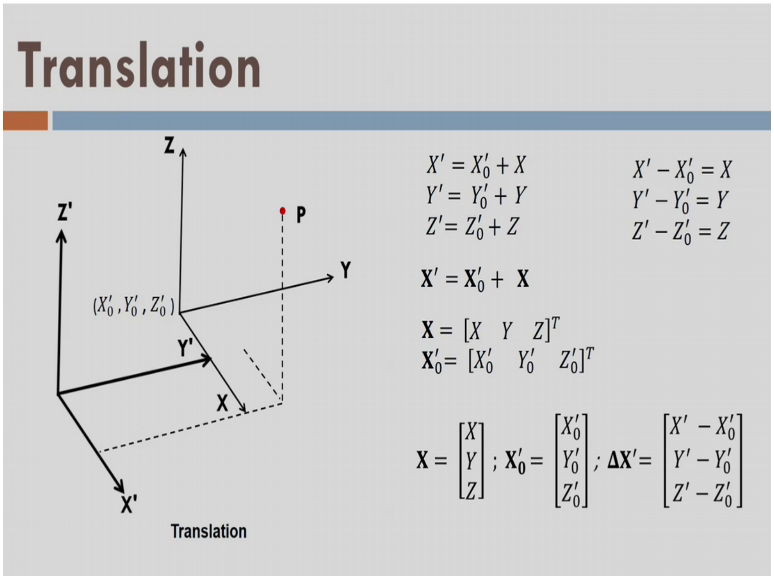
(Refer Slide Time: 15:44)



Now you see that I am writing X dash equal to R Y X more, over it is better to write R Y if the amount of angle phi, always desired. Otherwise you may think that by which amount of angle rotation has happened about Y axis or any other axis, so, better to use the proper notation. So, let us write R Y and then in the bracket amount of angle by which rotation has been done with respect to Y axis, ok; so, right again I am writing in an elegant fashion. Similarly, rotation about X axis by omega angle X dash equal to X, Y dash equal to Y cos omega plus Z sin omega; Z dash equal to minus Y sin omega plus Z cos omega.

So now you can see again the rotation matrix R X, and amount of rotation is omega. So, I am writing R X omega. And then capital X is representing the X, Y, Z coordinate system. So, rather it is representing X, Y, Z coordinate of point P. So, X dash is representing X dash Y Z Z dash coordinate of point P. So now, you have understood what is a rotation about 3 axeses, again they are orthogonal in nature. So, I am writing R inverse is equal to R transpose, same for the rotation about X axis also.

(Refer Slide Time: 17:11)



Now let us come to the translation. What is the translation? If the origin of the reference frame shifts without changing the orientation of the coordinate axeses it is called translation. So, let us see this is my X coordinate; Y coordinate and Z coordinate of point P in X, Y, Z with the reference frame.

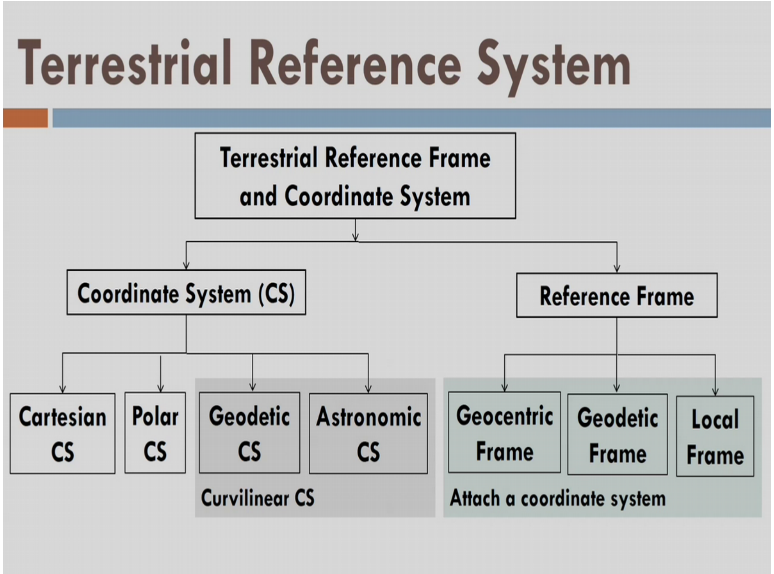
Now, it is translating without changing the orientation of the reference directions or reference axeses or coordinate axeses. Now let us look into this thing. So, this is my X dash Y dash Z dash. This is my new X, Y, Z. You can see it easily; it is very easy to understand, ok. Since, I have shifted my reference system in the left hand direction and if I want to find out the new coordinate of point P that is X dash Y dash Z dash. I need to add the shift between the 2 reference frames. Let us see that if this shift is represented by delta X in X direction, delta Y in Y direction, and delta Z in Z direction. Then I need to add let us see X 0, Y 0, Z 0 plus delta X delta Y delta Z.

Here so, these things are written like that. In the new coordinate system, X dash, Y dash, Z dash, I am adding X 0 dash that is this reference frame origin in the new coordinate system. Or in the new reference frame; that is, X 0 dash Y 0 dash and Z 0 dash is the coordinate of the origin of X, Y, Z system. But X 0 dash Y 0 dash and Z Z 0 dash I defined with respect to the new reference frame that is X dash Y dash Z dash. So, you can just imagine that the origin of reference frame X, Y, Z is one of the point in the X dash Y dash Z dash. And hence since I am doing the translation, I need to add X 0 dash Y 0 dash Z 0 dash in the earlier reference coordinates. And so, when I do it I will get these 3 equations, ok.

So, I can now write it X dash equal to X 0 dash plus X. So, X 0 dash is a constant value, right. So, again I am writing all these thing in a matrix notation where I am representing my X and X 0 dash and delta X dash by column vectors. So, you can just write it do it yourself on a paper, you will understand it very easily. What about the scale? Scale is something enlarging something, or shrinking something. If the scale is conformal; that means, shrinking is happening in all directions together or enlarging happening in all directions together; that means, scale is same in all the directions and each and every point.

So, that will be changing the scale of the coordinate axeses. Now, where it is used? You can imagine suppose there is a ellipsoidal surface or there is a ellipsoid. If I change the scale; that means, I am changing the ellipsoid itself. So, when we use the Datum transformation or we change the reference frame, this term of a scale comes into picture. We will talk about this thing in the coming slides, let us see.

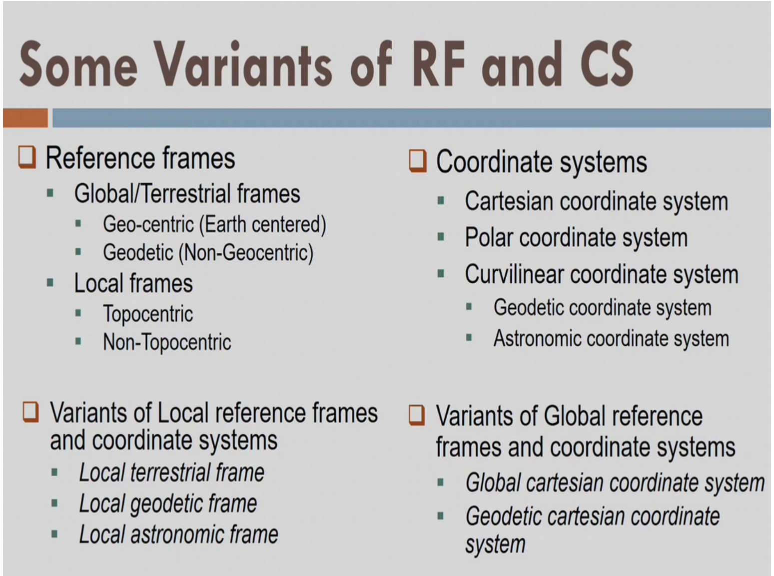
(Refer Slide Time: 20:38)



So, this is again the summary of the terrestrial reference frame, where I am defining coordinate systems in a 4 ways Cartesian polar geodetic and astronomic, reference frame in the 3 ways geocentric geodetic local. So, as a result we have curvilinear coordinate system, and we need to attach a coordinate system with the each and every reference frame.

Most of the time, it is convenient to use Cartesian coordinate system for the purpose of calculation. But for the purpose of visualization, geodetic coordinate system or the curvilinear coordinate system are much better. Coming to the reference frame, it is we generally use the local reference frame irrespective of any coordinate system. Let us go ahead.

(Refer Slide Time: 21:21)



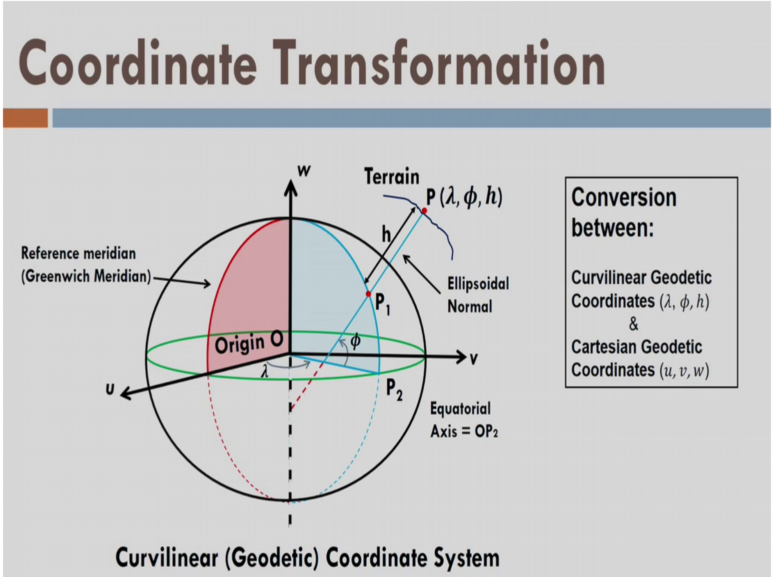
So, these are the variants as we discussed in last lecture also. So, these variants are related with the reference frame. For example, I can have global or terrestrial reference frame so, it could be geocentric it could be geodetic. Then we have local frame, it could be topocentric, it could be non topocentric. What is a topocentric frame?

A frame a local reference frame which has origin on the surface of the earth, so, that is called topocentric local reference frame or topocentric reference frame. Coming to the coordinate system, we have Cartesian coordinate system, polar coordinate system, curvilinear coordinate system, that is geodetic and astronomic. So, then I can make the variants of the local reference frame; that means, in a local reference frame, I can do a local terrestrial reference frame, I can also have local geodetic frame or local astronomic frame, ok.

So, what are the variants of global reference frame? Global Cartesian coordinate system, geodetic Cartesian coordinate system and so on. We will see all this variants in the coming slides we will where we will perform the coordinate and Datum transformations. Also, we are going to see the application called GEOTRANS which is a free application, a free software that will do your work in a very, very nice manner without significant efforts. So, let us see both things first the theory part and then the experimental part, ok.

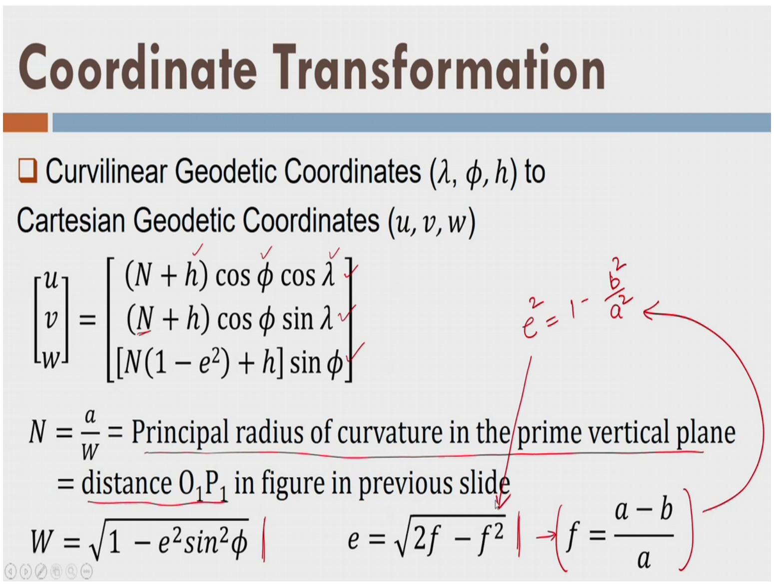
So, the coordinate transformation is there.

(Refer Slide Time: 22:46)



Let me explain, this is my ellipsoidal normal, passing through a point P with respect to the ellipsoid shown here. So, that is P 1. So, the point here ellipsoidal normal. That is my equatorial axis O P 2, fine. This is my lambda angle, and this is my phi angle, and they are called geodetic longitude and latitude. Well, I want to convert this geodetic longitude latitude and height h into u, v, w Cartesian coordinate system. Remember, I am not going to change the reference frame here. It is only I am expressing the coordinates of point P in 2 ways. One is geodetic way, and another is Cartesian way. So, I want to do this coordinate transformation. So, let us transform the geodetic longitude latitude and ellipsoidal height into Cartesian coordinates u, v, w, ok.

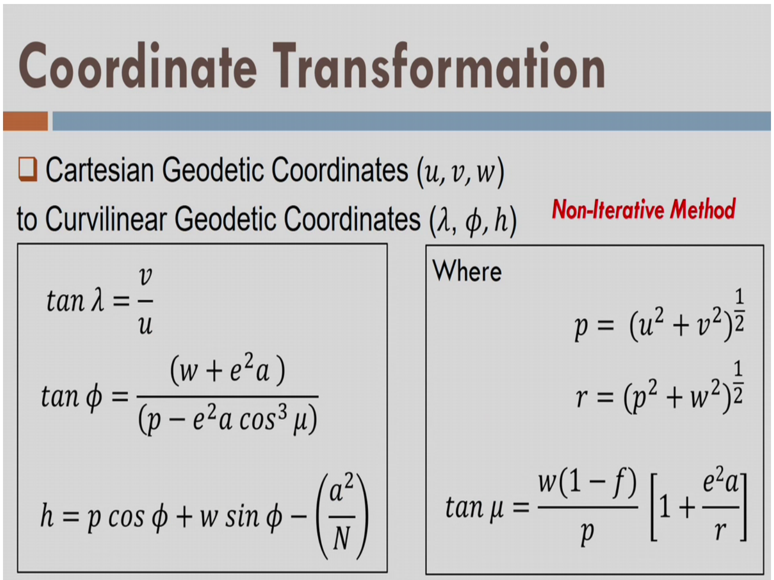
(Refer Slide Time: 23:48)



So now we are converting the lambda phi h or the curvilinear geodetic coordinates to u, v, w, ok. So, these are the 3 equations given here, fine. You can see easily here h is my ellipsoidal height, lambda is my longitude and phi is the latitude of the observers location. Now here we define terms N. What is N? N is the principle radius of curvature in the prime vertical plane. That is distance O 1 P 1 in figure in previous slide.

So, let us go back. So now, you can see that distance O 1 P 1 from this point O 1 to point P 1, it is called N, right ok. So now, you can use this expressions here. And this is the expression of the eccentricity of the ellipsoid, which is also given by like this, and now you can derive where if I define my f that is flattening of the ellipsoid given by this. So, using this here you can derive this expression, ok.

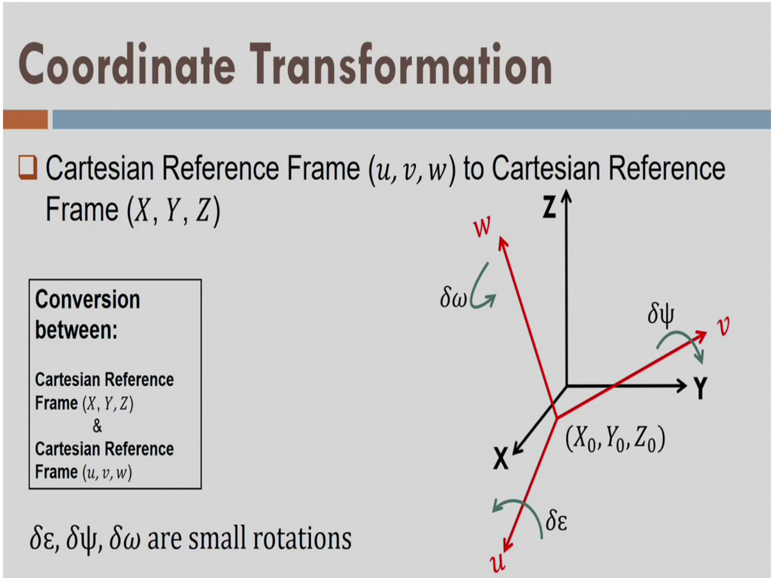
(Refer Slide Time: 25:10)



So, let us go ahead for the geodetic Cartesian coordinate, u, v, w to curvilinear this one; that means, I am doing the reverse transformation. Here, I will first find out tangent of lambda, there I will find out lambda value. So, tangent lambda equal to v by u, ok. Then I am finding out tangent of phi where small w plus e square a T minus e square a into cos nu where an h is given by p cos phi plus w sin phi minus a square by N. Here the small p is equal to the u square plus v square under root r v square plus w square, which is nothing but u square plus v square plus w square under root. And then we are defining tangent mu by this quantity.

So, once you find out p r and tangent mu, then you put this values on the left hand side equation, and then you will find out lambda phi and h. This method is called non iterative method. There are many iterative methods are there you will find it, ok.

(Refer Slide Time: 26:14)



Now coming to the Cartesian reference frame u, v, w to Cartesian reference frame X, Y, Z. Remember, we use u, v, w for a non-geocentric reference frame X, Y, Z for a geocentric reference frame.

So, understand in that way only; that means there is translation, because u, v, w reference frame is not on the geocenter of the earth. While X, Y, Z is on the geocentric of the earth. So, there is a clear translation between the 2. Also there is a rotation also, but this rotations are very, very small. And that is why we say that these rotation amount between the 2 reference frames are very, very small. And so, we are representing by amount delta delta omega delta psi and delta epsilon about different different axeses.

So, if I do these 3 rotations. I will be able to align u, v, w axeses into a capital X, Y, Z axeses. Or I will match the origin of 2 reference frame.

(Refer Slide Time: 27:15)

Coordinate Transformation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + (1+\delta s)\begin{bmatrix} 1 & \delta\omega & -\delta\psi \\ -\delta\omega & 1 & \delta\varepsilon \\ \delta\psi & -\delta\varepsilon & 1 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$\mathbf{X_0} = [X_0 \quad Y_0 \quad Z_0]^T$
= Coordinates of origin of (u, v, w) reference frame
in (X, Y, Z) reference frame

δs = Differential scale change

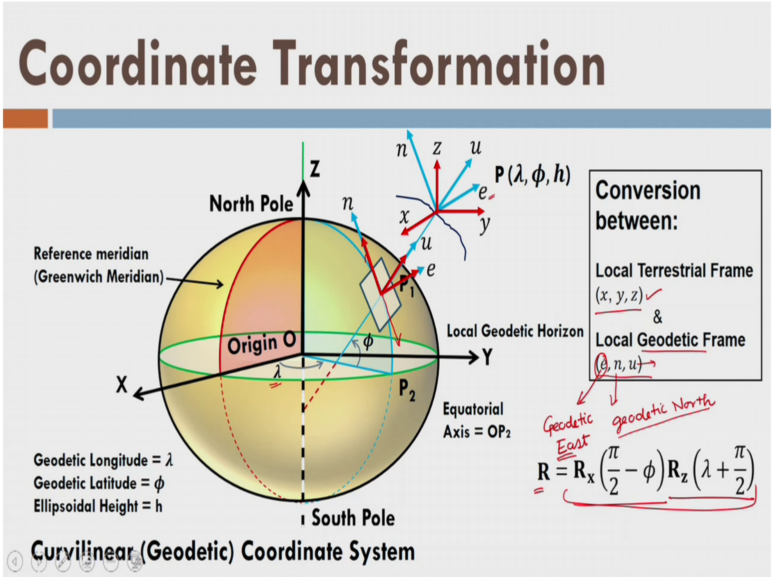
$\delta\varepsilon, \delta\psi, \delta\omega$ = Differential rotations between two frames (to align (u, v, w) reference frame to (X, Y, Z) reference frame

So, see this is the coordinate transformation and here, rather it is a reference frame transformation, I should not say coordinate transformation, but coordinate transformation is a kind of a general and generic term. So, we are using it, but it is a basically I am transforming one reference frame into another, ok.

So, this is my rotation matrix given in the delta omega delta psi and delta epsilon, ok. And then there is something called delta s plus 1. Delta s is a differential scale change ok. So, what is a scale change here? Scale change is if the reference axeses or if the ellipsoid of one reference system is changing, and the sizes increasing or decreasing. So, that in that case, in case of increase in the size I will have delta s positive. Let us see the one reference frame have on ellipsoid. And the second reference frame ellipsoid is 10 percent more. So, delta s will be something like that positive. And similarly it will be negative if the reference ellipsoid of second reference frame or transformed reference frame is smaller.

So, that is a factor is there, but in general delta s is considered very small. Here we are adding s X 0, Y 0, Z 0 and not the u 0, P 0, w 0. And the reason being since I am transforming u, v, w into X, Y, Z and hence I should know the coordinate X 0, Y 0, Z 0 which is the coordinate of origin of u, v, w in X, Y, Z reference frame. In the words it is difficult to understand, but if you draw the things on a paper and do the translation, you will come to know why we are adding X 0, Y 0, Z 0; when we transfer u, v, w to X, Y, Z. It is that simple a case. Let us go ahead.

(Refer Slide Time: 28:56)



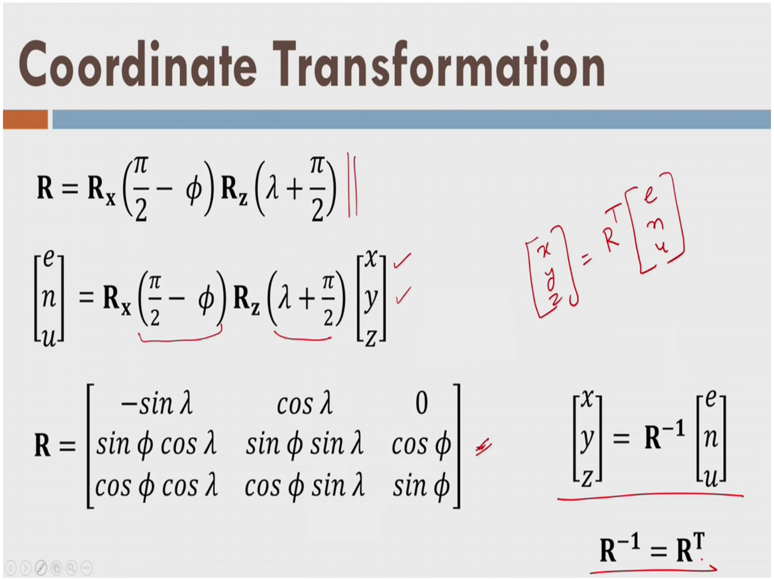
Then I am trying to do the conversion from local terrestrial reference frame to local geodetic reference frame. Both reference frames they have their origin at point P here. So, let us define x, y, z and e, n, u. So now, you know that e is in the direction of geodetic east. N is in direction of geodetic north. So, what does it mean? Here, this is your geodetic North here, indicated by the meridian line.

Now, in order to understand the conversion between the 2, let us define the local geodetic horizon at point P 1. As shown in the animation. And bring both the systems at point P 1, because it is very easy for the purpose of demonstration. So, we are just bringing them at point P 1 for the purpose of demonstration only. You guess because it is very easy to understand. Now you can understand that we are rotating the axis X in East and odd for the conversion.

So, this is my e, n, u at point P 1, and similarly this is my x, y, z at point P 1. And we know that small x is parallel to capital X. So, you can imagine that you are rotating the capital X axis, such that it will meet with the small e axis. Or you can do the same thing with this small x axis. So, for that purpose so, let us look into the animation. This is the rotation I need to do. So, you can see here the rotation is first about Z axis with amount angle lambda which will bring your x axis in this direction, right. And now if I rotate the x axis further by phi by 2 or 90 degree, it will be aligned with the e axis or the East axis here, right.

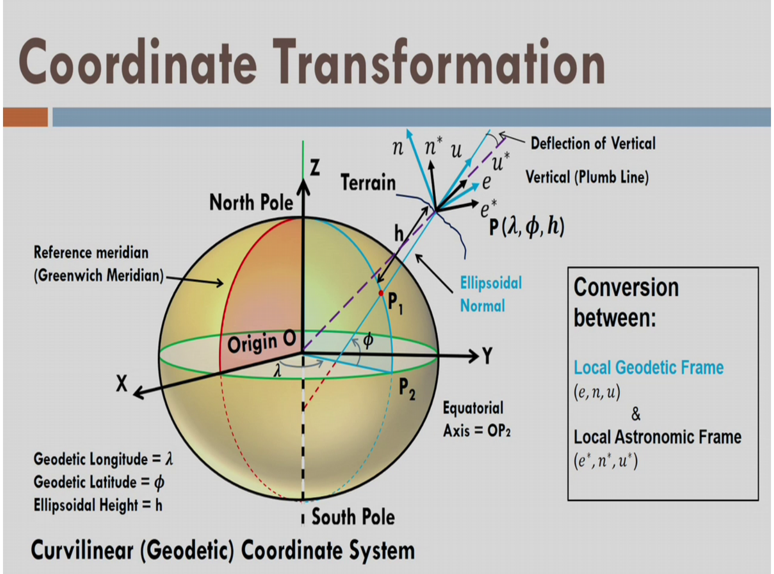
So, we rotate by lambda plus phi by 2 about z axis. So, it is indicated here, fine. What next now I need to align my y axis with the North and z axis with the u axis. So, for that purpose we need to rotate the system with respect to the new x axis which is already aligned with the East axis. So, let us do this rotation by phi by 2 minus phi. So, I am writing this thing that I am rotating about x axis by phi by 2 minus phi. So, we have aligned our small x, y, z system to the easting northing and up system. So, this is the way we establish the rotation matrix of this one. Now, this is my total rotation matrix.

(Refer Slide Time: 31:56)



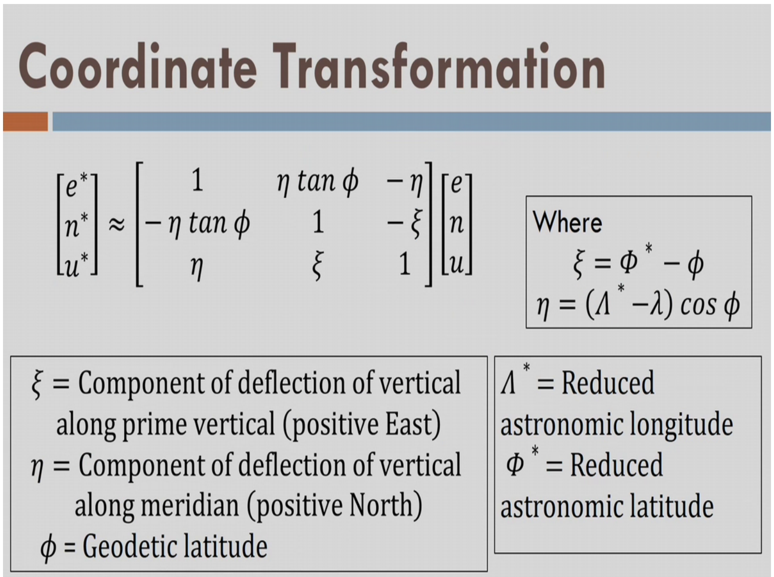
And I can write this one here. And now I am this is the formula I am converting my x, y, z to e, n, u. And there is no translation because both are centered at the point P, ok. What about the rotation matrix R? The rotation matrix R, if I put the values of phi and lambda here I will get this kind of rotation matrix here, ok. So now, you can see here that rotation matrix R is again orthogonal in nature, and as a result if I take it is transpose I can get the inverse of matrix R. So, even by taking just transpose, I can write that x, y, z is equal to R transpose e, n, u, like this.

(Refer Slide Time: 32:57)



Now, coming to the conversion between local geodetic frame; e, n, u to local astronomic frame, ok. So, let us look into this thing. Very surprisingly, if you see the both the coordinate system one is astronomic and one is geodetic. So, geodetic as we told earlier also the u axis of geodetic is aligned in direction of ellipsoidal normal. Whereas, in case of astronomic frame the u star axis is aligned in the direction of plumb line, or the gravity direction at the point P, on the surface of earth, well.

(Refer Slide Time: 33:36)

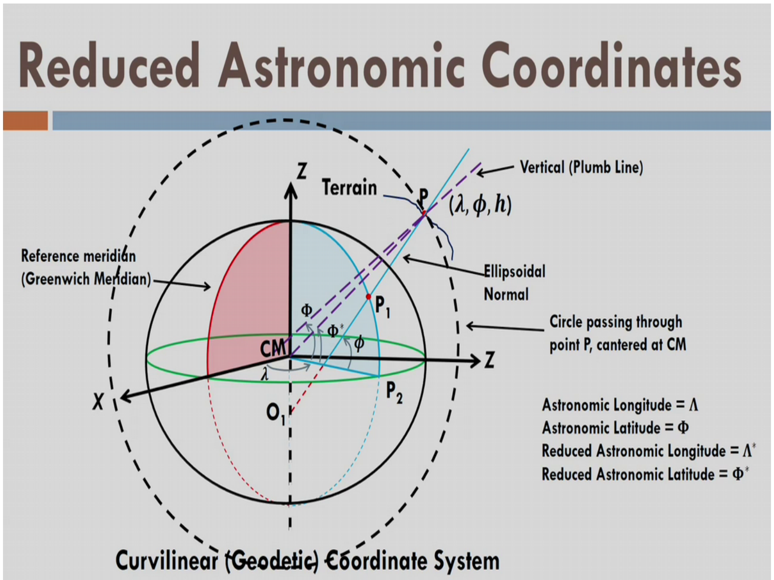


So, let us see what is required here? So, this is the standard transformation which is given. That is this, I am putting this matrix here, that is converting my e, n, u axis, with respect to e star into e star N star and u star. That is local geoid astronomic coordinate system.

Where, please note, that psi angle is phi star minus phi and theta equal to lambda capital star minus lambda small into cos phi. So, what is my phi star lambda star and phi star? They are reduced astronomic latitude and longitude, ok. Here the deflection of vertical in the last slide if you see, that is shown here; the angle between the vertical line or the gravity line or the plumb line and the angle between the ellipsoidal normal, it is call deflection of vertical.

So, if I resolve this deflection of vertical which is 3D angle into 2 planes; one is a East West plane, and another is the North South plane. Then I will get epsilon in the East West plane and I will get theta in the South North plane, right. If it is North wise I should take eta positive. If it is if epsilon or the that is component of deflection of vertical along prime vertical, it is positive, then it should be East wise, ok. With this we can do this transformation.

(Refer Slide Time: 35:02)



Now I would like to explain what is the reduced astronomic coordinates, ok? Let us see there is X, Y, Z reference frame which is having it is origin at center of mass of the earth. Now, this is my ellipsoidal normal passing thorough point P. So, I am defining it lambda phi h here as usual that is my lambda, and that is my phi angle. So, that is geodetic latitude and geodetic longitude. Now, it is with respect to some ellipsoidal coordinate system, ok. And let us assume that this ellipsoidal is a geocentric ellipsoid. So, that origin and center of mass of earth are matching exactly, ok. So, idea here is to show you reduced astronomic coordinate. What does it mean?

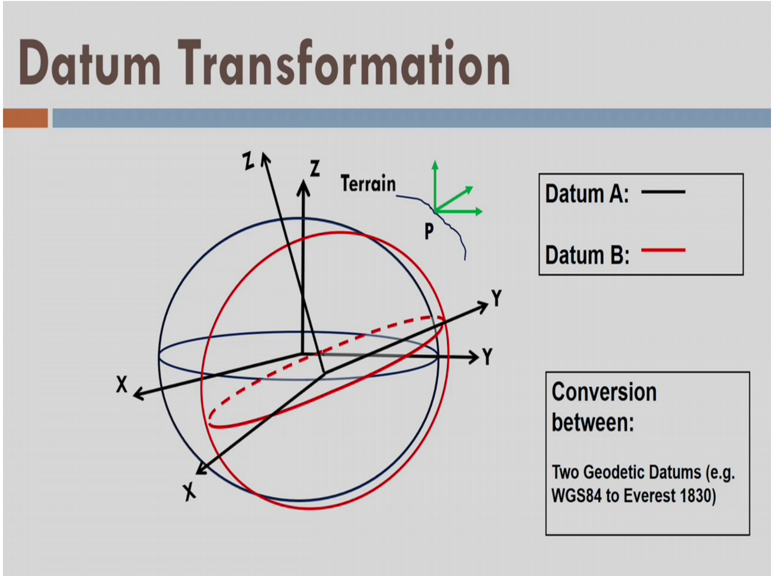
Suppose, if I pass the line that is vertical line or plumb line. In the last lecture I told that generally it is it should pass through center of mass. But remember it is not always necessary, because the gravity direction at point P depends on the gravity value, and the mass distribution at around point P. Ideally it should go through center of mass of earth, but it is not necessary. So, for that purpose let us see this is my angle and this is angle phi that what we call as astronomic latitude.

Well, I think still we are clear on this definition; whether the line passes through center of mass that is vertical line passes through center of mass or it is passes through somewhere little here and there or somewhere else. But we are very much clear what is the meaning of angle capital phi here.

Now, let us connect the point P with center of mass, right and let us draw a circle like this, ok. You can understand what is the radius? Radius is equal to CM, the distance between center of mass of earth and point P. Now, let me define an angle call here you see once again animation, I am putting it here this angle, which is with respect to the horizon; that is sorry I am sorry, not horizon it is equatorial plane, it is instantaneous equatorial plane and the line joining the center of mass to the point P. And this angle is called phi star and this is call the reduced astronomic latitude. I hope it is very clear, what is the meaning here, ok. Similarly, if I take the projection of this line and if I define the longitude of place P, then I will call it reduced astronomic latitude.

So, now I can define these 4 figures very easily, astronomic longitude and astronomic latitude and the reduced astronomic longitude and reduced astronomic latitude. Reduced astronomic longitude and latitude are generally used for the purpose of calculations. I hope you understand the meaning here now.

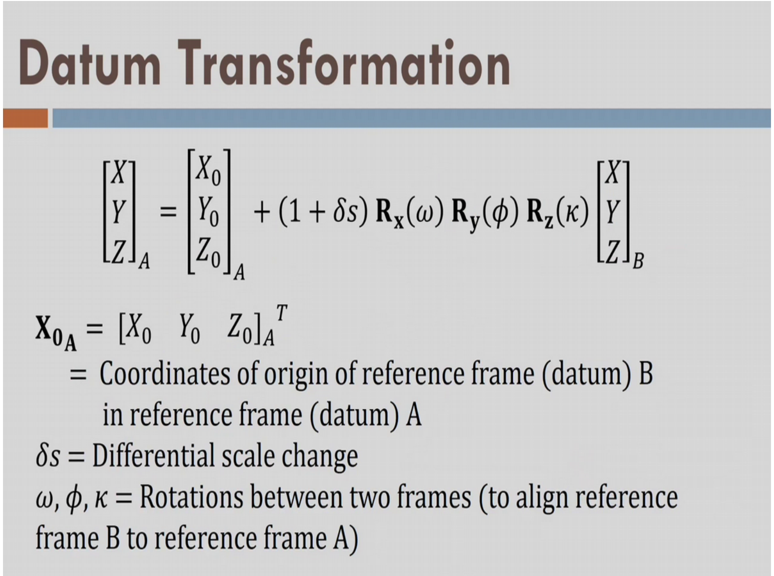
(Refer Slide Time: 38:04)



Again now coming back to the Datum transformation means I am going to change the ellipsoid itself. Well, there is a Datum A and Datum B. Remember, in case of Cartesian coordinate system, we define the Datum with respect to X Y plane, but now we are defining the Datum As a curvilinear surface. So, we are associating the Datum or ellipsoidal Datum with a reference frame. So now, I have 2 reference frames with 2 ellipsoids. So, one is shown by the red color the Datum and another is shown by the black color.

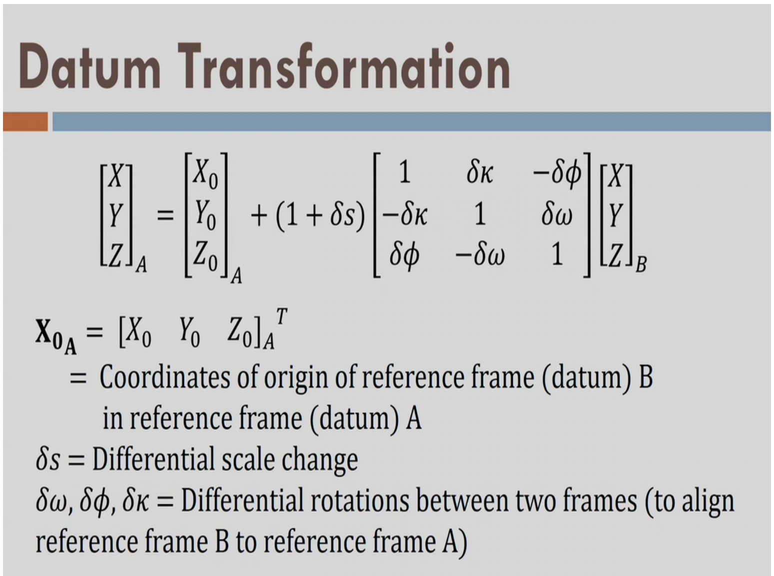
So, let us see the 2 geodetic Datums; one is WGS84. It is geocentric and other is Everest which is non geocentric. So, I am showing them like that. So, let us say this is a point P and local reference frame.

(Refer Slide Time: 38:49)



So now, I am converting Y 1 reference frame that is X, Y, Z in B to the X, Y, Z A and the equations are same. Now here if my rotations omega phi kappa are large in amount. I should consider the complete rotation matrix R X, R Y and R Z. Again delta is a scale factor and X 0, Y 0, Z 0 A is the coordinate of origin of B reference frame in the coordinate system A or in the reference frame A. Well, do it your this translation on paper you will understand it.

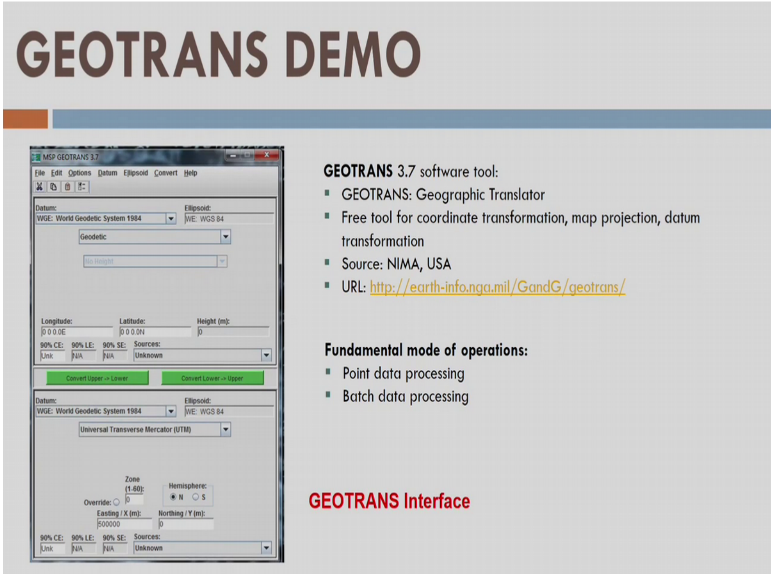
(Refer Slide Time: 39:31)



So now, if my rotations are very, very small, then I can take approximation; where I say that cos of small angle is equal to the 1, and sin of small angle is equal to the angle itself.

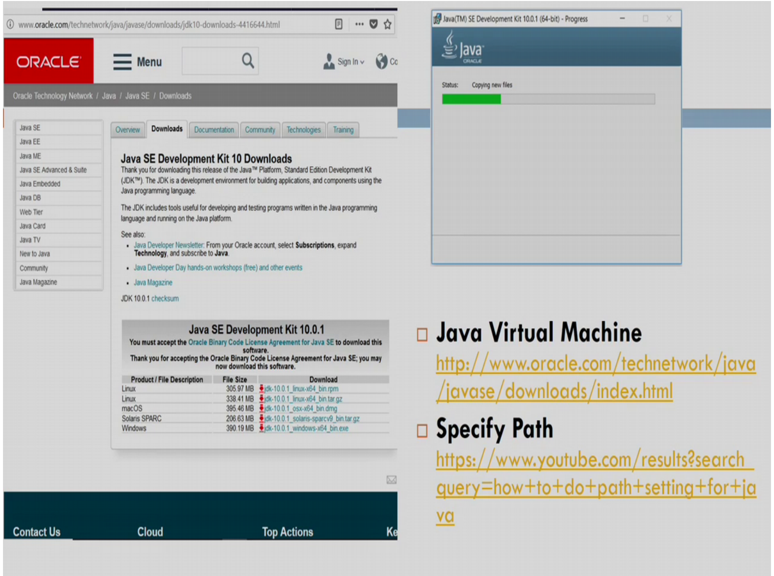
So, that means, if angle is delta omega which is quite small, I will say cos of delta omega equal to 1. And sin of delta omega equal to delta omega itself. So, with this approximation, I can write this rotation matrix with approximate small values, ok. So, I am assuming that delta omega delta phi and delta kappa are the very, very small angles. Now we are going to see the GEOTRANS facility. I am going to demonstrate you using the slides only, because it is very easy to learn system.

(Refer Slide Time: 40:14)



First of all, GEOTRANS stands for geographic translator, and it is a free facility created by nima, that is national emerging mapping agency in USA, ok. You can download from this URL this GEOTRANS, and then you will have once you download if you open the software you will have this kind of interface. So, I can do the point data processing as well as batch data processing. Batch means I have couple of points in a file I can use I can process that file data also. And I can convert the coordinates also.

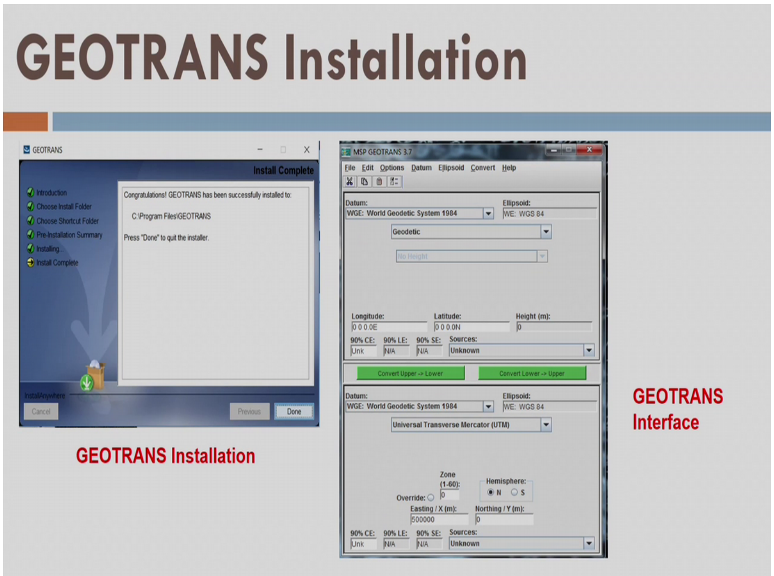
(Refer Slide Time: 40:48)



So, before that you should also understand that you should have the java virtual machine on your PC or Laptop. Well, in order to do the java virtual machine installation, you have to visit the site download it, and remember that you should specify the path in the environmental variable about this java virtual machine. So, all this 2 links are useful. For you do it read it and install the java virtual machine also, a specify the path also.

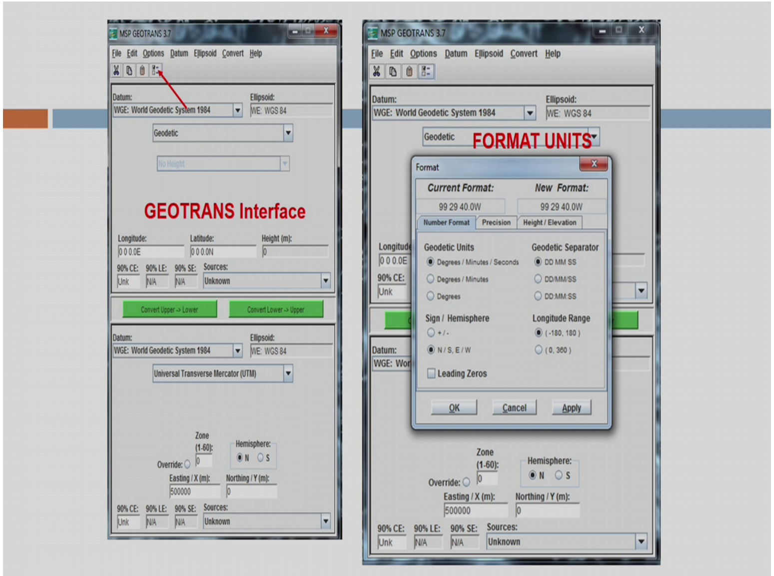
And then once you install the GEOTRANS it will be working for you.

(Refer Slide Time: 41:23)



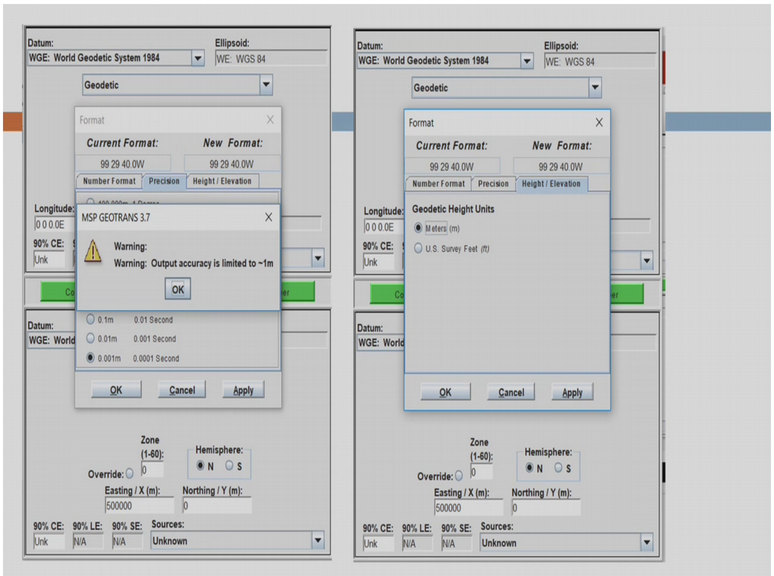
So, first thing you will get this kind of interface after the installation. So, the moment you finish the installation you will get this interface going ahead.

(Refer Slide Time: 41:31)



So, in the GEOTRANS interface if you click on this place, you will get the coordinates system and the format units of the coordinate system. So, here in the geodetic units it is shown degree minute second, degree minute second and degree minutes and degree, then you will have hemisphere. So, all this things are user defined preferences about the number format. So, you can choose and click on apply, ok.

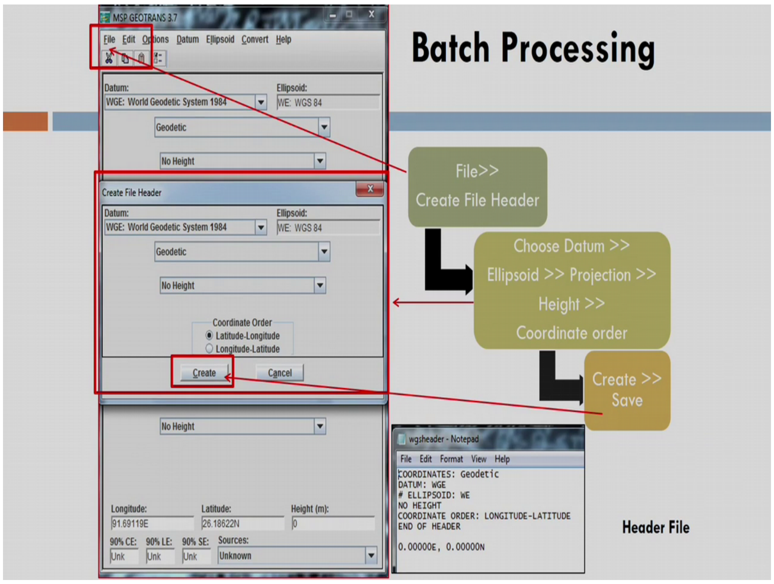
(Refer Slide Time: 42:01)



The another choice is about precision. You can choose any precision given here. That is, 0.1 meter 0.001 meter and so on. So, if we choose anything below point below 1 meter, then it will give you warning. Warning is the best accuracy possible is 1 meter. So, even if you chose something below; that means, something better than that you are not going to achieve that accuracy and there is no reliability of that.

Then third is the high television you can choose in feet or meters. So, all these things are there in this window called format, ok. So, you can choose correct format, you can define a new format and so on. So, but always click apply. So, the moment you click apply, it will be giving you the same kind of format for how long your session, right.

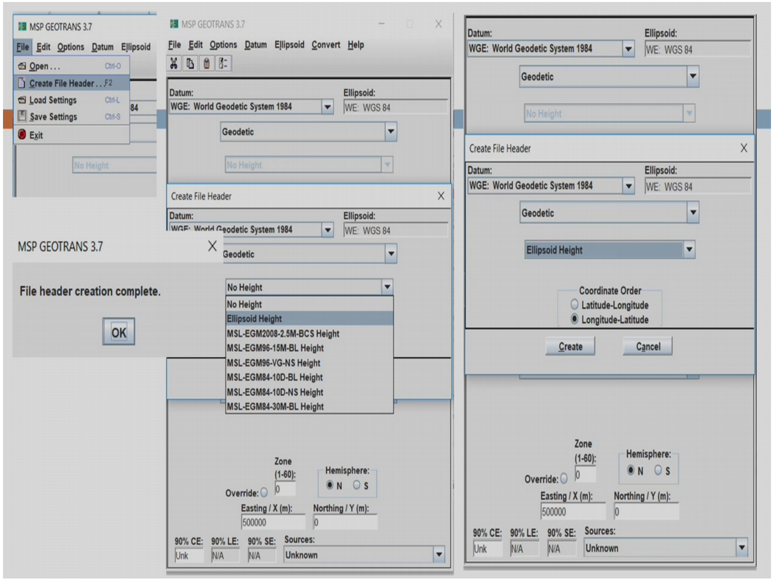
(Refer Slide Time: 42:50)



So, first will learn little difficult task which is batch processing. So, you will go to the file, you will click in the file, then you will get an option called create file header. Again you will choose that and once you click on the create file header you will have choice on data choice on ellipsoid, choice on projection system, and then you have choice to select height, no height or glipsoid height or orthometric height and so on.

Then coordinate order whether you want to choose latitude longitude or longitude latitude and Y Y and X Z X Y whatever. So, then you create save. The moment you create save it will automatically create a header file for you, and let us look what is a header. So, in the lower side of this screen, I am showing notepad file written as WGS header. In this header, I am showing coordinate as geodetic; that means, it is a geodetic coordinate system. Fine, next is Datum that is WGS84 ellipsoid is w is 84 again no height; that means, I am not going to choose any height, I can choose ellipsoidal height also then coordinate order is longitude, latitude and then end of the header. So, once this file is being supplied to the system, it will first read this data. And it will understand what kind of data has been given. Now you can populate your data in the ellip. This thing longitude and latitude in this file to make a input file. So, let us look into this thing.

(Refer Slide Time: 44:21)

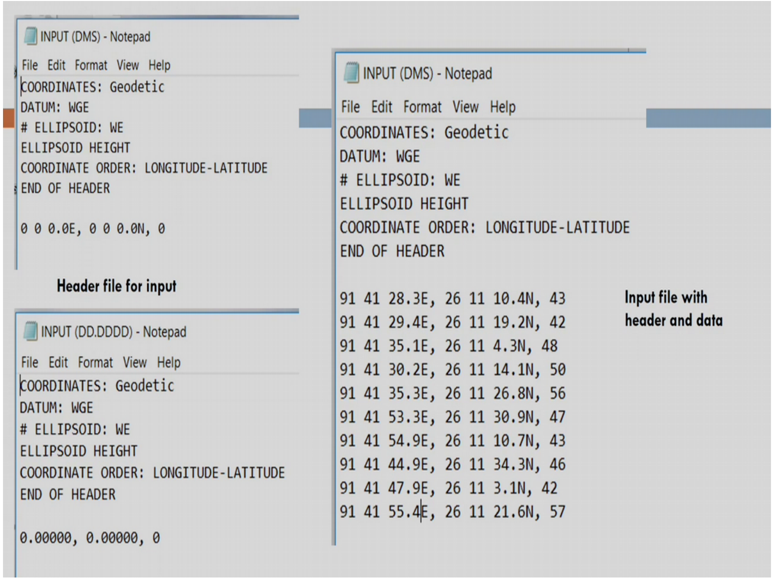


Here, now I am creating header file, ok. So, create header file, now I am choosing the Datum as WGS 84, ellipsoidal WGS 84, you can see on the screen and coordinate has geodetic. And in case of height I am choosing ellipsoidal height in case of no height, right.

So, I have 3 things here; Datum, ellipsoid and height. Well, I am choosing the (Refer Time: 44:44) northern hemisphere and moment I do it the create that means, before that I choose the order of coordinates as longitude latitude, not latitude longitude because generally we speak longitude and latitude. So, I am choosing the longitude latitude, right.

So, on the right hand side I am showing an interface. Once you create click on the create, you will get this message file header creation complete. So, it is showing you that you have successfully they have created file header.

(Refer Slide Time: 45:17)

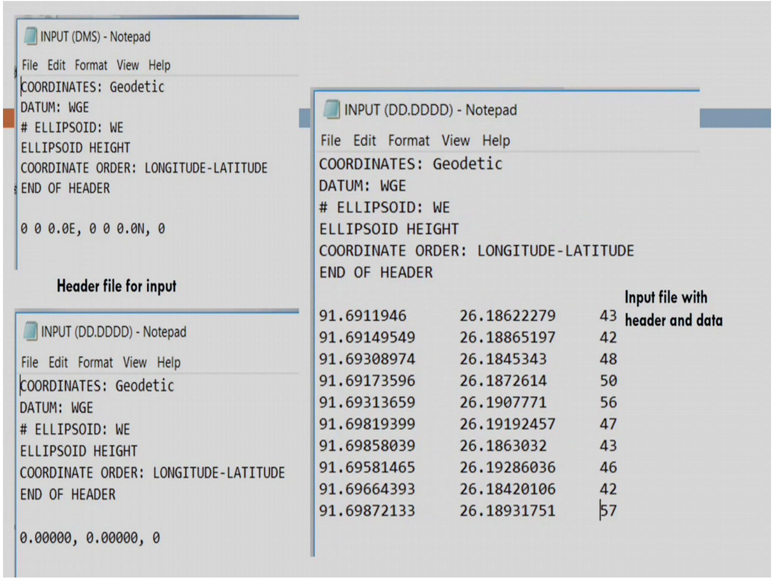


So, this is on the left hand side I have a file header, you can see, coordinates are geodetic Datum is w is 84 return s WGE, then you have ellipsoid WE that is again WGS84 only. Then you have ellipsoidal height, and coordinate order is longitude latitude end of the header then it is written 0 0 0 is 0 0 0 North and 0, showing you 3 coordiantes.

So, it is a header file for your input. Now you can define your own coordinates in this header file. So, you are showing here input file with header and data. So, I am writing longitude as 91 degree 41 minute 28.3 East comma latitude 26 degree 11 minute 10.4 seconds North comma the ellipsoidal height 43. Since this header is there, this system or the GEOTRANS software will first read the header, and it will go for the data.

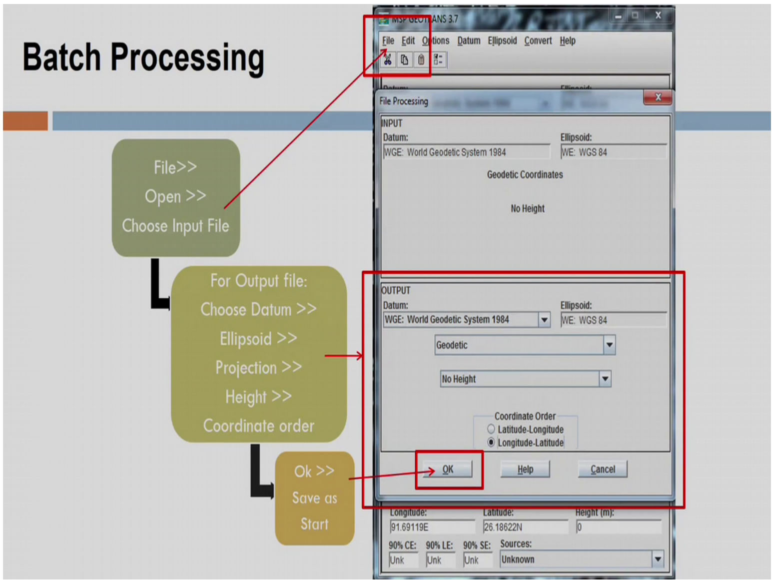
So, that there are 3 columns and the meaning of this 3 columns are they are basically longitude latitude and ellipsoidal height in WGS84 Datum and WS84 ellipsoid. That is the meaning here of this input file with the height of data. Let us go head. So, this is if I am taking degree decimal, not the degree made second for made degree decimal. So, I have chosen like that. So, I am getting this kind of, ok. So, you can define the coordinate system in degree decimal or the degree minute second whatever.

(Refer Slide Time: 46:51)



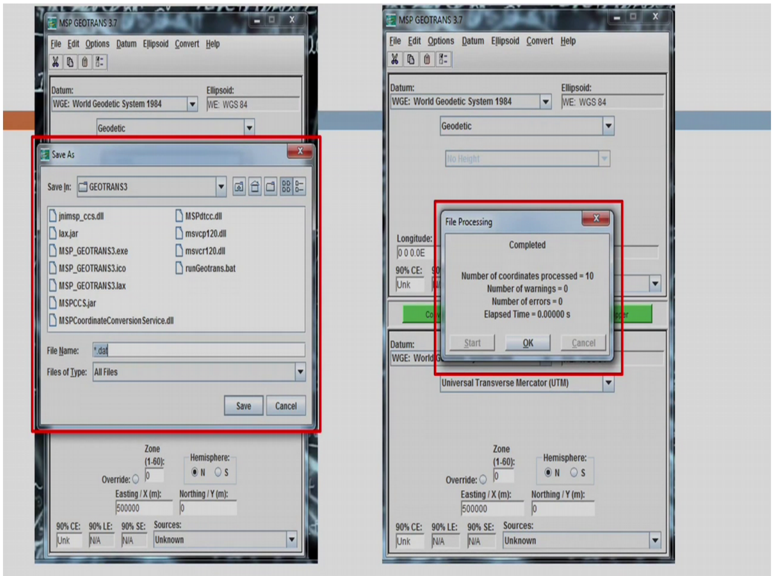
So, this is my input file now because I am defining in this fashion. And I am separating the coordinates 2 columns by tab.

(Refer Slide Time: 47:00)



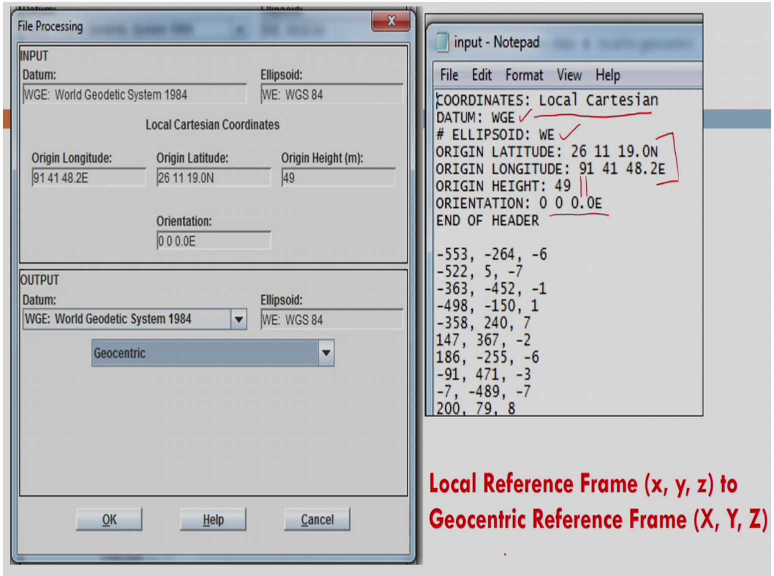
Let us go head batch processing. Now I have created my input file with 10 data points. I want to convert this 10 data points into different coordinate system, ok. Now I am choosing the header file for the output, ok. So, what is the meaning? Again I will follow the same process. I will go to input; I will upload the input file. And then I will choose the coordinate system ellipsoid projection height, and coordinate order for my output. And now I am choosing let us see the WGS84 geodetic height and longitude latitude, fine.

(Refer Slide Time: 47:38)



So, I am clicking, that means, let us see then it will ask me to save the coordinates, ok. So, what I have done here? I am showing you here that I am choosing the in output my Universal Transpose Marquetry UTM projection, ok. So, then I will put let us say number of coordinate process 10, it is going to show the moment I save it file it will show number of coordinate processes 10 number of warnings number of errors both are 0 because there is no arrowness point in my data. So, it is processing all the 10 data points and what is the time it uses to process this thing which is negligible or very close to 0. If you take very a large file where are some thousands of points millions of points billions of points there it will take some significant time. And that is why ellipse time will be more in terms of milliseconds of minutes and so on.

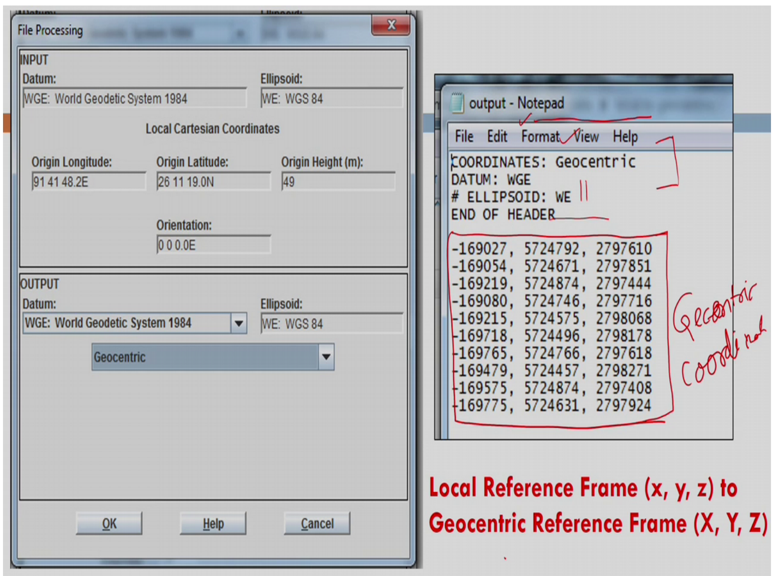
(Refer Slide Time: 48:29)



Now, I am doing the coordinate transformation between local reference frame x, y, z to geocentric reference frame. Here you see I am defining my input header here like local Cartesian; that is, the Datum I want to use WGS84 ellipsoid is WGS84. And this is my origin longitude and latitude, you see here, ok. And original height is 49 meters. Well, orientation will be this way, and then this is my and of header. And these are the coordinates; let us say I have measured in a local reference frame, ok.

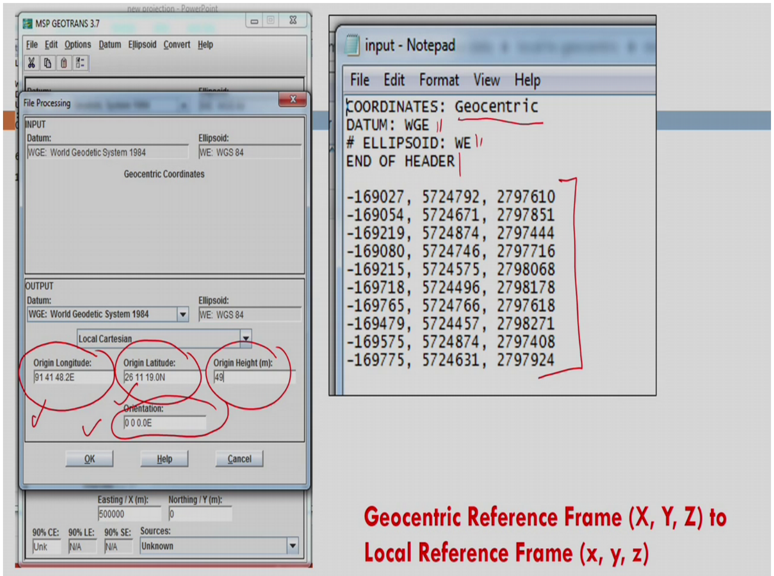
Now, I am converting into output is my W 84. And I want to convert this thing into a geocentric coordinates in the same reference frame, ok.

(Refer Slide Time: 49:15)



So, let us see this is my output here. In output these are my geocentric coordinates, right. So, these 10 points coordinates are there and they are geocentric, you can check yourself whether you are getting this.

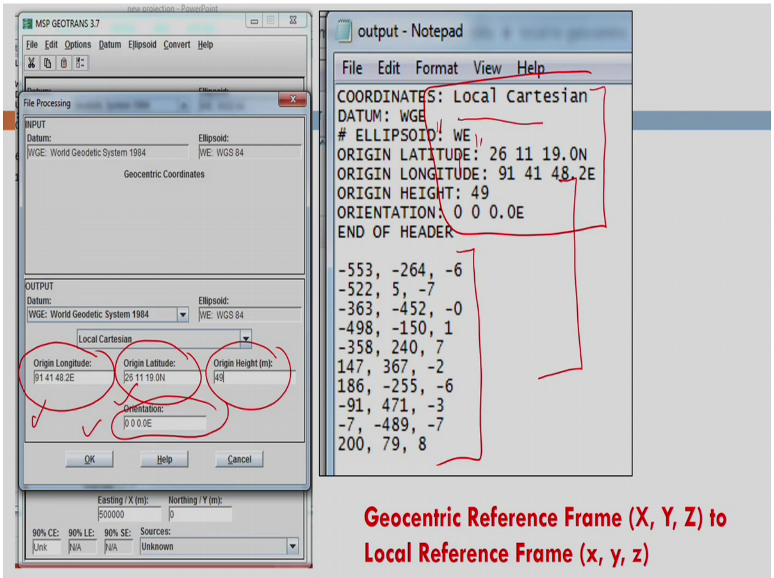
(Refer Slide Time: 49:37)



I am doing the now reverse transformation that is the header file, which is first created in the last slide also. That is showing the geocentric WGE84 this one and so. And these are my geocentric reference coordinates, ok. Now I am converting into the local reference frame, since I am giving this information about the local one this way, origin of longitude origin of latitude and where my origin of the local reference frame is lying.

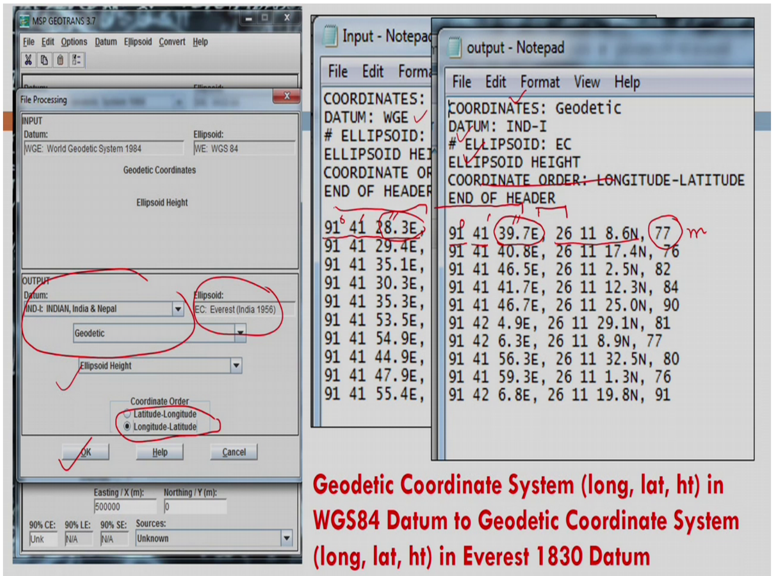
So, these are the locations. Now you can understand that it is my topocentric reference frame, which is at the height of 49 meter above the ellipsoid. And at this longitude and at this latitude and this orientation is 0 degree and so. Fine, that means, at this point, right it has some orientation like this. So, it is perfectly aligned with the geodetic north, geodetic East and the up direction. And that is why I am writing my orientation is this thing. Has the orientation different, I should this North axis of the local geodetic frame will not be aligned with the East or the North meridian axis here, right? It will something like that. So, I need to specify this orientation there, here. So, I will be specifying orientation here. So, I am be saying. So, you can assume here that is local reference frame is nothing but the local geodetic reference frame.

(Refer Slide Time: 51:08)



Now let us covert, so, you get this coordinates here. And this is my header file again made here. Now you can understand, how is it easy to use this kind of facilities for coordinate transformation.

(Refer Slide Time: 51:24)



Now, coming to the geodetic coordinate system, longitude, latitude in WGS84 to geodetic coordinate system in Everest Datum. So now, I am changing the Datum itself, you see how gradually we are progressing and this coordinate and Datum transformation. Now I am sowing you the Datum transformation. So, you see the input file here, that is my geodetic, that is my WC84, WC84 ellipsoidal height. So, it is longitude latitude order shown here longitude latitude here. And this is my ellipsoidal height in WGS84 coordinate system; that means, my reference frame is geocentric WGS84.

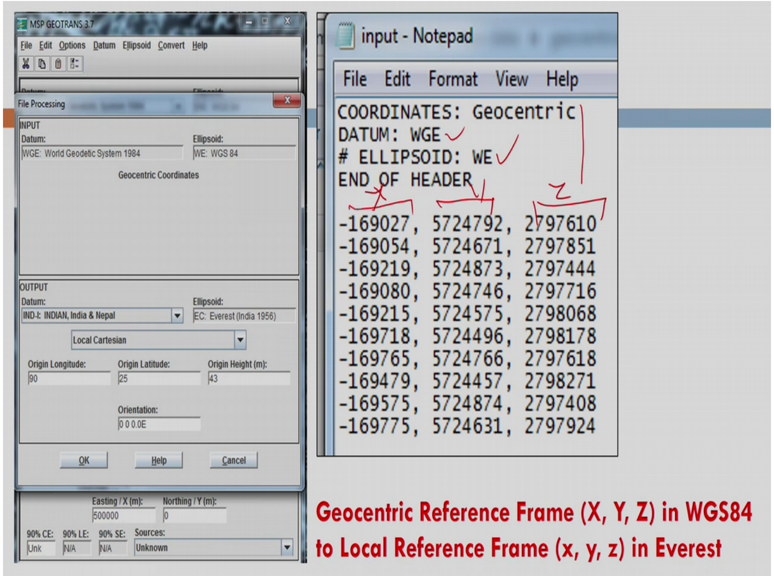
Now, I am converting into the Everest which is a non-geocentric reference frame. If I do you can see the details here, that is output details I am choosing this India one ellipsoid Everest 1956 and I am choosing the ellipsoidal height. And longitude latitude is my order in the output file and press, then the moment you press, ok. You will get this file you will save the file and this is the output here. See 91 degree here 41 minute 39 seconds

You can see easily here that because I have changed the ellipsoid, and as a result of that the reference frame is changed, I am getting different coordinates for WC84 I have this one, 91 degree 41 minute 28 seconds. Here I am getting 91 degree 41 minutes in, but 39.7, you see this is the difference between the 2. And I can now say that there is the difference of let say almost 10 seconds, which is quite small. And you know that the assumption of small rotations it is true here.

And now you can understand what is the meaning of that and I have said that maximum 3 minutes are there. The difference between the 2 reference frames as far as rotation is concerned. But as far as translation is concerned they are in order of 100 to 200 meters or little more also. So, be careful you can only take the rotations as a small rotations between the 2 reference frame; where one is geocentric reference frame another is non geocentric reference frame. But translation values are considerably high you should not take it small, ok. And that is the reason we never took any approximation for the translation, but we took approximation for the rotation only. The small angle rotation or the small angle approximation we have taken.

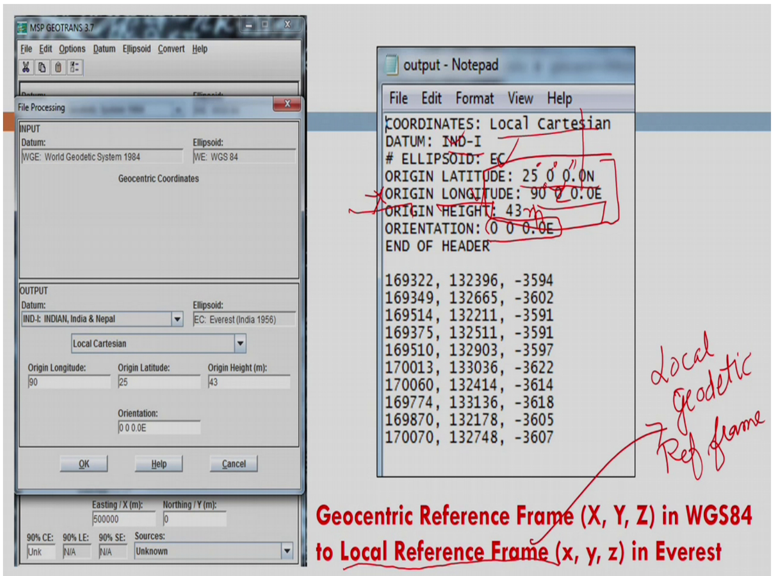
So, these are the heights above the Everest ellipsoid. So, here earlier it was almost 43 meters. Now I am having 77 meters, fine. This is the latitude value, and latitude values are also different slightly.

(Refer Slide Time: 54:11)



So, let us go ahead. Now, I am converting back; that means, geocentric reference frame in WGS84 to local reference frame in Everest. Well, I am becoming more ambitious now; that is, the geocentric coordinates in the WGS84. And this is my X, this is my Y, this is my Z, capital Z, here Y and X here. Now, I am converting into the, you see here.

(Refer Slide Time: 54:35)



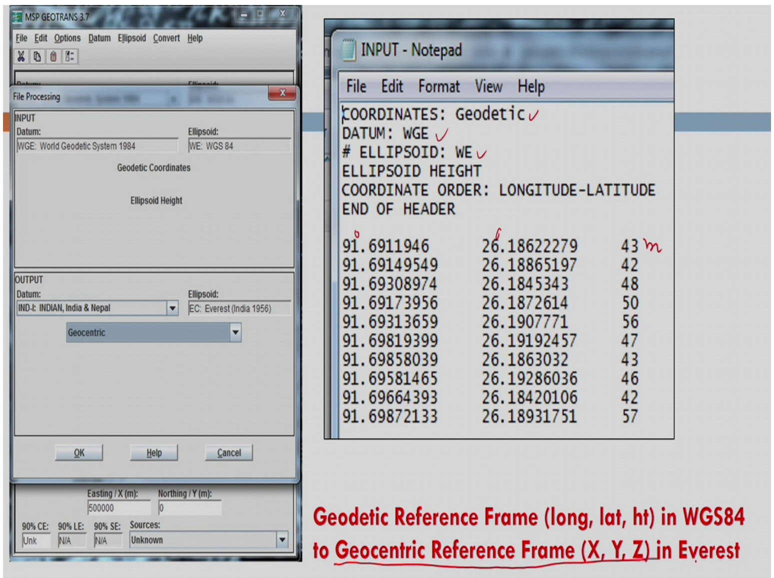
The local Cartesian, Everest here, and this is my origin, 43-meter height 25 degrees 0 minute seconds 90 degree 0 minute 0 seconds, ok.

Orientation I am using the same orientation; that means, it is my local reference frame is my geodetic reference frame, ok.

Now once I convert, it I will get this answer. So, this was my output here sorry, I have already converted it, ok. So, in the local reference frame, now you can read this coordinates why they are so negative. And so, well, it is because of the choice of the origin if choice is origin is far away from the local frame where we are working, what will happen? I will get this kind of high coordinates in meters. This kind of high coordinates, but the moment I take my origin of the local reference frame near to the site I will get very small coordinates as we got in the previous slides.

Now, coming back, I am doing the same thing, in the reverse fashion; so, local reference frame x, y, z in Everest to geocentric reference frame. So now, you can also see note down these details here, from here and here, and you can do it yourself also.

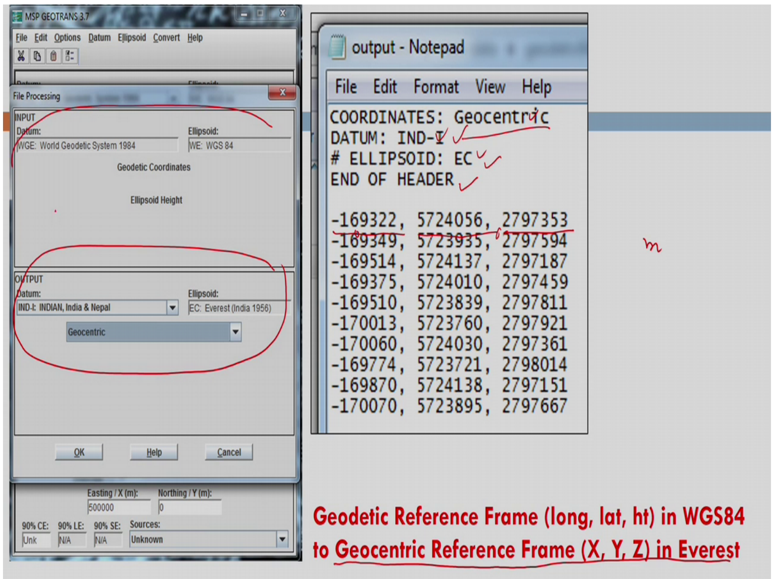
(Refer Slide Time: 56:11)



This is my output I am getting. Well, this is input and this is the output here, go ahead. So, geodetic reference frame, again I am converting geodetic to geocentric by changing the Datum.

Well, you can see longitude value here; 91 degrees 26 degrees and 43-meter height geodetic WGS84 and so on.

(Refer Slide Time: 56:34)



This is my input I am converting it into the geocentric. Everest, this is my output here you see. These X Y and Z geocentric coordinates. That is Datum Everest here and geocentric system. Well you can note down the same details here that is for output and that is for the input here.

I am doing it the reverse transformation. That is my input is Everest system. You see here, Datum and ellipsoid. And output is WGS84 Datum and ellipsoid and ellipsoidal height I am taking here, ok. Well, and I am changing the Datum here. So now, you are getting the change Datum here and the heights are 43 meter. If you go in the back slide that was the geocentric coordinate on the Everest, but now you are getting the ellipsoidal height in the WGS84.

So now it is very, very magical system. In a sense like, you are not doing anything effectively, you are not writing any soft here, you are not writing any your own module, but you can do it also, and you can check your work. So, using this facility whether your answers are correct or not; that means, suppose you are written script in python or may be any free interactive development environment. And you want to check whether it is performing well or not you take this coordinates as input coordinates, ok. Try to run your script, find out the coordinates and check the output with the output given by this software; that means, input is same output should also be same.

So, match the output if they are same you can understand that your script is correct in python or may be in the MATLAB as on, ok. That is the good use of this utility. Also if you do not want to write your own script you want to use this software. The limitation is one meter is the limitation. If you are happy with that, which is appropriate for most of the work, you can do it. Also in order to understand what could be a possible values, then you can also use this facility to do your job. So, let us go ahead I am doing now a geodetic reference frame longitude latitude height and WGS84 to geocentric reference frame in Everest, ok. So, again this is the coordinates here, and 43-meter height. You see and now I am converting into back where this output is defined like this, ok.

So, I got this one again you will surprise why this heights are. So, negative because the choice of the system or origin, local reference frame X, Y, Z in Everest to geodetic reference frame long let height in this one. So, this is my output you see, and this is my input again. And then this is the answer, where 91 degree 41 minutes 28.3 seconds 20 6 degree 11 minute 10 this thing. This my output you see the header, now you can easily read the header, you can understand Datum which is nothing but this Datum here, ellipsoid which is this ellipsoid here and ellipsoidal height which is this ellipsoidal height here. You see there that is the thing here.

(Refer Slide Time: 59:45)

So now, I have shown you some of the coordinate transformation and Datum transformation. So, before concluding I would like to say that we have learnt about coordinate transformation today. And Datum transformation we have also seen what is the difference between the Datum transformation and coordinate transformation. And then we have done theoretical understanding of various coordinate system how to convert for the same Datum.

And we have also learned that how to do with the change in the Datum. So, in the process there are 3 4 steps involved. Suppose, if I want to do Datum transformation and coordinate system in the both Datum's are different. One is Cartesian coordinate system one is the geodetic coordinate system. So, what will I do? First I will convert my geodetic coordinate system into Cartesian coordinate system.

Now, for this Cartesian coordinate system I will bring my coordinates into Cartesian coordinate system, and then I will from this coordinate Cartesian system. I will bring this system into the coordinate system desired coordinate system from the Cartesian to the other may be geodetic or local or whatever. So, that is the idea here, and this whole thing can be done in a single step in a facilities like GEOTRANS. So, I have demonstrated the GEOTRANS facility also. So, you should understand what is going on at the backend of GEOTRANS.

In the next lecture are going to learn about the projected coordinate system, or what we call is map, map projection system. So, map projection system are quite different from what we have learnt today. So, see you soon. Thank you. Thank you for attentive listening.

Thank you very much.