Hydraulics
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Module No. # 09
Hydraulic Model Study
Lecture No. # 01
Pipe Flow: Friction Loss

I mean, more detail about those numbers we shall be discussing first in this topic. And then, we will be gradually moving on to the hydraulic model study.

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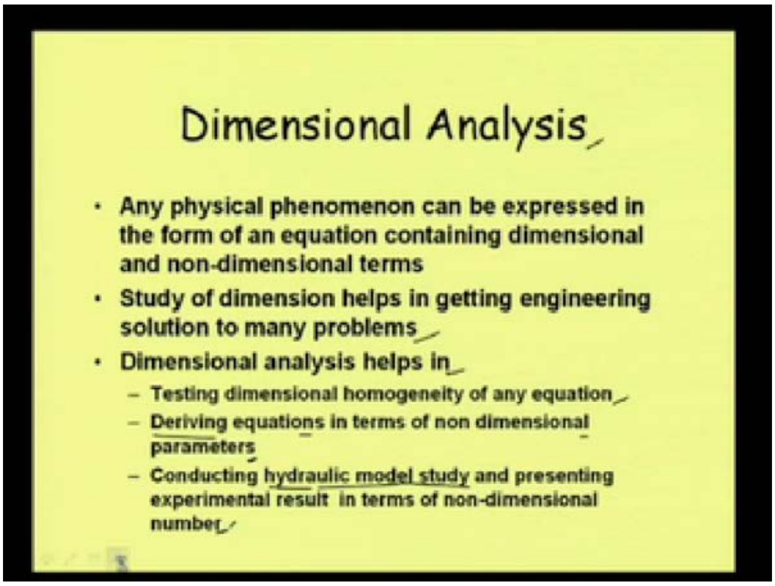

To start with dimensional analysis; what is that? Any physical phenomenon can be expressed in the form of an equation. That equation may contain dimensional and non-dimensional terms. So, after knowing the physical process or a phenomenon, understanding the physical process of a particular phenomenon, we can express that in terms of an equation. And, that is why, we are using lot of equations in hydraulic engineering. And, those equations will contain some of the terms, which will be having dimension. Dimension means… We know the very basic fundamentals of dimension already from fluid mechanics concept.

Say density – what is the dimension of that? Mass per unit volume – it will be mass divided by volume. And, volume – what is the dimension of volume? It is length cube. So, m by l cube will be the dimension of density. So, that way, for some of the term, there will be dimension and it may contain some constant term. And, if we combine some of the terms, that may become again a non-dimensional term. Like that, a equation will be containing some dimensional and some non-dimensional terms. A study of dimension helps in getting engineering solution to many problems. Our basic target is to find solution to our problem. And, if we do the study of dimension, then it helps us in getting solution to the problem. How? That of course, when we will be discussing further, it will be more clear.

And, dimensional analysis helps in… How it helps? The testing dimensional homogeneity of any equation. What we mean by dimensional homogeneity? Say, in an equation, we will be having some left-hand side and the-right hand side fine. Now, if the left-hand side and the right-hand side, if we just write the dimension of the left-hand side; say if it is q, discharge, then it will be having some dimension; or, suppose depth we are talking about; depth dimension is nothing but l, length dimension – simply depth. Suppose on the left-hand side, we have a quantity; this will be having some dimension. Again, on the right-hand side, we may be again having some quantity, which will be having some dimension. If the dimension of left-hand side and right-hand side are matching, then that equation we refer as dimensionally homogeneous equation; or, this equation have dimensional homogeneity.

And, by observing that, how it helps? See if an equation is having dimensional homogeneity, then we know that yes, it has an analytical basis and that way we are deriving this. But, if it is not having that sort of dimensional homogeneity, that means, it might be coming from some observation or empirical; that may be one empirical equation. So, that way, say when we talk about Manning's equation, we are not having dimensional homogeneity. So, if we do not know where from that equation is coming, what its origin, whether we can apply this equation directly in some problem, that can be understood more clearly if we check the dimensional homogeneity of that particular equation. Say, if the equation is not having dimensional homogeneity and it is containing some constant and say exponents; that sort of exponent and constants are there, then we should understand that this equation is developed from some observation; and as such, directly, we should not rather apply in any other situation if we do not know the original of that particular equation, how it was developed.

Then, next point that it helps in deriving equations in terms of non-dimensional parameters. Understanding dimensional analysis – it helps us in deriving non-dimensional equation, because if we recall our class of uniform flow, there also, we derived one equation, which was non-dimensional equation. And, that sort of non-dimensional equation we can derive, once we know that what dimension is, what dimensional analysis is. So, that way also it helps. Then, more importantly, it helps in conducting hydraulic model study. When we will be doing hydraulic model study, particularly, physical model we are talking about, then the knowledge of dimensional analysis is very important. And, for presenting experimental result in terms of non-dimensional number, in that aspect also, dimensional analysis helps us. So, say when we were analyzing hydraulic jump, then we were talking about that if Froude number is less than this, then what the Froude number is? Froude number is a non-dimensional term. So, we need to know these non-dimensional terms and that helps us in experimentation.

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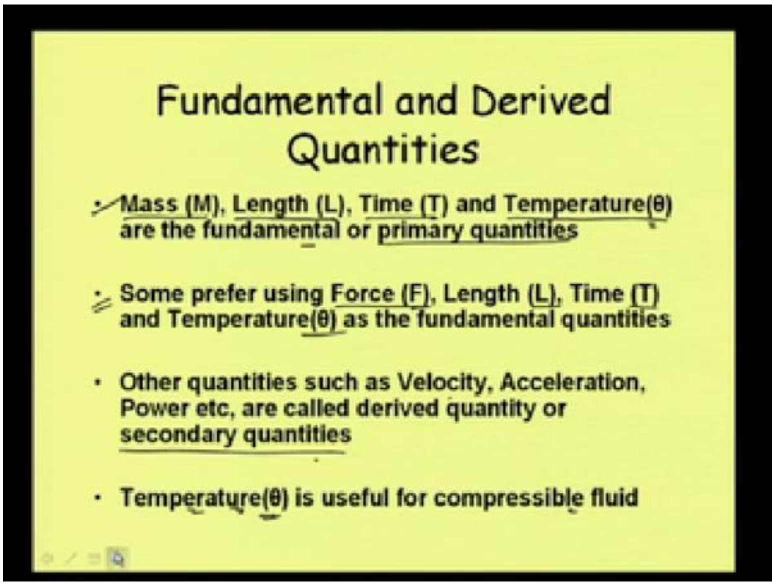

Let us move further. When we talk about dimensional analysis, then first we need to see what are the fundamental quantities; what are the fundamental quantities? When we talk about discharge velocity or depth mass, all those things, then we find that all these quantities can be expressed in terms of some fundamental quantities. And, those fundamental quantities are – if we see, it is mass. So, mass is one quantity, which is fundamental. Then, length – that is another quantity; time. Then, another quantity is temperature. So, these four quantities are normally considered as fundamental or primary quantities. Some people prefer to have force in place of mass. Of course, we know that force and mass; there is a relation that force is equal to mass into acceleration. So, if we have mass, then acceleration is nothing but length; length per unit time is velocity; then, velocity per unit time. So, it will become length by time square. That becomes the dimension for acceleration. So, once we know the mass dimension, then multiplying that by length by time square dimension, then we can get the dimension for force. And, some people prefer to have the fundamental quantities as force, length, time and temperature.

Normally, we go for this one, but some people prefer this one. And, other quantity such as velocity, acceleration, power; whatever it may be, say, viscosity, coefficient of viscosity – all these quantity are called derived quantity. So, these are called derived quantity or secondary quantities. So, that way, we should be clear about which part is fundamental, which part is derived. And, when we go for dimensional analysis, we express all other quantity (( )) which are derived quantities in terms of the fundamental quantities. And, temperature – that I have mentioned here; but, the temperature is useful for compressible fluid. If our fluid is compressible, then temperature has significant influence. And, in those things, those quantities, the temperature will also be coming. In hydraulic engineering, normally, we do not encounter this temperature very frequently; we do not encounter this frequently.

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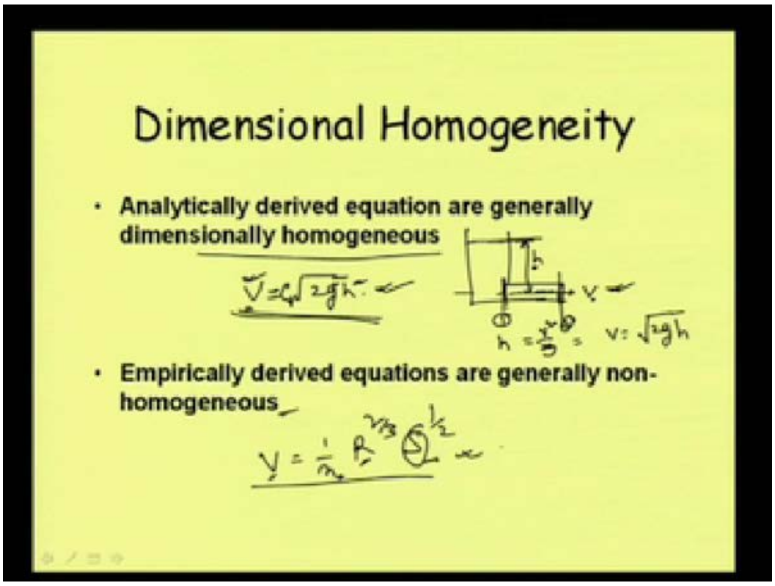

Then, dimensional homogeneity – as I have already explained that analytically derived equation are generally dimensionally homogeneous. Say if we write one equation, very simple equation; suppose say if we take this from a tank, say, water is flowing and this way; then, suppose this is a head up to this much, then we want to know what is V; then, we write that V is equal to root over 2 g h. That equation… And, there may be some coefficient in that; I am writing that coefficient is equal to C for actual velocity. Normally, we write the expression like that. So, where from this is coming; where from this is coming? Because we know that head at this point (Refer Slide Time: 10:25) is s h. At this point, that is just on upstream of this pipe. If we check, what is the head, total head? It will be h; z is same along this line. So, we are talking about say two points; one is this one; another is this one. This is 1; this is 2.

Now, if we equate the head, say, energy, at this point, head is equal to h. And, there is no velocity at this point; just on upstream. So, velocity head is 0; only the depth head is coming. So, h. And, z is same. So, here it is h. And then, I am writing it here. So, from this, what we can write? It is head; here is h (Refer Slide Time: 11:05). And, this is equal to… What is the head here? Here the pressure head is 0. 0 means actually we are not talking about absolute pressure; we are talking about atmospheric pressure. So, when we say pressure head is 0, we mean that pressure is atmospheric. Like that, here pressure – as it is open to the atmosphere, pressure will become 0. So, what is remaining is only velocity. So, head will be V square by twice g; and, z here is 0. So, from that, we can write that V is equal to root over twice g h. So, that is way, we can get the expression for velocity and this relation is say analytically derived equation.

And, in reality, there will be some contraction. As we were discussing some laws and all those things, were there in the pipe earlier in our discussion of pipe flow. So, some coefficient will be coming. This can be put as C V also – coefficient of velocity. Like that, we get this expression. And then, if we put say dimension of V, then h and g, everything here, (Refer Slide Time: 12:11) then we will find that this equation is dimensionally homogeneous. But, now, when we say about empirically derived equation or generally non-homogeneous, say we have used an equation several time. Like that what is the equation for normal depth – that for deriving the resistance flow formula rather? Resistance flow formula – we are writing V is equal to 1 by n; then, R to the power 2 by 3 and S to the power half. That equation we have used several time.

And similarly, chassis equation also, we have discussed several time. But, these equations – I mean this particular equation and the chassis equation as well – this equation – if we put the dimension, this is a dimensionless term. As we were saying, that in a equation, there can be terms which are dimensional which are non-dimensional; like the slope is one term, which is having no dimension, because slope is what? It is the ratio of how much distance it is falling in how much distance, how much depth it is coming down? Elevation – what is the change in elevation in particular distance? So, that way, this is not having any dimension. So, this is a non-dimensional quantity. This quantity is not having dimension. But, the radius – hydraulic radius is having the dimension of depth. Then, n is a coefficient and V is the velocity. This dimension is l by t. So, these equations – if we put the dimension, we will find that this is not a dimensionally homogenous equation; dimensional homogeneity is not maintained.

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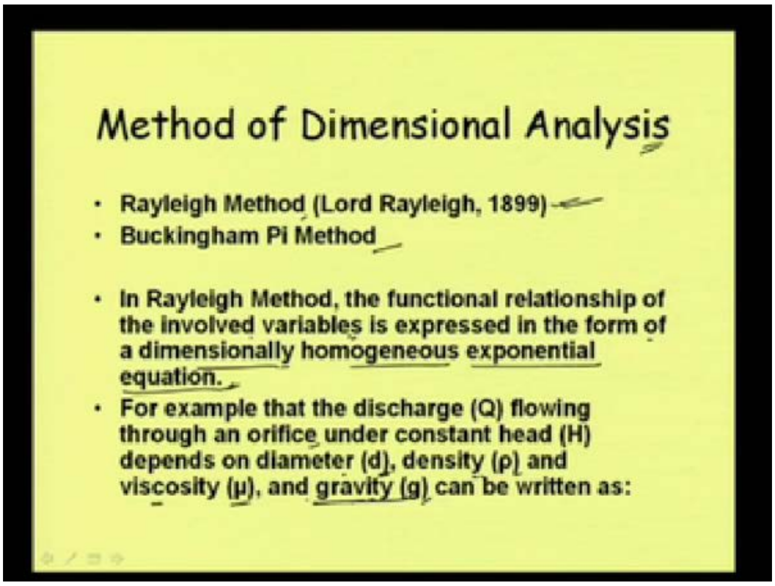

Then, next point what we want to discuss is the method of dimensional analysis. How do we do dimensional analysis? There are different methods; two popular methods are there, which are generally used: one is Rayleigh's method and another is Buckingham PI method. First, we will be discussing the Rayleigh method – that in Rayleigh method, the functional relationship of the involved variables functional relationship – what we mean by involved variables? When you talk about a particular phenomenon, then in that phenomenon, some parameters are involved. So, those involved variables we are talking about. So, function – and they will be having some relationship. So, that functional relationship of the involved variables is expressed in the form of a dimensionally homogenous exponential equation. So, we consider the equation to be exponential first. The exponent what we will be putting – finally, we can find; that suppose we are putting q is equal to some other parameters, which are related. And, for all those parameters, initially, we are putting an exponential term; then, later on, through dimensional analysis, we will try to see what the value of these exponents.

And then, we may find that some of the exponents are becoming one; that is also possible. But, initially, we will put this as a dimensionally homogeneous exponential equation. For example, let me take this one. For example, that the discharge (Q) flowing through an orifice under constant head depends on the diameter (d), then density (rho) and viscosity (mu) and gravity, that is, acceleration due to gravity (g). And, this can be written as… So, this relation. So, we know that through an orifice, suppose we are providing a constant head in a tank; and, from bottom of the tank, we are having an orifice; and, through this, we are finding that water is flowing. Then, how much water will be flowing through the orifice – that will depend on the constant head under which it is flowing; then, it will depend on density of the fluid; it will depend on diameter definitely of the orifice – diameter of the orifice. It will depend on acceleration due to gravity. What is the value of that? Then, definitely, viscosity mu. So, if a highly viscous fluid is kept, it will be flowing – I mean, smaller amount of that fluid will be flowing than a fluid of lower viscosity. So, all these parameters are influencing. Now, how we can write that in Rayleigh's method? We are talking about Rayleigh method.

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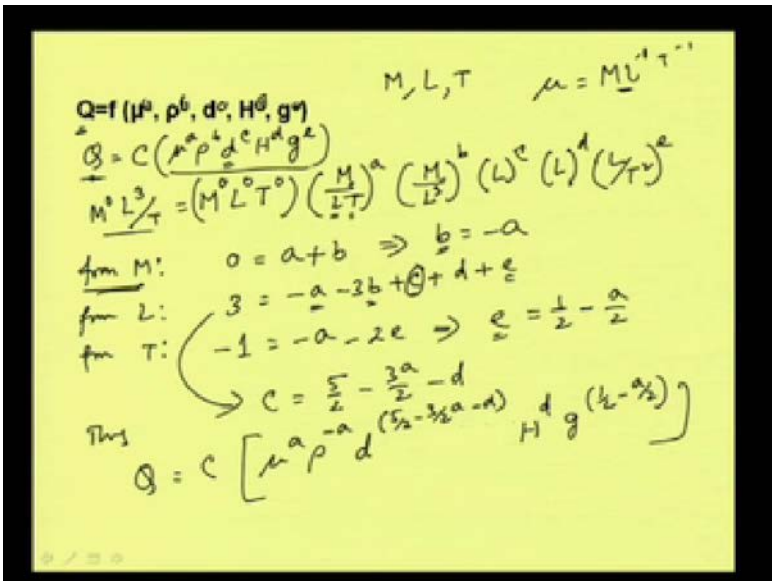

This can be written as say Q as a function. We know that Q; now, this discharge is function of all those parameters. And, this can be written as say Q is equal to. So, this can be say C, some coefficient and then we can write as a product of… Initially, we may not write this coefficient; we can just simply say… May not write this exponent; we can simply say that Q is a function of mu rho d H and g. I am writing these of course, but we may not write these things initially. Then, what we can do, that here we can put this thing that Q is equal to C and say mu to the power a into rho to the power b d to the power C H to the power d and g to the power e.

Now, this is our relation. Now this relation (Refer Slide Time: 18:01) – now, if we put the dimension of this relation; say, dimension if we put, then what we can write? What is the dimension of Q. This is equal to say volume per unit time. So, we can write L cube by T; volume per unit time. So, this is the dimension. Now, C is a constant. So, what will be the dimension of that? Constant means it did not have any dimension rather. And, that how we can write? We can write this as in terms of our fundamental parameters or fundamental quantities are, say, mass, length and time; temperature is not playing any role here. So, we are considering M, L and T. Now, C, which is a constant that we can write say M to the power 0. In fact, M is not there that means; then, L to the power 0 and T to the power 0. Then, this is the dimension for that.

Then, let us see what is the dimension for mu; if we recall, the our initial study in fluid mechanic, then we can just recall it that the dimension of mu is equal to mass. Then, M; that means, M L to the power minus 1 T to the power minus 1; (Refer Slide Time: 19:26) I am not going again back to that one, which we are supposed to know at this level. So, just we can just recall that mu is equal to M L to the power minus 1 T to the power minus 1. Now, just using that, we can write the dimension of this one as – let me write it as say M by L T to the power a. This is same as this one. So, this is M by L T to the power a. Then, rho – rho is nothing but density. What is density? Density is mass per unit volume. So, we can write this as mass; then, volume is L cube. This is dimension of volume; then, what is d? Dimension of d is nothing but diameter; diameter is nothing but L. So, L to the power C. What is the dimension of H? H is head. So, its dimension is again length dimension. So, H to the power d. And, what is g? g is nothing but acceleration due to gravity. Whether it is acceleration due to gravity or acceleration due to anything, but basically, dimension wise it is acceleration; dimension of acceleration. So, what we can write for g? Acceleration dimension as we know, that this can be written as length by say time square. This is the dimension of acceleration. So, this to the power e.

Now, if we just try to equate the dimensions whatever we are getting on left-hand side and right-hand side; first, let us equate; say from M, if we equate the dimension of M, here we do not have any value of M; that means, here we can write that M to the power 0 is there suppose. If we write even M to the power 0, there is no problem. So, what is the power of… If we just equate the exponent of this M, here it is 0. So, this implies 0 on this left-hand side from M; it is 0. We are writing the exponent; then, on this side, what are there? 0 is of course there; then M to the power 0 into M to the power a; what it will become? 0 plus a; and then, it is M to the power b. So, it will be a; M to the power a plus

b. On these, we do not have M. So, what is that? This basically, we can write that M to the power 0 is equal to M to the power a plus b. Here also we have M; here also we have M (Refer Slide Time: 22:03). So, if we combine, this is M to the power a plus b. So, 0 is equal to a plus b. Then, from L, if we compare, L here – we have 3. And, on this side, what we have? Here it is L to the power minus a. So, we will be writing minus a. Then, here we have minus 3 b. L is at lower position, at denominator means it is minus 3. So, it is minus 3 b. So, we can write minus 3 b. Then, it is plus c plus d and this will be plus e (Refer Slide Time: 22:42). So, this is also one relation we are getting.

And, from T, what we can get? T here is equal to minus 1. So, this minus 1 is equal to where we have T further. Let us search; this is T is equal to minus a. So, this is minus a; then, here we do not have; here again we have; minus twice e. So, we have three equations. So, by dimensional analysis from this relation; thus, from the functional relation we are starting. We are not using any other analytical basis; just from the functional relation, we are starting and we are arriving at three equations. Now, if we can somehow get the value of a, b, c, d and e, then our problem is solved; we are writing the equation, Q is equal to C into these things to the power these things (Refer Slide Time: 23:30). If we can get the values a, b, c, d, e, then we can put the value and we can get the equations straight away. But, there are definitely problems. What the main problem is that we have how many unknowns? 1 – a, b; 2 – c, d, e – means total five unknowns. How many equations we have? We have three equations. So, from three equations, we cannot solve for five values. And, that is why now; but, we can play with these equations; these equations can be used to express.

We have three equations; two quantities if we can express in terms of the other quantity, then we will be left with three variables unknowns. And then, we can definitely solve for that. So, what we can do, that from here, what it will be? Say this implies b is equal to minus a. So, b we can remove off. Now, in place of b, we can write minus a. Then, from this equation, what we can write, that say e we can write; if I take it to this side, (Refer Slide Time: 24:47) we can write e is equal to say half minus a by 2; that is, twice e equal to say 1 minus a; and, that we can write e is equal to 1 by 2 minus a by 2. So, e also we have expressed; b also we have expressed in terms of a. So, two quantities we have expressed. Then, c also we can express. Now, here also, because we have a and these things are there – b is there. So, if we replace these things, if we replace b and e in this form, then what we will be getting? That this will imply… First, we have done this; then, from here, this will imply say e is equal to… We can express b e; c – we want to express. So, c is equal to – that will become say 5 by 2 minus thrice a by 2 minus d. So, c – also we are expressing in terms of a and d.

Now, in this equation, if we put the value of all these quantities… Let me write. So, Q – now, how we can write? Thus, say Q is equal to – Q can be written as c; then, we can write mu to the power a; a we are maintaining – mu to the power a. Then, we have rho to the power b; but, rho – we will write as minus a, because b is equal to minus a; that we know. Then, diameter to the power c; in place of c, we will be writing this one; say 5 by 2 minus 3 by 2 a minus d. Then, what we are left with? H to the power d; d we are not disturbing – d let it be. Then, what else we have? We have g to the power e. So, e, v, l – we can replace as half minus a by 2. If anything is left? We have put all the five variables here – mu, rho, d, a, g. So, that is, we are getting now here how many variables? We have say a and then here we have d; these are there. And, a is here. So, now, q is being expressed in this term. If we now can separate those quantities, which are having only constant power, we can separate those; then, the quantities having power a, we can separate; then, quantities having power d, we can separate. Now, separating those quantities, this equation will take a shape; I am writing the final form; just we need to separate those and we need to write it in this form (Refer Slide Time: 27:46). So, this q, we can write in the form just combining those things and separating those exponents…

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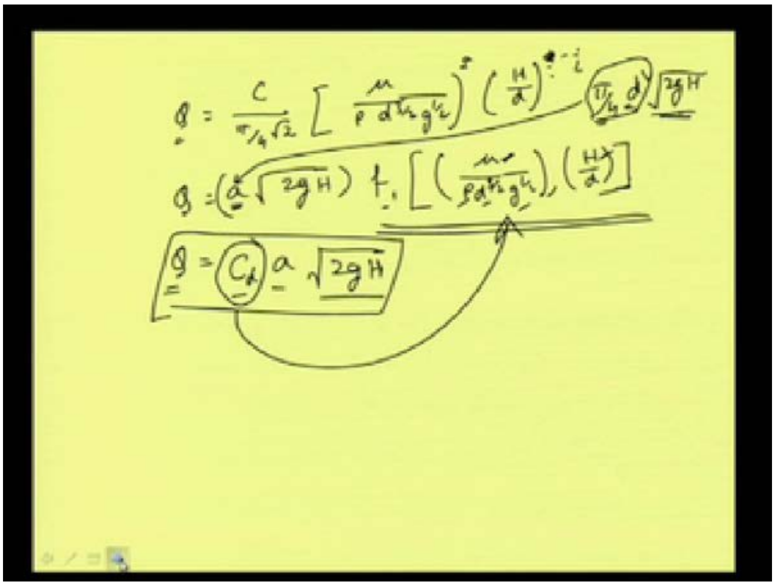

Having same exponent if we separate, then we can write it in the form that. I am writing the final form; Q can be written as Q is equal to C by say pi by four root over 2; then, we can write mu by rho d to the power 3 by 2 g to the power half; then, this is to the power a; that means, all a having a exponent, we have separated in this side. Then, we have say H by d – means H had the exponent d. So, H by d; then, d minus half; and then, pi by 4; then, d square root over twice g H we can separate. And, those which do not have the exponent, that we are bringing in one side and then we can write like this – root over g H.

Now, from this expression, what we can have? That Q we are getting in this form. And, we are not in a position to find the value of a and d at this point. But, we are having two terms. This should be like that (Refer Slide Time: 29:15). Now, we are having two terms, but we cannot get the exact value of this one; but, from this expression, we know that Q can be written as say – these are some constants; already these are some constants; then, these constant we can put again inside and we can say that Q is this one – say root over twice g H. These are also constant. So, what we can write – say Q is equal to – we can write say a root over twice g H. And then, we can write this as a function of… Other things we can put as a function; say constant will be there; then, it can go as function – function of say mu by rho d to the power 3 by 2; then, g to the power half; then, we can have H by d; and then, that way we can complete our relation. So, this pi by… Where from this a is coming? a is nothing but pi by 4 T square; that value we are writing as a here. So, this way a root over twice g H we are keeping here – Q is equal to… And, other things, which we do not know, because the power are there – a and d. So, exponents are there rather. So, we are writing it in this form.

And then, this equation (Refer Slide Time: 30:52) in fact is the equation for discharge – Q is equal to a root over twice g H. We started with one, say, dimensional equation at the beginning that V is equal to root over twice g H through orifice. And then, if we multiply it by the area, we are getting that; this is equal to a root over twice g H. So, as such, this equation can be now written as Q is equal to all these things – this function. Now, this function we can write in terms of coefficients say C d. And then, we can have C d a root over twice g H. So, the equation what we could write earlier – here we were writing in fact, this equation (Refer Slide Time: 31:37) that V is equal to C v root over twice g H; or, we can write V is equal to this means, if we multiply it by a and v; so, Q. This will become Q is equal to; then, the discharge – suppose I am writing as coefficient of discharge – root over twice g h; so, the same equation – Q is equal to C d root over twice g h. The same equation we are getting here, but what is the advantage of getting this equation? Because at this point when we were getting this equation in this form, then we were just knowing that discharge will not be equal to the exactly calculated value of root over twice g h; there will be some loss and all those things. And, because of that, there will be some reduction and that is why the coefficient of discharge is coming. And, that coefficient of discharge will be having some numerical value; that we can determine experimentally like say it will be 0.68 or something like that.

For orifice, it may be different; for weir, it may be different. So, like that, that is, coefficient of discharge will be having different value for say different object; but, we were not in a position to say that this C d will depend on what that was not known. But, now, when we do dimensional analysis, and then we are getting that q is – yes, this is say C d into a root over twice g H, then we can say that C d value will depend on these parameters, because we are writing that C d is nothing but function of this one (Refer Slide Time: 33:11). So, density, viscosity, then diameter, then acceleration due to gravity, what is the head under which it is flowing – those parameters are influencing the value of C d. So, that is the beauty of dimensional analysis. We are in a better position to say that what are the factor that will be influencing the value of C d. We can determine the exact value experimentally; that way this is what.

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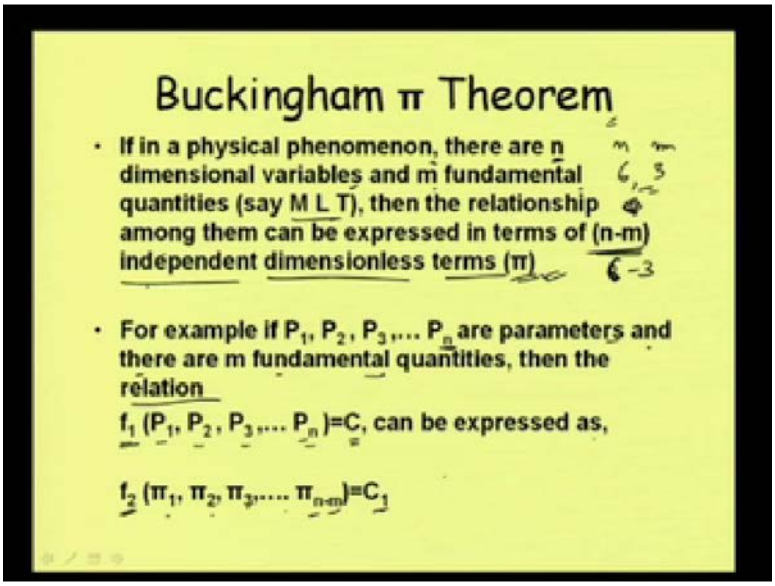

We have explained briefly the Rayleigh's method. Then, if we go for Buckingham PI theorem, let us just discuss that point. If in a physical phenomenon, there are n dimensional variables; n dimensional variables means – we have already discussed what is dimensional, what is non-dimensional variable; but, suppose in a particular phenomenon, we have n dimensional variables and m fundamental variables or fundamental quantities say M L T – these are fundamental quantities. And, in that phenomenon, involved fundamental quantities are suppose three; three means M L T; say sometimes if we consider temperature, there may be four. And, there are suppose n dimensional variables, then the relationship among them can be expressed in terms of n minus m independent dimensionless terms. So, that is what the Buckingham PI theorem. And, that is why this. Those independent dimensionless terms are referred here as pi. What we have actually explained by this one that if there are say six variables involved in that entire phenomenon and then suppose three dimensionless or three fundamental quantities like this, and then, we will be having three independent dimensionless terms, because n minus m is 6 minus 3; n is equal to 6; m is equal to 3. So, that is what we are referring.

We will be going further details into that. For example, if P 1, P 2, P 3 up to P n; say n number are the parameters; and, there are of course, we should mention here. These are the dimensional parameters or parameters having dimensions. And, there are m fundamental quantities; then, the relation – say our actual relation is say it is a function of… This can be related as say function of all the parameters and that can be equated to a constant c, because we know that these parameters are involved in that phenomenon. Of course, we can write say P 1 is equal to… We can write this in this form also (Refer Slide Time: 36:05) – P 1 is equal to a function of all these. That way also it can be written; or, if we just do alteration, then we can write it in this form. So, this can be expressed in the form of – say rather than writing this, we can have a different function f 2; and, this will be say pi 1, pi 2, pi 3. And, how many pi or how many dimensionless quantities will be there? It will not be m; it will be n minus m. So, up to n minus 1, say dimensionless parameter we can have and that is equal to say C 1; a different constant may be coming.

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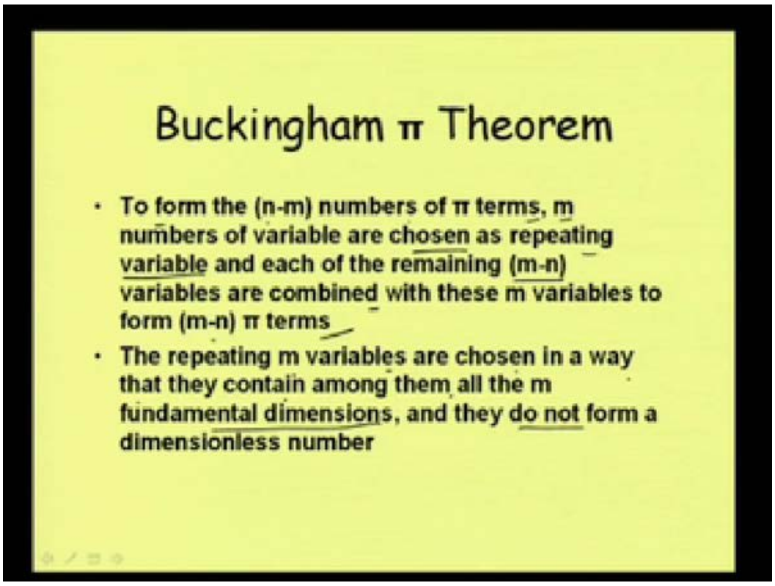

Now, this Buckingham PI theorem in it to form the n minus m numbers of pi terms, what we do, that m numbers of variable are chosen as repeating variable. We are saying that we will be getting n minus m number of non-dimensional term; but, how we will group them? How we will be grouping these non-dimensional terms? Because if a particular dimensional term is there, definitely, that will be dimensional. To have a non-dimensional term, we will have to group them in a combination that we get a non-dimensional term. So, two quantities having different dimensions; 2-3 quantities if we club, and then, these entire quantities may form a non-dimensional term. Now, how to form that? So, we have total m numbers; then, how to form that?

m numbers of variables are chosen as repeating variables. Some variables will be repeating in all these pi terms. And, each of remaining m minus n variables are combined with these m variables to form m minus n number of pi terms; that means, if we have six parameters or six dimensional variables, then to form the non-dimensional or to form the dimensionless pi term, what we will be doing? We will have to select three variables, because we know that our dimensional primary numbers are primary or fundamental quantities are M L T 3. So, our pi term will be 3; 6 minus 3; it will be 3. So, what we will be doing, out of the six variables, three variables, we will be choosing as repeating variables; that means, in each of the pi term, these three variables will be there; then, along with these three variables, the other remaining three variables – we will be putting one by one. In one pi term, we will be putting from remaining variables; remaining variables are basically n minus m remaining variables. So, we are putting one from that; then, we are getting one pi term; another in another. So, that way we are getting say first pi 1 and pi 2; then, pi 3 we will be getting by adding the third variable, which is not within repeating variable. Then, how many that sort of variables were there, which are not in the repeating variables that will be say n minus m. So, how many pi terms we are forming? That is also n minus m; and that way, we form the n minus m numbers of pi terms.

Now, the question is that out of six variables – I am just giving you a example – six variables; there may be even ten variables. Then, out of that ten variables, we need to take some variables; or, suppose if M L T are the fundamental quantities, then we need to take three variables as repeating variables. Now, which three variables we will be taking as a repeating variable; that is the important point. So, these three variables we should select in a way that these three variables will involve all the M L T; not necessarily that out of these three, one will have to keep M L T; it is not like that; but, if we see all three together, they will have to have M L T – all the dimensional parameters; all the dimensions they will have to have; all fundamental quantities they will have to have; that is M L T; this is one requirement.

Let us see what the requirement is. And, another requirement is that – first requirement what we have stated that the repeating m variables are chosen in a way that they contain among them all the m fundamental dimensions. Among them; not individually. Among these three, all the fundamental dimensions will have to be there. Then, second point – that they do not form a dimensionless number. See if these three variables if we combine and form a non-dimensional number or dimensionless number, then it will be a problem; I mean it will not serve our purpose. So, these three should not form a dimensionless number; that way this is the criteria of selecting these repeating variables.

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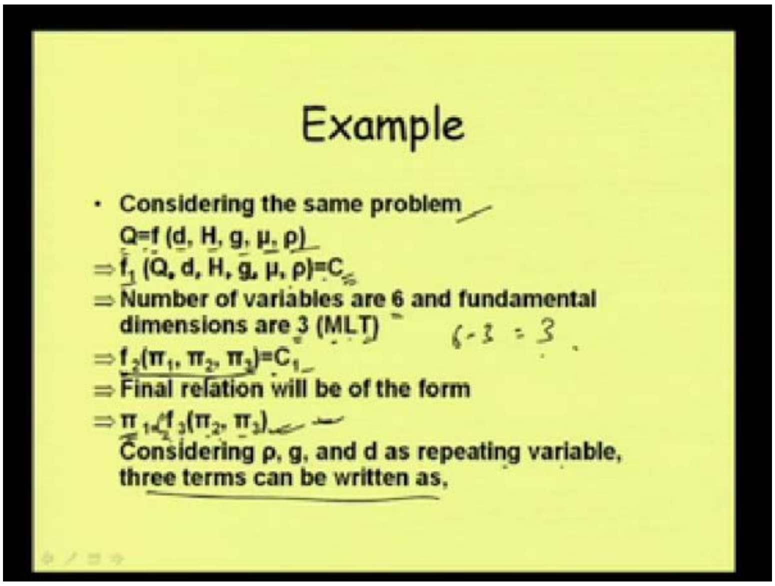

I will be giving one example quickly; that is, suppose let us consider the same problem what we have discussed; that Q is a function of… I mean discharge is a function of flow through orifice again we are talking about. So, Q is a function of d, H, g, mu and rho. So, as a function (( )) we can write this is a function of all these parameters and that is equal to C. This we can now write say like that dot – this product. And, what it indicates? That number of variables are 6 and fundamental dimensions are 3; that is, M L T. We do not have temperature parameter here. So, these are M L T. Now, we are sure that 6 minus 3; this is equal to 3; pi term will be there; and, that is why. This equation – let us write as f 2, a different function we are writing. And, pi 1, pi 2, pi 3 is equal to C. So, that way, we can write the function; and then, pi 1. Again, when we are writing like this with a different constant, we can write that pi 1 is equal to that; this pi can be related to other function also. If we just take it on that side, then we will be having a different function, but, it cannot pi 1 is equal to C 2 into f 3 into pi 2, pi 3. So, this way, we can write that expression. Now, which repeating variable we will be choosing? We are knowing fundamentally that this should be. Considering rho, g and d as repeating variable, three terms can be written as… So, how these three terms we are getting – pi 1, pi 2, pi 3.

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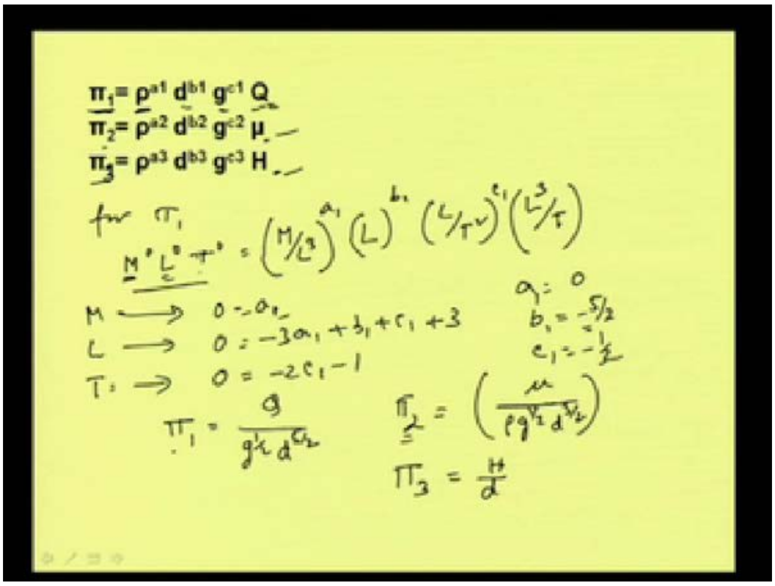

If we consider rho, g and d as repeating variables, we can write pi 1 is equal to rho to the power a 1 d to the power b 1 g to the power c 1. Then, we are adding rho, d and g – are our repeating variables; and, q, mu and h are non-repeating variables. So, with the first one, we are putting Q discharge; and then, pi 2 – again this is a 2; like that rho to the power a. So, all are exponents. Then, this mu we are writing here. Then, pi 3 – this should be pi 3 – is equal to rho, d, g; then, h we are getting here. Now, what we can do, from these particular expressions, we can move further; time will not permit me to go for the detailed analysis. But, just like what we did in case of the Rayleigh method, similarly, from this pi; pi is a constant term. So, what will be the dimension? We are saying that pi is a non-dimensional term. So, what will be its dimension? If I write in terms of M L T, it will be M to the power 0 L to the power 0 T to the power 0. And, we know the dimension of these terms. So, we can again proceed in the way that we did in case of Rayleigh method. Briefly, I am doing it.

Say writing for pi 1, what we can do – say for pi 1 first, I will write for one first; so, pi 1 what we can write, say M to the power 0 L to the power 0; then, T to the power 0. This is equal to… This discharge is the first term here. So, we can write that M by L cube to the power – first term is not Q it is rho. So, rho means M by L cube to the power – say a 1; then, d is the length dimension – say length to the power say b 1; g is the length by time square; so, this length by time square to the power say c 1. And then, Q as we know, this is equal to say volume; that is, L cube by time. So, we are writing like this. Then, again just like that earlier we do, here if we equate M value, that M will give us one equation; L will give us one equation; and, T will give us one equation. So, from these equations, what we can get, say for M, we will be getting 0 is equal to a 1, because in other times, a is not there. And then, L will give us a relation that 0 is equal to minus thrice a 1 plus b 1 plus c 1 plus 3. And then, T – this will give us one relation 0, because here all are 0 basically (Refer Slide Time: 45:54). 0 is equal to minus twice c 1 minus 1. And, from this equation…

Now, here this is little more easier, because we see we have three equations. And, from these three equations, we need to solve for a, b and c; that means, three quantities – we need to solve from three equations, which is quite possible; easy. So, we can get from here that a is equal to 0; b is equal to… a 1 is equal to 0; b 1 is equal to minus 5 by 2; and, c 1 is equal to minus half. So, that way we can get. So, what is our pi term? Now, pi 1 – we can write; say from that, just by putting these expressions, because pi already we are expressing. So, putting rho to the power a 1 means it is 0. So, rho is vanishing from that; Q is there. So, we are putting Q. And then, (Refer Slide Time: 46:45) g to the power what we are getting? c; c is minus half. So, g will be coming g to the power half here. And then, d to the power minus 5 by 2; so, d will be coming here – d to the power 5 by 2. So, pi 1 is this one. Similarly, going by pi 2 and pi 3, we can have expression for this pi 2 and pi 3 also.

Let us just write what is pi 2 and pi 3. We are not doing in detail, but pi 2 we will be getting in the form that it will be containing mu. So, it is mu by rho g to the power half; then, it is d to the power 3 by 2. Now, this remains as exercise; we can always try and we can see whether we are getting this or not. Similarly, pi 3 – let us take this also as our exercise; after the class, you can do that. So, pi 3 – if we just put it like that, then it will become H by d. So, that way we are getting… See H by d, obviously, this is non-dimensional term; dimensionless term rather we can say. And this is pi 2, is also a dimensionless term. If we check the dimension of this one, we will be finding that it is a dimensionless term, because we are starting with this principle that these are dimensionless term. So, we are getting these things.

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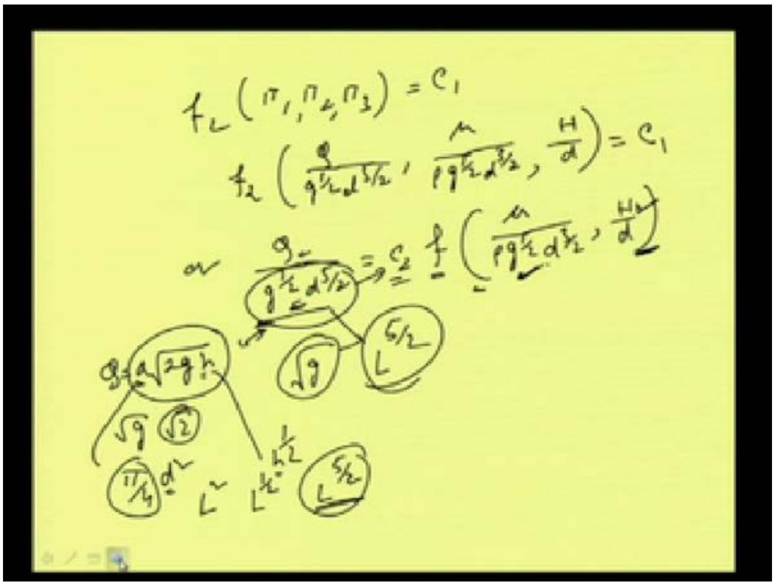

So, what our expression can be? Now, putting these values, we can write that f 2 pi 1 pi 2 pi 3. This is the function, is equal to c 1. So, by substituting the above value, what we can write pi 2? That we can write as Q by g to the power half d to the power 5 by 2; and say mu by rho g to the power half d to the power 3 by 2; then, we can write H by d. Whatever dimensionless term what we are getting, that we are putting here and that we can write that; this is equal to say constant c 1. This can be written in the form again that Q by g to the power half d to the power 5 by 2 is equal to c 2 and f. We can write mu by rho g to the power half d to the power 3 by 2; then, H by d. So, this way, we are getting a relationship, which is showing us a; that Q we are expressing in this form. Of course, this relation appears to be different from what relation we obtained earlier. But, this we can again change and then we can write it in a different form and we can have it in the original form also, which is very familiar form. So, this way, we can express the equation for Q.

This expression here – this we have already in the earlier method – Rayleigh method also, we got this one; this also we are getting fine. And then, this expression is little different, because earlier, we were getting say Q is equal to a into root over twice g h. So, that we were obtaining; velocity is equal to root over twice g h. So, area into velocity we are getting right. Now, if we can write Q is equal to… Taking this part to that side, we can have it in this form. Now, just verify whether this term and this term is identical or no. What is the dimension of this term? We can see that it is g; g to the power half of the… Root g is here. Here also root g is here. So, let us keep it. Then, here we have root 2, which is a constant. We are not bothering about that; that can be taken to other side. Then, d to the power 5 means we can have dimension of L and that is to the power 5 by 2.

And now, here what we are left with a and H. So, a can have a; the dimension of a; we know that pi by 4 d square and H to the power half. So, it is again h to the power half. So, what is the dimension of this if we combine? Say it is a constant part. So, d square means we can write L square and this H has the dimension of L. So, L to the power half. If we just combine, this becomes L to the power – say 2 into 2; so, it is 4 plus 1 5; so, 5 by 2. So, from here also, we are getting L to the power 5 by 2; here also, we are getting L to the power 5 by 2 – this. And then, root over g is there here. So, that way, dimensionally, these quantities are same (Refer Slide Time: 51:29). And, these are otherwise also same. That way we can have some more understanding of a particular phenomenon through dimensional analysis.

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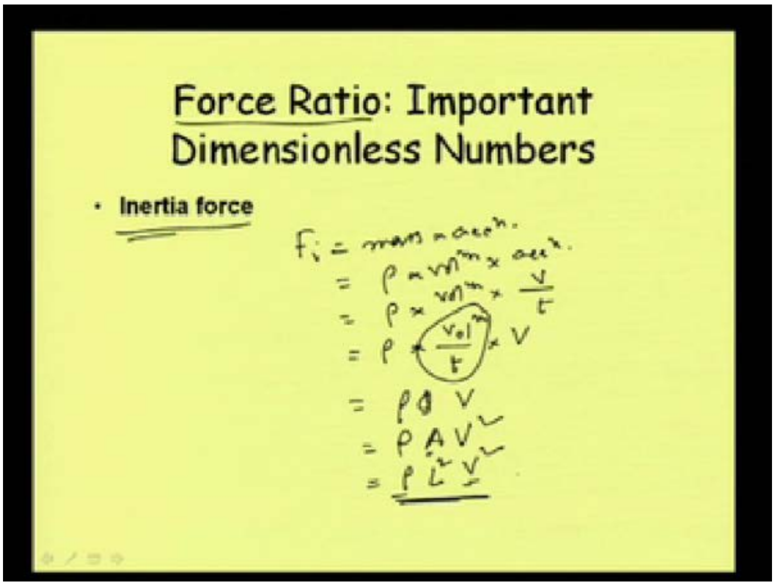

Then, some important dimensionless numbers are there. We know about those numbers little bit already, but we need to do more on that. And, that will be required in our hydraulic model study. The force ratio – if we take ratio of two forces, then definitely, there will not be any dimension; both are having dimension of force. So, there will not be any dimension. So, let us see, what is the expression for inertia force; we can write quickly say inertia force is equal to mass into acceleration. So, that we can write, say, rho density into say volume into say acceleration. And, that we can write in the form that rho into volume into acceleration – is nothing but V by t; this V is velocity. So, velocity by 2. And, that we can write as rho into volume by say time into velocity. What is volume by time? Volume per unit time is nothing but the discharge. So, we can write rho Q into V; or, this can be written as rho A into V square. So, dimensionally, how we can write these things? Dimension of this one we can write; that is, rho; and, dimension of say A, we can write L square and V square. That way also we can write this. Now, using this expression, we are keeping rho and V fine; and, this area, we are writing in terms of L square from the length dimension.

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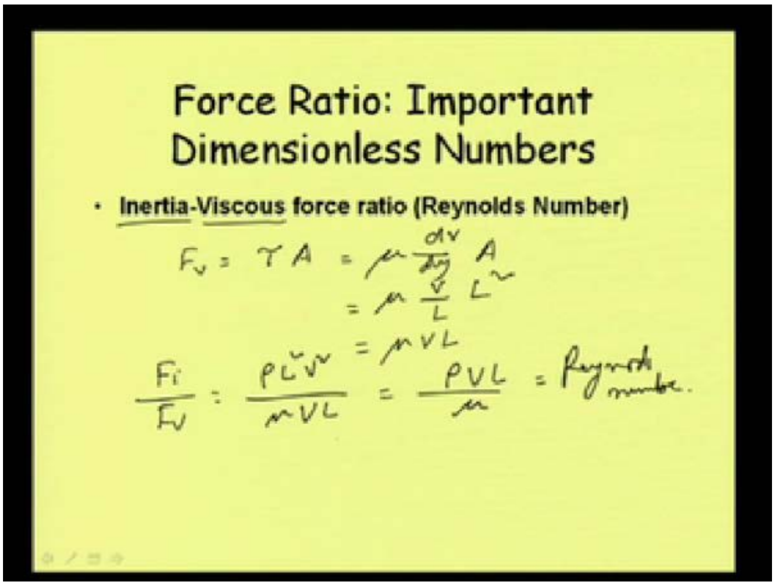

Now, let us see how we can use this to have a different force ratio. First, we will be talking about the ratio inertia-viscous force ratio. Now, what is the viscous force expression? Say F v we can write as, at any point, suppose shear stress is this one; then, this viscous shear is this one, tau. And then, if we multiply this by A, then we are getting say viscous force. So, this viscous force we can write. What is tau? We can write mu into d v by d y; then, we are having area. So, this can be written as mu; then, this d v by d y dimensionally… or, say value wise, you can write this is V; and then, d y is the length dimension. So, L; and, this area is equal to L square. So, we can write this in the form of mu V into L. And, just we should remember one point; this length dimension we are writing as say not exactly as L. Suppose area we are writing in dimension that L square;

that is why, these are called characteristic length. It is representing that way. And then, what is the ratio if we write say F i by F v; F i means this one (Refer Slide Time: 54:40) – rho L square V square and then this we are writing. So, this is equal to rho L square V square divided by mu V L; then, this will give us one interesting number; what we are using in all our earlier classes, that is, rho then V then L divided by mu. And, we know very well what this number is. So, this number we call as Reynolds number.

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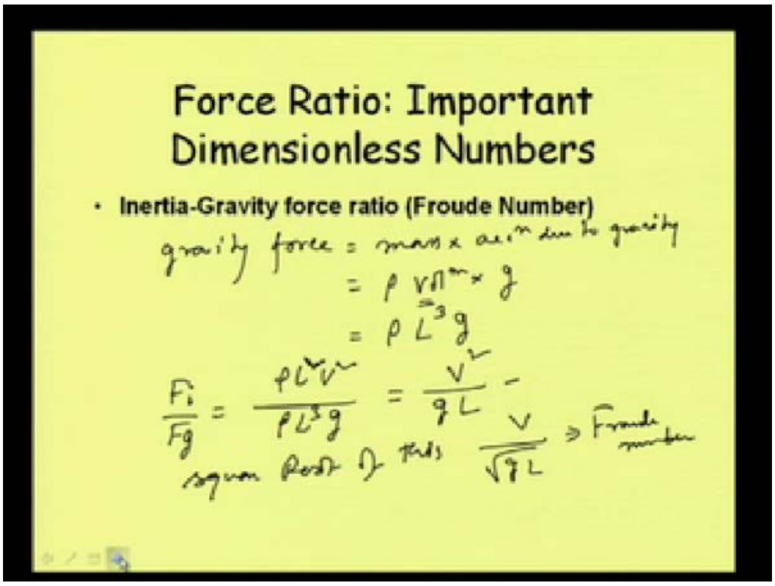

Then, similarly, let us see what is the… if we talk about inertia-gravity force ratio. So, gravity force – what can be the expression for gravity force? The expression for gravity force; we can write that mass into acceleration due to gravity. Again, whether it is due to gravity, as acceleration, it can be written; but, if we are interested to keep the g term here, then how we can write? Mass is nothing but rho into volume. And then, it is g. Now, what is the dimension of volume? Basically, here what we are trying, say, the quantity, which are which can be expressed in terms of length dimension, that versus geometrical quantity, we are expressing, say, rho into volume; means, we are writing say rho; this volume can be written as L cube and then we are putting g. So, we can now again write F i by F g, which is the dimensionless quantity. So, that will be again rho L square V square; that is the inertia force expression for inertia force.

And then, we are putting rho L cube into g. So, this will give us another interesting quantity, which we are using all. In our previous classes, this V square – that will remain;

and, this will become g and this is L (Refer Slide Time: 56:36). Again, that is why we call this L as characteristic length. And, for different situation, this characteristic length particularly in Reynolds number, we remember that when we were discussing pipe flow, our length was diameter. When we were discussing say open channel flow, this length dimension we took as hydraulic depth. So, that way, for different quantities, again, when we study boundary layer, then that Reynolds number; again, we take the length dimension as different. So, that is why; thus, dimensionally, we are writing this thing. So, this is V square by g L. And then, this we do not use as a number; this is a dimensionless number, but what we do, square root of this; say square root of this number of this; that is, V by root over g L. This is called Froude number. These two numbers, we are already getting. But, we are just trying to analyze how these are coming. Then, some number, Euler number, Newton number – that will be coming from inertia and pressure force.

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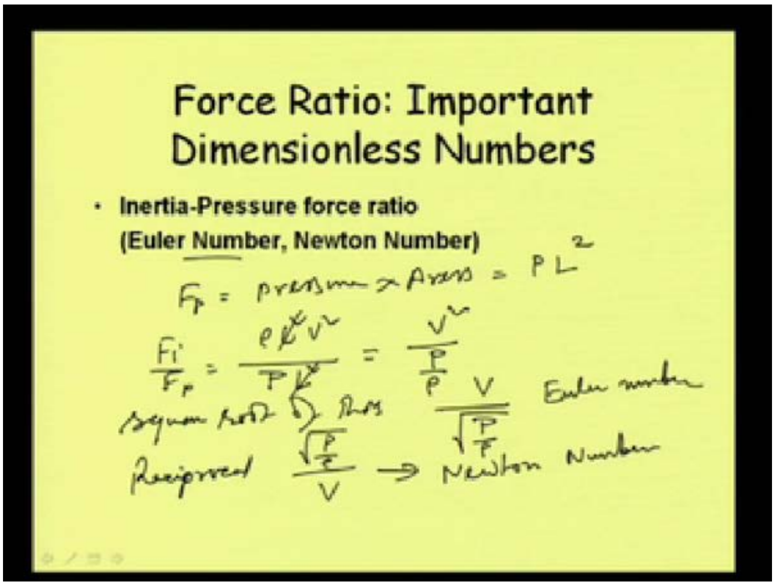

What is pressure force? Say F p if we write, this is equal to pressure into area; it is very simple. So, pressure we will keep P. And then, area we can write L square. So, what will be say F i by F p? That will be rho L square V square divided by say P (pressure) into L square. So, that will give us this expression as V square divided by P and rho will become – L square L square getting cancelled; so, p by rho. And, square root of this is called Euler number. So, square root of this number is called Euler number; that is, V by root over P by rho. This is called Euler number. Then again, there is another number; that is called Newton number – reciprocal of this one. So, reciprocal – say root over P by rho divided by V. This is called Newton number. These are some important dimensionless numbers that we use many a times in our hydraulic analysis.

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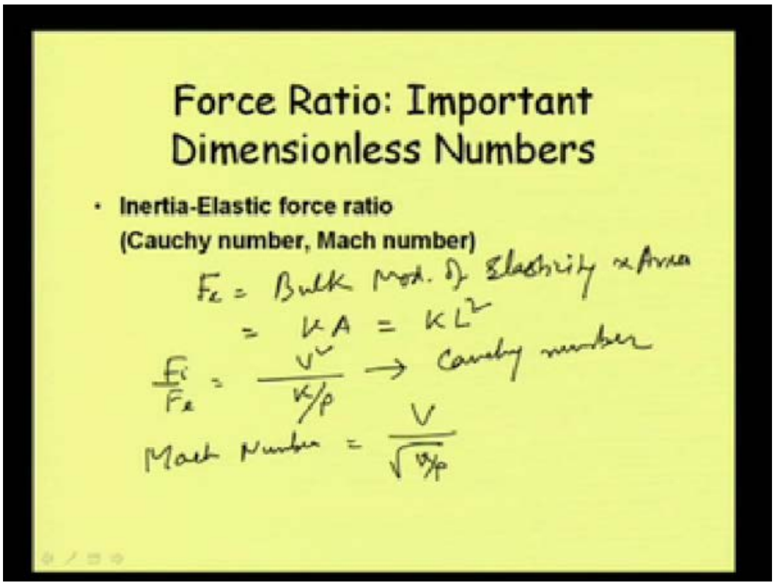

Another is say inertia-elastic ratio – inertia versus elastic force. So, what is the expression for… And, that is Cauchy number and Mach number. That is also very important in fluid mechanics particularly. So, say F e; what is the elastic force that we can write? Say bulk modulus of elasticity; that is, bulk modulus of elasticity into area. Strain is a non-dimensionless bulk modulus – stress by strain. So, stress is the pressure; and area – it will be force; and strain is a non-dimensional term. So, this F e we can write in this form. And, that is equal to K – is normally used as bulk modulus of elasticity; so, this is K into A or K L square. So, this ratio again, F i by F e, that if we write, we will be getting it in the form of V square by K by rho. I am not writing details of this one; just few more steps will be required. And, this number is called Cauchy number. And then, say this number – if we take square root of this number, this is called Mach number. So, Mach number – what is that? We can write Mach number is equal to V by root over K by rho.

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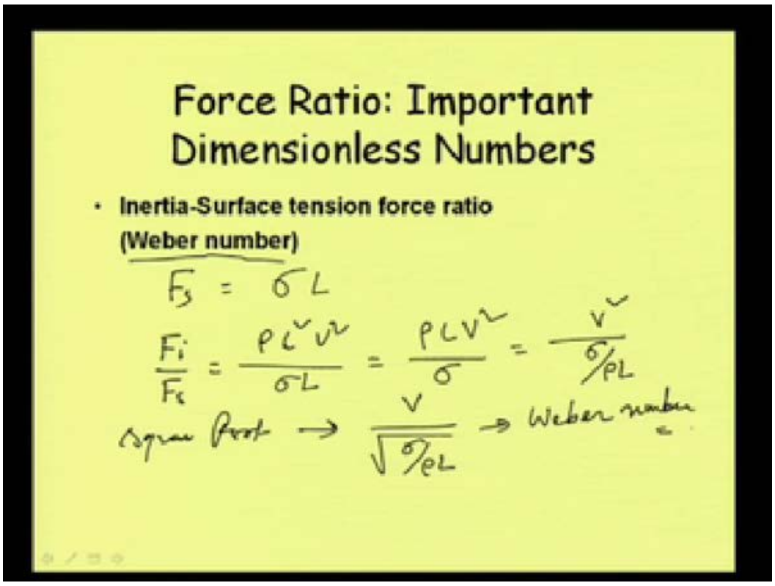

Then, one more number is popular; that is, for surface tension say F s. Suppose s represents surface tension; then, what we can write? We know that sigma – suppose surface tension value means surface tension force per unit length; what will be the force? It will be this multiplied by L; then again, if I take this ratio F i by F s, this will be say rho L square V square divided by sigma L. So, that we can write as rho L V square divided by sigma. So, this can be written as V square by sigma by rho L. And then, square root of this is equal to say V by root over sigma by rho L. And, this number is known as Weber number. So, this is Weber number. So, that way, we can have different dimensionless numbers. And, for hydraulic model study, these numbers are very important. How these are important and how these help us in developing hydraulic model? That we will be discussing in our next class.

Thank you very much.