Hydraulics
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Module No. # 07.
Unsteady Flow
Lecture No. # 02
Unsteady Flow Part-3

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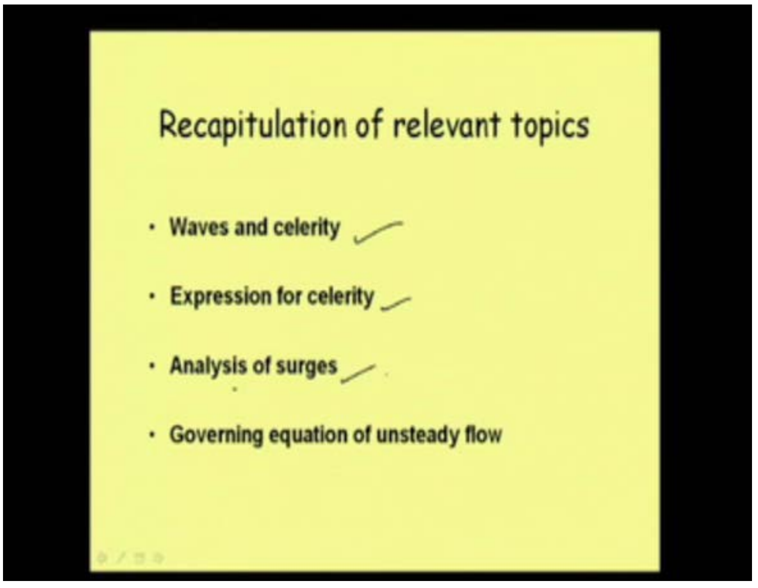

When the wave is moving, it is the moving wave front rather and that we call as surge and of course, this has some other name like bow, when it is in sea, I mean in astrory then we called that as a bore. So, then we were just starting the governing equation of unsteady flow, and when we did cover the gun, when we started with this unsteady flow equation, governing equation, that was particularly for gradually varied and shallow water wave equation of unsteady flow and we were talking about continuity equation. In fact, which we have discussed already in our class, when we are starting our governing equation, may be in our fifth class or sixth class.

Next equation, that is equation of motion, generally we need 2 equation for solving the problem of unsteady flow and that is why the continuity equation and equation of motion is used. An equation motion we can get again in different forms. We can develop this equation starting from different principles like from energy approach, we can develop it or starting from the Newton's equation of motion, we can develop it. And then here we were just starting to show how from the energy approach we can develop this equation of motion. And of course, here finally, we will be developing one form of the governing equation and we can have several different form of this governing equation and we were writing that equation for this sort of flows, section 1 and 2 say this is the datum and we were considering a small section, del x and then this is z 1 y 1 then v 1 square by twice g and this is z 2 y 2 and then v 2 square by twice g and then there is energy loss that we could get some expression for that. This energy loss we can divide it in 2 parts, s f loss due to friction and loss due to acceleration.

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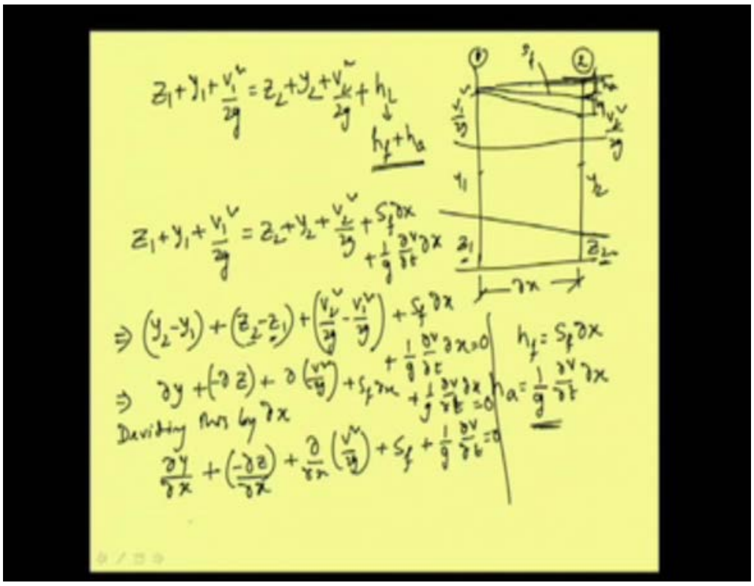

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And, then equating the energy between section 1 and 2, just we recall we did that in our last class. So, we could write that z 1 plus y 1 plus v 1 square by twice g is equal to z 2 plus y 2 plus v 2 square by twice g plus h L, and this h L that we can write as, h f this h L is nothing, but h f plus h a friction loss plus loss due to acceleration work done work energy loss due to producing that acceleration.

So, this part also we derived in our previous class and then from this expression, what we can write putting those expression for h f, if we remember or if we recall then, h f we got as this is from friction slope, if friction slope is s f, then s f into delta x is the total friction loss, if friction slope is s f, friction slope here if we write then this s f is equal to, this is s f then s f into delta x will give us this s f and then our this acceleration loss, loss due to acceleration, that we could derive as 1 by g, then del v del t into delta x, then this acceleration is per unit weight and we are writing all these head, you can see this energy we are writing in terms of per unit weight, that is why we did calculate this acceleration also in terms of per unit weight.

So, that expression, if we add here, then what will be our expression that will be, z 1 plus y 1 plus v 1 square by twice g is equal to z 2 plus y 2 plus v 2 square by twice g plus. This h h h f we can write as now s f into delta x, then h a we can write as a 1 by g into v del v del t into del x. Now, let me write this expression bringing all term in one side and putting 0 on right hand side. So, this will implies and let us write it in this form that, y 2 first we can write y 2 bringing on that side, y 2 minus y 1, because to have the expression, our main intention is to get a relationship. So, that we can have, how the y are varying, how surface is varying. So, y we are writing first y 2 minus y 1, then let me write this z z 2 is if we bring here, this will be z 2 minus z 1 and then let us write the velocity term. So, this will be plus say v 2 square by twice g minus v 1 square by twice g.

So, we are writing each term in a different way with a intention basically, we will write it in a very simple way and then what we have that is plus s f into delta x, we have then of course, that term is remaining same 1 by g del v del t into delta x. Now, this y 2 minus y 1, that is the difference between depth here and depth here, this difference we can write as a small depth change and that is why let me write this as del y, we can write it as del y, small distance and this change in small elevation, let me write as del z, then this we can write as del of v square by twice g and then other part will be remaining as it is.

So, plus s f into delta x plus 1 by g del v del t into delta x. Now, if we divide this expression by delta x, dividing this by delta x, what we will be getting? Del y del x, then this will be plus, here we can make a change, what is that? Z 2 minus z 1, that is z 2 here if it is a falling slope, if the slope is falling in the downstream direction, then z 2 minus z 1, that will be basically always negative because z 2 is smaller than that. So, if we consider that, then this small del z we can write as minus delta z, we can write it as minus delta z; that means, we are considering our slope to be falling, that is why we are putting this negative sign, as we know the z 2 is smaller than z 1. So, we are writing here then plus. So, this we will have to put here itself, so, minus delta z, so, delta z by minus delta z delta x and then this one will be del del x of v square by twice g and then this part delta x will be going off, then it is s f plus this delta x will be going out, then 1 by g del v del t, now from this what we can simplify further on this side as we brought all the expression to one side here itself, it is equal to 0 and here also it is equal to 0 and this is also equal to 0. We forgot to write that 0 on this side.

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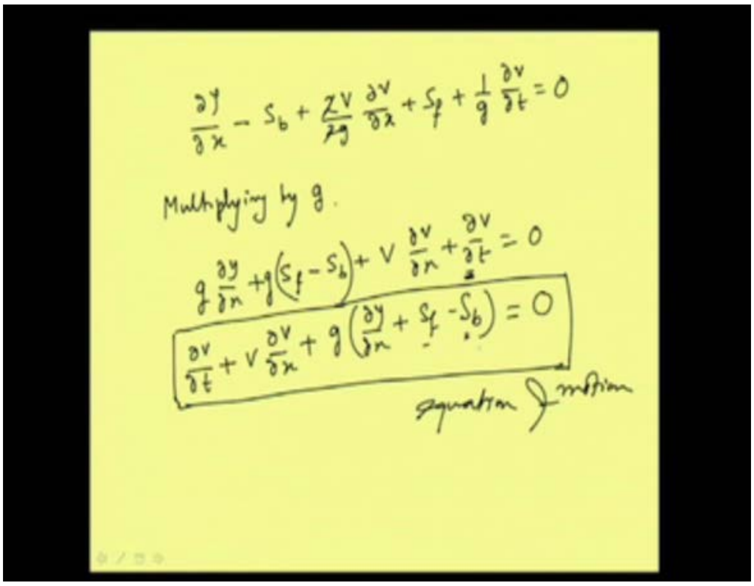

Now, this can be written as, let me write it here, del y del x is there, then minus s b plus, we had earlier, this expression we had del del x of v square by twice g. Now, this we can differentiate, how that twice v then del v del x that we can do. So, let us do that and we can write it like that, twice v by twice g was already there and. So, it will be del v del x plus s f plus 1 by g del v del t. This is equal to 0. Now, if we multiply this expression or multiplying by g, so, what we will get? G del y del x plus s f minus s b let us put it here, s f I am writing first, then plus 2 and 2 is going out here. So, it will be V g is already we are multiplying it. So, V del v del x, then it will be del v del t plus del v del t is equal to 0. This is what the equation for with s f and s b we did not have the term g, so, as we are multiplying it by g, this will be multiplied by g and this is the equation of motion and the standard we how we write this, we write the derivative of this t, now we are talking this as derivative, although we were writing this small increment earlier and we are following the notation considering the very basic concept that V will be changing partially with respect to time that is why we were putting the notation as del, they are to represent a small distance.

So, then we can write this as say del v del t plus V del v del x plus g del y del x plus s f minus s b, that is equal to 0. And this is the required equation of motion, this is a simple way of deriving this equation of course, as I explained in the last class, that we can derive this expressions starting from the Newton's equation of motion and that way also you can get the equation and so, that sort of momentum of equations are very popular and we can write it in metrics form also, combining this say continuity equation and momentum equation, we can write it in metrics form and that way it become more suitable for handling in computer programming and then this equation that we are writing is basically for one dimensional flow, that is we are considering the flow is one dimensional.

So, when we are trying to equate energy or when we will we suppose try to equate momentum, that we are doing in only one direction, momentum equation in one direction, but when it will be a 2 dimensional flow then we will be getting 2 momentum equations. So, that way our problem or the governing equation of this unsteady flow will be containing 3 equation, one is continuity equation may be in one direction x, then say momentum equation in x direction, then momentum equation in y direction means lateral direction that we may get 3 equations and solution of these equations are direct solution is not possible and because these are all non-linear hyperbolic type and so, we go for generally numerical solution and then when we go for numerical solution, lot of computer applications are necessary and then we write this equation in metrics form and that way, we get, it become more convenient for solution in a computer and sometimes we simplify this sort of equation, this is our equation of motion, this is our equation of motion.

And now in this equation, if we considered the bed slope to be very small, if we considered the bed slope to be very small and suppose it is negligible, then this s b will become 0 well. So, our equation from our equation this term is going out, then if we consider s f, that is the friction loss is very negligible, suppose it is a sort reach or somehow we are neglecting the friction, that will go out, then the equation will take a difference shape. Again in some case we may not consider some of this term and that without considering those term again s b equal to s f also we can get, without considering these terms. Then it becomes simple equation s b equal to s f that is the uniform flow we were getting that expression. So, like that we can have different forms in different way and then with this equation say finally, what we can write that this one dimensional governing equation, we can write in this form.

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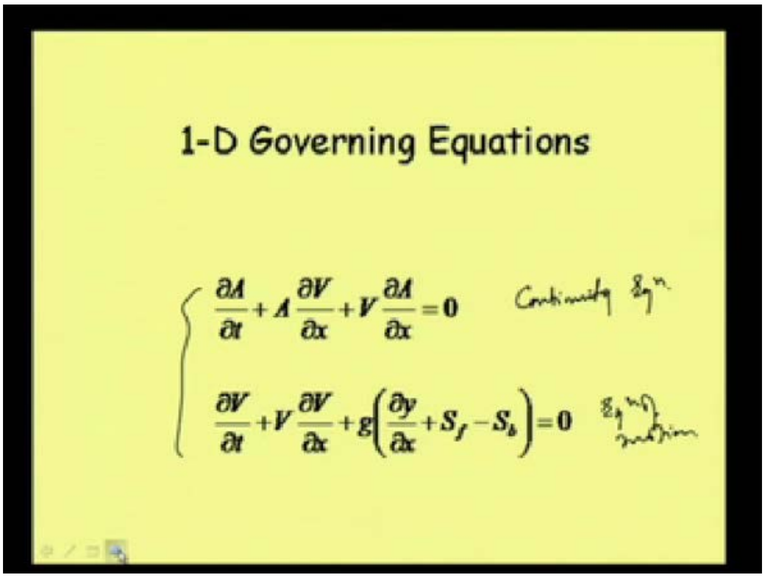

This is the continuity equation and this is equation of motion and as we have explained already that solution of this equation or these equations are not that simple, because of each form and. So, that fully dynamic equation, this one if you considered then, solution of this equation become complex and we need to take a recourse to a numerical methods and let us see how we can do solution of the governing equation.

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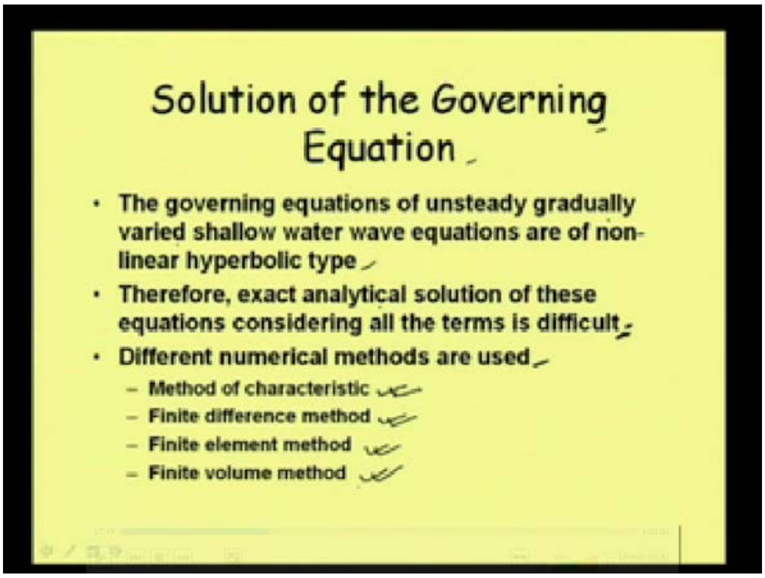

The governing equations of unsteady gradually varied shallow water wave equations are of non-linear hyperbolic type and therefore, exact analytical solution of these equations considering all the terms as I said that sometimes we can simplify this equation, but considering all the terms is difficult, I am not saying impossible, but it is difficult because, people are doing different type of solution and someday they may get solution for this one direct that different numerical methods are used.

So, what are the numerical methods? These are method of characteristic which was started long back. So, method of characteristic is one popular method which is used for solution of any partial differential equation of course, that was applied to this solution of open channel flow equation also and then that is why we are putting it here. And then finite difference method which is use popularly and finite element method which is also used for solution of these equations, then finite volume method is another method which is used. So, that way different method can be used.

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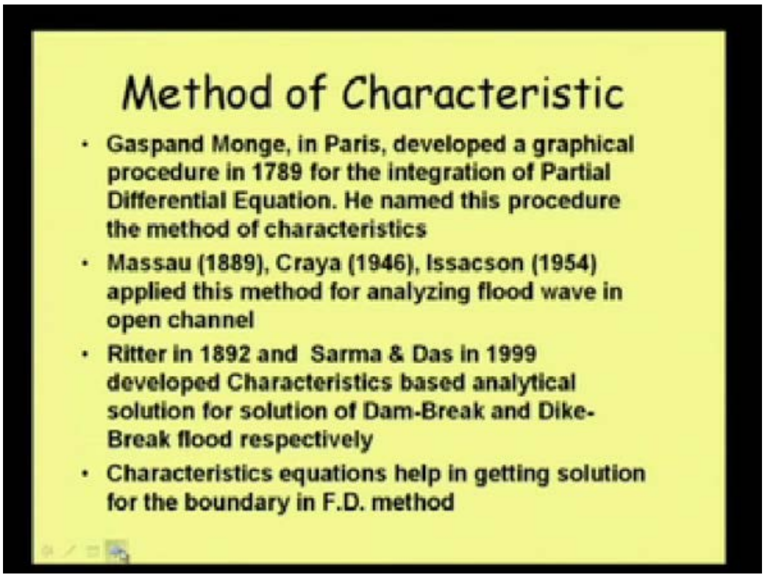

Now, we will be concentrating. So, far this particular course is concern, we will be just discussing some of the aspect of this characteristic methods, method of characteristic, not all, because these we are keeping for higher level of study. So, method of characteristic also will not be discussing in detail. So, Monge in Paris developed a graphical procedure in 1789, for integration of partial differential equation. In fact, this method of characteristic, initially it was given as a graphical procedure and then it was for solution of partial differential equation for integration of partial differential equation and he named this procedure the method of characteristic. So, that is, it came long back in mathematics then of course, the Massau and Craya and Issacson, they applied these methods for analyzing flood wave in open channel. Now, you may have some thinking that how then still it is remaining popular, it was long back that in 1789, it was started people have used this in 1889 and why still it is being used? Because starting from method of characteristic Ritter, he did one solution in 1892 long back, he did this solution for dam-break flood wave simulation, he did this solution, then we started this problem and we could see that for dike failure to simulate the dike failure flood, suppose the flood is moving from the river due to failure of a river dike, in that case, we do not have a solution or we require a solution for to fit into the initial part, later on, we can take it numerically, but the initial solution is important in some aspects. In that case we did use this starting with this characteristic method so, we provided a solution for dike failure problem. So, using this characteristic method, Ritter gave for dam-break and we gave it for dike break flood.

And then, another important why still it is gaining popularity, because or still it is in use because when we use numerical methods, finite difference methods suppose we are using, then always we find problem in solution of the boundary problem, we have a flow domain and we want to solve that flow domain, what are the value at, when we are talking about solution of flow in unsteady flow, we are meaning that what is the value of depth or what are the value of depth and velocity at a particular section or at a particular section x means space we are talking about, at a particular space x say special point x and then at particular time. So, with space and time these are changing. So, our challenge is to get solution for any point and at any time. So, when we try to do that by numerical method, we can get solution for some grid and that grid point, if we made very close, it become almost solution for that, it represent the solution for the entire domain.

So, in between the grid also we can solve sometimes and then, but when we apply those method of solution, then we find that at boundary we many time we find problem. And this characteristic method or method of characteristic, help us in getting the solution for the boundary. And that is why, we need to study this method and the very basic concept of this characteristic method. So, that we can, if we go to the higher level, other methods when we apply, then we will be able to use the advantage of this knowledge of characteristic method.

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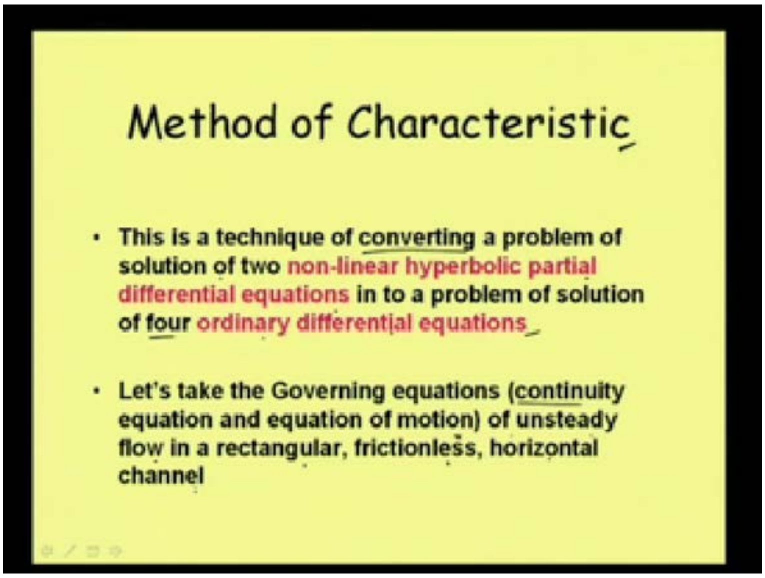

Then method of characteristic, what it is, its basic I mean, way of doing is that, this is a technique of converting a problem of solution of 2 non-linear hyperbolic partial differential equation into a problem of solution of 4 ordinary differential equations. So, far water problem is concerned, we have two non-linear partial differential equation, one is continuity equation, another is the momentum equation or energy equation. Now, these 2 equation partial differential equation non-linear hyperbolic type, we cannot have a direct solution by direct integration. So, if we can convert it these 2 equations into ordinary differential equation, then our task is simplified. So, that is done by characteristic method. And then you can plot those graphically and this gives us solution at certain point, we will be coming to those points.

So, let us take the governing equation, continuity equation and equation of motion of unsteady flow in a rectangular frictionless horizontal channel. Earlier we were writing the expression, as we can see here, this is our expression say del a del t del v del x v del del x and this is the equation. If this equation is considered as horizontal means, this is going and frictionless means this term is going s f and s b is going out and then our equation will be simple. So, let us take this equation for this one and we will be, as we are talking about that we are writing this for a prismatic rectangular channel, if we write it for a prismatic rectangular channel, then again this will be further simplified.

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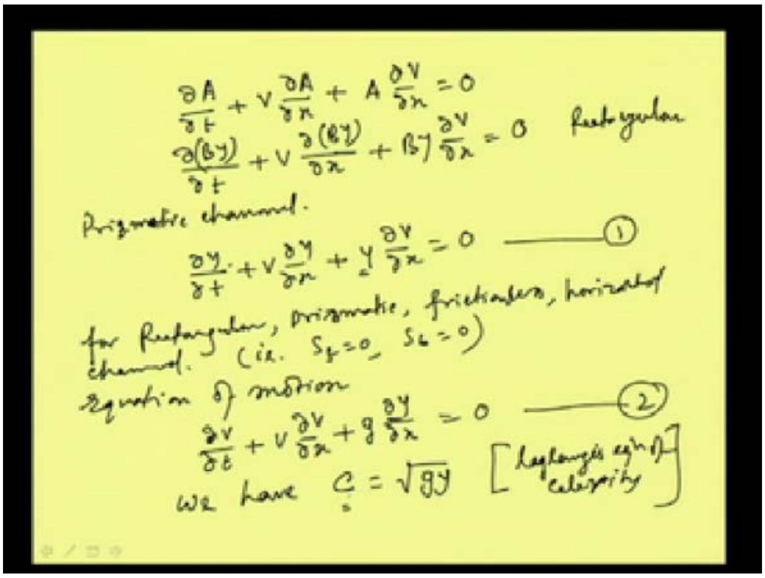

So, we will be starting with this expression, first let us take this expression, del A del t plus v del A del x plus A del v del x is equal to 0. So, this equation, continuity equation first and this we can write, del B into y B into y del t plus this v we can write B into y and then that we can write v is v a we can write their v and then a we can write del B into y del x plus, this area again we can write B into y and then it is del v del x, that is equal to 0. Now, if it is a prismatic channel. So, this part we are writing for rectangular and then for prismatic channel what is that for prismatic channel, B is not varying with x, means with distance our width is not varying because that very basic definition of prismatic channel is that the sectional dimension will not change or it does not change in the channel reach consider.

So, this B is not varying and as such what we can have that is B we can bring common and we can write it in a different way, that is we can cancel B rather, what we can write this is del y del t plus v del y del x plus y del v del x equal to 0. So, this is equation 1. Now, again for rectangular, because we will have to write for the same situation, rectangular prismatic as we are writing once. So, we will have to write it for all rectangular prismatic, then frictionless, channel frictionless, now we will be writing equation of motion frictionless, then horizontal, all these are there, horizontal channel. Now, we should remember that we are doing it for a very simplified situation.

So, it is a simplified solution of course, and then when we go for considering all these things then this sort of simple solution will not be applicable, but this gives us preliminary idea how will the flow will be or where the flow will move. So, this if we use, so, what this means, that is our s f is equal to 0 and s b is equal to 0, considering that the motion equation of motion can be written as, so, equation of motion, there is what we did derive earlier. So, in this equation, if we just write in a simple form, that will be del v del t will be coming as it is, then plus v del v del x is coming, then we had g del y del x and other term s f minus s b. So, here those term will be going out and we are having g del y del x equal to 0.

So, this let us give as equation 2. So, we are taking 2 equations in a simple form neglecting that there is some friction in the channel, neglecting that there is some definitely slope in the bed, but we are considering that slope to be very small or we are considering this to be horizontal. So, neglecting those things we can write the equation of motion in this form. Now, we know that for rectangular channel, celerity because wave will be moving in these sorts of channels, now because it is unsteady flow. So, celerity what we have, that we have it is a shallow water equation. So, celerity C, we can write is equal to root over g y.

So, starting from this, this equation as I did say, it is called Lagrange's equation of celerity, that we did mention in our earlier classes. Now, starting from this equation here, let us try to replace del y in this equation and y wherever is there, y is there, y is here, then del y is there. So, del y and y let us replace and let us try to have it in the form of C, we have a very basic intention why we are trying to do that.

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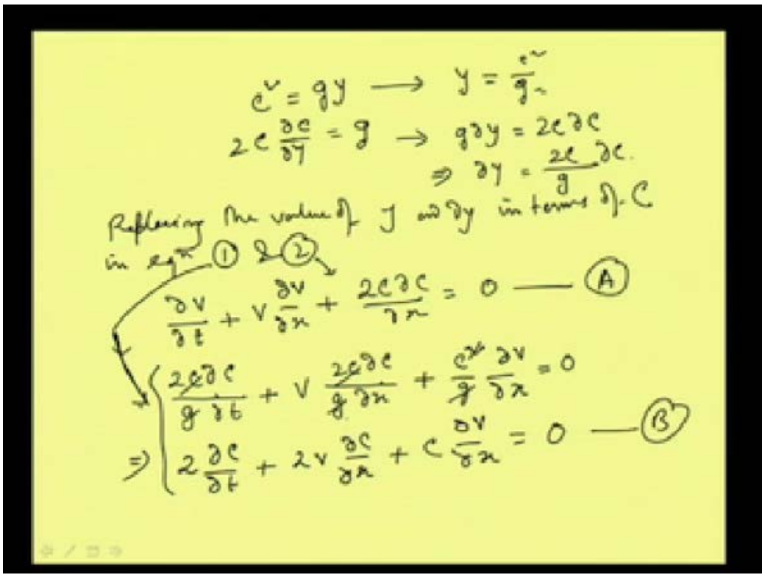

So, how we can do that, that from here we can write that C square is equal to g y. So, simply if we just do twice C del c del y means taking derivative with respect to y. So, twice C del c del y, this equal to g. So, from here what we are getting basically we are trying to get this will give us one relation, that is y is equal to c square by g. This is giving us that expression and our interest is to express this del y in this term. So, this will give us one expression, that is g del y is equal to twice c del c. So, if we want to replace this del g del y then we can replace it by twice c del c by g or somewhere if we have a term g del y, because in some places we have term g del y directly, I think this is g del y, that we can replace by that term directly and where we do not g, then we will have to divide it by g or this will that let me write keep on writing that one, del y is equal to twice c by g del c.

Now, putting or we can replacing the value of y and del y in terms of C celerity, in terms of celerity C in equation 1 and 2, what we can write, that equation 2, if we write that will become because here, we have only this term equation 2 will became del v del t plus v del v del x plus we have the term here, g del y del x that we can write as, twice c del c by del x equal to 0. And let us put this as equation A, let me put this as equation A, and then, this actually equation 2 and then equation 1, that let us see, what we can write, if we put del y and c value that is will become say twice c del c by g the term del y del t will become this one, because you can see this first term del y del t del y we can replace as del y is equal to twice c del c by g so, twice c del c by g and del t is remaining there plus V then it was the term was V del y del x again del y we need to replace by this term. So, it will become twice c del c by g del x plus then y. So, y is equal to c square by g, this term we had a term y, here y is equal to c square by g and then we have del v del x this is equal to 0.

So, after that, so, del v del x equal to 0, then we can further simplify this expression like, some of the terms we can just cancel and make it simple, if we multiply by g, this expression then g is everywhere. So, it will get cancelled and then we will be getting this term in the form that twice c, c is also there everywhere. So, we can cancel that, so, what we can write this expression as g we are cancelling here, then c we are cancelling here, and then here c square, so, this square will only get cancelled and this expression will become, twice del c del t plus, this will become twice v del c del x plus, this term will become c del v del x, that is equal to 0. So, this equation, let me give as equation B, let me give as equation B, now this A and B, here we are writing in the form, that in the equation A this is actually equation of motion, our this part is coming as equation of continuity equation, this part is continuity equation and this part is momentum equation. Now, here you can see that this equation b we have del v del t here and del c del t here. So, and here we have del x, so, here we have twice v del c del x and here we have v del v del x and here we have again del c del x here we have del v del x, if we sum up this term, then we can write a different equation.

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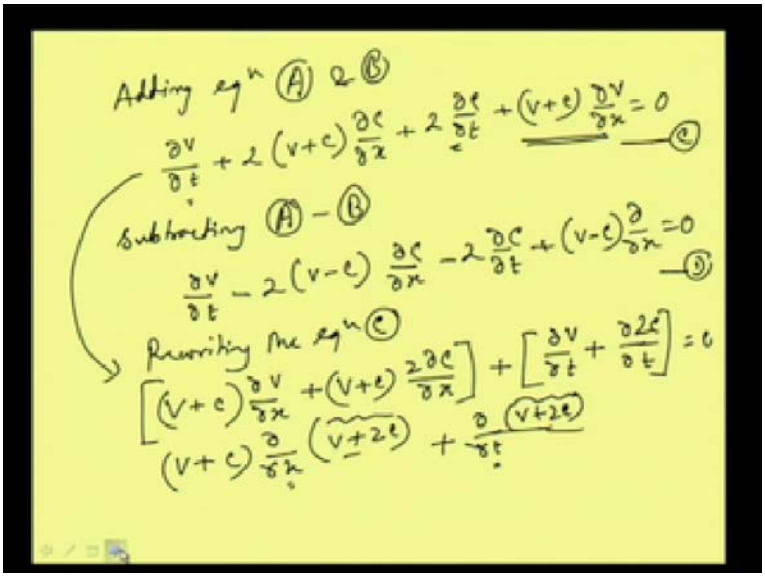

So, adding one A and B, so, what we can write that adding, equation A and B. So, if we sum up equation A and B, then we will be getting this as del v del t plus twice v plus c, then del c del x plus twice del c del t plus v plus c del v del x is equal to 0. So, we are just summing up these 2 expression equation and each of the term we are just adding t term being together, we are writing here and then here, this is the term. And then similarly if we subtract this equation from that is A minus B if we do, then we will be getting a different expression. So, subtracting A minus B, if we just do that, then we will be getting del v del t will be there, then minus twice v minus c into del c del x minus twice del c del t plus v minus c del v del x equal to 0.

Now, this equation we can give some name, this is equation C and this is equation D. This equation C, let us take first this equation C and let us see how we can express this equation C. So, equation C means the equation that we have obtained by adding this part. Now, how we can write this? Del v del x term if we just keep in one side and there will be del t term, if we just keep in one side and then let us see how we can write this expression. This part we can write as or rewriting, let us write rewriting, the equation C, what we can write that v plus c into del v del x; that means, this term v plus c into del v del x plus, again if we put this v plus c here, then we can write v plus c; that means, just we are trying to bring some similarity, here we have some intention, v plus c and then we can write twice del c del x or del twice c del x, that is also we can write. And this we are putting like that in one bracket then plus, then what are left say del v del t, del v del t is left plus this term is left, del v del t is left and this del v del t is left and then we left it twice del c del t or we can write this as del twice c del t, and this is equal to 0.

This term we can write it in a different form. How? We can write it like v plus c, we are bringing common, suppose and then it become v plus twice c of del del x. So, we can write del del x of v plus twice c, that is what our intention to write. So, we are getting it in this form and then plus this term we can write as del del t of v plus twice c. So, interestingly we are getting these expression here also as del del x of v plus twice c and here we are getting del v plus twice c. So, v plus twice c and v plus twice c is common and here we are getting in terms of derivative of this with respect to time t partial derivative and here we are getting it in terms of partial derivative of, this in terms of x, we know that this v and c, in fact, the velocity V and the celerity C will be always a function of x and t, will always be a function of x and t. Now, if we consider this as a single variable, then this particular expression is giving up us another option of simplification or another option of simplifying this, if we consider this as one expression just if we see the similarity, suppose a variable, this equation, we can further simplify as, let us take this example, suppose if we have a variable, any variable, let me write this as a the variable and which is a suppose a function of x and t, then what we can write, if it is a function of this then we can write d of this variable, we can write as this d of this variable, we can write as del of this variable divided by del x into d x plus, del of this variable by d t into d t. Now, what the similarity we have, if we take d v d t, this I mean, this v we are taking, and then what we can write, this of this variable x and then we are having d x d t plus this del t.

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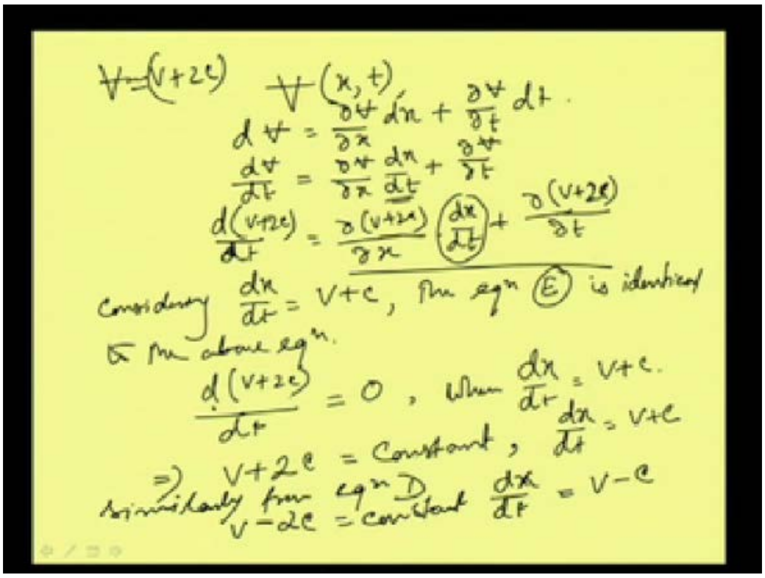

So, what this variable is? Actually, we are talking about this variable as, v plus twice c, suppose, this is equal to v plus twice c, if we considered this as v plus twice c, now you see, if we write this variable and we know that this is a function of x t and then, we can write it in this form. So, what we have seen finally, that in this equation, if we now put this part, then we can see this is d x d t. This expression is almost similar to that, del del x of this term and we are having del del x of this term. Let me write this here del of v plus twice c into del x and then of course, we have d x d t and plus, here we have del of v plus twice c into del t.

So, this expression is similar to this one, this expression is similar to this one, V plus twice c and del t, and this part is. So, what is not matching this equation and that equation, if we compared then what is not matching that here with this term, we have d x d t, and here we have v plus twice c, that is the only difference between this 2 forms. So, now if we can have, the d x d t is equal to v plus c, then these 2 equations will be identical. So, let us put let us do that. Considering d x d t equal to v plus c; that means, if we put d x d t is equal to v plus c, then the equation, let me give a name to this equation then it is E, the equation E is identical to the above equation means this equation.

So, what we can write this is what we are getting basically, total derivative of this with respect to time t, total derivative of v plus twice c respect to time t. So, that we are getting? So, it is identical. If it so, our expression is actually equal to 0, we are getting this expression is equal to 0, this is equal to 0. So, this is also equal to 0. So, this expression is equal to 0 means, here we can write that this expression is equal to 0. So, what we can write that; that means, d of v plus twice c by d t is equal to 0, but this will be valid and that expression we are writing like that, this will be valid when d x d t is equal to v plus c. So, we are getting and this will be actually when d total derivative of v plus twice c is equal to 0; that means, what, this implies v plus twice c is constant.

So, v plus twice c is constant and along with that we will have to have this equation, d x d t equal to v plus c. So, these are the combine equations, actually when d x d t is equal to v plus c, on those points, where d x d t is equal to v plus c, there only we get v plus twice c is equal to constant. And similarly if we take the other equation, that is the equation D, by subtracting what we got this expression. So, similarly from equation D, what we can get, v minus twice c is equal to constant, when d x d t is equal to v minus c. So, these equations are basically, the ordinary differential equation that we are getting, we started with partial differential equation, but we are getting ordinary differential equation and we are getting 4 equation in this form and then this contain ordinary differential equation only here, and this is of course, straight equation and difference equation only. So, this gives us a better opportunity for solution of the problem.

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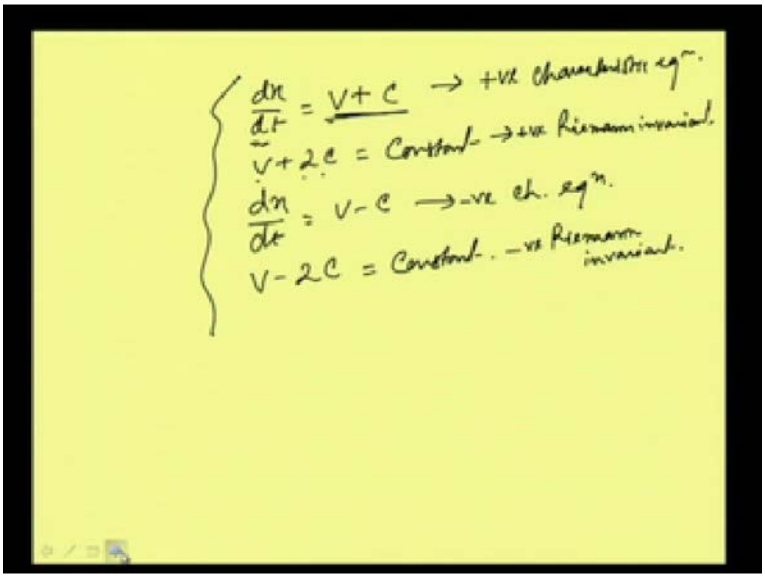

Now, let us see these equations can be written as, d x d t is equal to v plus c and then d x, along with that I will be writing the other equation, that is v plus twice c is equal to constant, this is another relation. And then, we are having d x d t is equal to v minus c and here we can write, v minus twice c equal to constant. And this equation is called characteristic equation and this equation is also called characteristic equation and this equation as it is, v plus c, so, this is called positive characteristic equation. And this equation is called negative, I am writing in brief characteristic equation and v plus twice c is equal to constant, that is called positive Riemann invariant and this is called negative Riemann invariant, because it is not variant, it is remaining constant.

So, now let us see understand this point, that we can consider, v plus twice c as constant only on those point where, which follow d x d t is equal to V plus c. Now, if we draw a plot or if we make a computational domain of x and t, then we can have d x d t a line and then that line at d x d t, that line is equal to actually v plus c, the slope of that line we will be getting by v plus c, so, v plus c will give us some value that will give as a slope of that equation and then accordingly we can draw a line and on that line whatever point lies at any point, if we take the v and twice c at one point at and other point, if we take v one plus twice c one say this all value will remain constant. So, taking that advantage we can solve for the, I mean this characteristic equation can be used for solution of the governing equation.

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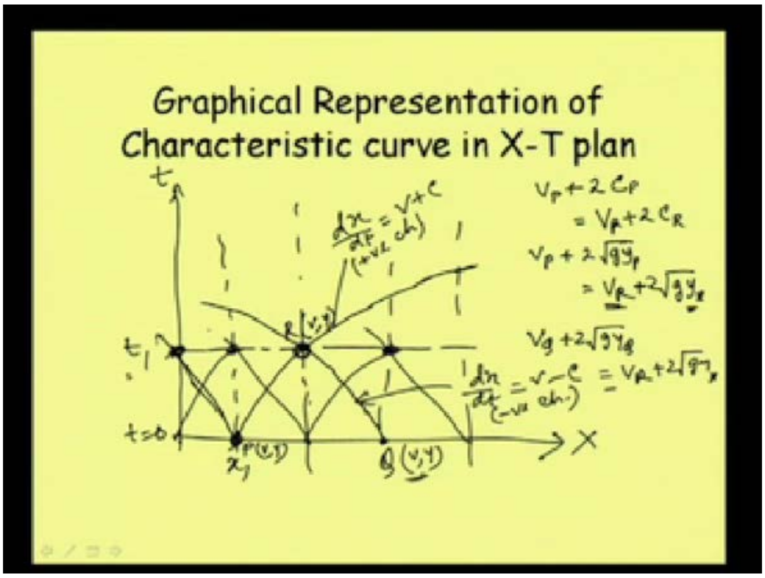

Now, let us see how we represent this graphically. So, graphical representation means, if we just take x t plan. So, this is our x and this is the time t, in x t plan, we are having a line d x d t equal to v plus c. So, from that we can have a line like this, this is suppose representing d x d t is equal to v plus c then we will call this as a positive characteristic. So, on that line, if we take suppose x at a particular section this one and if we just make some division here, this is a point and this is another point, another section, these are some section here and this is going characteristic line. Now, suppose if we know at time equal to 0, may be the steady state time equal to 0, may be the steady state then unsteady has started. So, with time this flow will be changing. Now, at initial at unsteady flow, at steady flow condition, suppose we know this value, at x equal to suppose some value x 1, we know the value of the variable, what are our unknown values, unknown is our v and t. Our unknown is the depth and velocity. So, at suppose this point we may be knowing, this is p, let this point be p and suppose we know this value v and y, these are known here and we know that this is the v plus c. So, v plus c is equal to d d x d t, so, accordingly we are drawing this line.

And then at another time level, suppose t 1, we do not know, because unsteadiness has started and we want to get solution, what will be happened at time t 1. At this time level we have this point. This point is Q, let me write it as r, because I will be writing another curve from that side, suppose this is r and at this point, what are the velocity and what are the depth that is unknown. That we do not know at time level t 1, but we know one relation that if I put this as v r and y r, so, what relation we have, because along this line we know that v plus twice c is equal to constant.

So, v at p plus twice c, c is nothing, but twice root over g y p, this is nothing, but c this is equal to v r plus twice c r, let me write it as c r first, then. So, that you may not confuse this is, c P and this is c R, but what is c P, that c P is nothing, but v P plus twice root over g y p our unknown quantity is actually y this is equal to v R plus twice root over c root over g y R. So, as these now these equations are there and our unknown quantity is this one v and y, but we need to solve for two unknown quantity one is that v R, one is that another is that y R, but two unknown quantity we have only one equation. Now, if we use the negative characteristic curve, then we can have that negative characteristic curve in this side. This will be coming like this. How it is coming like this? That I will tell again, because this will be coming like this, for shallow for subcritical flow. Why? That I will be explaining when in the next slide, but if this is Q then this is v y and v Q and y q is also known, because it is at the time equal to 0 initially we know. So, now, if this negative characteristic line is passing through this one, we can write this as d x d t is equal to v minus c, that is, what is negative characteristic line.

Now, from this again, we can get one relation between this and that. So, what we can have? That another relation we can have is that v Q plus twice root over g y Q is equal to v R plus twice root over g y R, because it is passing through the same point. And then using these two equations, now we have two unknown we can solve for this particular point. So, that is what the beauty of characteristic method and we can solve this once we know these value then similarly to solve what is the value here we can get the solution here. Similarly, we can proceed to solve this one we can get here and here, but to solve this one we need some boundary solution or boundary relation. What is the some condition here of course, we will be getting one negative characteristic line. This will be giving us one relation between this and that, but we need one more relation.

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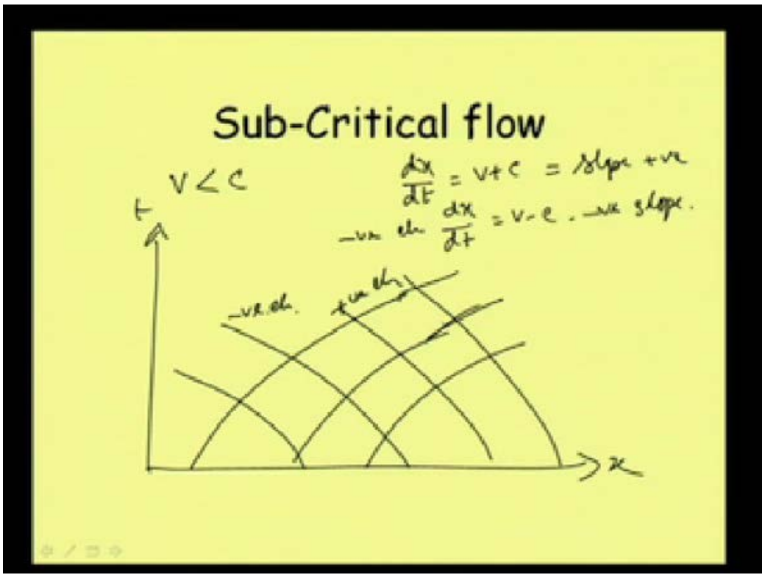

Now, let us see for subcritical flow, for subcritical flow, as we know, that v is less than equal to c, when v is less than equal to c d x d t equal to v plus c, that is the positive characteristic line, this will be slope will be positive. So, this will indicate that slope is positive and d x d t, what is negative characteristic line, negative characteristic d x d t equal to v minus c. This will be having, as v is less than c, so, this will be having negative slope and that is why when I was drawing for subcritical flow, if I draw the x t plan, this is the time, this is x, then for supercritical and subcritical for subcritical, we will be getting the positive characteristic line, this negative characteristic line, this is the positive characteristic and this is negative characteristic and that is why earlier we were drawing this is positive, this is negative. Then this is not the only line, we will be having several set of line like this and several set of negative characteristic line, because at different point, we will be having different situation and that will lead us to several line.

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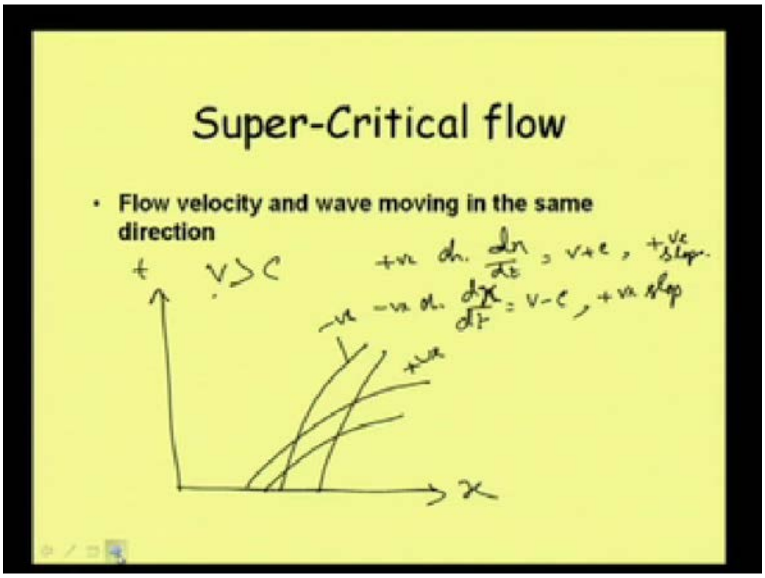

Then, in case of supercritical flow, v is greater than c. So, positive characteristic, d x d t equal to v plus c will be definitely positive, but negative characteristic, d x d t equal to v minus c, but that negative characteristic line will also have positive slope. So, this positive means, we are talking about positive slope in the x t plan. So, this we are drawing in x t plan because v is greater than c. So, v minus c will also be having positive slope. So, if we just draw it here, then we will see that, but which one will be which slope will be greater that because v and c it is plus. So, this positive characteristic line slope will be greater and so, positive characteristic line slope will be going like this. So, greater means, it is we are writing d x d t, not d t d x. So, t is in direction, x is this slope movement it will be from the t line. So, it is going the positive characteristic and the negative characteristic will be going like this, it has less slope**.**

From our general concept, you may have a feeling that this is having higher slope, but this is with respect to this line, we are talking about slope. So, this will be having less slope. So, it is negative characteristic. So, that way we will be getting series of curve like that and then this is the case, when flow velocity and wave are moving in the same direction.

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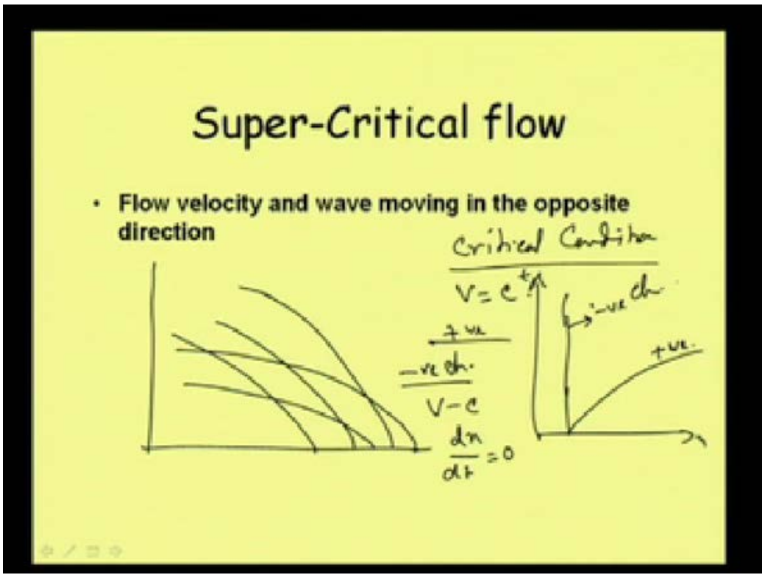

If the flow velocity and waves are moving in the opposite direction, then v become negative, then our curves become both are becoming negative slope. So, both are becoming negative slope, positive characteristic are also becoming negative slope because v is this time negative and with respect to c. So, v minus c means minus v minus c, that will be steeper and the other one will be say flatter. So, like that we will be getting positive and negative characteristic curve, both are sloping upstream. And this sort of things become more important, when we go for numerical solution of this one. Now, just to finish this one or to complete this one, if we say, that for critical condition what will happen, for critical condition what will happen? At critical condition v is equal to c, so, when v is equal to c, our positive characteristic line fine, this will be having a positive slope, but what about negative characteristic line, positive characteristic line will be having this one, but for negative characteristic line, v minus c. So, as v and c is equal to 0 equal, so, d x d t is become 0. So, d x d t 0 means with respect to this axis we are talking about t and x, so, this line will be just vertical. So, this is the negative characteristic line for negative characteristic line for critical slope.

With this limited time, we have just tried to give you some idea or rather we have discussed about some idea of this characteristic method and how these things can be helpful in solving some of the problem and that way, we could cover at least very preliminary of this part and lot of things to be discussed in this line and then how these are applied for actual solution, this also need discussion that are of course, we are keeping for higher level of study and for unsteady flow in our this course, let us conclude here and from next class onward, we will be moving to a different topic, that will be related to pipe flow. We are still till today we are doing with the open channel flow, from next class onward we will be moving to pipe flow. Thank you very much.