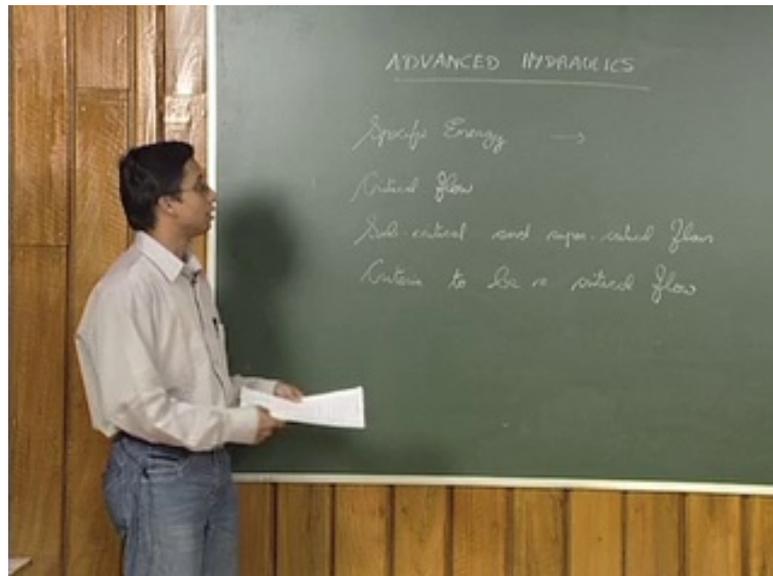


Advanced Hydraulics
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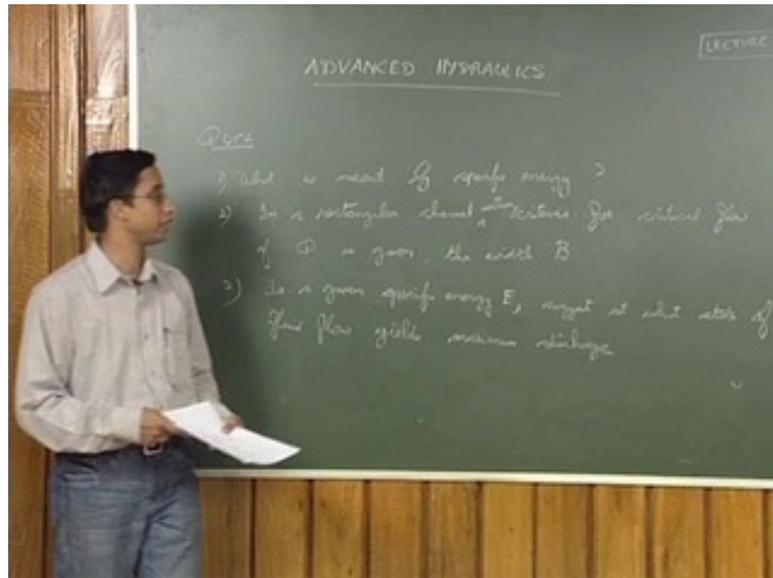
Module - 1
Open Channel Flow
Lecture - 7
Energy, Momentum and Specific Force

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So, today we are into the seventh lecture, and still we are in the first module itself. Last class we discussed on the concepts of specific energy. We have given the definition of specific energy; the equation for specific energy was mentioned there in the last class. Subsequently we have discussed on critical flow in the section; what is meant by critical flow and all those things were also discussed. We have suggested subcritical and supercritical flows as well, we have also discussed on the criteria in a section to be critical flow when this was also discussed in the last class. At the end of the lecture we suggested that the quiz component for the topic will be given today, and the quiz let us start with the quiz itself; then we will continue with the next topics of our lecture.

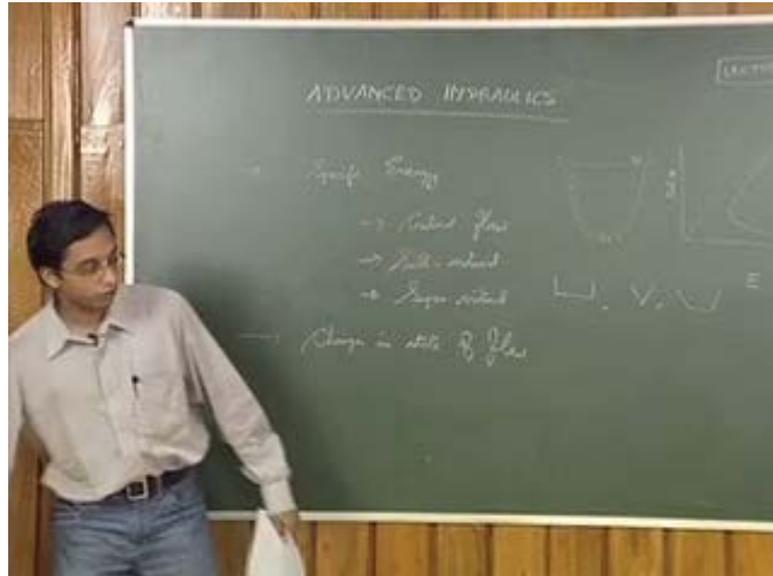
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So, our quiz; the first question is what is meant by specific energy? You have to give the equation of this as well as you have to describe in one sentence, one or two sentences as well. The second question - in a rectangular channel, what is the criteria for critical flow if discharge Q is given you have to determine the criteria for critical flow in a rectangular channel section, please note that in a rectangular channel section. What is the criteria for critical flow if Q is given also width of the channel B ; this is also given to you. You have to determine the criteria for critical flow in this rectangular channel section.

The third question: For a given specific energy E , suggest at what state of fluid flow yields maximum discharge. For a given specific energy E , suggest at what state of fluid flow yields maximum discharge; whether it is subcritical, supercritical or critical. So, I will give you three minutes to answer this. It is a very simple quiz; one of the easiest quiz in our entire course. I hope you have answered the quiz questions.

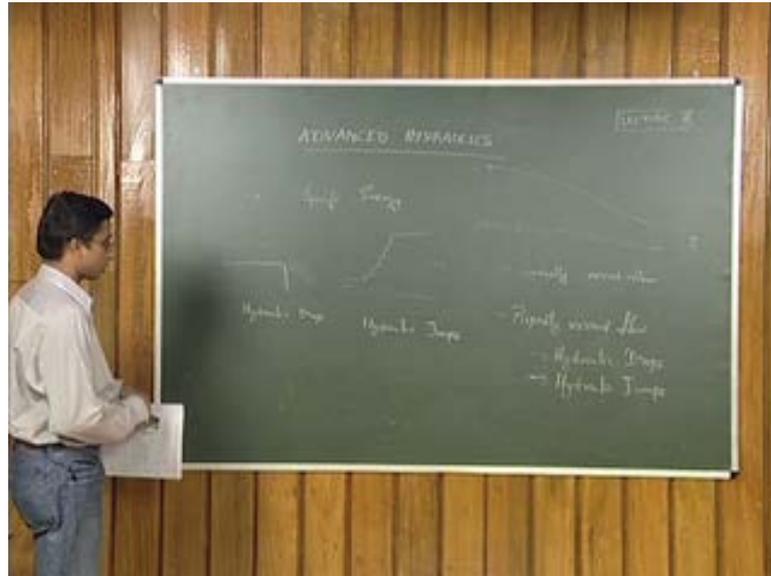
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Now, let us continue with our lecture. You know based on the specific energy concepts, we have discussed critical flow, subcritical and supercritical. You can say for any arbitrary cross section of a channel, you were able to determine the specific energy curve. If you draw E versus y , you had got such curves and all; you can use the various criteria to determine critical flow and all, critical or to measure the critical depth and all; you can use the criteria suggested.

Now for various cross sectional channels, whether it be rectangular, whether it be triangular, whether it be trapezoidal; anything it is up to you to engage in this, means, you take any arbitrary cross sectional channels, try to compute their critical depth or criteria for critical flow and all. What we want to suggest here is now, if there are changes in the state of flow; that is if there are changes in state of flow that is from subcritical to supercritical or from supercritical to subcritical. This phenomenon they occur very much in a common for the open channel flows; they are encountered in many type of fluid flows in open channels. So, you have to understand what are the changes occurring or you should be able to solve such problems and all...

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Let us see for example what is meant by changes of state of flow? If there is a long stretch of a channel, there is a long stretch of channel and if there is a fluid individually subcritical state, then in the long reach it reaches a supercritical condition. So, this is the supercritical state; this is the subcritical state at this section. The flow in this long stretch, it changes from subcritical state to supercritical state. These things are commonly encountered in many of the open channel problems and all. This is a case of gradually varied flow. Suppose if the change from one state to another occurs in over a long stretch then such things are gradually varied flow.

Suppose if this change of state of occurs in a small stretch, they are called rapidly varied flow. Rapidly varied flow you may encounter in hydraulic drops or hydraulic jumps. You are not going to do much technical details right now; at the present state we are not going to describe much on that hydraulic drop means, say if the channel is there and if the bed of the channel suddenly dips, then the flow here, two free surfaces appears. There will be two free surfaces now and such type of fluid flow, this is a hydraulic drop because whatever specific energy you have computed here and whatever specific energy you will be going to compute here and all based on your energy conservation and all; you will see that is from either from subcritical to supercritical flow and all that occurs in this case of hydraulic drop.

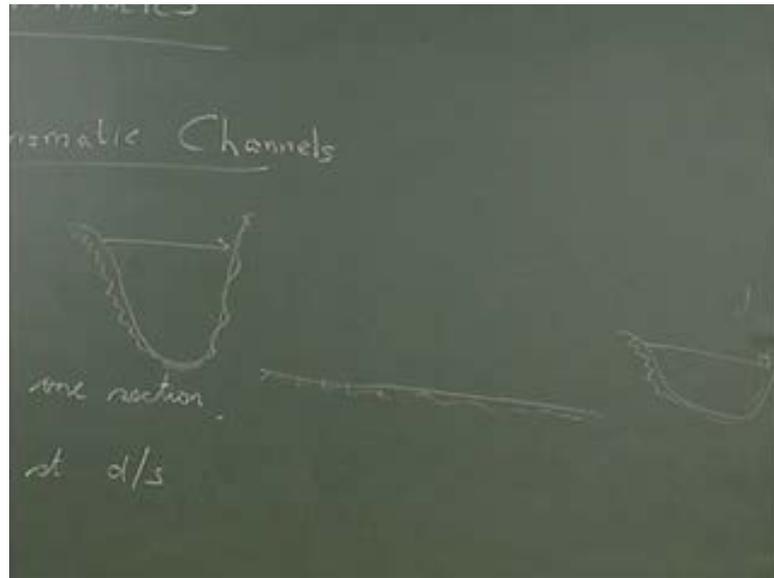
This is a free fall hydraulic drop. Hydraulic jump here from a supercritical flow in a short span itself it just becomes a subcritical flow; from supercritical flow state all of a sudden in a short span it converts into a subcritical flow state. Such things, such rapid changes, they are also a rapidly varied flow; they are seen in hydraulic jumps. Here from supercritical it is changed to sub critical. So, we have other modules to discuss on rapidly varied flow as well as hydraulic jumps. So, we will not discuss much of that issue here.

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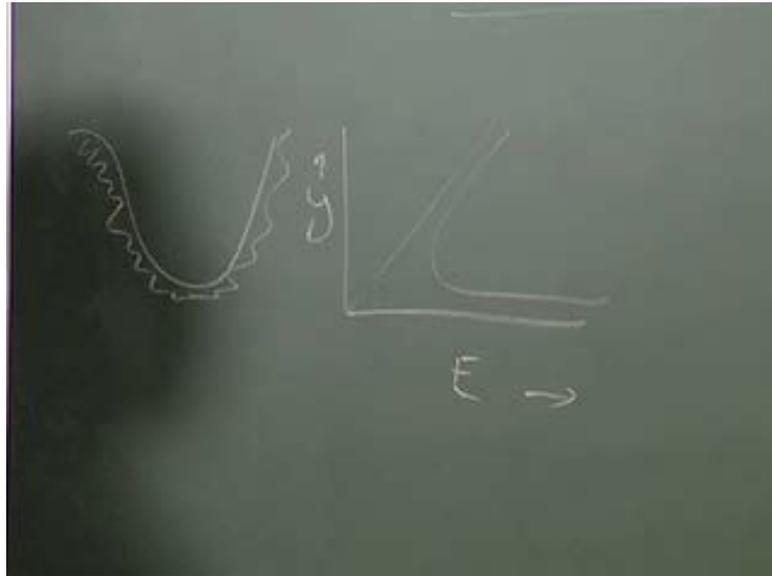
So, we will discuss on energy in non-prismatic channels. So, till now we were discussing on how to measure energy in prismatic channels; that is over a stretch of the channel, the cross section of the channel remain same; that is what is meant by prismatic channels. What happens if the cross sectional area, if the cross sectional area of the channel if it varies continuously between two sections and all, they are called non-prismatic channels and how do you compute energy in such channels. They will be quite difficult compared to prismatic channels. In prismatic channels you have seen that if you measure energy at one section, then you will be able to easily measure or easily compute energy at downstream ends is quite possible; that is it is due to from the conservation of energy equation and all, you will be easily able to compute energy now at the downstream if the upstream energy magnitude is given to you.

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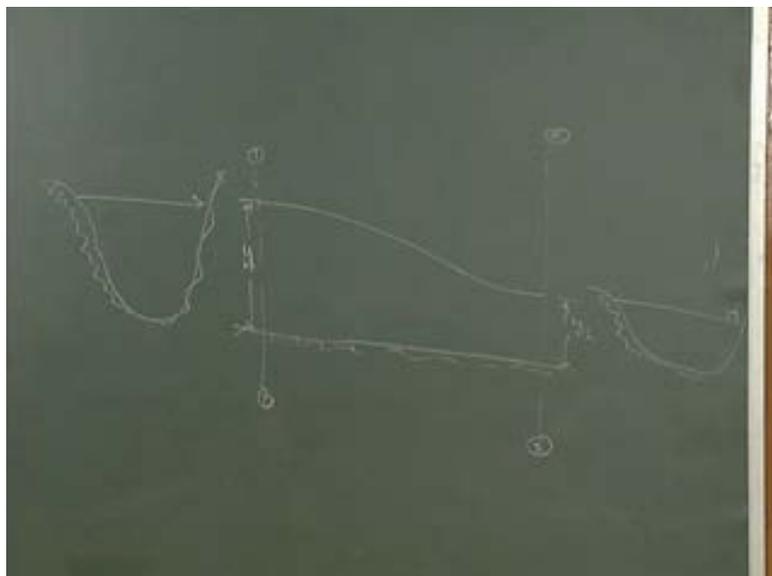
What happens in the case of non-prismatic channels? In the case of non-prismatic channels the complexity arise that say if the upstream channel section is like this and there is a reach like this and the downstream cross sectional details are like this. There is a change in flow type from the upstream to downstream end. You will see that you will see such situations here. In such situations what you have to do is that normally in non-prismatic channels, one has to measure energy frequently in several intermediate sections. Then you need to understand what type of change in flow is occurring, whether from subcritical flow to supercritical or from supercritical to subcritical and all; you need to identify those things. So, what do you in non-prismatic channels in such situations and all?

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So, as mentioned earlier for prismatic channels, any cross section will yield you, for a given discharge yields you E versus y curve. Based on this curve one can now identify the flow depth or other parameters in the downstream as well in prismatic channels.

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Now in the non prismatic channels if you identify such a flow, see here in this case if this is section 1 1, section 2 2, say if a flow is occurring in this way; that is the depth of the flow is changing in the non-prismatic channel. You have to compute or that is say, if this is a subcritical flow and if this is a supercritical flow, you have to draw the energy depth

curves for this cross section as well as this cross section; how will it appear now? Use the same formula, say, in this channel if I try to draw the energy versus depth curve now; depth is already given to you here, say, this is y_1 , this is y_2 at section 2 2. If I draw in this direction the energy magnitude, say, then you can easily identify in the following form.

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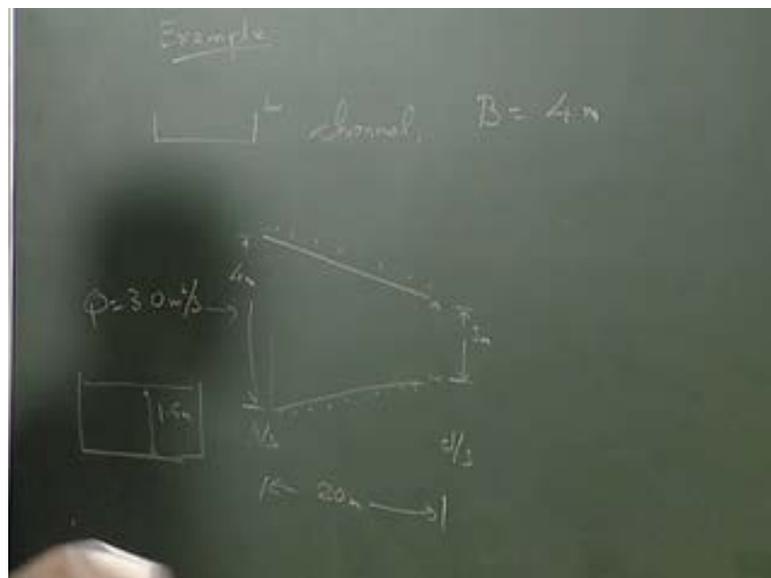
I will just redraw the things procedures here in a better way. The upstream section, depth of flow, you can draw the corresponding E versus y curve. So, the upstream from this particular curve it is quite understood the upstream flow stage. It is a higher flow stage from the E versus y curve. We have suggested that for same energy there will be two alternate depths; one the low stage and one the higher stage. So, definitely the height of water as given here that corresponds to the higher stage here. So, we suggest that the flow is subcritical. You extend the diagram now to the downstream y_2 ; this is just a hypothetical extension like this. We do not know how the bed it is varying, it is a non-prismatic channel.

So, if you draw the energy curve here; same energy curve energy versus y curve, you will see that in this particular case your energy curve E versus y may be something of this type and this particular depth correspond to some height in the lower stage. This height is a lower stage for this particular energy value and you will get a corresponding curve like this. So like this, you need to compute energy versus depth curve frequently in

a non-prismatic channel. If the flow changes from subcritical to supercritical, definitely there will be a section between section 1 1 and section 2 2. This is your section 1 1 and this is your section2 2; between these two sections there will be some section in between where the depth of flow corresponds to the critical flow in that section. That is quite understood from this graph or this diagram.

So, one may ask you, how to then identify the critical section if in the non-prismatic channel and all. So, where you have to measure both the conditions, then subsequently you have to identify the critical condition or critical flow. If the alternate depths whichever are given in the downstream when as given in the earlier figure, if in the downstream end if it happens to be the lower stage and if in the upstream end it happens to be the higher stage from the energy depth relationship or energy depth graphs, definitely you can suggest whether it is a subcritical or supercritical flow, and at any location in the energy graph if there is only one depth that is coming into picture that correspond to the critical condition.

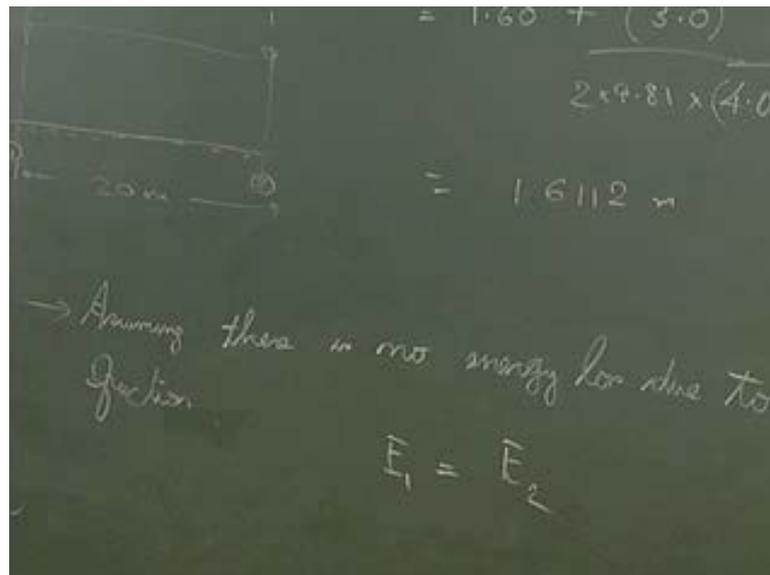
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Let us do an example. Suppose if there is a rectangular channel, if there is a rectangular channel of width 4 meter. Now this 4 meter width channel if you view from the top, it is now narrowing down to or it is passing through a constriction and this 4 meter wide channel narrows down to 3 meter; this is just an example. These are the sidewalls of the channel. The rectangular section is maintained here as well. This is the upstream and this

is the downstream. If the upstream discharge is given, not only upstream discharge, we are dealing with the steady state problem. So, the discharge is constant. If discharge is given as 3 meter cube per second and if this constriction length is 20 meters; if the upstream depth of flow, if the depth of flow at the upstream if it is given as 1.6 meters, you are now requested to find the water surface profile.

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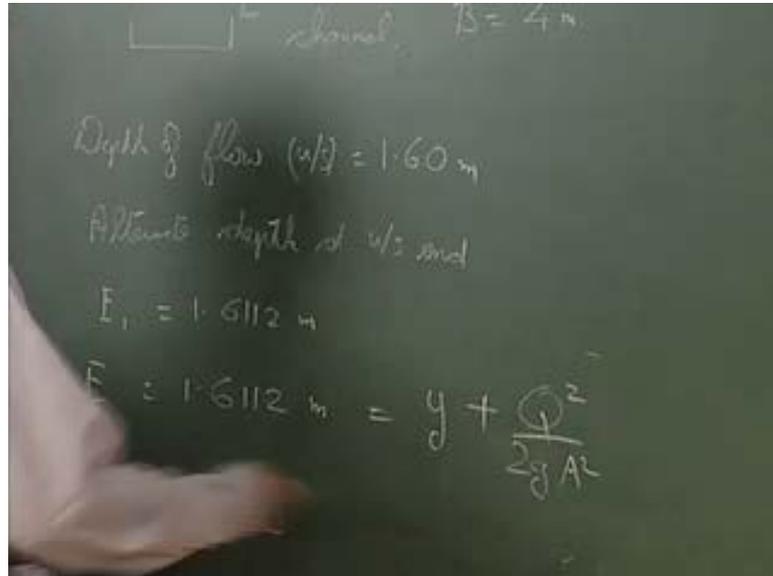


You see from this 20 meter as it passes through the constriction from here to here, what could be the water surface profile; you are requested to compute that. How will you approach the problem or how will you try to solve that? At the upstream end, let us give the section as 1 1 at the downstream 2 2; one one, 2 2. So, energy at the upstream section E_1 in the rectangular channel, this can be given as y_1 plus Q_1 squared by $2g A_1$ square. Substitute the corresponding values given here. You are given the depth of flow one as 1.6 meters, discharge is given, area you can easily compute 1.6 plus 3.0 whole square by 2 into 9.81, A_1 is your rectangular channel width is 4 meters.

So, we can suggest now; 4 into depth of flow 1.6 whole square. You calculate them; you will see that you will get the value as 1.6112 meter. Now in this problem we are assuming, please note that, assuming there is no energy loss due to friction. We are assuming there is no energy loss due to friction. Based on this assumption it will be seen that your energy at section one will be same; specific energy at section one and specific

energy at section two they will be same. Now you know the magnitude of energy at section one this is 1.6112 meter.

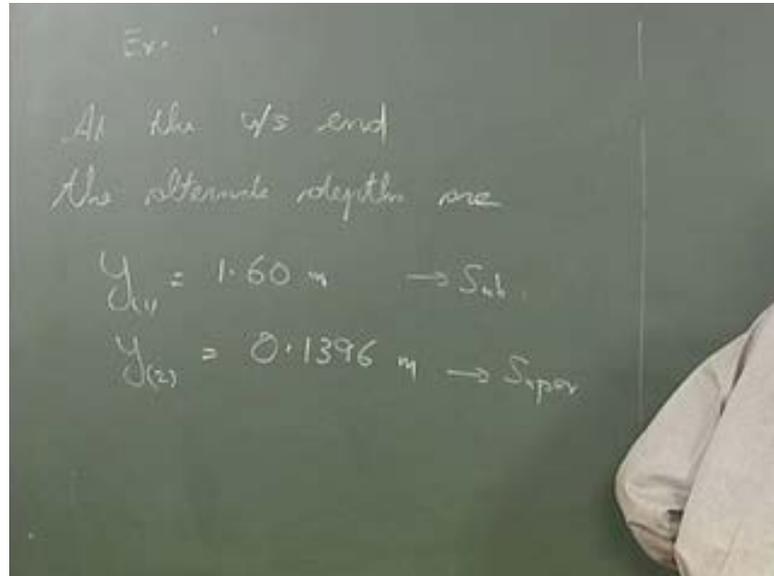
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You also know depth of flow at section 1 or you can say at upstream this is equal to 1.6 meter. What could be depth of flow in the downstream end when the flow passes through this constriction? You can find alternate depth at the upstream end; how will you find the alternate depth at the upstream end? E₁ is equal to 1.6112 meter or to avoid confusion let us say at the upstream end E, I am just substituting it as E. This is equal to 1.6112 meters and the equation for energy is nothing but depth of flow plus Q square by 2g A square.

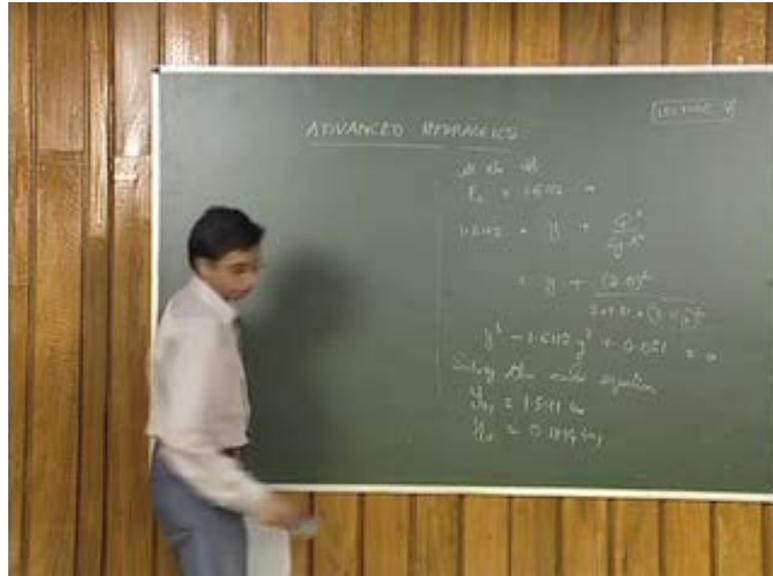
to 1.6 meters and y is equal to 0.1396 meter. So, these two are the alternate depths; I can write it in that way.

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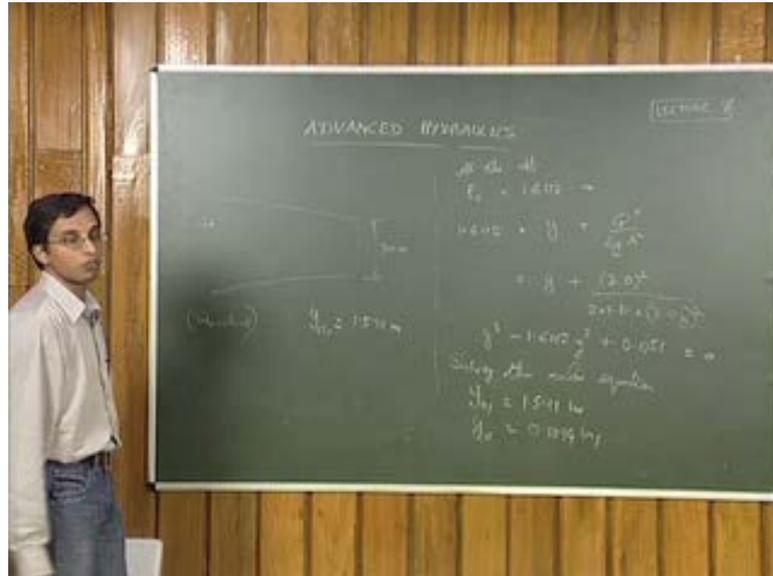
So, at the upstream end the alternate depths are y_1 is equal to 1.6 meter and the corresponding alternate depth is equal to 0.1396 meter. This is a subcritical flow and this is a supercritical flow. The flow passes through the constriction and we have not provided any hydraulic drop or jump in between the constriction. So, we need to compute what is the depth of flow in the downstream section and based on that whether it is at supercritical situation or subcritical situation or critical situation that also you need to identify.

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At the downstream section E 2, we are neglecting the losses is same as 1.6112 meter. Again use the same relationship at downstream y plus Q square by $2g A$ square; this is equal to y plus the discharge is same and area at the downstream the width of the channel is 3 meters. So, 3 into y whole square; you will get a corresponding third degree polynomial. On rearranging the terms, you will get the third degree polynomial of the following form. y cube minus 1.6112 y square plus 0.051 is equal to 0. Solve this; solving this cubic equation, we get two feasible solutions. The third one is a non-feasible solution which I am not going to write it here; 1.591 meters and the other depth are 0.1894 meters. So, in the downstream end we do not know which depth will be coming into picture; which depth is coming into the picture, can you give a guess in the upstream end and downstream end.

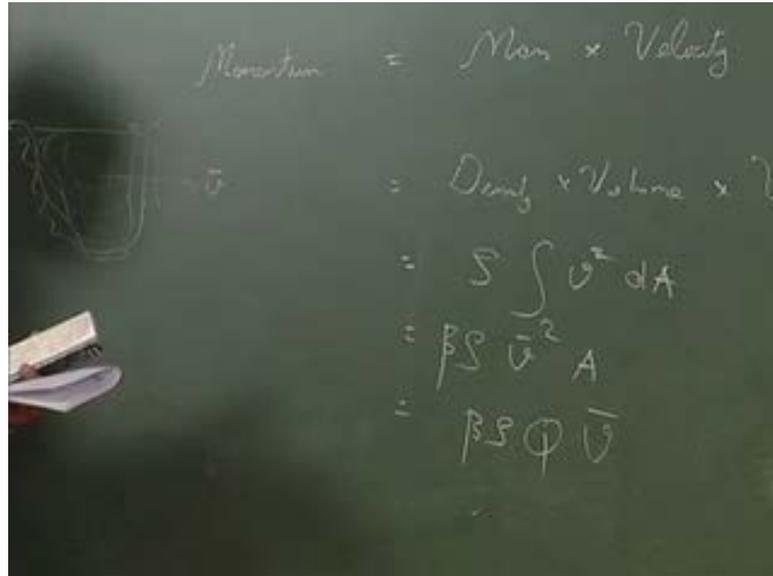
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As the flow occurs through a constriction, definitely at the upstream end the flow is subcritical and in the downstream end, we have two alternate depths 1.591 meters and 0.1894 meter which could be the depth if there is no structure provided in between. When the flow occurs through this contraction, you are not providing any hydraulic drop or any hydraulic jump or any structures like that or any phenomenon. Therefore, the subcritical flow should be maintained and hence at the downstream end your depth of flow will be 1.59 meter seeing the magnitude.

You will see that the flow profile or the flow surface it is almost horizontal as subcritical flow is maintained; of course, the velocity will be different. Till now we have discussed on the energy equation, conservation of energy in open channel flows and all. Let us consider about the momentum equation. In one of our earlier lectures and all we have discussed on momentum and momentum correction factor in open channel flows and all. Let us again come back into that; how do you apply the conservation of momentum and how we apply various loss of physics related to momentum in the fluid flow in open channels.

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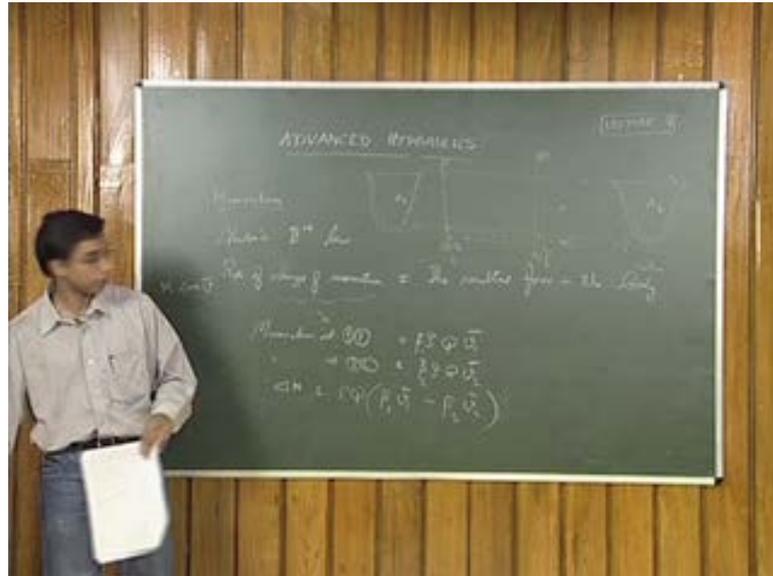
The image shows a chalkboard with handwritten equations. On the left, there is a simple diagram of a rectangular cross-section of a channel with a velocity vector \bar{v} pointing to the right. The equations on the right are:

$$\begin{aligned} \text{Momentum} &= \text{Mass} \times \text{Velocity} \\ \bar{v} &= \text{Density} \times \text{Volume} \times \bar{v} \\ &= \int_S \bar{v}^2 dA \\ &= \beta \rho \bar{v}^2 A \\ &= \beta \rho Q \bar{v} \end{aligned}$$

Momentum is nothing but mass into velocity of an object. So the same phenomenon if you apply in open channels, you can use the same principles; try to calculate the momentum and also conserve, means you know from Newton's second law and all, the rate of change of momentum is nothing but it is equal to the net force wherever in whichever direction you are dealing, the net force in that direction will be equal to the rate of change of momentum in that direction. So, let us come into that. If you take any arbitrary volume of liquid, mass is nothing but density into volume, then into velocity will give you the momentum; that is $\rho \text{ del } v$ or $v \text{ volume}$, for volume I can write $\text{del } v$ or like that, into velocity.

We suggested that if you are using for any cross section, if you are taking the average velocity term, then to compute the momentum you need to incorporate the momentum correction factor β into the computations. So, that is what we are doing it here. This is nothing but $\rho \int v^2 dA$ if the velocity profiles you know like this. This is nothing but integral of $v^2 dA$. This can be given as $\beta \rho \text{ average velocity into area}$. Now most of the time we are dealing with constant discharge, so $\beta \rho Q \text{ into } \bar{v}$; like this also we can write.

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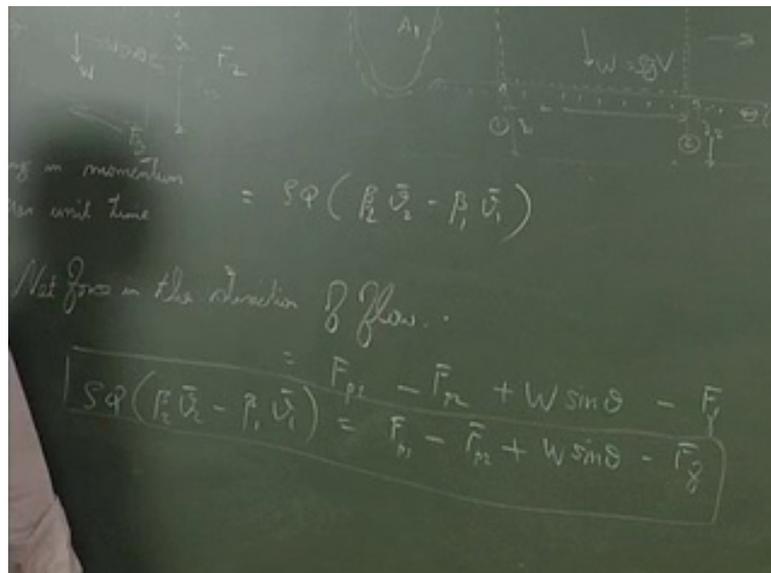
So, from Newton's second law that suggests that rate of change of momentum is equal to the resultant force in the body. So if you consider an open channel section, if you consider a stretch in the open channel, take two cross sections. So, we are suggesting channel; suppose if it is having a slope with the horizontal angle theta, we are considering two cross sections. Section 1 1, this is having area A 1; then section 2 2, this is having area A 2. What do you mean by rate of change of momentum? As we are dealing with the steady state flow whatever momentum is there from the mass of water stored between these two sections.

So, let us say if I mark the thing as a dotted line between two sections; if between these two sections this entire stretch and if it is at a length l and if you have, say, from any point datum, if it is located at z 1 height, this portion is located at a z 2 height. Consider this dotted portion. So, now this acts as a volume of water. We can consider the volume of water within this reach now. You have the cross section here, you have a cross section there in this side also and this much stretch is there. The mass or the amount of water that is stored here that gives you the mass of water in this entire portion. Now the momentum flux, whatever momentum flux that comes into the section here from this left hand side and whatever momentum that crosses through the section at this side; that gives you the change in momentum.

See the change in momentum, you can suggest that whatever momentum comes here and whatever momentum goes out, the difference between those two momentums gives you the change of momentum. And now the rate of change of momentum means the rate at which the momentum gets changed, that is the change in momentum you can say ΔM . If I am defining M as the momentum, if M is equal to $m v$ beta $m v$, then the change in momentum can be given as ΔM , ΔM between these two sections; that can be now computed, that gives you the net force that is there in the body now. So, you know that force occurs or the net force will be in the direction of flow. The direction of fluid flow is here in this particular direction. As open channels we are predominantly considering one dimensional fluid flow where there is no fluid flow in the vertical direction; that is not being considered here only the flow in the direction or it is along the bed of the channel.

So, how will you compute the change of momentum here? I can say now ΔM this is equal to or first let me compute momentum at section 1 1; this is equal to $\rho Q \beta_1 v_1$ bar. Momentum at 2 2 is equal to $\beta_2 \rho Q v_2$ bar. The change in momentum will be ΔM is equal to you can take ρQ out $\beta_1 v_1$ bar minus $\beta_2 v_2$ bar. So, this will be the change in momentum. So, the rate of change of momentum or the change in momentum per unit time, this will give you the net or the resultant force of the entire control volume. Whatever dotted line if you consider it as a control volume, whatever force will be there that leads the flow; that will give you or that can be correspondent with respect to the change in momentum or rate of change of momentum.

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So, we suggested that change in momentum unit time is equal to $\beta_2 v_2^2$ bar minus $\beta_1 v_1^2$ bar; that is the momentum coming out and momentum coming in. The difference that gives you the change in momentum and the change in momentum per unit time is taken as same. This is equal to $\rho Q \beta_2 v_2^2$ bar minus $\beta_1 v_1^2$ bar. What is the net force in the direction of flow? If you look into this control volume, as mentioned earlier, there are no flows in the vertical direction and assuming predominantly it is in the direction of the inclination of this bed itself. So, the direction of the fluid flow is in this direction. So, we suggest that the net force will be in this particular direction; how will you compute that? You know the weight of the liquid stored within this volume or weight of the liquid from this volume of water, you can give it this as ρg into some volume v ; ρg into this volume of water. So, we do not know what is the volume of water we can compute? We will come into that.

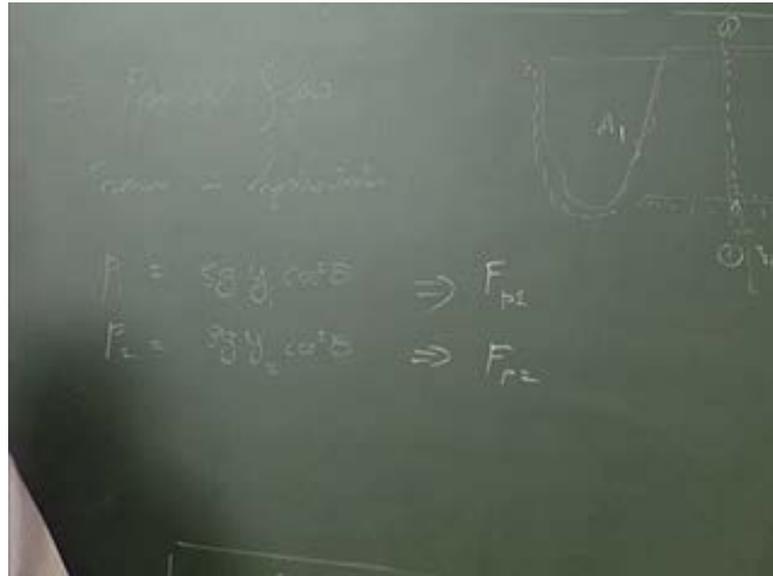
If you take this section out; I can just take that section out now. This is how your section will look like. Here the depth of flow is y_1 ; here the depth of flow is y_2 . So, if you take the components out, as discussed in our earlier lecture, pressure you know whatever pressure is there that pressure will now act on this particular plane because we are taking this particular volume out now like this. So in the downstream, the pressure force will be in this direction; in the upstream it will be in this direction, because we are taking a plane perpendicular to the board now. So, the pressure will be in this particular direction here and in this here. So, the force due to pressure I can give it here in this direction F_1 let it be here, F_2 this again can be made in to components.

I can make it in to components; one in the direction of flow, one normal to the bed, here also like this. So, I can give this as $F_p 2$, $F_p 1$, this one also vertical direction in the components will be there. So, we are interested only in the direction of fluid flow; that is along the bed only the net force in that direction we are interested. So, let us consider whatever is there in the direction of flow. So, w its component will be there $w \sin \theta$ in this direction. So, I can say this thing as, say, F_{net} force now will be equal to $F_p 1$ minus $F_p 2$ opposing the flow $w \sin \theta$ minus frictional force. There will be definitely frictional force opposing the fluid flow.

So, these quantities now you can equate it here. So, I will get this as $\rho Q \beta_2 v_2^2$ bar minus $\beta_1 v_1^2$ bar is equal to $F_p 1$ minus $F_p 2$ plus $w \sin \theta$ minus F_f . Once you equate this thing, you can now subsequently suggest various simplifications, say, if the

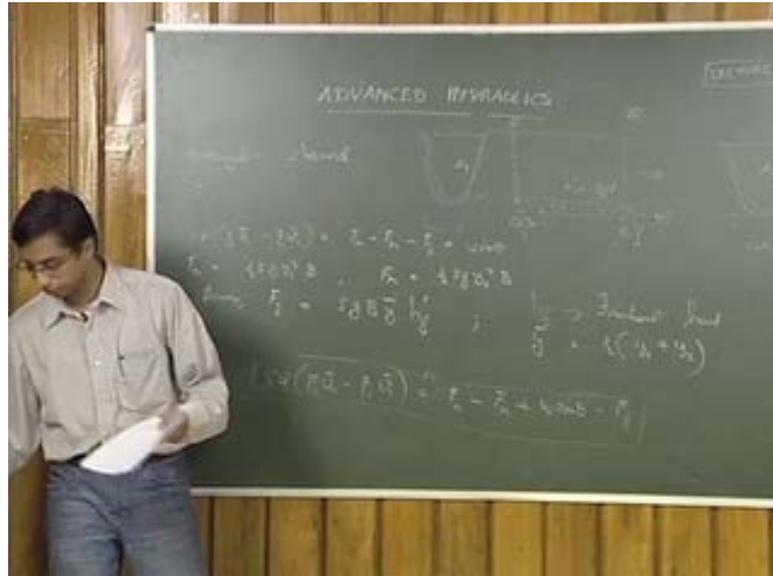
channels slope if it is not that much, you can see that the component of weight in the direction of flow will be negligible. If there is no frictional resistance or if the frictional resistance is negligible, then even that component can be eliminated. But this is the momentum equation in general; this is the momentum equation in general for such an open channel fluid flow conditions.

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Generally for fluid flow, if you have parallel flow or gradually varied flow between two sections 1 and 2, then you can suggest that your pressure is hydrostatic or in section 1 at any depth y , this will be depth of flow y at any depth of flow y , p_1 will be equal to $\rho g y_1 \cos^2 \theta$; p_2 is equal to $\rho g y_2 \cos^2 \theta$. These were derived earlier; therefore, you can suggest that from this pressure you can easily measure the force due to this pressure. You can easily measure them; you know pressure into area that gives you the force in that corresponding direction. So, that can be computed; you can substitute those things subsequently if it is parallel flow or gradually varied flow.

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Suppose what happens if I suggest that the same thing what happens if I suggest a rectangular channel width B if I suggest that thing, what happens if this is a rectangular channel? That is this section will look like this and this section will look like this. You can now write the corresponding equation $\rho Q \beta_2 v_2 \bar{y}_2 - \beta_1 v_1 \bar{y}_1$. This is equal to $F_p 1 - F_p 2 - F_f + w \sin \theta$. Your $F_p 1$ this can be given as $\frac{1}{2} \rho g y_1^2 B$; I hope you should have understood what do you mean by this one, $F_p 1$ is to $\frac{1}{2} \rho g y_1^2 B$, $F_p 2$ is equal to $\frac{1}{2} \rho g y_2^2 B$. So, you substitute this here.

Another thing is that suppose if you assume the frictional resistance F_f this is nothing but something related to the weight of water here in this volume and its component; that is you can relate it to something with respect to weight as well as the frictional head. So, I can write it now the corresponding way. This is $\rho g B \bar{y} h_f$, where h_f is the frictional head. It is the frictional head units of length and \bar{y} this is the average depth of flow between the two sections 1 and 2. I can easily suggest that this is nothing but $\frac{y_1 + y_2}{2}$. If you can incorporate this relationship now subsequently here; all of the terms you just incorporate it in the equation, I will get the corresponding form. Also you can suggest the $\sin \theta$ term here; $\sin \theta$ is nothing but $\frac{z_1 - z_2}{L}$.

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$$\int_{z_1}^{z_2} (\rho u^2 - p u) dx$$

$$B(z_1 - z_2) + \frac{1}{2} \rho g B (y_1^2 - y_2^2) - \rho g B \bar{y} h_3'$$

$$\frac{u^2}{2g} = (z_1 - z_2) + (y_1 - y_2) - h_3'$$

$$z_1 + y_1 + \rho \frac{u^2}{2g} = z_2 + y_2 + \rho \frac{u^2}{2g} + h_3'$$

$$(\rho u^2 - p u) = \rho \frac{u^2}{2g} - \rho g \bar{y}$$

You can also see that as Q steady discharge is there, Q will be same throughout. This Q I can compute it as related to the average depth of flow now. For that I can write it as Q is equal to average velocity between two sections, you see v_1 bar plus v_2 bar half; that is this is the average velocity between these two sections into the average depth into the width of the channel. So, if the width of the channel is not changing here. So, you can correspondingly write it as well. So, we will get the thing as Q is equal to the following form; our momentum equation becomes this particular equation becomes half rho into half of v_1 bar plus v_2 bar B y bar. This is equal to rho g y bar L into B z 1 minus z 2 by L plus this momentum equation.

Now for this particular case of rectangular channel having width B, we can write it in the following form. Please note that this is for any general area and for the rectangular channel of width B, this can be represent in the following form. You can cancel out whatever terms are repeating, you see rho B y bar, rho B y bar; they are repeating on LHS and RHS. You can cancel it out; you can also substitute appropriately the terms using y 1 here, half into y 1 plus y 2 term is coming, that you can give it as average depth y bar; like that you can eliminate the terms. This can be written as beta 1 v_1 squared by 2g plus beta 2 v_2 squared by 2g is equal to z 1 minus z 2 plus y 1 minus y 2 minus h f dash or I can write rearranging the things z 1 plus y 1 plus beta 1 into v_1 squared by 2g is equal to z 2 plus y 2 plus beta 2 into v_2 squared by 2g.

You have seen a similar sort of relationship already in the energy; equation another this thing is coming here now. I just forgot to write h_f here. A similar sort of relationship as found in energy case is again found here when you computed through the momentum equation. The h_f the friction head h_f that is computed using momentum equation; if this is quite different as compared to the one computed using energy equation. Here you are using the average depth term \bar{y} ; using those things, the things are coming here. So, little bit difference is there from the energy equation and momentum equation for the rectangular channel; however, for gradually varied flow or uniform flow and all, you will see both are same.

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Specific force

$$S \rho Q (\bar{y}_2 \bar{v}_2 - \beta_1 \bar{v}_1) = F_{p1} - F_{p2} + W \sin \theta - F_f$$

If slope = negligible

$$S \rho Q (\beta_2 \bar{v}_2 - \beta_1 \bar{v}_1) = F_{p1} - F_{p2}$$

$\beta_2 = \beta_1 = 1.0$

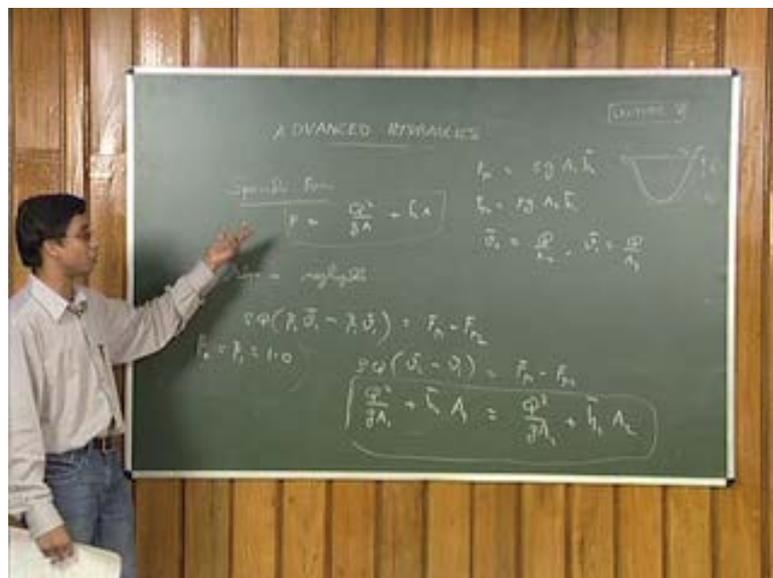
$$S \rho Q (\bar{v}_2 - \bar{v}_1) = F_{p1} - F_{p2}$$

The diagram shows a channel cross-section with a slope angle θ . It indicates two sections, 1 and 2, with average depths \bar{y}_1 and \bar{y}_2 and average velocities \bar{v}_1 and \bar{v}_2 . The weight W of the water between the sections is shown acting vertically downwards.

Now we will discuss on the concept called specific force. What do you mean by specific force? What was the momentum equation you have seen earlier? It was same as $\rho Q \beta_2 \bar{v}_2 - \beta_1 \bar{v}_1$. This is equal to $F_{p1} - F_{p2} + W \sin \theta - F_f$; this was the momentum equation discussed earlier. So, in this particular case for any arbitrarily cross sectional channel need not be rectangular or need not be triangular in the each channel. If your slope is negligible and you are taking a stretch of flow say section 1 and section 2 such a way that, if there are small stretch what happens is that the effect of friction can be neglected. So, if F_f friction if it is neglected, similarly any of the weight component due to the slope neglecting the weight component is also neglected, you can write your corresponding equation now $\rho Q \beta_2 \bar{v}_2 - \beta_1 \bar{v}_1$ is equal to $F_{p1} - F_{p2}$.

So, please note that we are talking about prismatic channel. So, in the prismatic channels if you neglect these following quantities, you take a particular stretch such a way that a short stretch where friction force can be neglected as well as the weight is now neglected due to the slope is also negligible. You can now write the following quantities. Your equation will become $\rho Q \beta_2 v_2 \bar{v}_2 - \rho Q \beta_1 v_1 \bar{v}_1 = F_p 1 - F_p 2$. Most of the cases in such prismatic channels we can approximate, say, β_2 is equal to β_1 is approximately equal to 1 in the prismatic channels. If that approximation is also true, then you can write this as $\rho Q v_2 \bar{v}_2 - \rho Q v_1 \bar{v}_1 = F_p 1 - F_p 2$. So, what is $F_p 1$?

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What is $F_p 1$ from here? Your $F_p 1$ is nothing but $\rho g A_1 \bar{h}_1$, say, a prismatic channel. The area is A_1 and the centroid of this area from the free surface; that is the term called \bar{h}_1 here. So your pressure force, definitely the force due to pressure will act at the centroid; we are considering it to be acting at the centroid. So, that centroid point is \bar{h}_1 from the free surface $F_p 2$ this is $\rho g A_2 \bar{h}_2$. So, the \bar{h}_1 is at the centroid; the height or the distance of the centroid from the free surface and \bar{h}_2 is the distance of the centroid from the free surface in the downstream section, this is in the upstream section. You substitute the following quantities; you will see that your momentum equation this particular momentum equation this becomes you know $v_2 \bar{v}_2$ is nothing but Q/A_2 , $v_1 \bar{v}_1$ is equal to Q/A_1 .

Substitute the following quantities, you will get $Q^2/gA_1 + h_1 \bar{A}_1$ is equal to $Q^2/gA_2 + h_2 \bar{A}_2$. If I get the following equation the momentum equation in this particular simplified case where slope is negligible, friction is negligible and all; in such a prismatic channel where momentum correction factors are also one, you are getting the following relationship. This is nothing but if you see here this is the momentum flux that passes any cross section of the channel. This is the momentum flux that passes any cross section of the channel. This one the corresponding term that is related to pressure force, what will you get now? If you summate these or if you take this particularly, I can write this as I can define this as a quantity called specific force where F is equal to $Q^2/gA + h \bar{A}$.

So, if I can specify this quantity or define this particular term where you have seen that for a small stretch or any stretch both the upstream and downstream quantities are same. If you can define this term as specific force, you will see that the specific force is same in the upstream and downstream irrespective of the type of flow here. We will deal with much more aspects of the specific force in other classes and all. So, let us conclude it here. The quiz of the class will be taken in the next lecture.

Thank you.