Advanced Hydraulics
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Module - 2
Uniform Flows
Lecture - 4
Uniform Flow in Compound Sections
Concept of Normal Slope

Welcome back to our course on advanced hydraulics. We are in the second module on uniform flows. So last few classes, we were dealing with various topics related to uniform flow.

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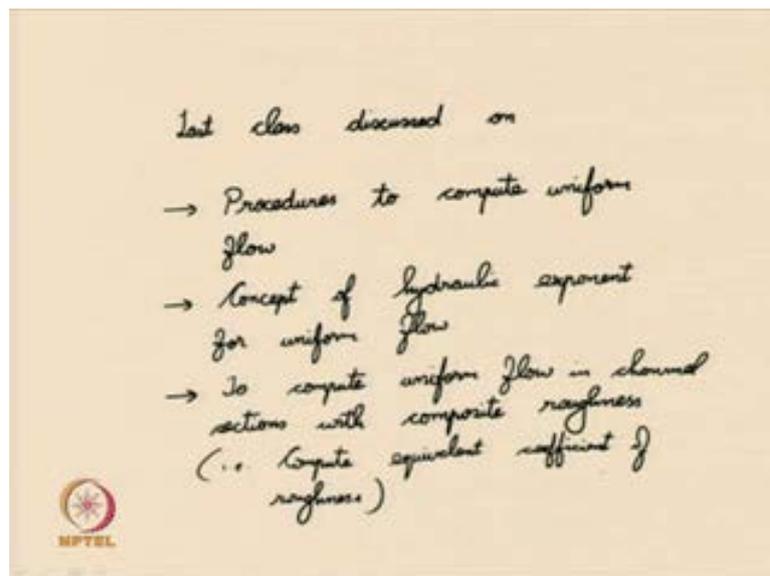
In the last class, we discussed on the procedures to compute uniform flow. We also discussed on, the concept of hydraulic exponent for uniform flow. Also we suggested how to compute uniform flow in channel sections, with composite roughness; that is how to evaluate the equivalent coefficient of roughness for various situations.

So today we will be going with the following topic, we will be dealing with uniform flow, how to understand the uniform flow in compound section, and to introduce the concept of normal slope.
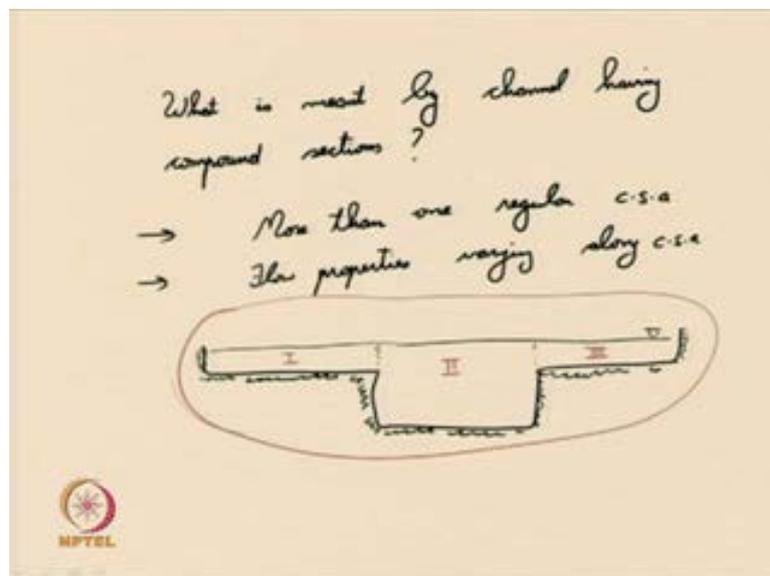
So, what is meant by channel having compound sections? So can any one of you tell, what is meant by channel having compound sections? A compound section is one; you can see that, a compound section we generally call as compound, if it has more than one regular area. If it has more than one regular cross sectional area, then along the channel

cross section, if the channel and flow properties are vary, then also we call them as a compound channels cross section. That is flow properties varying along the cross sectional area. If these two conditions appear mainly, we usually call them as compound channel sections. So, this is quite difficult to compute flow in compound channel sections, and if you imagine how to compute, how to approximate uniform flow for the channel compound sections and all, it is quite tedious, compare to the normal regular section, whichever you have dealt earlier. If you want see some examples or some types of compound sections; for example, here I am just drawing a very simple compound section.

Now, just to make you understand what is meant by compound section, as mention in the definition of the compound section, it is this particular channel is having, more than one regular cross sectional area. So, if you see here, it has; say if you just demark it this portions, this you can consider as area number one, this you can consider as area number two, this you can consider as area number three. You can now easily see three rectangular cross sections in this compound section already given to you. Like that, if many cross sectional areas appear in a channel cross section, many type of cross sectional area appear in a single section that such type of channels are called compound sections, or compound channels. So very simple situation is that, you can individually compute flow in each of the areas, it has been mentioned, and like that finally add it up; that is a quite a possibility, but to approximate the entire thing, with the uniform flow for the entire section; that it requires certain theoretical concept, as well as certain approximation, so let us see that.
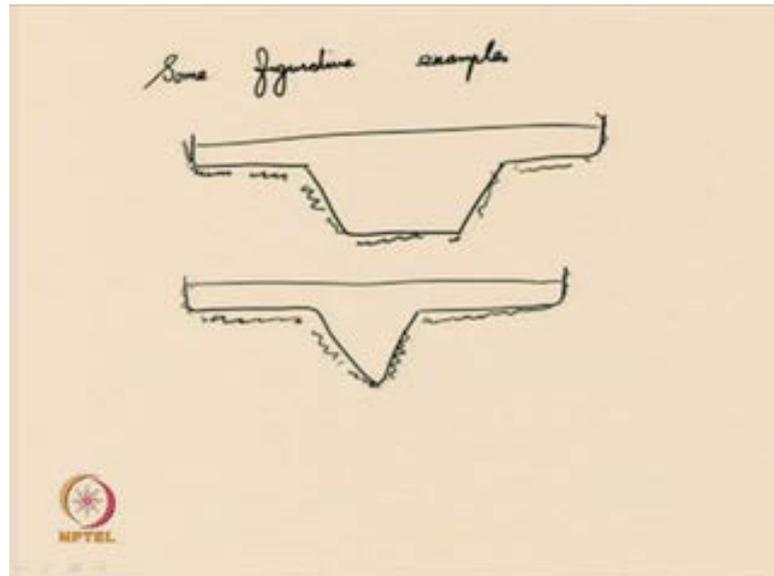
Some figurative examples of compound cross section, now here say; see this is a channel cross section, having a trapezoidal main portion channel, and rectangular, or you can say parallel flight cross sectional flood plain channels, or side channels. So this is a compound section, another example you can imagine of cross section is that. So this is having a triangular cross section as the main channel, and rectangular portion as the side channels, so why do we deal with, why do you want compound sections to analyze open channel flows and all, why it is required.
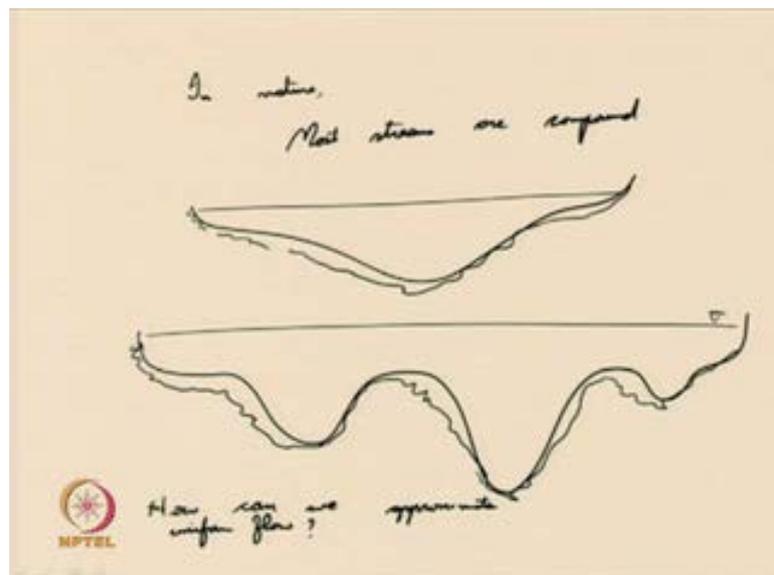
In nature, most of the streams are, most streams are compound in nature. So how the channels of, or the river cross sections may be; say it may be a following type, some of the river cross sections may be like this. Now you see, this is a compound cross section, it is having some side portion, and some main portion, which the bulk volume of water is being carried in the main portion, side portion is also having certain discharges, or you may also see channels, where multiple main channels, or multiple main channels, or you can say body channels are there, for example in this particular type of cross section. Say there can be cross sections of such nature, cross sections of river bed, in the following form, so this are typically witnessed in meandering rivers and all.

For example, if you go through the Brahmaputra basin, Brahmaputra rivers system and all. If any one takes the bathymetry survey, or bathymetric study of this bed, using hydro graphics survey and all. One way witness such type of cross sections, where more than one main carrying, channel will be present, then some site channel may be present and all, it may be quite possible. So to analyze the flow mechanisms, or to approximate uniform flow in such situation, such type of compound sections, we require as I mentioned earlier, certain knowledge, just we will see how we can approximate. how can we approximate uniform flows, how we can approximate how can we approximate uniform flows in compound cross sections, can you guess, one method of analysis, in the compound section.

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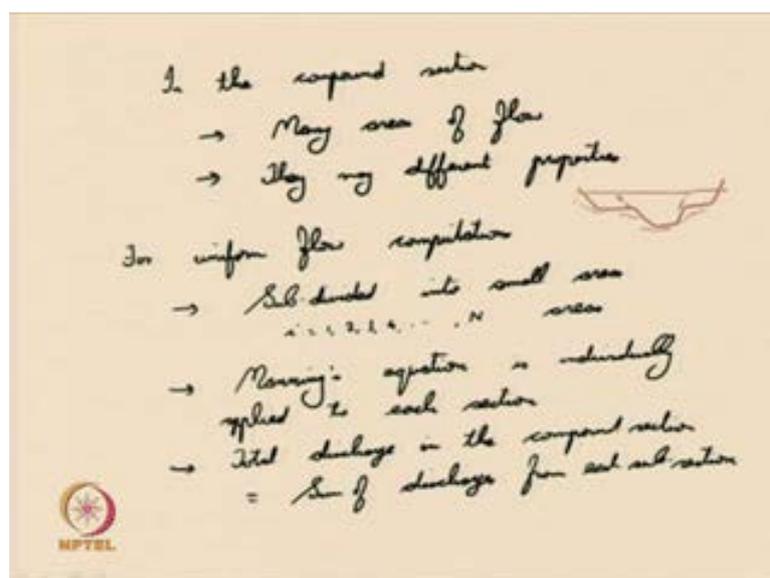
In the compound section, you have many areas of flow. They may have different properties; each area may have different properties, so you have to device the strategy accordingly. So for uniform flow computations, what we do is that, for uniform flow computations, the entire compound cross section of the channel, is now sub divided, the entire area is sub divided into small areas; say may be n areas, i is equal to one two three four etcetera to n areas. Manning's equation is now, manning's equation is individually applied to each section, each sub section. Then we suggest that, the total discharge in the compound section, is equal to some of discharges from each sub section. So, whatever I have written it here, if you want just figuratively show that, say a very small figure I will draw on the sides here. You can have say such compound section, it is now divided into small areas, say one two three like that, n small areas, it is being divided. Then manning's equation is then, apply to each of this small sections individually. You can compute the total discharge in the entire section, compound section. The entire compound section the total discharge in the entire compound section can be, obtained as summation of discharges from each of these small sections, like that one can device the strategy. So for the demonstration, as I mentioned earlier, let us sub divide the area into n cross sections; that is the entire area.

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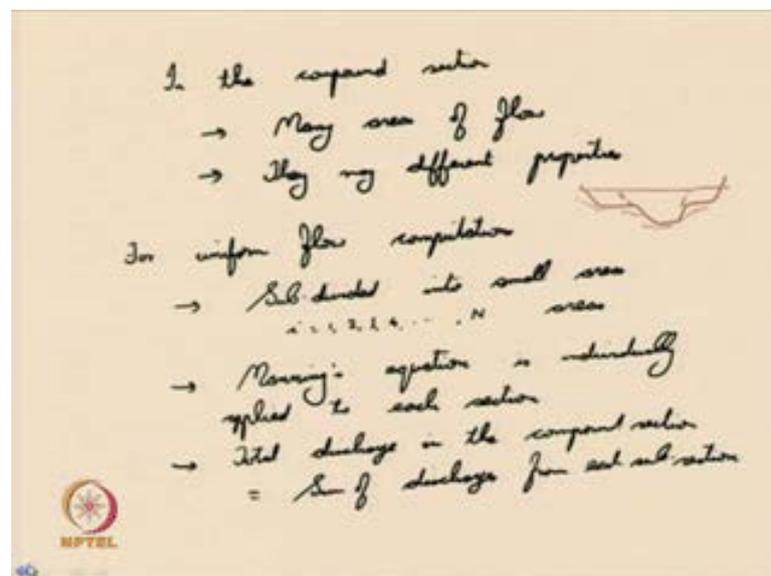
The entire area cross section, entire area of the cross sections divided into n sub areas. So the channel sections, see here, if you take the bigger figure here, or here if you see here. As we divided into smaller areas, the entire compound section, if you divide into smaller

area. Now here in this particular portion, the depth of the channel is shallow, here it is very deep, again here it is shallow, so various flow properties are coming. Then as we know depth of flow is also have property, so various properties are coming into picture, so definitely the flow characteristics changes in each of these sections. So, you need to compute velocity in each of the channels. As you see the depth of flow is different in different channel sections, different parts of this compound section, definitely the velocity in this portions also will be different, so you need to compute velocity independently using manning's equation, so for that case. Again if you suggest that, if you compute the quantity independently, if I take this channel independently, this channel independently, this channel independently, if I compute the velocity using the manning's equation and all. So now you will be having several one dimensional module.

So, if that several one dimensional module, if you try to club and all what happens is that, you may have to take into account, how much amount of mass and momentum, mass flux as well as momentum flux, how amount of them, they are getting transferred in between the sections; say adjacent section. If some mass from this section get transferred to this section, from this section to this section, like that if it gets transferred. You need to take into account all those quantities also, or if one decides to think in a multi dimensional way and try to analyze in multi dimension way, then also the problem gets complexed. So you need further approximations and further knowledge, of whatever way we can do the uniform flow of approximation, so let us see that. So, my question is, flow in open channels, we have the top surface of water, it is level surface, you would not see some fluctuated surface and all.

Usually the top surface of water is a level surface; that is pressure is same along the surface of water. So if pressure is same along the surface of water, the level of the water surface becomes constant, or means level, it will be a uniform for uniform flow. You know that usually the depth of the flow is considered the uniform, and water surface, the top surface of the water; they will be in uniform width, so pressure along the surface of water; that is also the same quantity here. When you compute total head or energy head, you know that it has components of pressure head, velocity head, and datum head. So your total head there is having the following three components, that these things we are dealing with, from our earlier classes onwards itself.

So along the channel cross section, along the channel main compound cross section, you have different sub sections now. And also we suggested that the velocity in each of the sub section may be different. So that means, all though the presser is same along the surface of the water, datum head is also same, means for the same thing we are the taking the same datum. Then the velocity head, or the component of velocity, the velocity as it is changing, along the portions of the section, not along the section we are talking about, along in the same section, along the various of portions of that particular section, velocity is changing, it creates different energy head along the parts of the section. So this is quite abnormal, if there is different, means if the, along the flood plains of the channel; for example, again let me refer that figure; say here this portion, you are having the certain energy line, or another energy head.

Here you are having different energy head; here you are having different energy head. Although the water surface at this location they are same, and pressure head is same. So pressure head is same, water datum is same, but energy, the velocity head is different in all the three portion. So, that creates different energy levels, along the different sections. So, you need to incorporate some correction factors, or some coefficient in the velocity, which we had already studied. We have already studied the energy correction energy correction coefficient factor for the velocity, momentum correction factor, or momentum coefficient for the velocity. So you need to take into account those factors also, when you compute uniform channel, or when you try to approximate uniform flow for the compound channel sections.

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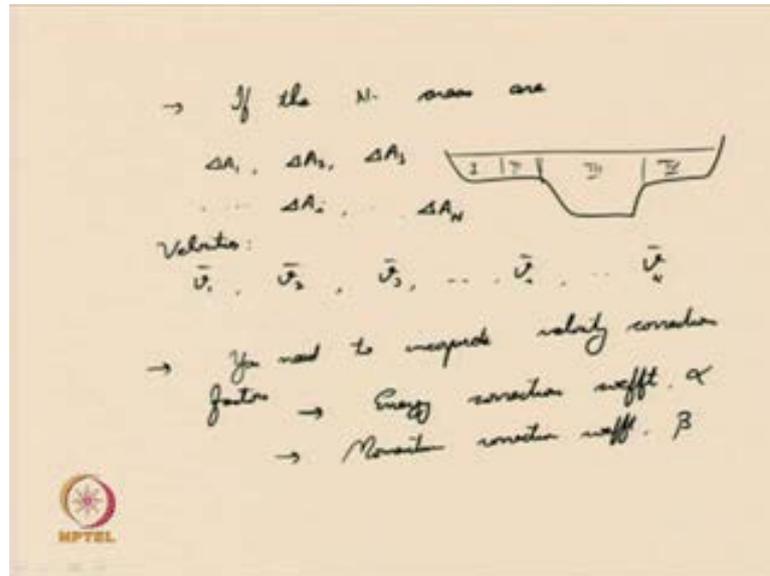
So, if the n areas are, I will just again one two three four as in the figure, but generally you can divide the entire cross section into ends of areas; delta A 1 delta A 2 delta A 3 delta A i, similarly up to n sub areas if you can divide them. Then the velocity of flow in each of this sub areas, it can be given as say v 1 bar v 2 bar v 3 bar v i bar v n bar. I hope you, why we put the bar here. We are generally using the bar notation suggesting that, you are averaging the velocity for the, entire cross section. You know along the cross section, the velocity has further variations, and we gave you how to average the velocities and all. Now when the compound cross section, for each of this channel sections, you are having the average velocity for that particular section.

For the area del A 1 you are having the velocity v 1, for del A 2 you are having the average velocity v 2, for del A 3 you are having v 3; similarly, you have different velocity components. So as we mentioned earlier, you need to incorporate velocity correction factors appropriately. For example, if you are computing the total energy head, for wherever the velocity head is coming into picture, you need to incorporate energy, correction coefficient for velocity. If you recall the simple, we have given it alpha, if you are. If momentum fluxes are coming into picture, subsequently you need to incorporate momentum correction coefficient, if you recall them we had given it as symbolically as beta. Then how do you compute, means I hope there is no need to tell you how to compute alpha and beta and all. Anyhow we will be coming in our subsequent portion of this lecture also.

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So, discharge in any sub area; say Q y, this is nothing but, v i into del A i, so if you are applying the manning's equation, you had already studied conveyance factor for uniform flow. Therefore, for the i'th section, you can have the conveyance factor k i, you can represent your discharge Q i is equal to k i into s naught of the ith section; that is bed slope of the ith section, its square root, so if you obtain it like this. Now you know v i bar this is equal to Q i by del A i right. So, this is nothing but, k i s naught to the power of i to the power of half divided by del a i.

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That means v 1 bar is equal to k 1 by del A 1 s naught at the first section to the power of half, v 2 bar is equal to k two by del A 2 s naught of the section two to the power of half like that. For the n'th last section, is k n by A n s naught to the power of half. So, how will you compute uniform flow velocity now. So you have now the uniform flow velocity each of the sub areas; that is you are having the uniform flow velocities in sub areas, you are having them. So based on that how will you compute the uniform flow for the entire compound channel cross section.

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So this you can do the velocity averaging, you have already studied that. So let me suggest this as v m bar; that is the uniform flow, which the average uniform flow velocity for the entire channel section, this is summation of v i del A i. So you have the method to compute the average velocity, or the uniform flow velocity for the entire compound section, so what is this quantity now. You already have they expression for v i bar, so you substitute that. So v m bar is equal to k i by del A i s naught i'th section by half into del A i divided by summation of the individual areas; that will give you the total area A, i is equal to 1 to n k i. So through this way, one can compute the uniform flow for the entire, means uniform flow velocity; please note that one can compute the uniform flow velocity, for the entire compound channel cross section.

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So as we motioned earlier, one need to incorporate energy and momentum correction factors, when you do such averaging. When you do such averaging you need to incorporate energy and momentum correction factors, so how do you incorporate them. So do you recall that, your energy correction factor for the velocity, it was given as v cube d A by v cube A. Your momentum velocity coefficient, momentum velocity coefficient was given as v square d A by v squared A. So you use the same relationship use, use similar relation for compound section. Now what happens, for the compound section, there may be a chance that for each of the sub sections, you may be having individual energy correction coefficient; say alpha i, each sub sections, it may be present. Similarly, beta i for each of the sub section it is may be present.
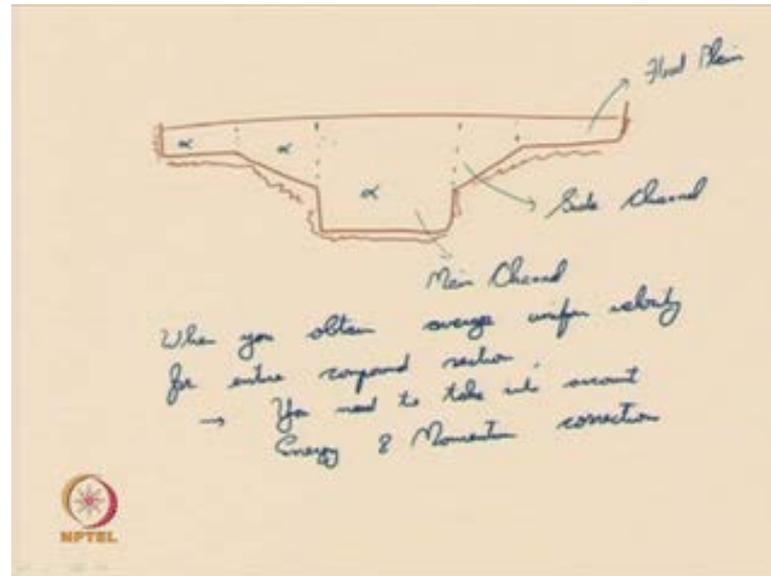
For example, in the earlier figure drawn, or say here; say for such an example of compound cross section, this portion may be flood plain, this may be the side channels, and this is the main channel, so you can suggest like this. This is the main channel, this can be your flood plain, this can be your side channel. Like this you can distinguish, or you can just suggests that you are, you can the entire compound section can be now sub divided, and you can describe some of the properties, that this is are flood plain portion of the channel, this is the side channel, this is the main channel, or the main body of the channel like that, if you are able to suggest. Now for each for example, there may be some correction coefficient for the main channel here. There may be some coefficient moment energy correction coefficient for the sub channel, for the flood plain channel.

They may be quite different also, that can be, there are definite chances, because for example, the flood plain, normally it is in a dry situation and in the flood arrives its get wet. So the roughness of the location may be quite high, and this may lead to different type of energy correction coefficient. In the side channel, the water may be existing, but during flood it may be in a different way and all. The main body it is always, mostly it is carrying the water, its energy correction coefficient may be quite different. So you can have individual energy correction coefficient for each of the sub areas. So you need to take into account that portion also, when you obtain average velocity. So when you obtain average uniform velocity for entire compound section, you need to take into account energy and momentum corrections.

You know that momentum, fluxes mass fluxes, all this occurs between the sub areas. So that those correction you require, if you substitute, if you when you try to compute for example, if you try to compute mass flux along, between this two sections. If you try to compute the mass flux, and if you see that, and if you incorporate, as some portion of the (()) main velocity, along this sections, some portion of this is. You need to incorporate the corresponding coefficient correction; you cannot take the entire velocity. Similarly, momentum, if it is getting transfer from this section, from this area to this area. You need to take into account appropriate correction factors. Similarly, in the stretch of channel also, you need to take into account the corresponding correction factors.

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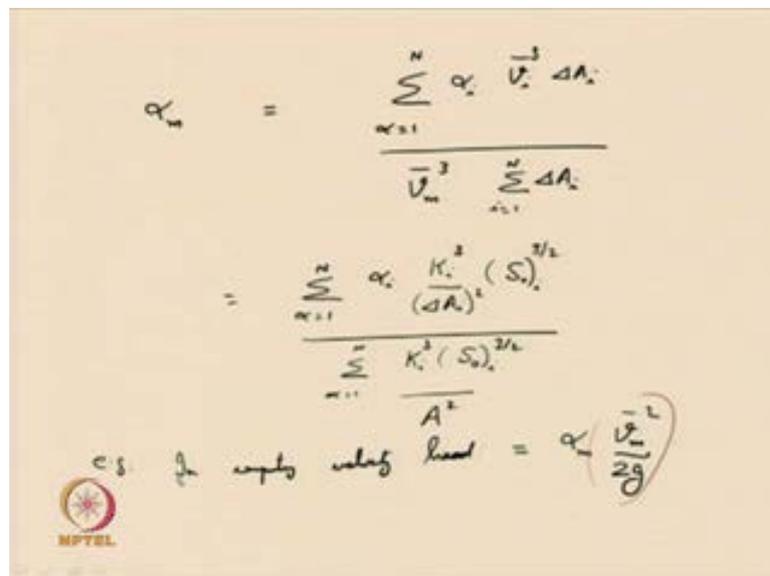
So, I can now suggest, the energy correction factor, is of the following form alpha equal to one to n, alpha i v i cube del A i. This is the equation, you just expand it, substitute v i individual v i, and also you can tell this summation of the sub areas as, the total area A, like that you can represent it. I can write the following form now, alpha i k i cube by del A i whole square s naught of the i'th cross section divided by k i cube s naught of the i'th cross section whole is to 3 by 2 by capital A square. You can if possible rearrange the terms if you required, you can rearrange them or you just note it in that way. So for example, in computing velocity head, using the average velocity of the compound section, average uniform velocity of the compound section, if you want to compute velocity head. This is given as alpha m into v m square by 2 g, so it is not simply v m square by 2 g, it is alpha m times v m squared by 2 g.

Similarly, the momentum correction factors, you can give that, give this as, beta is equal to sigma, beta is equal to sigma i equal to 1 to n beta i v i square into del A i divided by v m square A, this is the form. So you substitute v i is equal to k i by del A i s naught to the power of half, you substitute this thing here. You will get the following relation now i is equal to 1 to n beta i k i k i square by del A i s naught of the ith cross section, it is 2 by, raise to 2 by 2, so that is 1 divided by k i square s naught of the ith cross section divided by the capital total area A, so this relation also you can keep it in mind.

Now, if we suggest that, if we suggest the beds slope is same, for all the sub area. Then s naught 1 is same as s naught 2 for all the n sub areas, the bed slope will be same. That situation, you can write your energy correction coefficient as sigma i is equal to 1 to n alpha i k i cube by del A i whole squared by sigma i is equal to 1 to n k i cube by capital A square.

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$$\beta_m = \frac{\sum_{i=1}^{N} \beta_i \frac{K_i^2}{\Delta A_i}}{\frac{\sum_{i=1}^{N} K_i^2}{A}}$$

Similarly, the momentum correction factor also you can write it 1 to n beta i k i square by del A i by to n k i squared by capital A. So like this you can obtain in your energy and momentum correction factors, for the entire compound section, and you can subsequently incorporate them, while computing the velocity head, momentum fluxes, whatever fluxes or whatever relations you want to use, in those things, you need to properly incorporate them.

So, as we mentioned the discharge along any of the sub section, using your manning's equation, it was given as k i s naught to the power of half, so that means, your bed slope, the root of your bed slope of any ith section, this is equal to Q i by k i. So if your s naught i is same for all the sub sections; that is as mentioned earlier, s naught 1 is equal to s naught 2 is equal to all the n eth cross all the n sub section of the cross section of the compound channel. Then you can devices the strategy that, your Q 1 by k 1, this is also equal to Q 2 by k 2, this is Q 3 by k 3. Similarly, it becomes a property this ration that is Q 1 by k 1 Q 2 by k 2 Q 3 by k 3, they all become a property. And it is we can consider that this property is some constant C, as this is constant value, or some value, let us approximate let us give it as capital C. So as you know that C is s naught to the power of half.

So I can now write Q i is equal to k i s naught to the power of half, or k i C. Your total discharge q, this is equal to Q i; that is it is equal to k i times C, or your this constant C can now we suggested as Q i by k i or you can suggest that, the bed slope s naught is nothing but, summation of discharges, the whole square and this is equal to Q square by k i whole square. So, this way you will be able to get the bed slope of the channel, you will be able to get the bed slope of the channel directly for the component section, you just need the channel property. If you have the conveyance factor for the each of the sub sections, based on that you are able to compute the bed slope also.

You can see that as an home work, you can just think of doing the following problem home work , you can see say, a natural stream, having a main section and flood plain flood plain section; two section is there in natural stream. The areas, you can give the following data, area in meter square for each of them. Say the main section is having 500 meter square, the flood plain is having 535 meter square. If you observe the wetted perimeter; say it is having 70 here, 125 here. If the manning's coefficient is given, say here it is 0.035, 0.040. And alpha for the main section, it is given as 1.10, beta for the main section, it is given as 1.04. Similarly, alpha for the flood plain, it is given as 1.11; beta for the flood plain is given as one point same, 1.04. You compute the alpha m and beta m coefficients, for such a cross section, compound cross section, you compute them as a home work.

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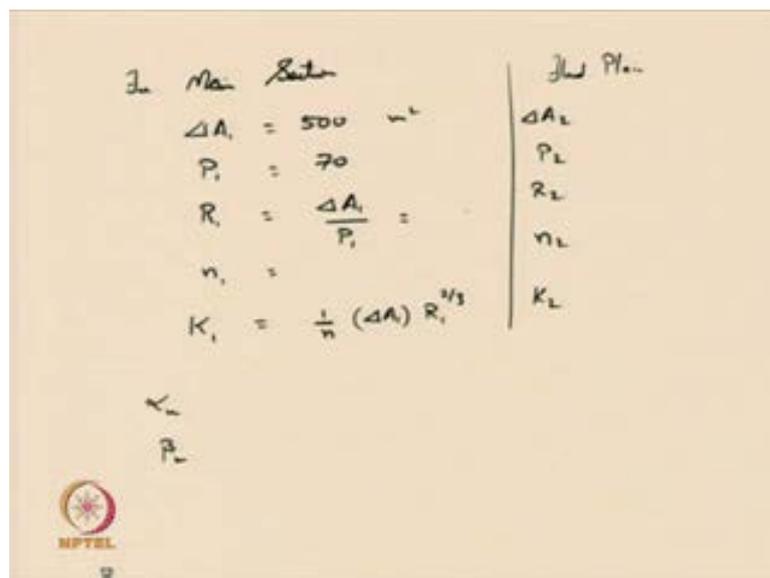
So just I will give you the hint, how to do, how to start it. You know that for main section, del A 1 is given as 500 meter square, wetted perimeter is given as 70; therefore, your hydraulic radius R, this is given as del A 1 by p 1, is equal to seven point one four meter. I just computed it for you, you can do it your own, I am just rubbing it, you compute it; n 1 is equal to given. Subsequently you can compute the conveyance factor for the main section k 1, this is nothing but, 1 by n del A 1 R 1 to the power of 2 by 3. Similarly, for the sub sections also, for the flood plain del A 2 p 2 R 2 n 2 k 2, you compute them, or give them substitute them in the expression for alpha m, you will get

them, substitute them in the expression for beta m, you will get the corresponding coefficient.

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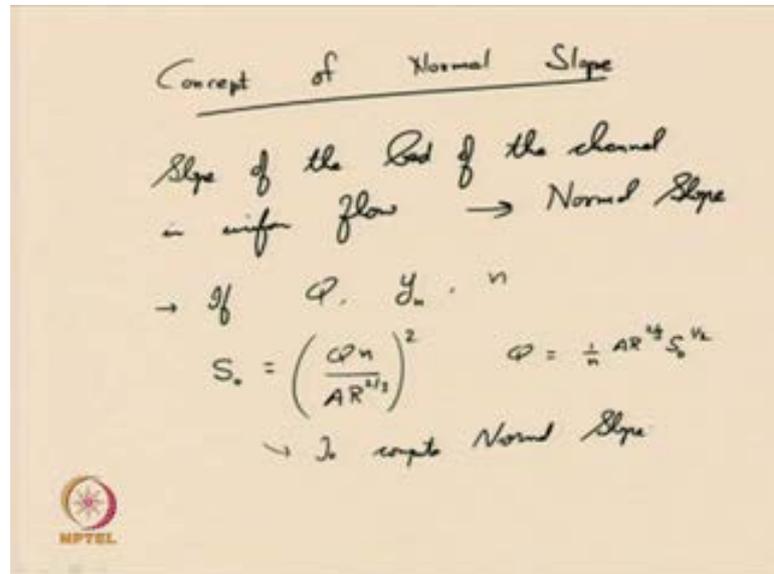
So, let us start, let us introduce you the concept of normal slope. So what is meant by normal slope. As we have already mentioned for uniform flow, you have the quantity call, or you have the depth of flow, named as normal depth. Similarly, for the uniform flow in the channel, the slope of the bed of the channel, is given as normal slope. So slope of the bed of the channel in uniform flow, it is called as normal slope. It is defined like that, it is quite useful, you can one can easily compute, suppose if the discharge in the channel is given, if the normal depth of the channel, if it is given, if the channel cross sectional details are given, and if manning's roughness coefficient if this is also provided to you. One can easily compute the bed slope, how do you compute that. You see Q A is equal to 1 by n A R to the power of 2 by 3 s naught to the power of half, for uniform flow.

So if the depth of the channel given means, and the cross sectional details if they are given, and these quantities A and R is available to you, so s naught is equal to Q n by A R to the power of 2 by 3 this whole quantity raise to 2, this is the way to compute normal slope. So why we this normal slope is discussed is that, normal slope is available only for uniform flow. If you vary the slope of the channel for the same depth, whichever depth of flow, whichever normal depth you have been initially using for the uniform flow, for

the same depth, if you just vary the beds slope, you may see that at a certain situation, the flow will just become critical. So that variations and all, one can see in laboratory, you can just do the experiment, you can check it, you can just change the bed slope, and see for the same depth of flow, the flow becomes critical, or it will changes its course this thing, from critical sub critical or super critical and all, it may change, so that way one can compute the things here.

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Just see an example here, if you have a trapezoidal channel, its bottom width is given as 6 meters, side slope is given two is to one; so that is according to our notations and all, small b is equal to 2, manning's n is if it is given as 0.025. If the normal depth of flow is 1.5 meter, and it is having a discharge Q is equal to 12 meter cube per second. If such a data is given, compute normal slope, so how will you do that, how will you compute the normal slope. Your solution is, you know b 0 is equal to six, all the quantities are given to you, A is equal to y n times b 0 plus b in to y n, substitute the quantities appropriately, I am getting this as 13.5 meter square. Your hydraulic radius is equal to A by p, I am getting this quantity as 1.062 meter you just substitute it; therefore, your bed slope s 0 is equal to Q n by A R to the power of 2 by 3 whole squared. Substitute the quantities whichever are available; that is 12 into 0.25 by 13.5 into 1.062 to the power of 2 by 3 the whole quantity squared.

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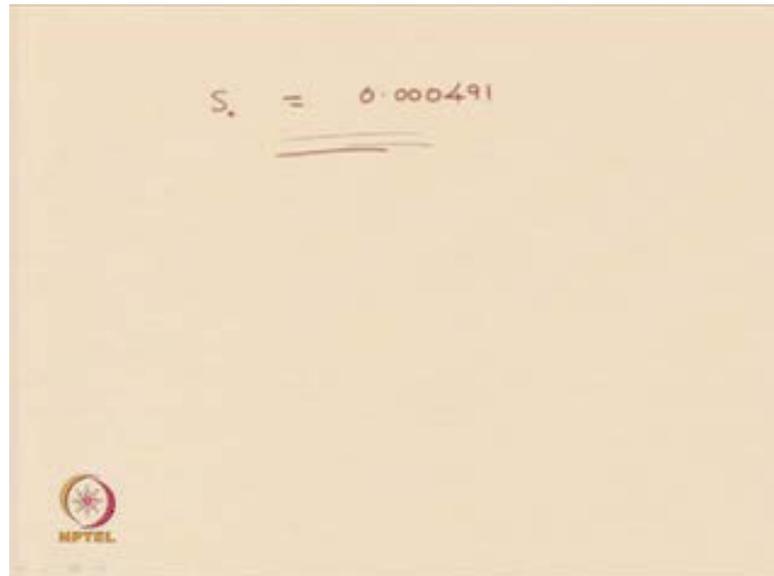
$$S_o = 0.000491$$

So you will get this s naught as 0.00049, so this is your normal slope. You can also see that, at the same slope if you want to have critical flow, for same bed slope if you want to have the critical slope, you can compute the corresponding depth of the flow; that makes the flow as critical flow for the same discharge; that is also quite possible, you have seen how to compute critical flow and all, you can just evaluate those things. So let me conclude today's lecture, we will just have a brief quiz related to today's lecture.

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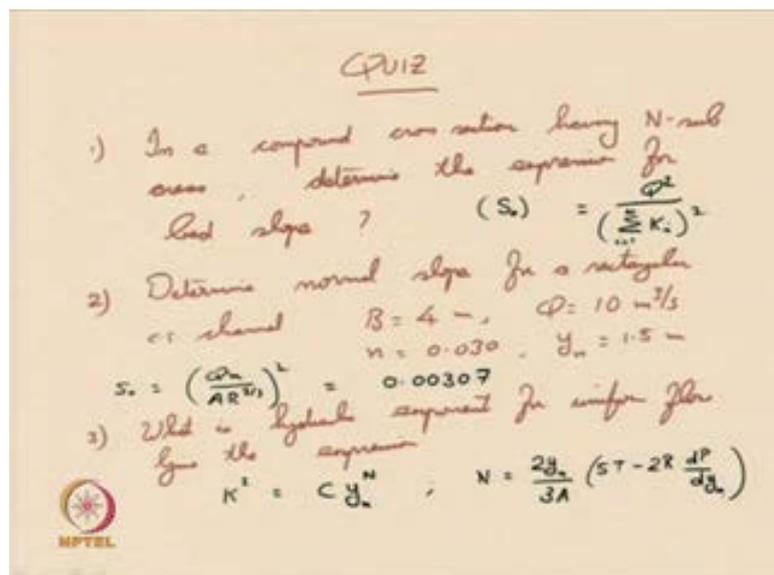
The first question is, in a compound cross section, having n sub areas, determine the expression for bed slope. Just few minutes before we have done that, so you just determine that expression, you just give the expression for the bed slope. Your second question - determine normal slope for rectangular cross sectional channel, the width of the cross section is 4 meters, discharge in the cross section is 10 meter cube per second, manning's coefficient given as 0.030, and the normal depth of flow is 1.5 meter. So for a rectangular cross section channel, it is quite easy, you just determine it, it is hardly takes the thirty seconds for you to solve them. So the third question is, what is hydraulic exponent for uniform flow, give the expression for the hydraulic exponent?

So, today we will have these three questions, fourth that is I hope you have solve this things, the solutions for this thing, I can just give it in the slides, the written you have already seen, s naught to the power of half for compound cross section, it is, or you can avoid this half s naught is equal to Q square by sigma i is equal to 1 to n k i whole square, this thing we have already derive it. The solution for the second question, it is given as s naught is equal to Q n by A R to the power of 2 by 3 whole square, I am evaluating the quantities, I get the thing as 0.030, so please verify it. If there is any wrong, please let me know that. So what is the hydraulic exponent for the uniform flow, so you recall the expressions for conveyance factor k square is equal to some coefficient C into normal depth times, raise to an exponent capital N. So this capital N is called hydraulic exponent for uniform flow. The expression for capital N, it was also derived in the class, it was given as, twice y n by 3 A 5 T minus 2 R d P by d y n.

Thank you.

Then, so we will continue with same module in the next lecture as soon.