Fire Protection, Services and Maintenance Management of Building
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Lecture – 21
Water Supply System: Constant Demand

So, we will continue with the storage and as I was mentioning in the last class that we you know we optimize the size of the storage.

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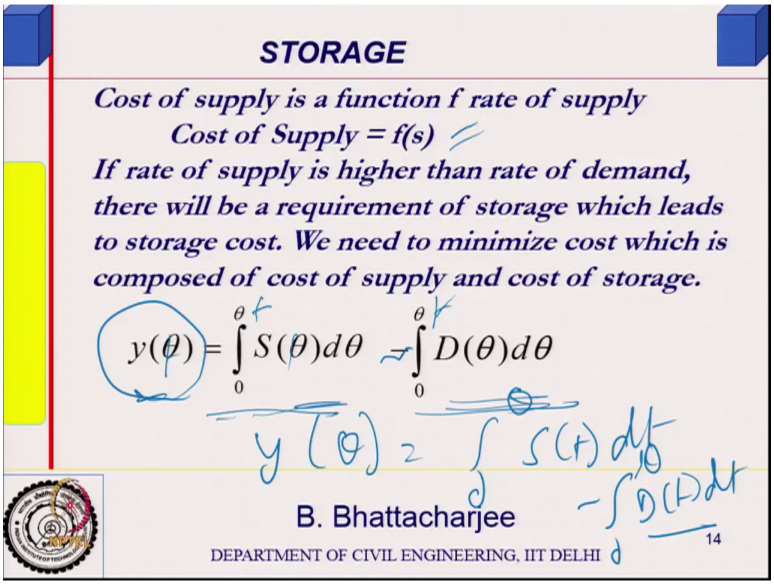

We take say supply cost we write it as f s, where f you know it is a function of supply rate. So, supply cost is a cost of supply is function of supply rate and basically we must maintain this condition that total supply over the period time period because this is a periodic 24 hour evaluation must be equals to total demand otherwise either it will have overflow or you will not be meeting the demand.

At any point the difference between the cumulative supply and cumulative demand that would give you the kind of a storage right. So, this is because this is supply is more than now in this case, supply is more than the demand up to a time theta I mean this should be some t small t maybe up to time t so, y is time t or this should be written as d t, whatever it is.

So, you know it can be written like this y theta is equals to 0 to theta S t d t minus 0 to theta D t d t something like this 0 to yeah; so you write it. So this is the difference. So, this difference is what gets accumulated and if this is bigger and this is smaller, in other words one can write it in form of difference.

If this is bigger this is smaller in that case, you must only store it to maintain the requirement of demand. So, this y theta is basically the storage, y theta is basically the storage, y theta is basically the storage.

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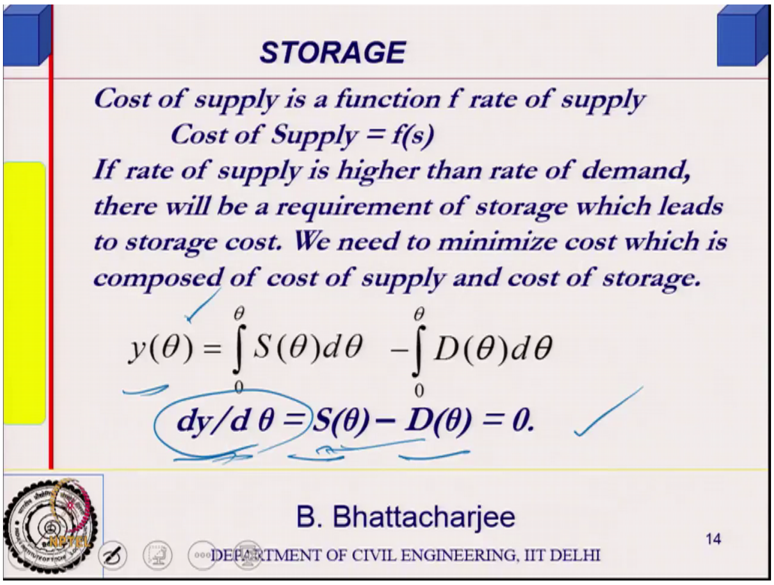

So, yet if I differentiate this with respect to theta, you know where it is maximum say y is maximum or minimum maximum or minimum is negative side; S theta will be equals to D theta. So, the maximum comes where this you know S theta is equals to D theta, S theta is equals to D theta. So, that is from that we can get actually find it out, it should be. Let us see how we do it; I think last class we are talking about this.

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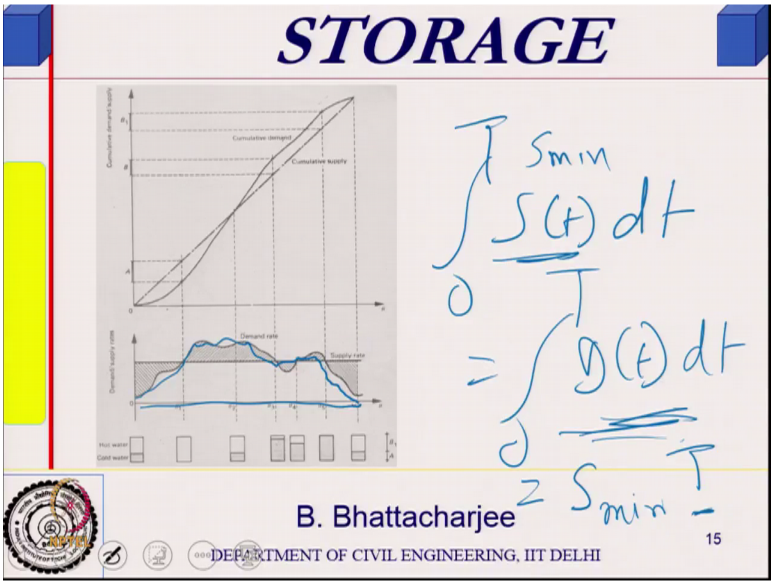

So, one can actually plot this once; for example, this is a time period and my demand requirements are something like this. Now as I said that cumulative demand must be equals to over the time period cumulative demand must be equals to cumulative supply. Otherwise, I will have over flow and you know storage etcetera; so as we said or I will not be specifying the demand.

So, this has to be periodic over the time period that is 24 hours in our case, this must be condition must be satisfied. Now I if I want to find out the minimum supply rate which I require to satisfy this demand profile which is given by this.

And let us say we keep it constant, usually supply rate we keep it constant. Why? Because this is difficult to vary it with time which would means that you have a controller and feedback system and depending upon what is a demand you try to which is not easy. I mean possibly with very good control system you can do one can do it today, but generally this is not done. The simplest thing is to have a constant supply rate, constant supply rate.

So, minimum constant rate supply rate let us say if it is n, this must be equals to S min into T, because then this is constant S min and this is T so you if you integrate it you get S min into T.

So, minimum supply rate is simply the area under the demand curve divided by T; in other words this is nothing but, the average; it is nothing but the average. You know so, this S min would be given by 1 by T 0 to T that is all it should be.

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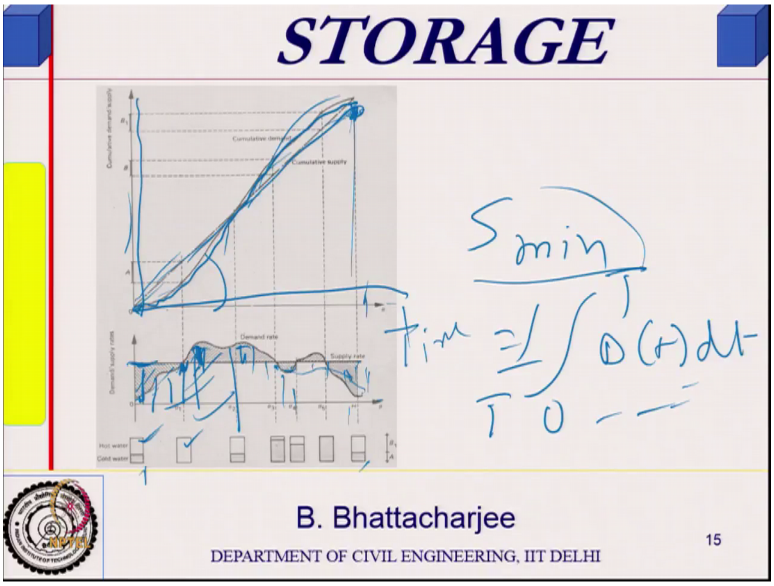

In other words this is it is nothing but is this height average basically. So, if you have sum total of this area under this curve divided by the base, this is what it is. So this rectangular area you know so this is the minimum constant rate.

And if I plot it in the cumulative form that is this is my time and this is the you know cumulative this term cumulative demand or cumulative supply, then cumulative demand would look something like this right.

So, because this is a summation of this area, so up to this is this area total area. Similarly, now it starts reducing the slope would change. So this is cumulative demand would be written in this manner. Now this one when you have a constant supply rate, cumulative supply would be a simply a straight line; cumulative supply will simply be a straight line right, cumulative supply will be simply a straight line.

So, they must match here because after the cycle, after the period the total cumulative demand must be equals to total cumulative supply. So, all you can do is you can even plot this cumulative demand and just last go to last point join them the slope of this line will gives you the supply rate, slope of this line gives you supply rate.

Now, if you look at it what will happen? At this point of time, let us say we were somewhere here right and I have got a supply rate which is higher. So it will go on storing, because supply rate is higher than the demand. So, you know it is go on storing. Let us say this is my hot water case and this is white color is the hot water.

So, you know this is the situation from previous scenario, now it will go on storing till this point of time. After that demand is exceeded, so it will go on consuming them. You know it will go on consuming and the consumption will become again possibly maximum you know it will whenever it goes down below this line, it actually demand increases; so, storage reduces and again storage additional storage and then again there is a you know, the supply right its more storage increases actually here.

So, this is a how it progress, this diagram and this diagram is same. So we want to find out what is the capacity of the storage I require. Now, capacity of the storage will be maximum of these differences, because this height gives me what is the difference between cumulative supply and demand. So, whatever is you know this plus this maximum of this and this; this is what I will require is a storage.

So, so, storage would be given by maximum of y or plus absolute value of y minimum, absolute value of you know, so this differences would give me the storages yeah.

Student: Sir, in that third rectangular storage.

This one.

Student: Yeah this one.

Student: Sir, here also the demand is more than this supply rate.

Right, but what has happened is demand rate reduces slightly. So, it stored a little bit.

Student: But still the supply is less than that.

Yeah. Supply that is why you have got still lot of you know that is it is a accumulating actually, it is accumulating; I mean not accumulating, it is consuming sorry; accumulation is white color yeah. So the accumulation so here the maximum accumulation is actually occurred here from here, because this is supply is here, then here also supply is higher here.

Student: Accumulation is the white.

Accumulation is y and the consumption is a black, white means hot water, the hatched or the darker one is cold water. So, you are heating it, you know so hot water is withdrawn, the water supply I mean water supply is there, but at the same time heating is also there.

So, you know your heater is operating, so when that is how it is basically. So, hot water as the this is all hot water and this is cold water and this is a heating period. So, that is how it is.

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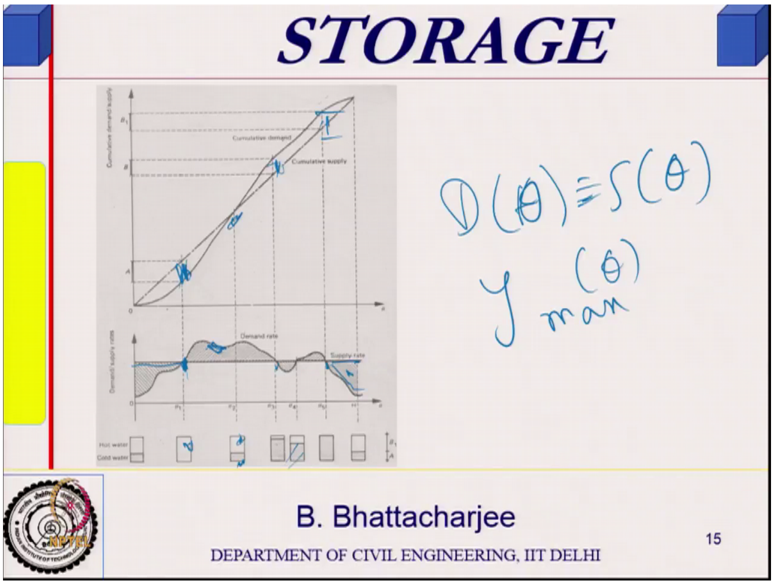

Now, this therefore, these are the cumulative if you look at it, at this point as I said when D theta D theta is equals to S theta that is where you get you know the difference is maximum, y either get y theta max or mean whatever it is. So, this is when this is equals to this so when they are equal this difference is maximum. Again you will have somewhere they will be equal say you will find there is a difference. And whenever they are equal you find the difference is maximum or negative side maximum.

So, the storage is given as a sum total of maximum of these two, because this is in the cycle, whichever is higher that is what it is plus this one. So, total storage is sum total of this, maximum and minimum because you need that you know you need that whenever this is higher; that means, you go to store it and this is the cases where you have to draw it, you will consume all the storage, whatever you have stored, the consumption will finish there.

Then you must have adequate available so that you can withdraw more. So, that is why the maximum these two. So that is how you actually find out.

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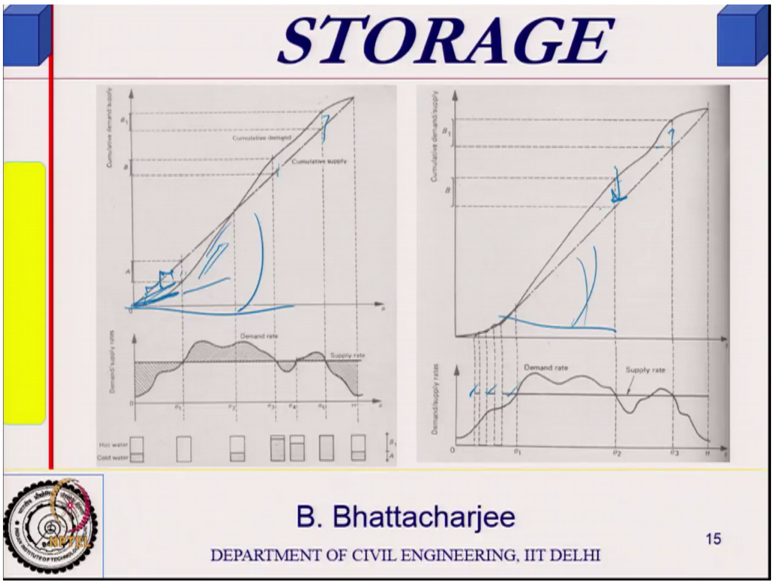

So, what I can do? I can actually see that this gaps becomes minimal. How can I do it? Supposing, I you know instead of this you know what I do is you know I do i, I stop it here and then go I stop it here go possibly like this, try to come you know this is my demand is infinite.

So, initially I started the rate like this then stop fixed rate, rate has to be fixed I cannot change this slope of this line. So, maybe I go something like this stop. Then again start from somewhere there, go at the same slope and so on. So this is what is shown in the next diagram. What I have done is, now I have increased my slope; this value has increased, this value has increased, but not I am trying to match it as close as possible.

So, operational you know the periods I do not have continuous now, what I do is I supply sometime here, sometime here, sometime there and then of course, I might some sometime supply something like this. Such that the maximum these values maximum of these values is you know minimum or maximum of this negative values this is practically 0 very small. So, my storage actually would get reduced on because, earlier it was sum total of this plus this, now it is only this value.

So, I can match it in such a manner with intermediate supply, so that my storage reduces down, but at this, but what I am doing I am increasing the slope; that means, my supply rate is increasing. So, when supply rate is increasing; that means, my cost of supply will increase, either the heater capacity, the pump, everything; something in increase actually all right, and I can do it further.

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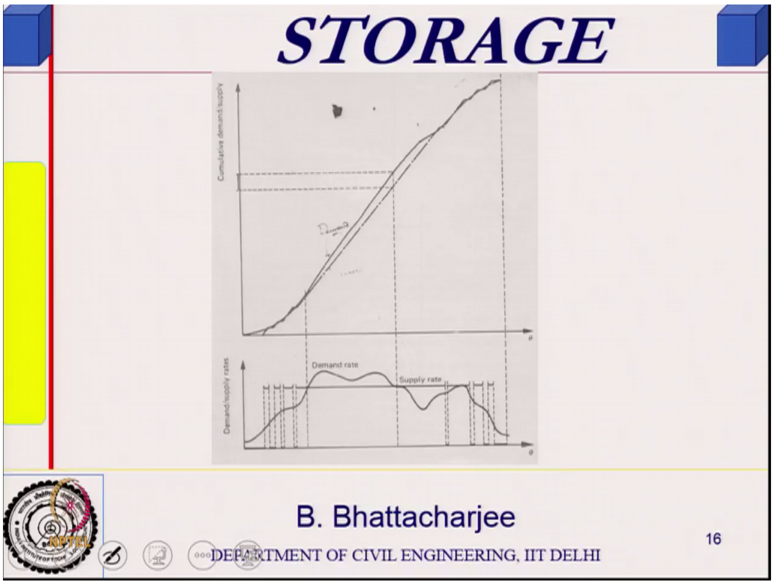

Say if I did further, it will be something like this, you know here the storage reduced significantly.

I have operating schedule I have fixed it in such a manner that this storage has reduced, but this supply rate has increased; slop, this slope has increased right slope has increased. So, this slope has increased all right. So, I can have several supply rates and several you know storages corresponding to a demand profile, the demand profile is same.

So, supply, so, I will have a series of supply rate and storage which will satisfy the demand profile; which will satisfy the demand profile right; which will satisfy the demand profile.

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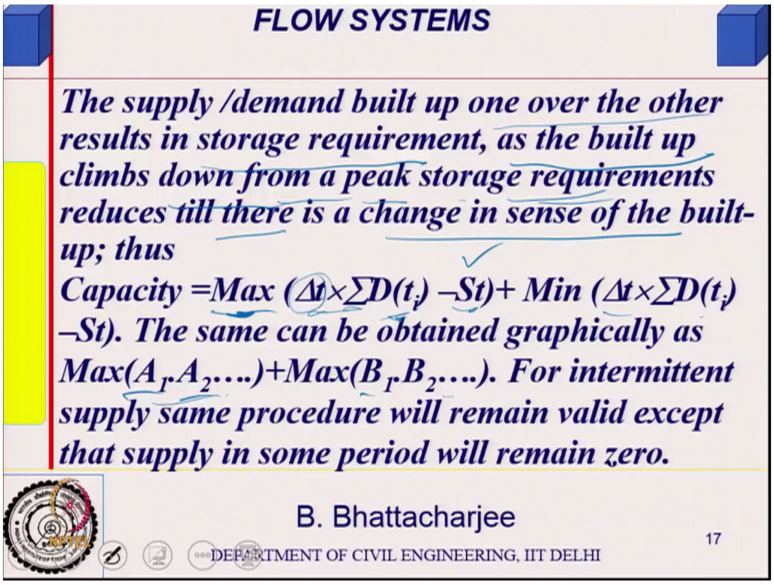

So, what I can do is the supply demand built up one over other results in storage requirement as you have stated, as a built up claims down from peak storage requirement you know built up claims down from a peak storage requirement reduces still there is a change in sense of built up; either the positive sense of built up or withdrawn negative sense of built up.

So, capacity therefore is maximum, delta t into delta t i, if I am taking delta t time interval minus S t if it is constant and minimum plus minimum of this one you know so as I said. So, maximum of A 1 A 2 or y 1 y 2 and maximum of B 1 B 2 that is what I am saying; maximum of you know this equals A 1 A 2 or B 1 B 2; B 1 B 2 is here. So, maximum of or you know y 1 y 1 etcetera I can call it.

So, I am now differentiating them as A 1 A 2 B 1 B 2, where, A 1 is a difference between cumulative supply and demand right and positive and here the same negative, that is now you know so whichever is maximum take absolute value of them and sum them up. So, for intermediate supply same procedure will remain valid, except that supply is some period now is 0.

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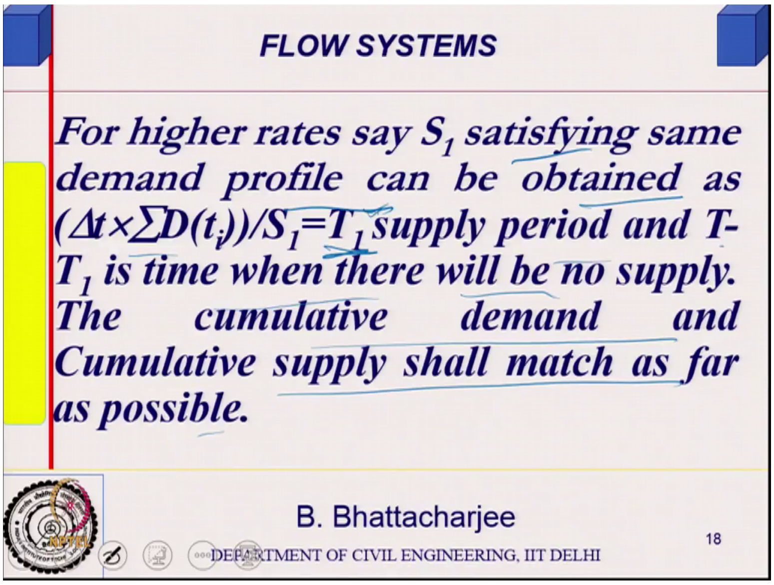

So, for higher supply rate satisfying the same demand profile one can obtain, this is the constant one; simply the time period you know is the time when there be no supply. So, cumulative demand and cumulative supply shall match at the end. And this is that you know this is the period, so S 1 is the supply rate; so the one can that formula already I have told you.

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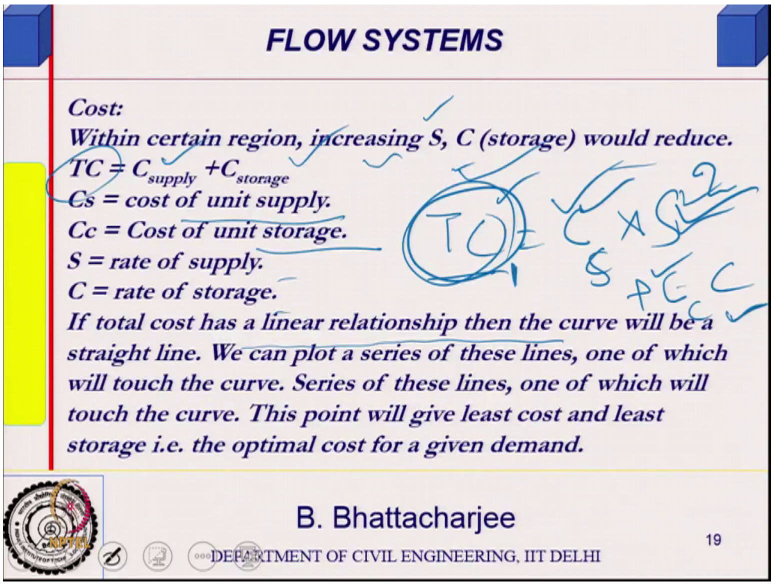

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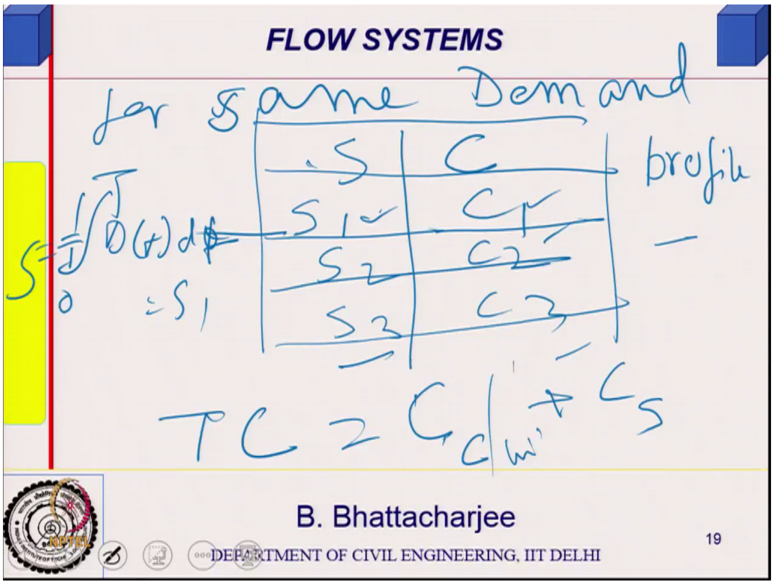

Now, you can find out the total cost, because you have now I can draw it like this table tabular form; my demand profile is fixed, so supply rate and storage. Now, if I call it I think I am using a nomenclature; you know C is the storage and S is a; so I will have set of S 1 C 1 S 2 C 2 S 3 C 3 etcetera for same demand profile, demand for a given demand profile I will have set of now, this would be simply you know as I said average the minimum, this is the d theta d theta 0 to I mean D t 0 to T, 1 over T that is equal to the first one S 1; S 1 that is equal to S 1; S 2 would be now higher supply rate, but this will go down, S 3 will be still higher this will go down and so on.

So, for the same demand profile I will have set of these values, therefore, total cost I can find out; total cost will be cost of the supply and cost of the storage cost of the storage.

So, you know I can so the since this one I know, this one I know I can find it out. So, that is total cost is if I call it cost of storage as C C total cost and cost of supply is S C S this total cost I should be find out. Now, supposing I know the unit cost unit cost of unit per unit if I know per unit then simply multiply this and then you can find out.

So, that is what we can do let us see how it goes. So, S and C would you know as S increases C would reduce. So, cost of supply, cost of storage, cost unit supply, cost of you know rate of supply, rate of storage. So if a total cost has a linear relationship ok, now this cost total cost I can for example, if I know the value of supply and it is you know linearly related for example, T C is simply C C C S multiplied by S plus C multiplied by C plus C multiplied by C right.

Then in that case it is a linear relationship, I can chose a value of S; chose a value of S and keep this value constant, then I can find out C; I can find out C right. cChose a value S and chose a value of you know chose this will value is constant corresponding C value I can find out. See the thing is that I can now plot for different T C values choose 1 T C 1. Now this cost is known to me, this cost is known to me. For any given S, I find out the corresponding C which will satisfy T C 1.

Similarly, I can have T C 2, so I can go on finding out several curves, several curves I can go on finding out several curves, several TC lines; I can find out several T C line right. Now there will be line, but if it is function of this kind that it is S square or something like that then it will be kind of non-linear parabolic curve.

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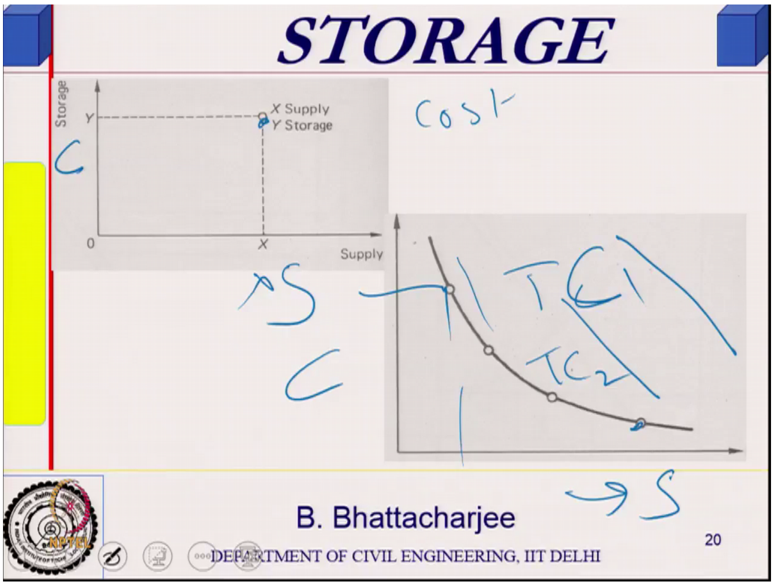

So, one can actually obtain something like this; X is let us say supply or what I am calling as S and this is my C. So, for each X and I can find out the cost; so, this is the total cost, cost I can find out. And I can plot this cost versus so this is basically constant T C value; constant total cost value and combination of C and S; so this is S this is C, so this C and S satisfy.

So, this is a curvature this showing that it is not linear non-linear, but had it been linear it would be something like this, for given S value I have a T C 1 value, T C 2 value which is smaller and T C 3 value etcetera are smaller.

So, I can have set of curves for different T C values, for different T C cost total cost values so I will have set of curves right.

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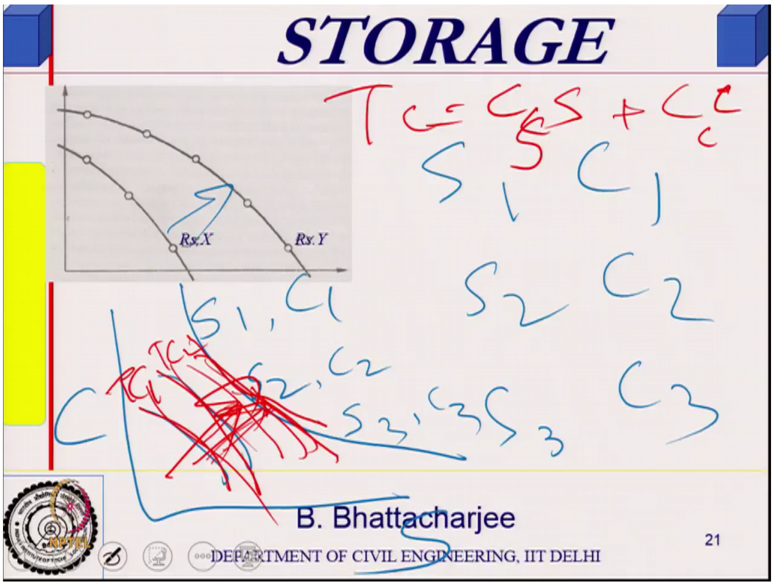

Now, you know this will go on increasing with cost; T C curves will go on increasing with this cost and I already had a costs you know demand supply because, I said I had S 1 C 1 S 2 C 2 S 3 C 3 which satisfy my demands.

So, I can plot them, this would look like this typically this will look like this and cost curve will increase in this manner, cost curve will increase in this manner, cost curve will increase in this manner. As my cost increases, T C value T C curves which could be straight line also. Wherever the touch because this is the profile like I had S 1 S 2 S 3 etcetera.

Now, this curves is what we call ISO demand curve, demand remains constant ISO demand curve and ISO you know a demand for a given demand profile, these are the combination of S and C. So this is S 1 C 1 S 2 C 2 S 3 C 3. So, this is ISO demand curve right, demand remains constant for that I will have set of S 1 C 1 and etcetera etcetera and then I just think in terms of the cost, I think in terms of cost, think in terms of the cost right; this is the ISO cost curves, ISO cost curves right, these are the ISO cost curve.

So, cost increases T C 1 T C 2, because if it is you know simply as I said T C is a function of some unit cost multiplied by S could be a square or linear if it is then and C C into C S into S and C in to C, if it is linear.

If it is parabolic or something then because it maybe may not be linearly increasing with supply rate, it might vary depending upon because it will depend upon the pipe sizing, motor size maybe supply rate you want to increase motor size, pipe sizing has to increase, storage means total storage capacity has to increase; so they maybe linear or may not be linear. So, somewhere this will touch as you go on increasing the cost somewhere it will touch, somewhere it will touch and that is the optimal point for us, you know this is the minimum. So, this point somewhere it will touch.

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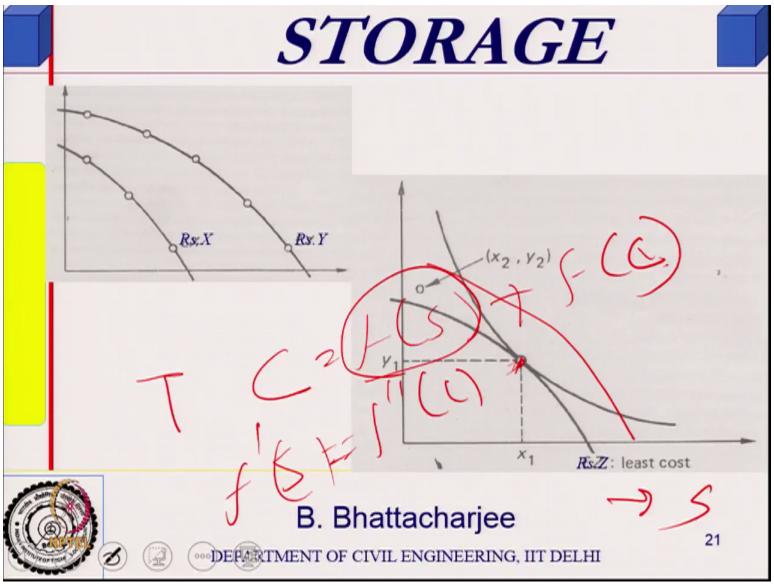

So, this is the least cost, this side is your source you know this side is your S, this side is your C; somewhere this will touch and that is a optimal point, because that is this curve satisfies the demand and supply and this is a cost curve, minimum cost curve because, any further cost means it will increase in this manner.

So, this one can find. In fact, one can simply formulate this is an optimization problem right, but then you know this you can formulate as an optimization problem, because then you have to know the relationship between supply rate right relationship between supply rate, I mean explicitly know the relationship between supply rate and the cost right, also explicitly know the relationship between cost and storage, storage per unit cost and storage that you have to know explicitly.

And you have to know the relationship between the supply and cost satisfying the given demand profile; you know a relationship between so, this relationship between this should be known.

So, once these are known then you can actually formulate this is an optimization problem; formulate this is an optimization problem and you can optimize these you can optimize these right you can do you know (Refer Time: 23:54). For example, you can minimize that T C if it is a function of S and function of C function of S and C if it is function of S and C you can minimize this, but then these values you must know explicitly.

And relationship between C and S or constraints, it could be in the form of constraints that S and C should be related in such manner. That you know S is some a function of f 1 tends of S is equals to f double dash of C, because there is a relationship between f and C exists, which will satisfy the given demand.

So, if explicitly mathematically these are known you can actually arrive at a you can do that optimization, but graphically this can be done then this one.

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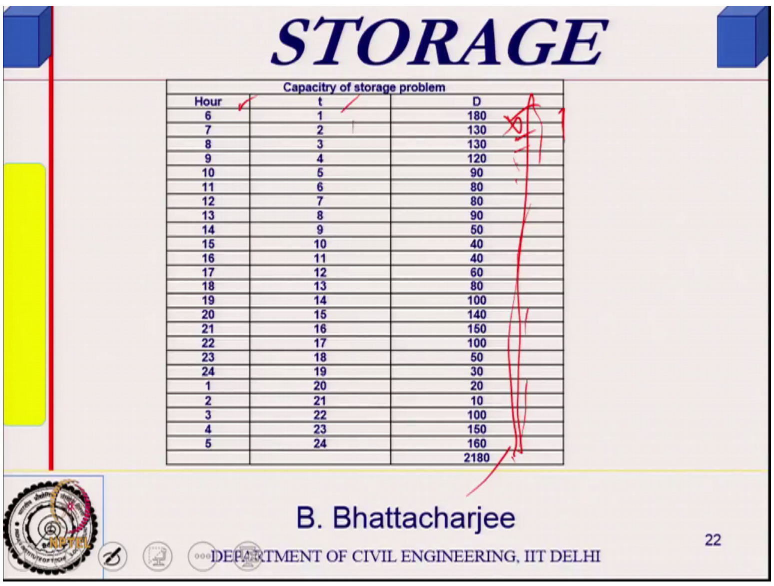

Capacitry of storage problem

Hour	t	D
6	1	180
7	2	130
8	3	130
9	4	120
10	5	90
11	6	80
12	7	80
13	8	90
14	9	50
15	10	40
16	11	40
17	12	60
18	13	80
19	14	100
20	15	140
21	16	150
22	17	100
23	18	50
24	19	30
1	20	20
2	21	10
3	22	100
4	23	150
5	24	160
		2180

So, that is how we determine. Let us see as an example problem, simple example problem. So, let us say my supply rate is you know this is hour, this is my time 1 2 3 4 up to 24 hours; so hourly demand is known to me. So, it is 180, 130 etcetera; this is known to me for whatever unit it could be; it could be if it is hot water supply or hot water supply cannot be like that, may be cold water supply or something whose rate is known to me in some unit.

I want to find out the optimal storage and you know best supply rate. So first thing is you accumulate this sum total of all these demand because, 1 over so 180 into 1 etcetera so you get 2180 units. So, minimum supply rate would be this divided by 24 because that is the constant supply rate and it has to be minimum; minimum constant supply rate would be simply 2180 divided by 24.

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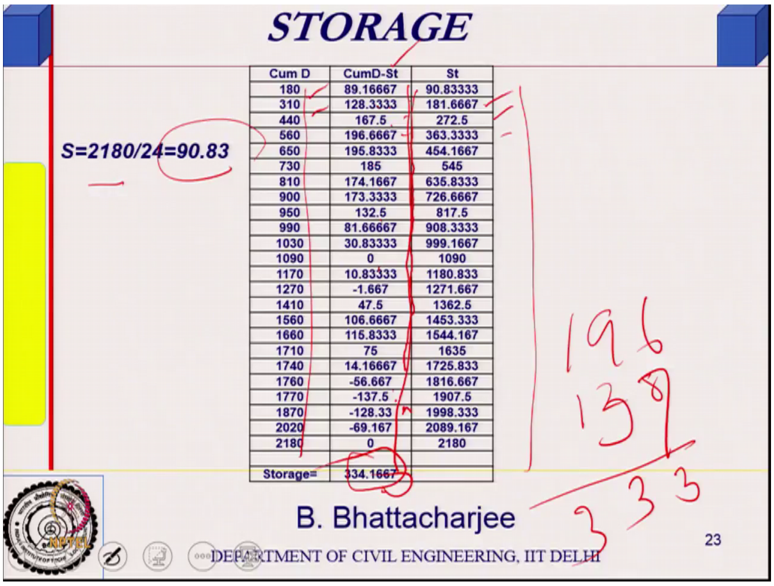

Cum D	CumD-St	St
180	89.16667	90.83333
310	128.3333	181.6667
440	167.5	272.5
560	196.6667	363.3333
650	195.8333	454.1667
730	185	545
810	174.1667	635.8333
900	173.3333	726.6667
950	132.5	817.5
990	81.66667	908.3333
1030	30.83333	999.1667
1090	0	1090
1170	10.83333	1180.833
1270	-1.667	1271.667
1410	47.5	1362.5
1560	106.6667	1453.333
1660	115.8333	1544.167
1710	75	1635
1740	14.16667	1725.833
1760	-56.667	1816.667
1770	-137.5	1907.5
1870	-128.33	1998.333
2020	-69.167	2089.167
2180	0	2180
Storage=	334.1667	

Now, you can find out the difference between supply and demand, cumulative demand minus supply you can find out. For example, this was the cumulative demand is 180, next hour it would be 180 plus 130 cumulative. So it would which will make it 310; next will be 310 and S into t 90 is a supply rate.

So, first one is 89.16 etcetera, second hour it will be double of that because this constant we are talking about. So, S t is 90, 181 etcetera. So, demand difference I am finding out; 180 minus 90 is this. So this is the difference; now, difference takes positive values then negative values as well. Find out the maximum positive value and maximum negative value, absolute value of that negative value will give me the storage capacity.

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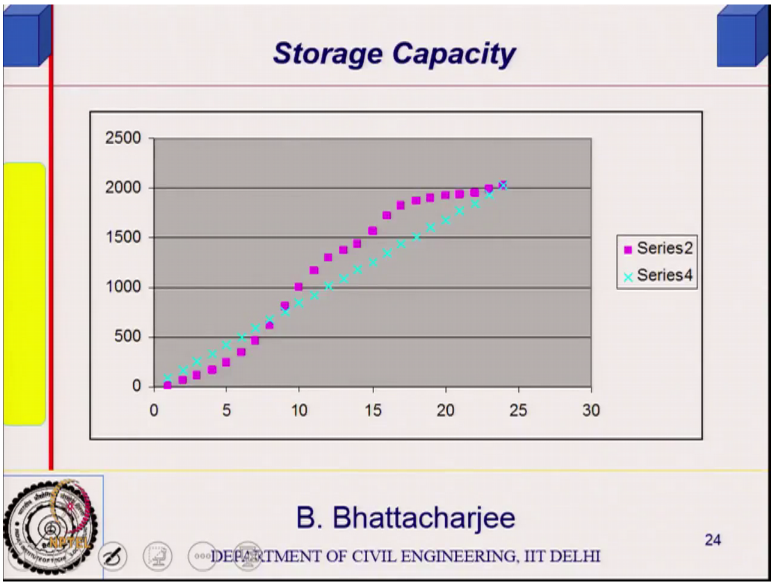

Absolute value or you can plot it in this manner also. So, this is how it can give us you can actually. So, if you plot it the you know the same one curve just I plotted these values and these values they would appear like this, they would appear like this finally, they would match and the gap maximum gap will be here somewhere and here somewhere. So, therefore, you can actually calculate this out and this would turn out to be you know this is and you want to improve this all right.

So, you can find out actually the storage capacity; which comes out to be sum total if you take it 334 because the maximum of positive is I think 196 and minimum value negative value maximum negative value is 128.

So, 196 plus 128 something like this, just see if you see the maximum value is 128, 167 196.6 and that is it, there is a negative value is coming as 56 137 sorry 137.

So, yeah that is right 334 point something that is how it comes. So that is the storage capacity I need and my supply rate is constant here. Now what I can do is I plotted this I can do the same thing, change the supply rate in such a manner that it matches most of it. So, that my storage has reduces down, storages reduces down, but then I may not I may stop do it interminably do may not supply it constantly.

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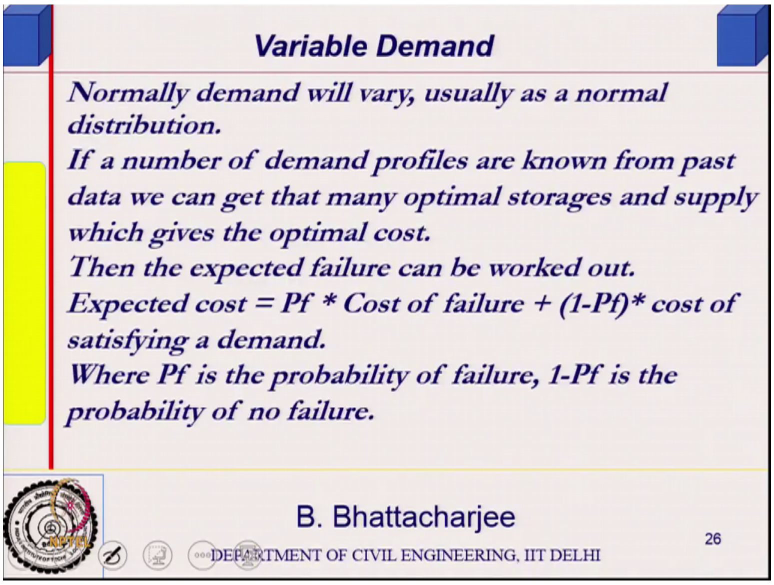

And you go on doing this, get new supply rate you will get new storage. So that is what you can do and to find out you know to find out the optimal scenario, optimal scenario. So, that is how it is that is how it goes about getting supply and storage.