Energy Efficiency, Acoustics & Daylighting in building
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Lecture – 16
Heat Flow in Buildings (Frequency Domain)

So, we start from where we stopped.

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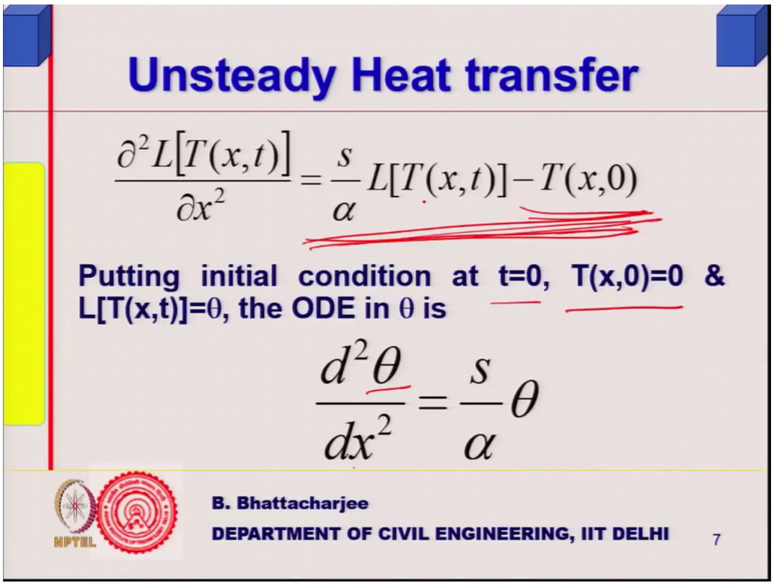

We said that this is you remember last time we discussed putting the initial condition in this one, you know if you recollect we said that after Laplace transformation this part I am just repeating the last what we said Laplace transformation this part becomes the PDE comes ODE. So, this we are calling as you know this at t equals to 0 initial condition T x initial time T equals to 0 is equals to 0. So, this term goes to 0 and we follow from there we call this term Laplace transformation or dx dx theta. So, this becomes d square theta dx square s by alpha. This term is not there simply this is theta. So, I am repeating this.

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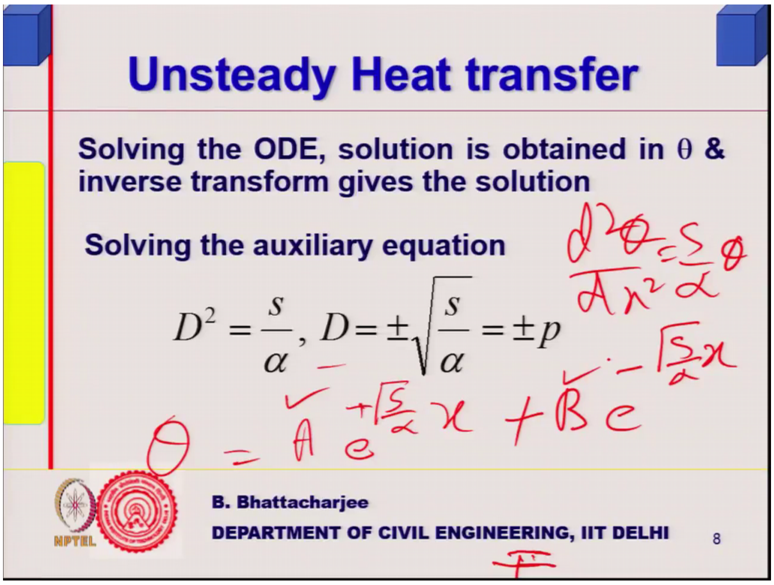

So, this ODE would be solved by solving the auxiliary equation D square is equals to s by alpha D is equals to, so, the solution would be solution of theta would be actually you know A e to the power under root s by alpha plus right and this was x plus B e to the power minus under root s by alpha x, solution of this one, because if you differentiate this twice it will again come back to e to the power this term only, but twice differentiation will be first time differentiate you will get under root s by alpha second time of differentiate will guess.

Student: S.

S by alpha so that coefficient you know coefficient was it was d square theta dx square is equals to s by alpha theta. So, if you differentiate it as similarly minus e to the power minus you different you know in differentiate it once you will get minus s by alpha into e to the power under root 2 s alpha x and you differentiate it again it then will that minus s under root x by alpha become plus. So, both are the solutions of this equation. So, the general solution is the sum of these 2, this you get usually from the boundary condition. This you get from the boundary condition.

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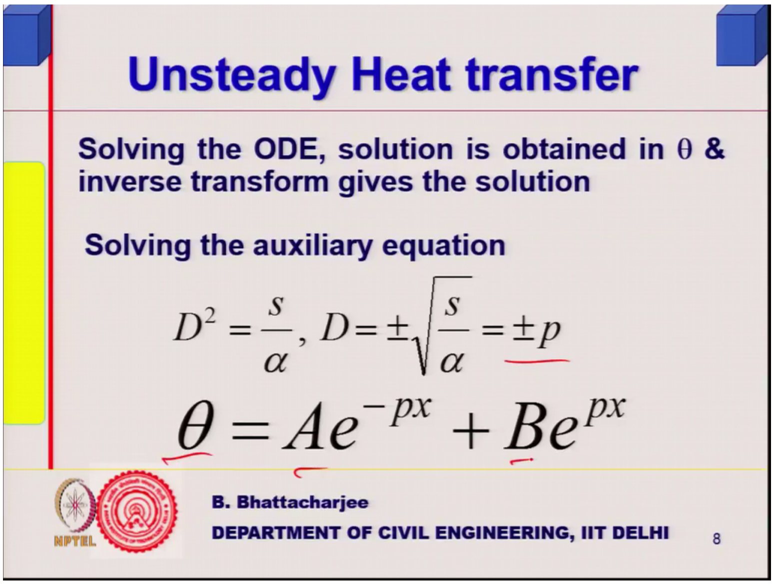

So, let us see how we go about it. So, we write this as plus minus p. So, this will be written in this manner, theta is equals to A and B we get from boundary conditions when x equals to 0 theta 0 equals to A plus B when x equals to 0 theta 0 is equals to A plus B.

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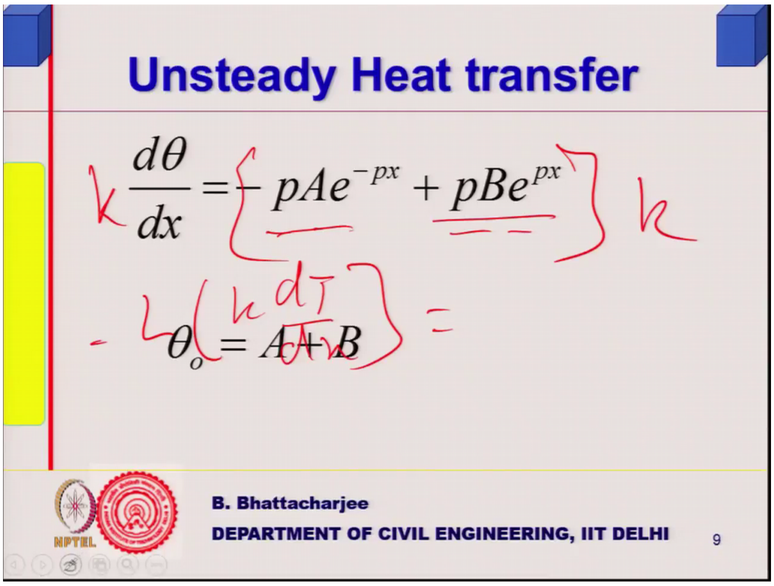

And, that is what it is and if I differentiate this I get pAe to the power of minus px plus pBe to the power plus px because it was you know e to the power I am just differentiating this theta. So, d theta dx will be A minus e to the power minus px B p into e to the power px right. So, that is that is what I am writing here.

Now, k multiplied by this is nothing, but Laplace transformation of k dT dx that is right. So, that is nothing, but Laplace transformation of the heat flux we call it phi with a minus sign of course. So, we call it phi.

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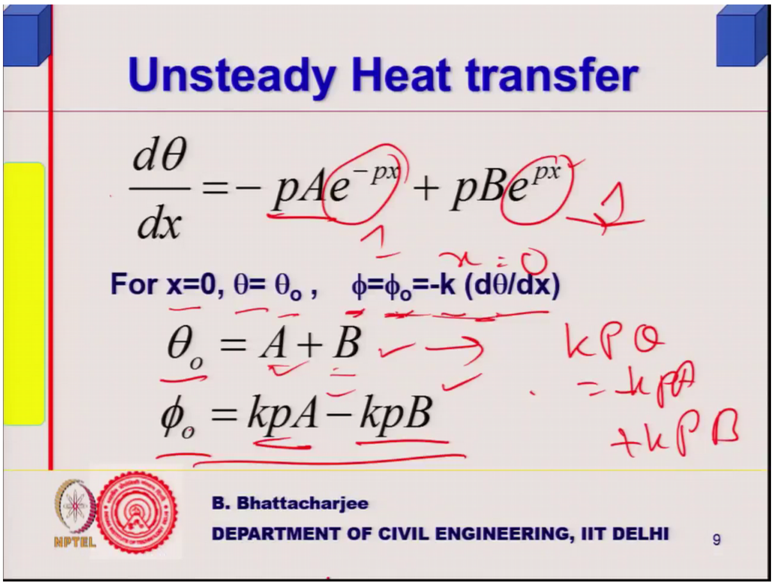

So, first term is theta 0 at x equals to 0, I call it theta 0 theta 0 is goes to a plus B for x equals to 0 theta 0 and then I am defining phi which is k minus k d theta dx there is a minus sign comes because the heat flows from higher temperature direction of heat flow and you know from if I higher it from lower temperature minus the gradient is negative. So, there is a negative sign comes k d theta dx.

So, therefore, this is nothing, but Laplace transformation of heat flux and if I put this you know this one at x equals to 0 I call it phi 0. So, multiplied by this minus kpA because this term will go to 1 and this term will also go to 1. So, therefore, kpB, this is how we can write as the minus sign this becomes plus and this becomes minus.

So, I get theta 0 like that putting 2 boundary condition theta at x equals to 0, theta equals to theta 0 phi is equals to phi 0. What are the values of theta 0 phi 0 that we need not look into right now, but supposing the boundary values of temperature and heat flux is known and I can take Laplace transformation of those then this will be the you know from this now I can find out actually A and B because there are 2 equations 2 unknowns. So, I can solve them and I can find out B at any length l, because I have taken x equals to 0. So, x equals to l I can find out anywhere value I can find out. So, just I am trying to do that find out express A and B in terms of theta 0 phi 0 and what will you do in that case multiply this by kp.

So, kp theta 0 will be equals to kpA plus kpB and if I sum this up this term will cancel out I will get A. Similarly, if I want to find out B what will I do I will multiply this by minus kpA and add this up. So, I will get you know another equation from which I will get B, A will get eliminated this term will get eliminated and that is what I am doing next. So, that would result in let us do that then for you.

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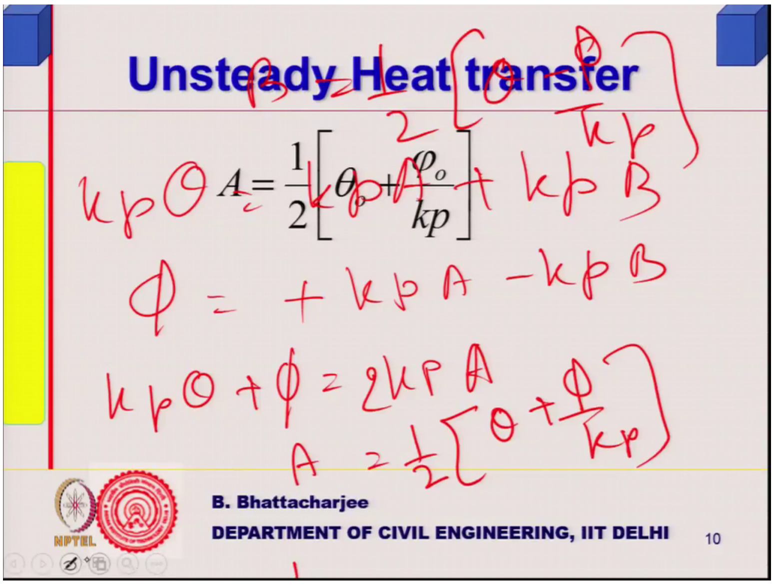

So, you know theta 0 kp is equals to kpA plus kp it was plus or minus yes kpB and phi is equals to minus kpA am I right minus kpA plus k pa minus kpB . So, plus kpA minus kpB add this up. So, you will get kp theta plus 5 equals to kpA and therefore, a is equals to simply half you know theta plus phi by right. Similarly, if I subtract you know repeat this process I will get B equals to half theta minus phi by.

Student: Phi by k p.

That is it. So, these are the 2. So, I get A and B like this and this is what it is, A and B like this.

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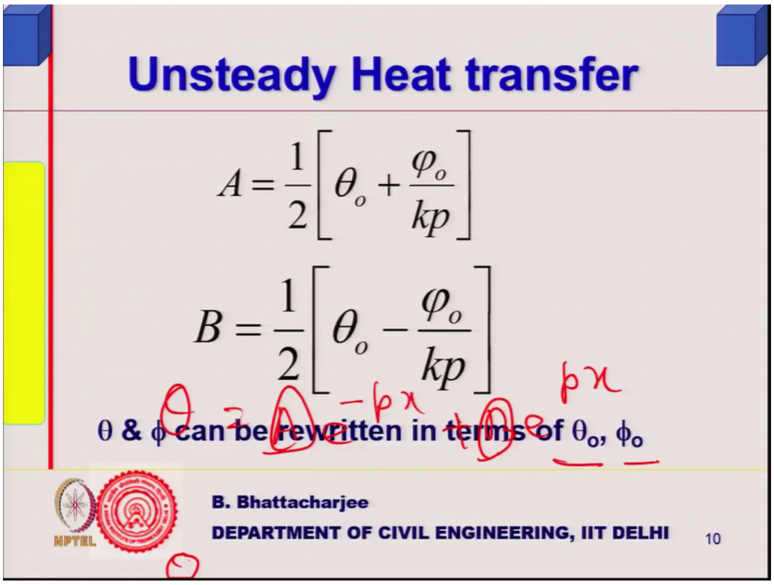

So, is like this B is like this and I can put them back in the equation that I had what I had I had theta is equals to I had theta is equals to what was it A.

Student: E to the power minus px.

Right.

Student: E to the power minus px.

Right px plus B e to the power. So, put A here B here and similarly put for phi I can do the same thing which will be kp minus kpA e to the power px. So, I can put this let us see let us do that. So, if I do that you know phi can be rewritten in terms of theta 0 phi 0. So, let me just write it. So, let me write it now this is what we saw.

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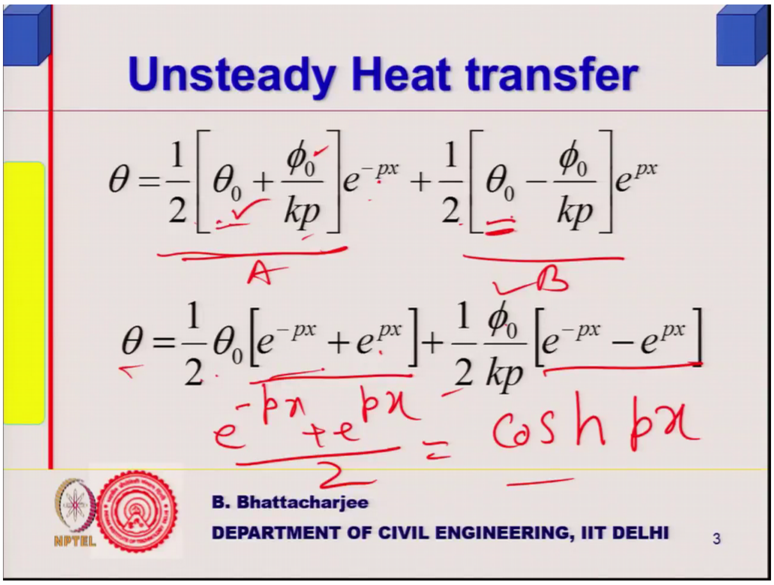

So, A and B is like this and I can get right theta and phi in terms of. So, if I write it put a here half theta 0 phi 0 by k p plus half theta 0 minus you know e to the power minus p x here e to the power plus px here am I right this is fine. This B, this is A, this B I just put them here.

Similarly, let us simplify this a little bit. Similarly I can do for phi, but let us simplify it a it a little bit here I isolate the terms of theta 0 and isolate the terms for phi 0. So, this will be half theta 0 and from here comes half theta 0 again and e to the power minus px plus e to the power plus px and here half still remains and I have phi 0 by kp and again have to the power minus px minus e to the power px and what is this this divided by 2 this term e to the power minus px plus e to the power px divided by 2 is nothing, but cos hyperbolic px. Cos hyperbolic ps what about this by 2 sine hyperbolic.

Student: P x.

So, that is what we do. So, you can see that theta can be expressed in this manner half theta 0 cos hyperbolic px and plus half phi 0 by kp sine hyperbolic px. So, let us write it that way.

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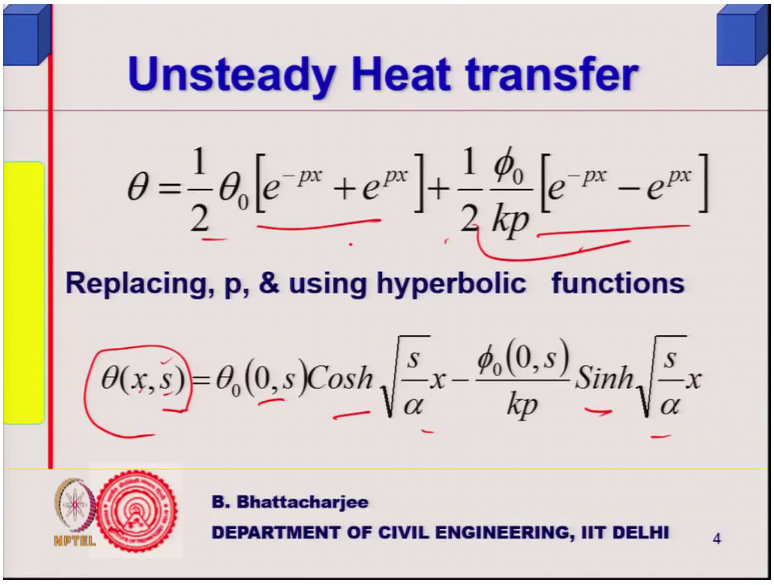

You know. So, this is what is written replacing p using hyperbolic function, so, I can write it like this theta at x equals to s at any value x equals to you know any value they say theta at I mean x equals to at any value and yes because it is a function Laplace transformation. So, it is a function of s, t is gone s has come, cos hyperbolic x by alpha minus phi x at x equals to 0 because phi 0 corresponds to x equals to 0 sine hyperbolic. So, this is this is how it is we express you know we have just written sine and cos hyperbolic replace them this terms you know this including this including this by cos and replace got back the p again under root case.

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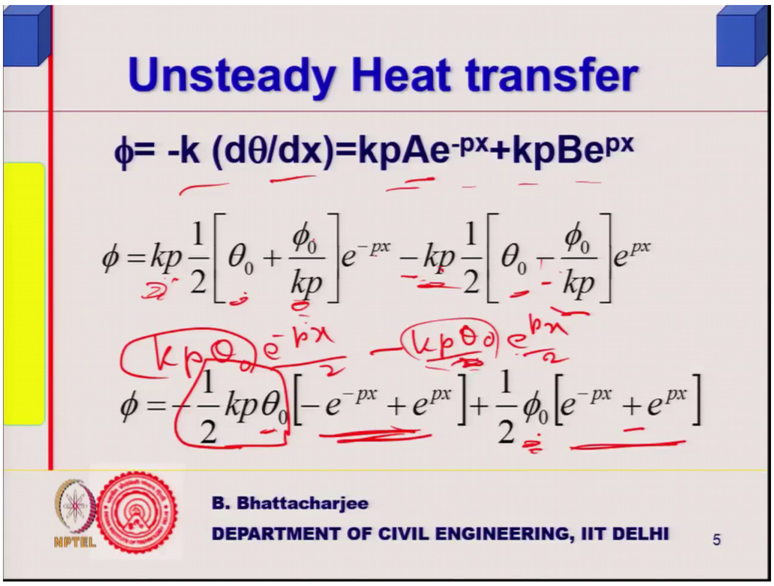

Because phi was k, d theta dx kp you know this was the case. So, I am just putting it here kp half a. So, phi 0 etcetera e to the power px and kp theta 0. So, again separate the theta 0 and phi 0 terms and you will get something like this phi is equal to kp half e to the power minus px e to the power px plus px theta 0, you know coefficient is theta 0 here and similarly, for this one you will have kp half e to the power minus px minus and minus this will make it plus e to the px. So, this is minus this is minus e to the power px and phi 0 you know.

So, this is kp by kp theta 0 and phi 0 by kp after again now since there is a kp you know both the cases there is a kp kp you will cancel out and we will have left to phi 0 because this kp divided by this kp and this kp divided by this kp and this kp divided by this kp. So, I am trying to find out the coefficient of phi 0. So, we will come something like this.

So, simple algebra because if I write expand it kp theta 0 e to the power minus px by 2 minus kp theta 0 e to the power this was minus ps this is a px by 2. So, this is common in both of them, separate it out, half now 2 you take this way and 2 you take this way this is common. So, if you know do this you get e to the power minus px and this is what is this sine hyperbolic x, sine hyperbolic px and this is what is this is cos hyperbolic p x. So, phi can be written as half you know minus half k p theta 0 sine hyperbolic x and plus half phi 0 cos hyperbolic x.

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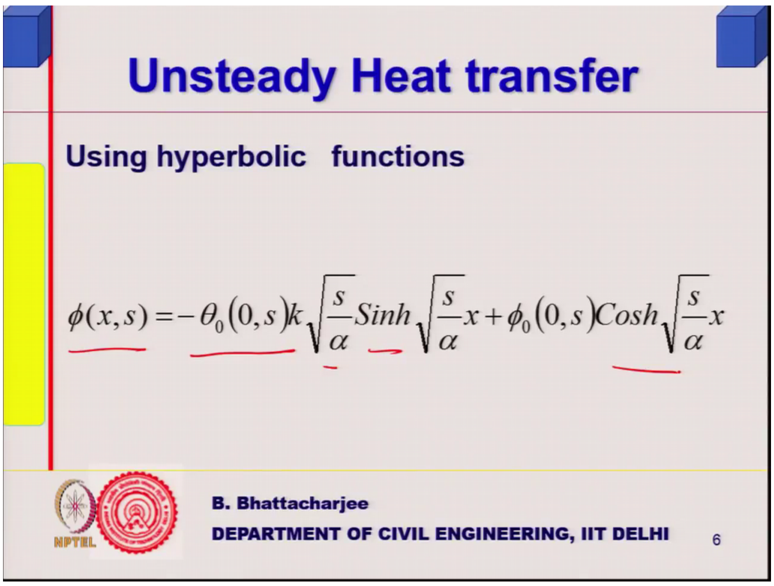

So, just writing this again using hyperbolic function phi x, s can be written as theta, 0 k this is p, sine first term and then is cos term. So, if I rewrite this 2 equations again you know if I rewrite these 2 equations again then I will get something like this.

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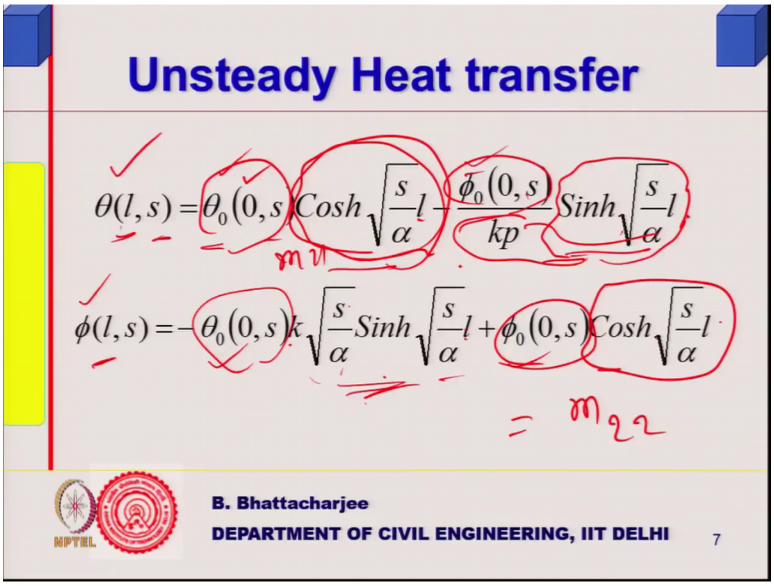

Theta at any l at any distance you know homogeneous isotropic material where k is not changing and I am dealing with one dimensional you know unsteady state heat flow I have not used any kind of boundary condition. So, far only I said that it is unsteady state initial condition I have used that at t equals to 0, temperature is also equals to 0 and this is in terms of Laplace variable. So, theta ls is equals to at x equals to 0 s and similarly you know sin s l this is what we got and for phi I can write it like this at any distance l replacing x by l in both the cases x by l you know l and this is coming because p was that kpA, so, replace p by s alpha this is I get.

Now, you can see that this can be written as 2 by 2 matrix in terms of theta 0 and phi 0 because these coefficients I can take it out. So, Laplace transformation temperature at any distance l and the flux at any distance l can be written in terms of some constant which are actually part of the material constant you know sectional constant all this these are l is nothing, but the length alpha is the diffusivity and of course, there is a Laplace variable is very much there. So, this can be these are basically material, but this is the flux Laplace transform of flux and Laplace transform temperature.

So, this can be written in matrix form.

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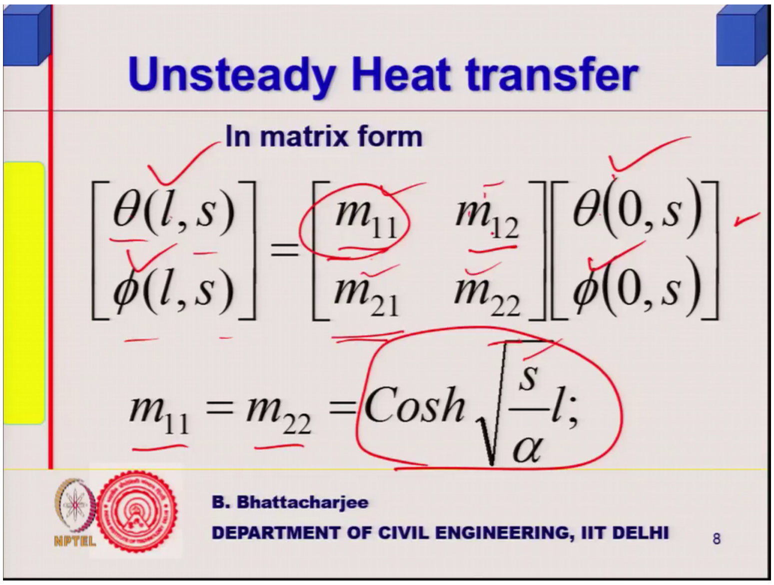

This we can write in matrix form and we write it like this theta l, s, phi l, s is goes to sum m 11 m 12 m 21 m 22. This I just write this in matrix form you know phi 0 phi 0 this I take it out this is come here. So, these are the coefficient of the matrix this is. So, I can write in terms of boundary condition theta 0 in other words I have I am now out of this I am saying that theta is at 0 and phi 0 is known it can also be known that you know theta l can also be known or phi l can be known, any 2 of them known out of this 4. This is related to the length of the section and k is thermal property, alpha is diffusivity, p is related to the frequency p would be you know p is basically what was p? p is I mean p what was p is s under root s by alpha. So, is I will not frequency it is related to s and diffusivity. So, these are properties are essentially related to the material properties and Laplace variable of course.

So, m 1 m 22 if you can go back and see m one see this value and this is same. So, m 11 is equals to m 22 and these are this is you know these 2 are common, but there is some similarity. Here, k this is multiplied and here it is divided. So, I can write this expression for m 11 22 etcetera. So, m 11 equals to m 22 and this is what it is. m 21 is this there is a relationship between m 12 and m 22, but I can just have I am. Let me write this full thing.

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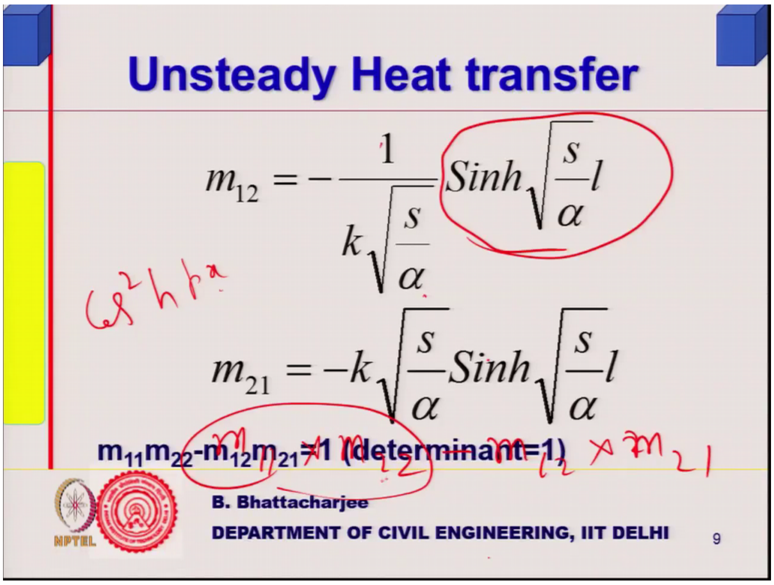

m 12 is sin hyperbolic this 1 by k p minus 1 by kp, m 21 is minus k p. So, p is this. So, sin hyperbolic this. Now, if you take the determinant of this matrix you will find the determinate comes out to be one determinant because it will come we will just do that right now because this will get multiplied by this m 11 will be multiplied by

Student: M 22.

M 22 minus m 12 into m 21 and this you know this is will be there are minus signs 2 of them plus and it remains minus. So, this will be cos hyperbolic cos square hyperbolic px I mean if I may write this and minus say which is equals to 1. So, determinant is 1, determinant of this matrix is 1. So, it makes sense. So, this sorry this can be written. So, this one can we can write thi,s we can write this. So, this is what it is right m 11 minus m 22 is equals to 1 this is equal to 1. So, determinant is equals to 1.

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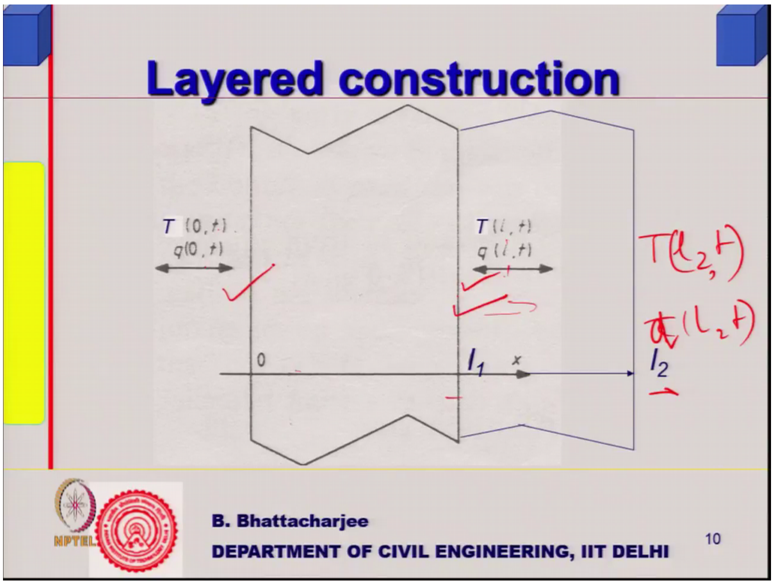

So, now let us look at a construction I have T and q and T and q going out here at distance l 1 and there is l 2, layered section. I did it for a homogeneous isotropic single layer material I said l. Now, I can extend this idea to layered section like we did for you value calculation here we can do the same thing for layered section this is 0, l 1, l 2 etcetera.

So, you know like T 0 q 0 t t q 0 you know T q T l q l 1 and this is l l 1 and this is T l 2 T and it is a function of you know, q flux here is q l 2 t. So, at any time, but space wise this is how it is. So, this I can now write theta l 1 s you know I can write theta l 1 s in terms of Laplace transformation theta l 1 s right and phi l 1 s here can be written in terms of this because we wrote it in terms of theta 0 and phi 0.

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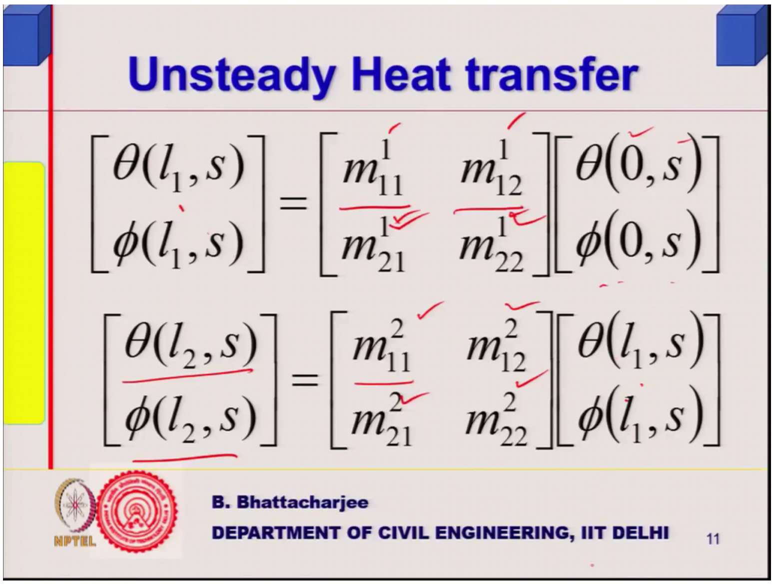

So, I can write it in terms of this theta l 1 s phi l 1 s is equals to m 11 I am putting a superscript. Now, for the first layer this is the superscript for first layer right. So, theta 0 s theta phi 0 s. Similarly, I can write now theta l 2.

Student: S.

A in terms of theta l 1 s and l 2 s. So, just that is what I am doing l 2 s phi l 2 s this is. So, each individual matrix if I know I can find out. Now, replace this here, put this here right put this here theta l 1 s and phi l 1 s and.

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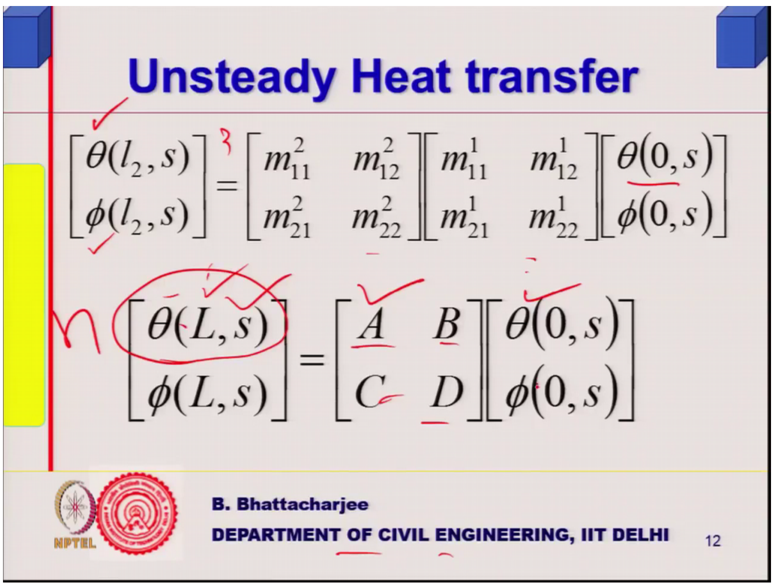

This can be written like this theta you know in terms of theta 0 phi 0 I can write it. So, if I have n layer first layer will come here, second layer, third layer, nth layer where this will be the Laplace transform temperature at the nth layer and there is a flux at nth layer you know because this is 2 layer I am showing 1, 2, 3, 4, 5 etcetera.

So, in fact, I can write it like this finally, I call it say A B C D matrix which could be a product of several matrices where L is the overall thickness of the construction. So, I can write A B C D matrix. Now, what will be A B C D matrix? It will be m 11n m 12 you know all this nth matrix then n minus one-th matrix, up to first matrix you know last and then clusters. So, I can write it in this minus. So, there is a solution of matrix this is a matrix from solution.

Now, if I want to get the actual temperature in flux I can 2 of them being known their Laplace transformation being known I can find out them in terms of s and take inverse Laplace transform and you get that temperature at you know heat flux at the other boundaries. So, that is possible we really do not need that.

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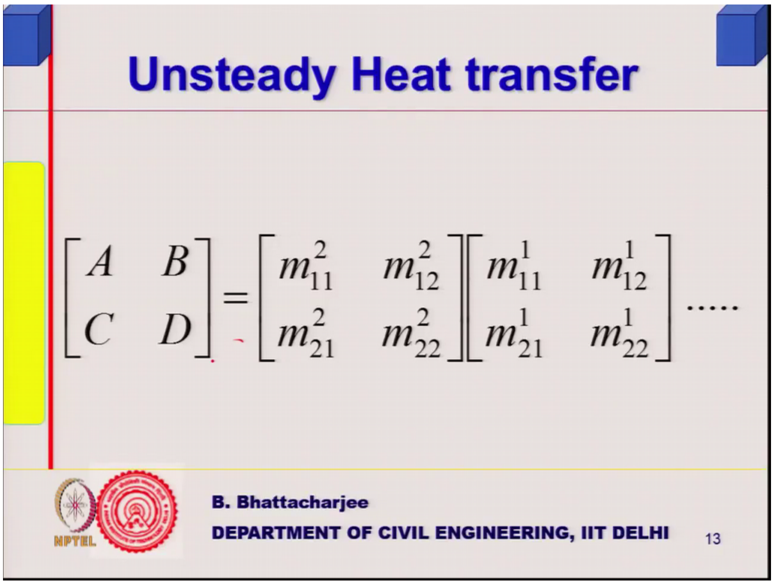

A B C D is matrix of this kind, as I said in there is 2, there are m there will be normal number of them and one can find out to this one. So, that is the kind of solution now I can keep the boundary in different manner. I showed you frequency domain treatment where we said that we can separate it into number of harmonics. I am talking only of a fluctuating part, steady part I have already taken care of, so, it is a fluctuating part I am taking you know looking at and basically I can I am not assumed any kind of I mean I can one ways frequency domain take sinusoidal boundary condition or co sinusoidal boundary condition as we have done. One can take even pulses single pulse like they do it in you know like signal processing and all they take a pulse and find out the response of this pulse to the other side transfer functions. So, anyway we will do the simplest one in our class, but explain you some of them other way.

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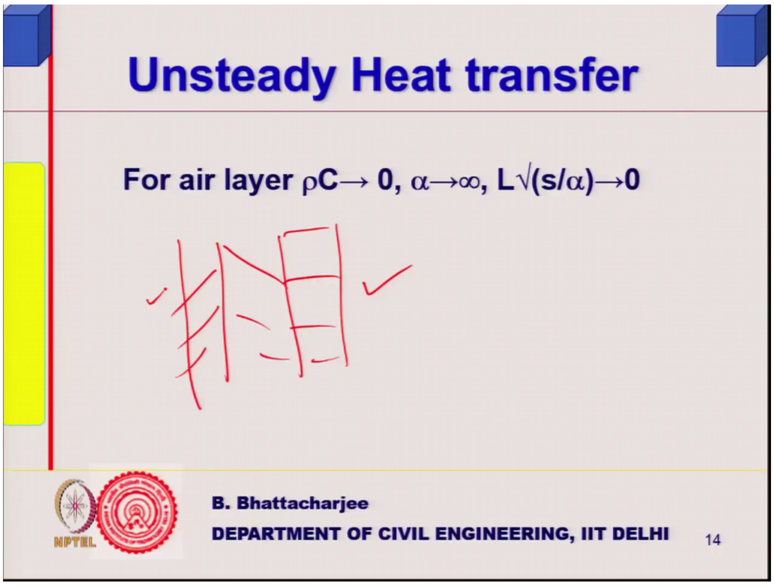

Now, one thing remains what about you earlier you know because we always said we said that we have solid boundaries maybe there is one layer, another layer etcetera third layer, but there will always be two boundary layer. So, I got to take care of this boundary layer like I did in case of u value. We did in case of u value we take 1 by h over plus 1 by h s etcetera. So, I got to take care of that in some manner.

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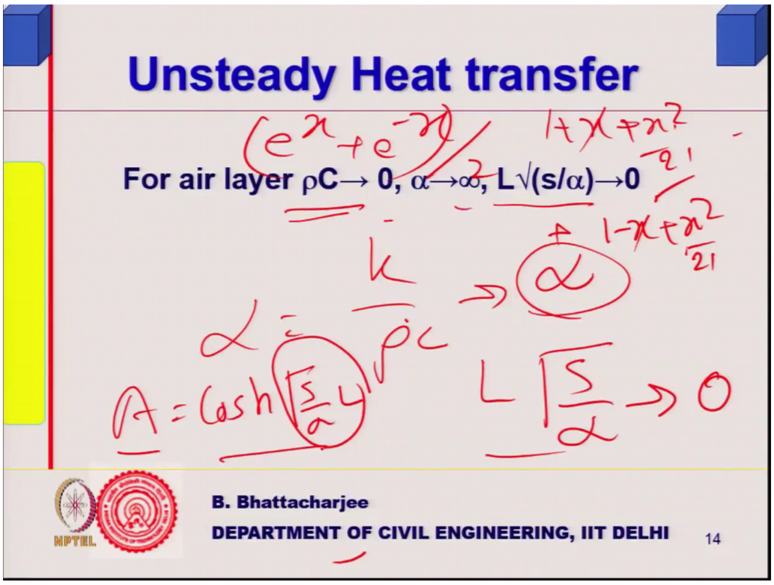

Now, for air layer, thermal capacity of air is very small compared to the solids. So, I can assume rho C tending to 0 therefore, k over rho C who tend to infinity alpha. So, alpha which is k over rho C will be infinity, that is, thermal diffusivity is infinity, what does it mean? It will take very long time to diffuse the heat to diffuse through such a layer because storage is you know 0 storage.

So, this is basically alpha is alpha is something a meter square per second. So, sorry it will not store allow you mean instantaneously to go it will not take very long time if it will allow instantaneously to go and followed by it is L s under root you know L under root s by alpha, this will also tend to 0 because alpha tends to 0 this tends to 0. Why this is needed, because in my cos hyperbolic L you know if you remember A was nothing, but cos hyperbolic under root s by alpha L.

So, I want to find out this A for air layer, B for this air layer. Now, if this tends to 0, what will be value of limit this tending to 0 it will be 1. Cos 0 cos hyperbolic 0 also tends to one it is not very difficult to even derive e to the power x plus e to the minus x divided by 2 was the case and you can expand this e to the power x is 1 plus x plus x square by factorial 2 etcetera and e to the power x is 1 minus x plus x square by factorial 2 etcetera. So, if I have plus sign in this two this terms all odd terms will cancel out and this can be neglected higher terms can be neglected.

So, I will be led to from e to the power minus you know sine hyperbolic x again you can show that it will tend to 0, because there is a minus sign involved here. So, you know it will tend to 0. So, this tends to 1 and what about A you know A 12 or m 12, I mean you know second term of the matrix which at sign hyperbolic they will tend to.

Student: 0.

0. So, let us see, what there is a little bit of catch.

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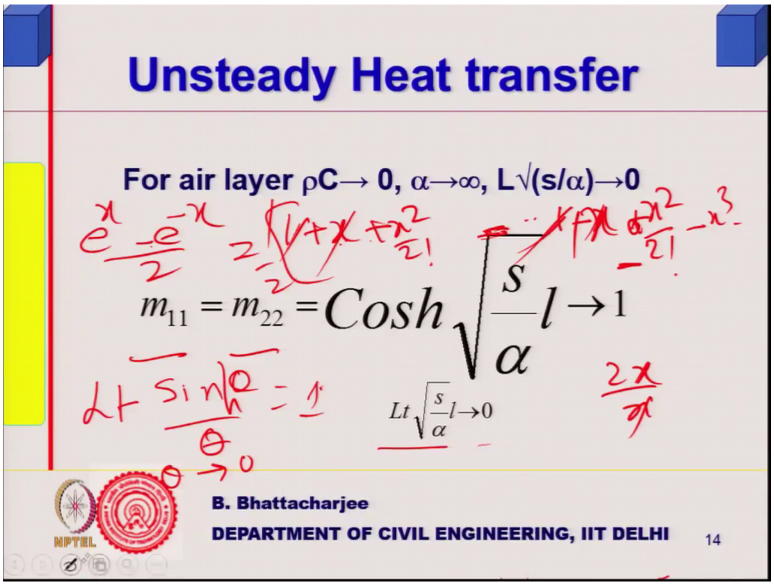

So, m 11 m 12 will always tend to 1 for the air layer, for the boundary layer this will be one because this tends to 0 this tends to 1, but m 21. You see, remember this limit trigonometric function I am talking about then I come to hyperbolic. Remember, this sine theta by theta tending to 0 what is the value of this.

Student: 1.

1, because sine theta becomes nearly saying as theta and you know x equals 1. Similarly, I can show that sine hyperbolic we can do that because it was e to the power x minus e to the power minus x divided by 2 and I have now 1 plus x plus x square by factorial 2 etcetera anyway half is outside and 1 minus x plus x square by factorial 2 and then x cube and so on those terms. So, this will cancel out and there is a minus sign here sorry there is a minus x remain sorry there is a minus sign here because between them was a minus sign. So, this will cancel out this will add up 2x will add up.

So, you know this is plus, this is minus, so, this minus. So, this will also become minus this is plus because there is a minus sign in between minus 1 plus. So, this will cancel out with this will remain. So, 2x by x will cancel out divided by 2. So, again this will tend to sine hyperbolic sine and let me let me just do it again for you let me just do it again for you know.

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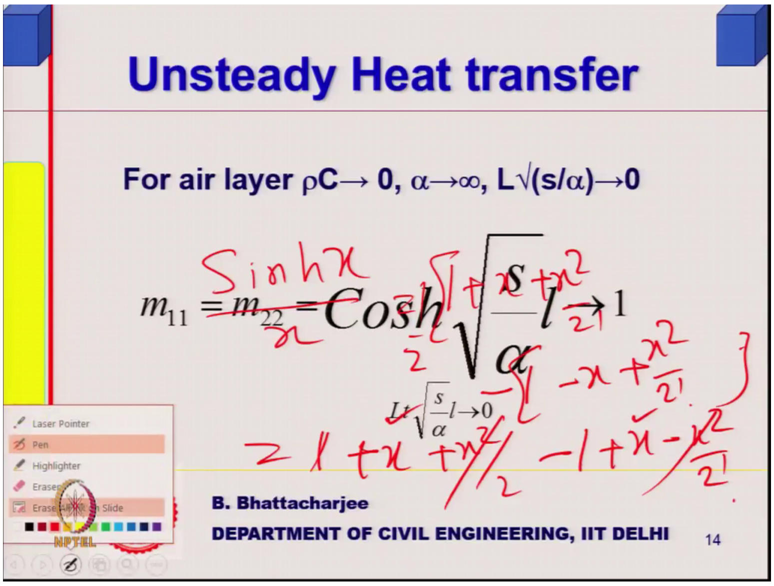

Sine hyperbolic x by x this would be 1 plus x plus x square by factorial 2 blah, blah, minus 1 minus x plus x square by 2 and now, this can be written as 1 plus x plus x square by factorial 2 minus 1 plus x minus x square by factorial 2. So, this term will remain and this will cancel out, leaving you 2x divided by x it will remain 2 divided by 2 because half was there, so this was there, was a half here outside. So, this will tend to again 1. So, that is what I am saying. So, you can see that this term tends to 1.

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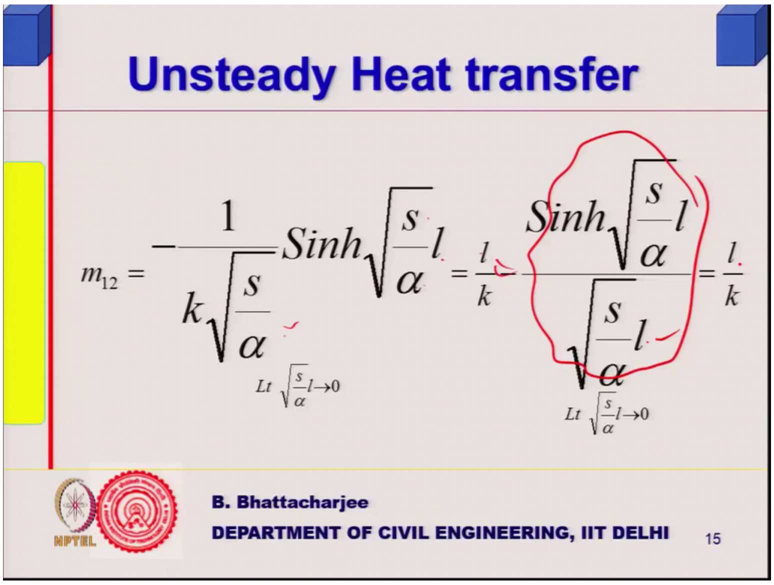

This term divided by m 12 or something like this there is a dividing term. So, s by l s by alpha l and I had k s by alpha. So, what I do is, I multiply this by l you know take 1 by k out multiply this by l here and divide by also l. So, this algebra is fine, I am just doing this. So, this will tend to now 1 and I will be left with minus l by.

Student: k.

Minus l by k what is l by k? l by k for the boundary layer is equivalent as h naught. So, 1 by h naught you know l by k is equivalent to is that 1 over resistance, I mean sorry 1 over conductors this is nothing, but the resistance of the equivalent resistance of the boundary layer. So, that is how I write.

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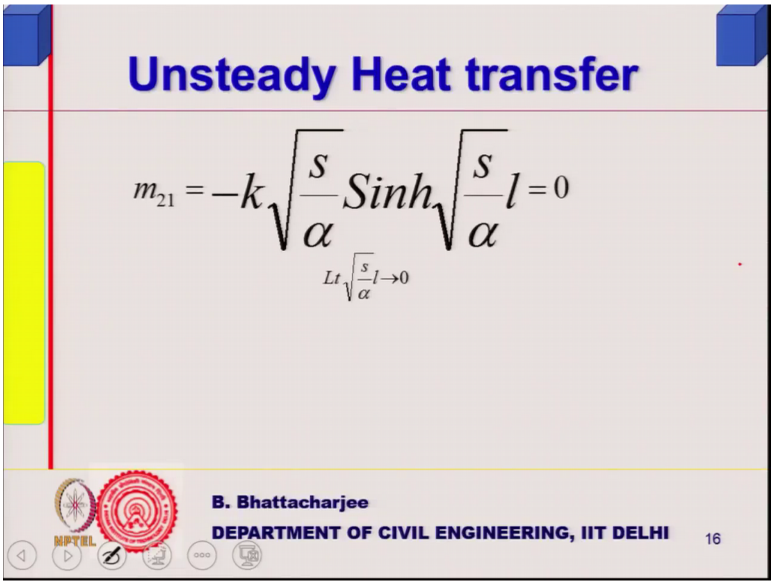

So, two 1 this tends to 0.

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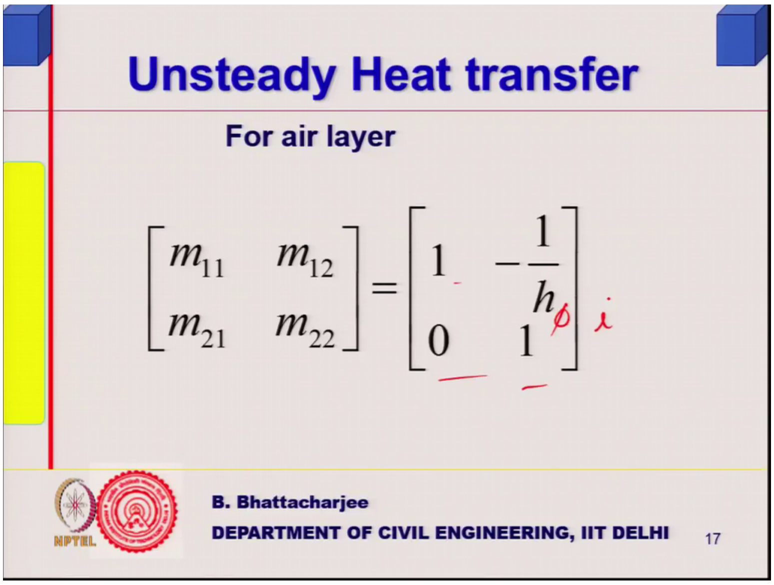

So, therefore, for a boundary layer I can write like this for a air boundary layer 1 minus 1 by h and if it is outside then 1 by h 0, if it is inside then this will be h i. So, now I have got any homogeneous layer also will be 3 layer, 3 matrices minimum will be there. 3 matrices will always be there. So, you know nth layer was coming, how was it coming sorry how was it coming, if you recollect how was it coming if I recollect go back there.

Student: Nth layer.

You know. So, nth layer was here. So, one external extreme layer will be coming fast which would be the outside I mean inside if the heat is coming from outside to inside first layer will be the inside boundary layer. Next, will be the actual solid layer if there are more than those layers will have take them and the last layer would be the outside boundary layer outside layer, because heat is flowing from outside to inside if it is flowing more inside to outside matrix will be different it is not same matrix will be different because the product of those matrix also could not be same. So, so far so good, let us follow it up from there a little bit more.

So, we will go to the next one now, next case where we will follow it up.