

**Energy Efficiency, Acoustics & Daylighting in building**  
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**Lecture - 14**  
**Heat Flow in Buildings (Frequency Domain)**

So, we continue to solve this problem, right?

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**Transmission Matrix**

**Example:** *3/sec*  
 Let  $k = 1 \text{ W/m}^\circ\text{C}$ .  
 $L = 100\text{mm}$  ✓  
 $\rho = 2000 \text{ kg/m}^3$  ✓  
 $h_o = 15 \text{ W/m}^2 \text{ }^\circ\text{C}$   
 $C = 1000 \text{ J/kg}^\circ\text{C}$ .  
 $h_i = 10 \text{ W/m}^2 \text{ }^\circ\text{C}$

*$F = \sqrt{\frac{W}{2\alpha}}$*   
 *$W = \frac{2\pi}{P} = \frac{2 \times 3.14}{3600}$*   
 *$\alpha = \frac{k}{\rho C} = \frac{1}{2000 \times 1000} = 5 \times 10^{-4}$*



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So, first you find out f. So, what was f? Omega by?

Student: (Refer Time: 00:27) omega.

Omega by 2 alpha. And this is for first harmonic. So, what is the value of omega? Omega is twice pi by.

Student: P.

P. Now your all everything is in watt, which is joules per.

Student: Second.

Second. So, period must be expressed in second. And our periodicity is 24 hours; so 3600. And 2 if you are using radians, calculating the angles you must be get using radians. If you use 2 into 3.14 etcetera, etcetera. Then must you must use a radians, you

know in a calculator change it to radians, right. If it is in degree, then we will have all kind of erratic results coming in 180, it would be instead. So,  $2\pi$  by 3600, that is your  $\omega$  and  $\alpha$  would be how much here?  $\alpha$  is  $k$  over  $\rho c$ . So,  $k$  is 1,  $\rho$  is 2000 and  $c$  is 1000. So, this is simply  $5 \times 10^{-7}$ , right?  $1/2 \times 10^{-6}$  to the power minus 6. So,  $5 \times 10^{-7}$ , right?

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**Transmission Matrix**

$\alpha = 5.0000e-007$  ✓  
 $\omega = 7.2722e-005$  ✓  
 $F = 8.5277$  ✓  
 $FL = 0.8528$  ✓  
 $z_1 = 0.8528 + 0.8528i$   
 $m_{11} = 0.9120 + 0.7229i$   
 $m_{12} = -0.0982 - 0.0242i$   
 $m_{21} = 3.5168 - 14.2882i$

$\frac{2\pi}{3600 \times 24}$   
 $\alpha$

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And find out  $f$  so,  $\alpha$  is  $5 \times 10^{-7}$ ,  $\omega$  comes out to be  $7.27 \times 10^{-5}$ , because that was coming to be  $2\pi$  divided by  $3600 \times 24$  sorry,  $1$  over  $24$  hours. So, into this is what?

It is so, this will come out to be it is this and  $f$  is  $\omega$  by.

Student:  $2\alpha$ .

$2\alpha$ , and  $\omega$  is you know if you calculate this out divided by  $2\alpha$  minus  $7$  minus  $5$  it comes out  $8.5$ . How much is the  $l$ ?  $100$  mm in meter it will be  $0.1$ . So, comes out to be  $8.528$ . You know  $8.5$  so now, the question of finding out  $z$ .

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### Transmission Matrix

$\alpha = 5.0000e-007$   
 $\omega = 7.2722e-005$   
 $F = 8.5277$   
 $FL = 0.8528$   
 $z1 = 0.8528 + 0.8528i$   
 $m_{11} = 0.9120 + 0.7229i$   
 $m_{12} = -0.0982 - 0.0242i$   
 $m_{21} = 3.5168 - 14.2882i$

$FL + i FL$   
 $\approx 0.8528 + j0.8528$   
 $\cos h[0.8528 + j0.8528]$

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One which is  $FL + i FL$  can be written as  $0.8528 + i 0.8528$ . And now you want to find out cos hyperbolic this complex of this complex number  $0.8528 + i 0.8528$ , right? Now cos of  $x + iy$  I do not know whether I have it here.

(Refer Slide Time: 03:23)

### Unsteady Heat transfer

$m_o = \begin{bmatrix} 1.0000 & -0.0667 \\ 0 & 1.0000 \end{bmatrix}$

$m_i = \begin{bmatrix} 1.0000 & -0.1000 \\ 0 & 1.0000 \end{bmatrix}$

$M1 = \begin{bmatrix} 0.9120 + 0.7229i & -0.0982 - 0.0242i \\ 3.5168 - 14.2882i & 0.9120 + 0.7229i \end{bmatrix}$

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So, let us have, let us do it. So, cos hyperbolic  $x + iy$ .

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**Unsteady Heat transfer**

$$\begin{aligned} \cosh [x+iy] &= \frac{e^{x+iy} + e^{-(x+iy)}}{2} \\ &= \frac{e^x \cdot e^{iy} + e^{-x} \cdot e^{-iy}}{2} \\ &= \frac{e^x [\cos y + i \sin y] + e^{-x} [\cos y - i \sin y]}{2} \\ &= \frac{e^x \cos y + i e^x \sin y + e^{-x} \cos y - i e^{-x} \sin y}{2} \\ &= \frac{(e^x + e^{-x}) \cos y + i (e^x - e^{-x}) \sin y}{2} \\ &= \cosh x \cos y + i \sinh x \sin y \end{aligned}$$

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You know because it is not the same kind although x and y are same. Now this will be what? This you can write like this, e to the power x plus iy minus e to the power minus x plus iy and whole thing is divided by 2. So, this will be written as e to the power x into e to the power i y minus e to the minus x into e to the power.

Student: Minus iy.

Minus iy something like this. Oh, is plus here is plus. So, I can take actually what I can take common? I do not take common so far, I will write the e to the power x. What is the e to the power I y? Cos y plus.

Student: I sin y.

I sin y and what about this e to the power minus x cos y and this was this fine? Or this is wrong?

Student: Sir, (Refer Time: 04:39) in the third blank, this is plus.

This is plus. Minus i sin y, right? This is fine minus i sin y. So, you see cos x, cos y, cos y can be taken out. So, if I take cos y common. I get e to the power x plus e to the power.

Student: Minus x.

Minus  $x$  and; obviously, there is a 2-half appearing everywhere. So, there is  $1$  by  $2$  everywhere. So,  $1$  by  $2$  is here also, right? Everything has to be divided by, similarly  $\sin$  one  $\sin y$  I can take common. So, it will be minus or plus if I write it  $i$ , right? And  $\sin y$  I take it out. Then I will get  $e$  to the power  $x$  minus  $e$  to the power minus  $x$  divided by  $2$ . So, this becomes  $\cos y$  and what is this?  $\cos$  hyperbolic  $x$ . And this becomes  $\sin y$ ,  $\sin$  hyperbolic  $x$ . So, this will be written as  $\cos y \cos$  hyperbolic  $x$  plus  $i \sin y \sin$  hyperbolic  $x$ , right?

And then I can erase out most of it this this derivation is simple not very far you know. So, sorry this is wrong this is  $\cos i$ , right. So, that is what it is this is the final result. So, you see  $\cos FL$  plus  $i FL$  you can now calculate out. So, that is what I am saying it is amenable to  $n$  calculation. So, you can do  $n$  you can write a program for this also that is not a problem just writing simple MATLAB program is very easy. We were doing it in our good old days Fortran 4, or Fortran you know 77 c or whatever it is.

So, this is how it is. So, this will be  $\cos$  hyperbolic  $FL$  plus  $i FL$ , or we simply  $\cos FL$ , all in radians, plus you know and then  $\cos$  hyperbolic  $FL$  in this case because  $x$  and  $y$  are same plus  $i \sin FL \sin$  hyperbolic  $FL$ , right.

(Refer Slide Time: 06:53)

The slide contains the following handwritten text:

$$\cosh [x + iy] = \cosh x \cos y + i \sinh x \sin y$$

$$\cosh [FL + iFL] = \cosh FL \cos FL + i \sinh FL \sin FL$$

At the bottom of the slide, there is a footer with the NPTEL logo, the name B. Bhattacharjee, and the text DEPARTMENT OF CIVIL ENGINEERING, IIT DELHI. A small number 14 is also present in the bottom right corner.

So, this is this will be your  $a$ ,  $a$  can be easily found out in this manner, right? Let us just do the  $\sin$  hyperbolic  $x$  plus  $i y$  and after that I suppose you should be able to solve them easily, all right. So, this is this, this is again you available in any basic maths keying

Erwin keying and engineering maths that is a good book, you can look into it pretty old book, but good one. You can look into it. So, you know I will come to now sin hyperbolic x plus iy.

(Refer Slide Time: 07:56)

**Unsteady Heat transfer**

$$\begin{aligned} \text{Sinh}[x+iy] &= \frac{1}{2} \left[ e^{x+iy} - e^{-x-iy} \right] \\ &= \frac{1}{2} \left[ e^x e^{iy} - e^{-x} e^{-iy} \right] \\ &= \frac{1}{2} \left[ e^x (\cos y + i \sin y) - e^{-x} (\cos y - i \sin y) \right] \end{aligned}$$



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Do the same thing e to the it will be e to the power x plus iy minus e to the power half would be there, x minus iy. And again, I write like this half e to the power x into e to the power I y. Minus e to the minus x e to the power into e to the power minus iy, and e to the power I y this is all there half e to the power x, this is cos y plus i sin y. (Refer Time: 08:47) theorem minus e to the minus x, and this is cos y minus i sin y. And take coefficient of cos y again you will get half. So, if I take coefficient of you know, just erase some of them out, just erase this out, all right. And rewrite something write this again.

(Refer Slide Time: 09:19)

**Unsteady Heat transfer**

$$\begin{aligned} \sinh[x+iy] &= \frac{1}{2} \left[ \cos y (e^x - e^{-x}) + i \sin y (e^x + e^{-x}) \right] \\ &= \frac{1}{2} \left[ e^{x+iy} - e^{-x-iy} \right] \\ &= \frac{1}{2} \left[ e^x (\cos y + i \sin y) - e^{-x} (\cos y - i \sin y) \right] \end{aligned}$$



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So, I will have half still remaining there, and cos y into e to the power x minus e to the power x minus x this is minus x, minus x, this is one term. And then there is a sin term, sin term will be i sin y plus plus i sin y. And inside would be e to the power x plus e to the power minus x divided by 2.

So, you find that this becomes what is this? If I just rewrite it like I did earlier, it will be cos y.

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**Unsteady Heat transfer**

$$\begin{aligned} \sinh[x+iy] &= \frac{1}{2} \left[ \cos y (e^x - e^{-x}) + i \sin y (e^x + e^{-x}) \right] \\ &= \cos y \sinh x + i \sin y \cosh x \end{aligned}$$



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Sin hyperbolic.

Student: Yes.

Yes, sin hyperbolic x plus i sin y cos hyperbolic x. So, that is it use this formulae to find out. You know, instead here I will have FL. So, in my case for  $m_{12}$  and  $m_{21}$ , I will have to do that. And this will be FL. This will be also FL, this is FL and this is FL. So, you can calculate this out if you do this exercise, then I get values of I get values of if I do this exercise, I get values of  $m_0$ , ha  $M1$  first for the layer solid layer this values comes out.

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**Unsteady Heat transfer**

$$m_o = \begin{bmatrix} 1.0000 & -0.0667 \\ 0 & 1.0000 \end{bmatrix}$$
$$m_i = \begin{bmatrix} 1.0000 & -0.1000 \\ 0 & 1.0000 \end{bmatrix}$$
$$M1 = \begin{bmatrix} 0.9120 + 0.7229i & -0.0982 - 0.0242i \\ 3.5168 - 14.2882i & 0.9120 + 0.7229i \end{bmatrix}$$

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One can try this out there may be mistake in this one has to check this do this calculation oneself, and then you know you get it get you get at right. This is the arithmetic I am not doing calculators everybody. Can do it  $m_0$ ? What will be  $m_0$ ?  $1 - \frac{1}{h}$  naught, now  $h$  naught was 15. So, this is  $1 - \frac{1}{15}$  0 1. And  $m_i$  because it boundary layer remember this was  $1 - \frac{1}{h}$  no  $h$  0 1. So, I am just doing this. So, that is what it is and my solid portion matrix is here solid portion matrix is here, solid portion matrix is here, solid portion matrix is here.

And from this you know you have to do this exercise I think I explained you unless, you do it you would not get it please do it yourself, and you will get it. So, overall transmission matrix should be multiplication of this matrices, and there to give the something like this.

(Refer Slide Time: 12:16)

**Overall Transmission Matrix**

$$M = m_i * M1 * m_o$$
$$M1 = \begin{bmatrix} 0.9120 + 0.7229i & -0.0982 - 0.0242i \\ 3.5168 - 14.2882i & 0.9120 + 0.7229i \end{bmatrix}$$

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You know one can multiply the matrix easily and you can find this out, right. So, that is that is one of the example calculation you do it.

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**Overall Transmission Matrix**

$$M = \begin{bmatrix} 0.5603 + 2.1518i & -0.2268 - 0.2399i \\ 3.5168 - 14.2882i & 0.6775 + 1.6755i \end{bmatrix}$$

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So, you get it and finally, after multiplication you get something like this. So, that is the overall transmission matrix, overall transmission matrix. One has to do this calculation and verify that is important otherwise you do not understand how it is being done. Now we still do simplification. Matrix calculation will have to do will have to find out abcd. But if I am interested in finally, finding outlet us say internal temperature of the room,



have a set of alternatives for example, long axis parallel to east west another case long axis parallel to north south.

Student: North south.

I calculate by a method which is relatively approximate. And it gives me that north south is you know is better than east west long axis parallel to east west. Now if I do an accurate methodology use more accurate mythology, and it might still give me the it will give the same result. As long as my approximation do not affect my relative, you know relative outcome. It chooses still the best that would be chosen by the most sophisticated mathematical treatment problem remains. The other problem is you can mathematically, mathematically make use of very sophisticated tools. But the pattern of variation of temperature every year is not same.

Anyway, you are going to use a design temperature profile, heat flow profile and so on so forth. So, long as your objective is to design the envelope, even approximation limiting, limiting yourself to fast harmonic is good enough. Because whatever result you get using first harmonic if you take 6 harmonics you will still get the same results, error would be there in the first one itself. Or by any other more sophisticated mathematical tools. So, approximate solutions or approximate treatment is as long as you are designing. But supposing you want to calculate out the energy, energy consumption in a given year.

You cannot do it. You know, energy consumption in a given year. You must have right kind (Refer Time: 17:01) verify energy audir you want to do yearly energy audit you want to verify. Actual temperature measurements are available with, and you had electricity bill or something of that kind. Now in that case you need really much more accurate mathematical treatment because now you are comparing with that real thing. So, long as design is the objective, then you have n number of alternatives available, I mean n can be very large as large as 10 1000 or so. Or maybe more as we shall see later on, but you got to choose the alternative which is which will you know show best performance. So, they are the relative error as long as they are same, it does not matter. And this is the kind of treatment we do in other civil engineering scenario also. You know when you do a structural design. You would not find a beam which is simply supported in real life.

Because if you put it on a pedestal put a beam you always assume that it is simply supported, but always there will be some.

Student: Friction.

Friction; so fixity or friction there will be horizontal resistance. So, we actually make system amenable to mathematical treatment for a specific purpose. Here our purpose of this in this course is to look into thermal design of building. I am not interested in calculating the energy consumption per annum. For that the robot softwares are available, and if you want to relatively compare one solution with the other for yearly annual consumption the design temperature again design temperature is taken. Which is there in most of the most of the software will have temperatures available for major cities of the world. They have a data bank they would you can actually pick them up or you can give your own input of temperature yearly temperature profile and so on so forth. So, that you can do. So, here our purpose is design therefore, we will do only use first harmonic and not beyond, right. Will use only first harmonic at not beyond. And in that case it becomes fairly relatively simpler right.

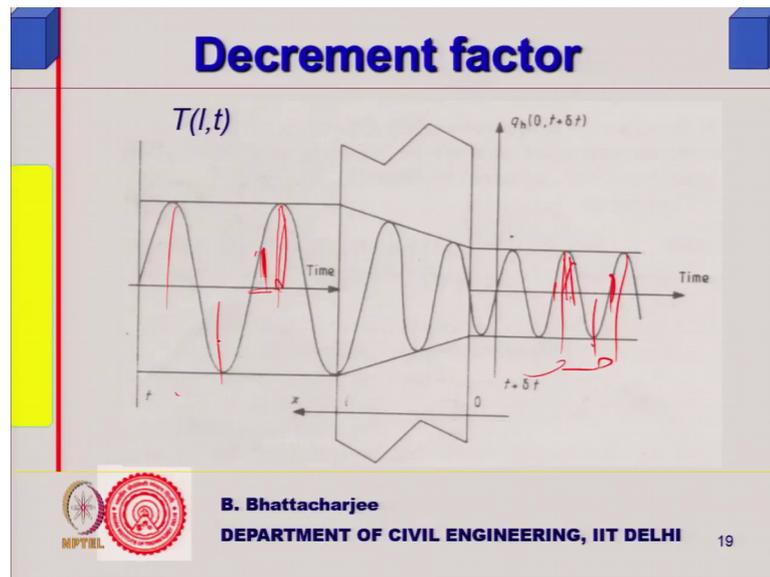
So, we use certain response factor in that situation. One of them is called decrement response factor. You know, what is the idea, you see if I have a sinusoidal pulse of temperature outside. It will cause heat flow inside, you know I have a sinusoidal temperature outside, it will cause some heat flow here; obviously, the time if I look at same time you know like this is time is 0, this is  $t$ , period  $p$ , this is this is 0, and  $p$  will be somewhere.  $P$  will be somewhere there. Actually, it will reach the heat flow will reach somewhat later. Peak of the heat flow will be later than this time, right. There will be a phase lag as we call it or a time lag. There will be lag after which will be result.

And also, this peak to peak ratio, this will be less than 1. You know, this is usually less than 1. And this is what we call as decrement respond response factor; so a decrement response factor, as a ratio of cyclic flux transmission. So, flux transmission to the steady heat flux transmission. So, this is what we do is we divide by steady flux. So, cyclic flux is this flux cyclic flux is applied to fluctuation about mean in external temperature or flux harmonics impinging on exposed tropic surface undergoing transient temperature, the heat transfer.

So, if I have a opaque body like this, and it is going undergoing transient heat transfer, I can talk in terms of this, this decrement response factor, either from temperature to heat flux or flux to heat flux. But usually we would be using temperature to heat flux right. So, I have a unit sinusoidal temperature fluctuation outside. Now, why unit because if it is unit you know so for that any other for any other one I can find out. And we actually divide this ratio by this we define, how we define  $u$ ?  $U$  value if you remember  $q$  capital  $Q$  was  $u$  into  $a$  into  $\Delta T$ . So, small  $q$  will be  $u \Delta T$ , small  $q$  is a heat flow per unit area heat flux. And that is the steady heat flow for unit temperature difference, right. So, here we define a fluctuating  $u$ , fluctuating  $u$  is nothing but  $q$  peak divided by unit  $t$ . Now supposing this has got some  $T_0$   $T$  value.

And here it is  $q$ , then  $q$  by  $T$ , you know  $T$  amplitude  $q$  is the amplitude  $q$  by  $T$  will give me for unit fluctuation, what is the  $u$ ? So, we define  $u$  fluctuating or  $u$  swing as  $T$  swing versus you know  $T$  swing stands for amplitude divided by or (Refer Time: 22:01) around no, no  $q$  swing sorry.  $Q$  swing divided by  $T$  swing. Supposing I have at  $T$  amplitude outside, and due to that heat flux inside is  $q$  peak of the heat flux, then this ratio of  $q$  flux  $q$  peak this  $\sin$  is for fluctuating, because I am using mean and fluctuating. So, this is nothing but  $u$  swing. And decrement factor is ratio of  $u$  swing divided by steady  $u$ . Decrement factor usually denoted by mean  $\mu$   $u$  swing by  $u$ . You know so, this is defined as a ratio of cyclic heat transmission to steady heat flux transmission. And mathematically defined like this, mathematically defined like this, mathematically defined like this, but first let us understand a little bit more.

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I can have a flux here, and due to this, this flux will change. And this ratio of this by this if it is flux or it can be temperature as well, or it can be temperature as well, it can be temperature as well, you know it could be temperature instead, instead not only the flux. But temperature and flux swing here corresponding to this the swing here would be reduced, the peak would shift. So, this height is less. So, this height by this height I call as  $u$  swing. And that would be same as any point corresponding height. Because they are all I am assuming 24 of period is not changing, that is not as an assumption. Period is remaining same 24 hourly, and there also 24 hourly.

So, any point if I take this ratio this ratio should remain same. You know, ratio said any corresponding point. Corresponding point I mean that distance from the peak; so corresponding point if I take this ratio. So, it will remain same, because these periods are I am measuring in 24 hours, here also 24 hours, only amplitude is changing. So, at any point if I take for example, this to this ratio or this to this ratio is same. So, that is what we are assuming and this ratio we defined as  $u$  swing and  $u$  swing divided by  $u$  steady we call it decrement factor, right. Decrement factor is an important one. So, consequently mathematically defined as  $q$  by  $u$  s wing is defined as  $q$  l t divided by you know this is how it is.

(Refer Slide Time: 24:18)

**Decrement response factor**  
 Consequently mathematically defined as

$$\tilde{U} = \frac{q(l,t)}{T(0,t)} = \frac{q_i}{T_o}$$

= heatflux / swing of outside temperature

$$\mu = \frac{\tilde{U}}{U}$$

$$\tilde{Q} = \mu U A \tilde{T}_o$$

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Heat flux per unit swing of outside temperature, or unit swing of temperature outside. For unit swing of temperature outside, right. Or unit swing of temperature outside. So now, I can use the matrix to find out this. Now I can now these are all complex by the way. Because T is complex periodic T is complex q is complex, I am dealing with the non-steady state situation, I am talking about the amplitude, but abcds are all complex.

So, there will be you know abcds are complex. So, this ratio will turn out to be complex number actually. Because ratios will ratios will turn out to be, because you know this ratio will be complex number, why? Because abcd, these were all complex number cos FL plus i FL etcetera, etcetera. And mu is defined as u swing by u mu is defined as u swing by mu. Consequently, if I know this mu, the absolute value of capital Q swing part of it I can find out. In fact, it will be mu into u, which is u swing into T swing outside. Mu u into area into T swing outside. I can find out; the whole idea is to get this. Mu multiplied by u multiplied by the T swing will give you the inside, heat flux swing in heat flux at a given temporal time. So, this is you know this is this ratio. So, that is how it is and let ussee how you find out u swing.

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## Decrement response factor

$\mu$  can be determined from Transmission matrix, assuming constant inside temperature, i.e.,  $\tilde{T}_i = 0$

$$\begin{bmatrix} T_i \\ q_i \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} T_o \\ q_o \end{bmatrix}$$
$$\begin{bmatrix} T_o \\ q_o \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} T_i \\ q_i \end{bmatrix}$$

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Mu can be determined from transmission matrix, assuming constant in constant inside temperature. So,  $T_i$  by  $q_i$  that is what we are trying to find out,  $q_i$  by  $T_o$   $q_i$  by  $T_o$ . So, I take inverse of this matrix, then I get  $T_o$   $q_o$  is equals to  $abcd$  inverse  $d$   $iq$  I you know if I take inverse of this matrix, right. Inverse of this matrix, right; because I want to find out what did I do I want to find out? I want to find out  $q_i$  by  $T_o$  for  $t_i$  being constant, because I am saying it is only the fluctuating part.

So, if the inside temperature is constant, how much is the?

Student:  $Q_i$ .

$Q_i$  that is what by definition, this is how anywhere. So,  $t_i$  swing is equals to 0, this is 0.

(Refer Slide Time: 26:56)

**Decrement response factor**

$\mu$  can be determined from Transmission matrix, assuming constant inside temperature, i.e.,  $\tilde{T}_i = 0$

$$\begin{bmatrix} T_i \\ q_i \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} T_o \\ q_o \end{bmatrix}$$
$$\begin{bmatrix} T_o \\ q_o \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} T_i \\ q_i \end{bmatrix}$$

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I want to find out the ratio of  $q_i$  divided by  $T_o$ . So, that is why if you know one way of finding out will be  $q_i$  would be  $c T_o + d q_o$ , but simpler way would be take inverse of this one,  $T_o$  is equals to  $a b c d$ , right. And this you write maybe some other notation we use. Do you know, if what will be inverse of this matrix,  $a b c d$  inverse is what?

Student: (Refer Time: 27:24).

So, basically it is leave this out, right this one you want to find out, and to the power this would be  $d$ , simply  $d$ , this will be simply  $d$  because  $2$  to the power minus  $1$  you know plus  $1$  plus  $1$ , right. And when I come to this, this will be minus  $c$ . And this will be  $a$ .

Student: A.

And this will be  $b$ .

Student: Minus  $c$ , minus  $c$ .

Minus  $b$ .

Student: Minus  $b$ .

Minus  $b$ , right; so  $d b c$  was is written like this correctly  $a b$  minus  $b$ , why is minus  $b$ ?

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**Decrement response factor**

$$\begin{bmatrix} T_o \\ q_o \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} T_i \\ q_i \end{bmatrix}$$

For  $\tilde{T}_i = 0$   $T_o = -Bq_i$

$$\tilde{U} = \frac{q_i}{T_o} = \frac{1}{-B}$$

*img*

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Is it correct?

Student: Of course, this.

Then transpose yes then transpose yes, yes. So, d minus b minus c, right fine that is. So, that is what it is. So, therefore, for  $T_i$  is equals to 0 this is equals to 0,  $T_o$  is equals to  $d - b q_i$ . So,  $T_o$  is equals to  $b q_i$ . Because I am assuming there is no fluctuation of temperature inside. You know, outside there is a given simple pulse, and inside to the inside the wall, it is all steady, there is no change. So, that  $T_i$  swing part I am assuming it to be 0, and that is how I define as b.

So, u swing will be  $b q_i$  by  $T_o$  was minus 1 by b, and b is complex. So, it will have an absolute value, and it will denote the time lag, because you see it is in complex plane real imaginary. So, this is your real if you know  $T_o$  is real  $q_i$  will be somewhere here,  $q_i$  will be somewhere here in a complex form, right; because it would because if this is real this ratio is complex form. So, somewhere will be there. So, this will have some sort of a phase lag time lag time difference right. So, that is it and mu will be given like this.

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### Decrement response factor

$$\begin{bmatrix} T_o \\ q_o \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} T_i \\ q_i \end{bmatrix}$$

For  $\tilde{T}_i = 0$   $T_o = -Bq_i$

$$\tilde{U} = \frac{q_i}{T_o} = \frac{1}{-B} \quad \mu = \frac{1}{U(-B)}$$



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So, absolute value of mu we can find out, and use for our calculation also.

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### Decrement factor

$M^{-1} = \text{inv}(M)$   
 $1/-B = 1/M'(1,2);$   
 $U = (1/(1/h_o + l/k + 1/h_i))$   
 $c_{dec} = 1/-BU$   
 $\mu = \text{abs}(c_{dec})$   
 $\phi = \text{angle}(c_{dec}/U) \pi * 12$

$$\frac{1}{U(-B)} = X + iY$$

$$\mu = \sqrt{X^2 + Y^2}$$

$$\tan \phi = \frac{Y}{X}$$




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So, inverse of m is so, if you see that, u is given by this, and complex the implements factor will be given by this, mu is absolute of complex decrement factor phi is angle that you can find out, timeline you can find out or phase difference you can find.

Now, I think I have the here x plus iy mu is equals to under root x square, plus y square you know u by u by minus b is something like this and 10 phi is written in this manner. So, that is what it is. So, this is this is how we can find out the decrement factor. Now I

think I have got an example. Perhaps, I have got an example calculation for example calculation for this to find out this. What is the actual value of this one. It will depend upon  $\sin$  of  $y$ . And  $x$  whether  $10$  you know  $\sin$  I mean whether it lies in if it is rise in first quadrant both are positive  $10$   $\phi$  is positive. If it is if one of them is you know because in real and imaginary scenario, everything is positive here, here, this is positive this is negative. So, real term is this  $x$  is in real term is negative in this one both are negative, in this one  $y$  is negative, from that you can find out the angle.

So, I think again will break.