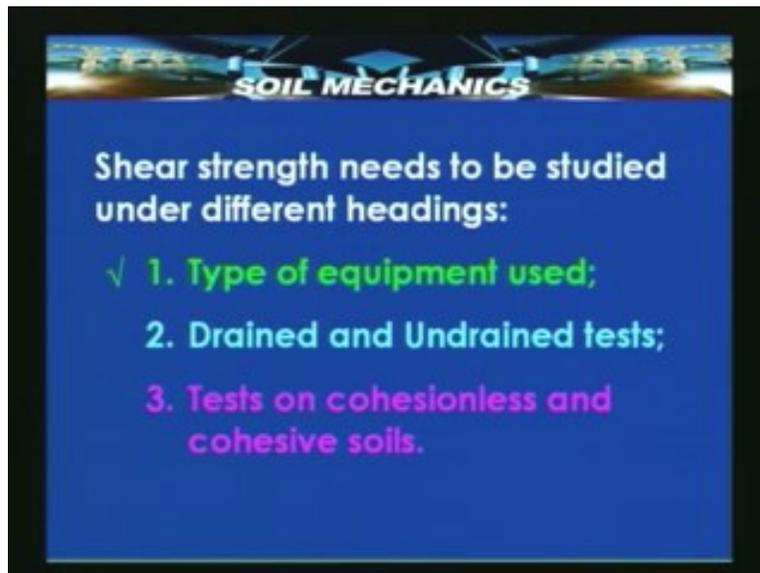


**Soil Mechanics**  
**Prof. B.V.S. Viswanathan**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Bombay**  
**Lecture – 46**  
**Shear Strength of Soils**  
**Lecture No.4**

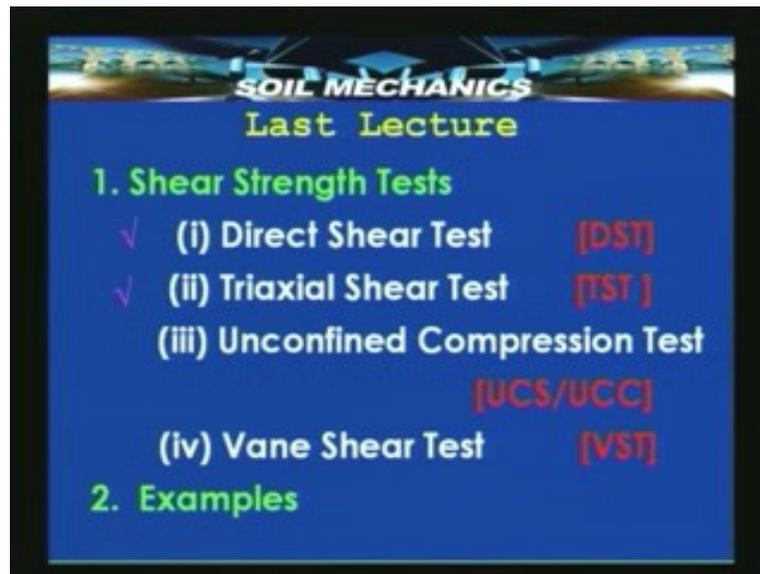
Students we had 3 lectures so far on this topic of shear strength of soils. Today we will go on to the fourth lecture. Let us take a look at the first slide which describes what we have seen in the past. We have seen basically that this topic of shear strength of soil needs to be studied in essentially 3 stages; number one is understanding the behavior under shear depending upon the type of test or depending upon the type of equipment that is used for testing shear strength.

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Secondly, the drainage conditions used in the test depend very much and influence very much the behavior of the soils under shear and lastly we need to study the shear strength according to the type of soil because behavior of cohesionless soil is quite different from behavior of cohesive soil under shear. Having classified this topic of shear strength into these 3 basic categories, we had started looking at the first category that is the type of equipment or type of test that is used for determining the shear strength of soil. Under that if we go to the second slide we will be able to recapitulate that we have already gone into complete detail of the shear strength known as the direct shear test DST. We had also seen an overview of triaxial shear test TST of course in one of my early lectures the first one perhaps I had also given you the general idea about the next 2 test, the unconfined compression test and the vane shear test.

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Today what we shall be doing will be to quickly recapitulate briefly what we saw under direct shear stress test. Then again recapitulate what we have studied under triaxial shear stress and then go on further to get in to the **nitigrity** (Refer Slide Time: 3:10 min) and details of the triaxial test and also do some examples.

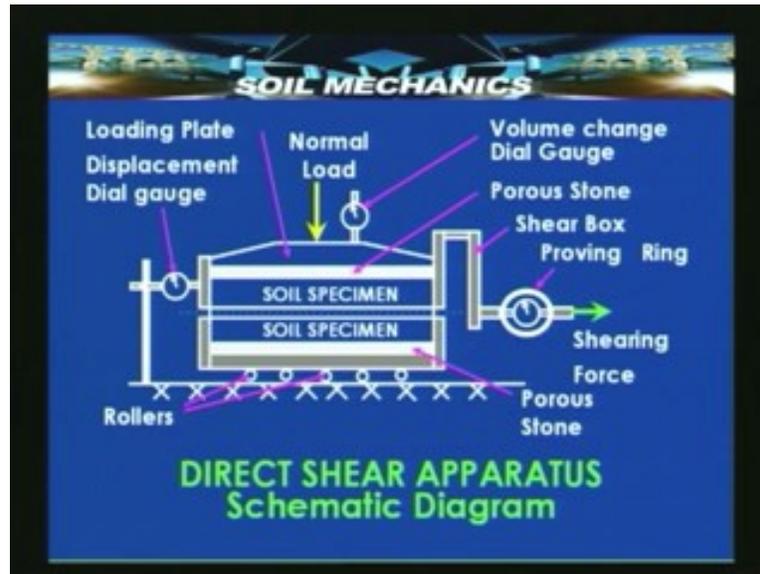
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If you remember this was the basic schematic diagram of a typical direct shear apparatus. You can see there that there are 2 parts in this.

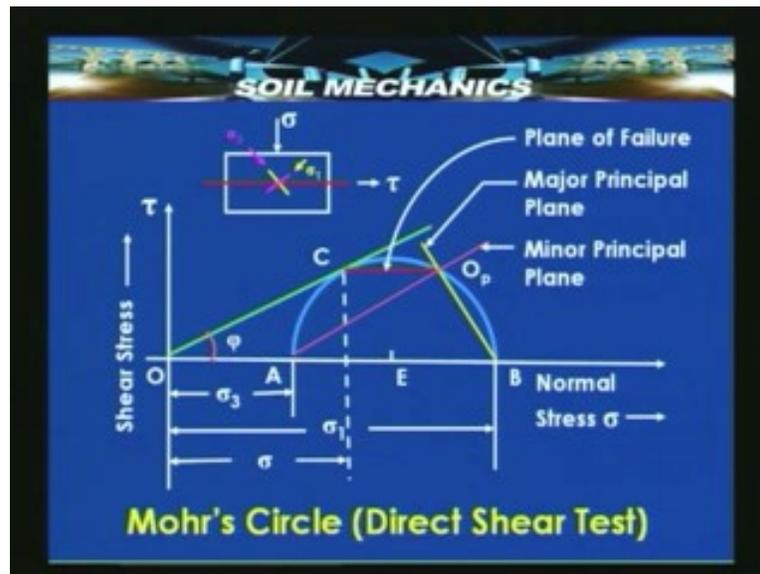
The lower part and an upper part separated at the junction here like this and the soil specimen is also a specimen and this specimen is sheared by applying a tangential load, a shearing force here. Of course a normal load is applied to keep this specimen confined.

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Then there are of course other small additions such as the dial gauge here to measure the volume change, the dial proving ring here to know the force that is applied, porous stone here to permit drainage and then the displacement gauge to know the displacement that the sample undergoes. All these together constitute the entire shear strength apparatus known as the direct shear apparatus.

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Then we also saw that the results of a direct shear test can be analyzed and can be interpreted with the help of a Mohr circle. The principle of the Mohr circle was explained to you in very great detail by me in one of the earlier lectures. So today we will quickly see a typical Mohr circle particularly the way it looks for the direct shear stress. You know that a Mohr circle is this, the point A corresponds to the minor principle test, point B corresponds to the major principle test and the point OP which is obtained by drawing a line parallel to the plane of  $\sigma_1$  gives the pole OP because by definition O is a point on the Mohr circle which has coordinates sigma and tow which in turn or equal to those on a plane passing through that OP and the point A.

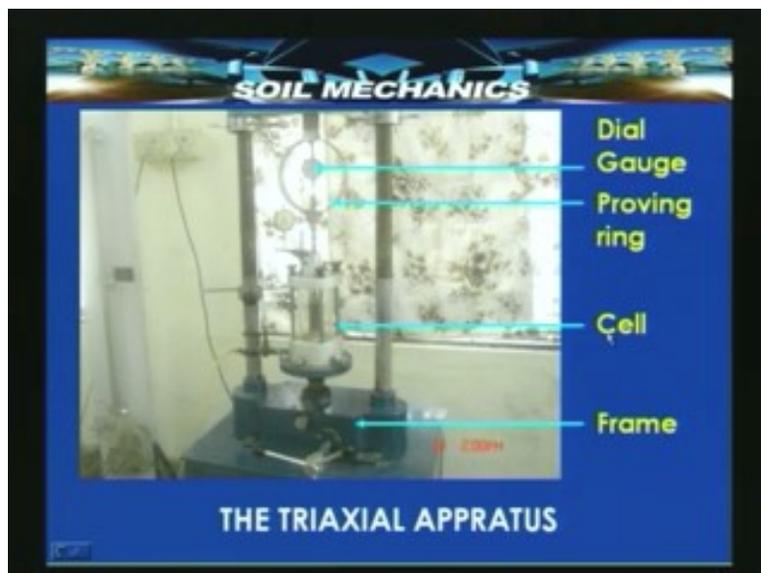
It's so happens that this red line which passes through the A and OP is nothing but the minor principle plane and therefore point OP represents the stresses which act on the minor principle plane. And now from this OP if you draw a line horizontal OPC then since it is horizontal it will be nothing but the representation of the failure plane. In a direct shear stress as you remember this failure plane is an imposed plane, a horizontal plane and C will then be having coordinates which are equal to normal and shear stresses on the failure plane. Now from this, one can find out the values of cohesion and friction being a direct shear test which is drained where drainage cannot be prevented, you only get an angle of friction phi, cohesion is zero.

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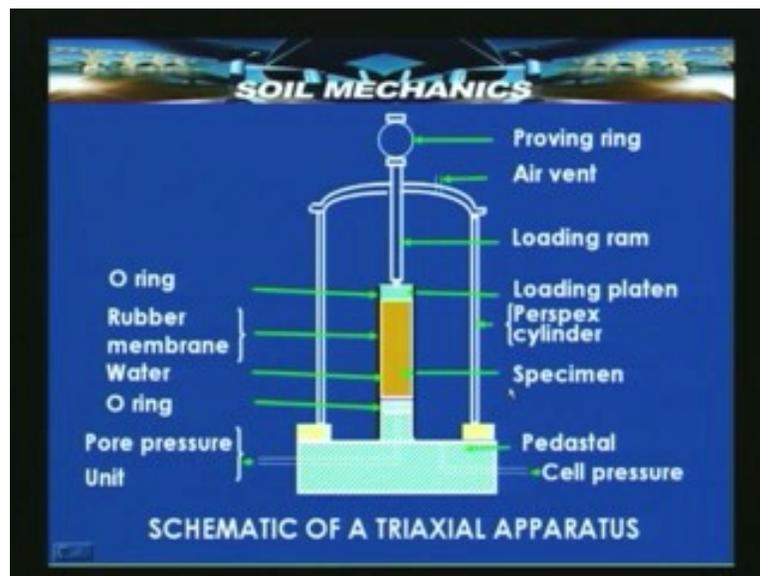
Now let us take a quick look at the triaxial test, the way we have seen it in the earlier lecture. This is what a typical triaxial apparatus looks like. There is a loading frame, there is a cell which contain the triaxial specimen where the so called triaxial stress state is achieved, below that there is a motorized unit which drives the platen upwards, the base upwards thereby imposing a vertical load and then of course there is a proving ring to measure the vertical load and dial gauge and so on.

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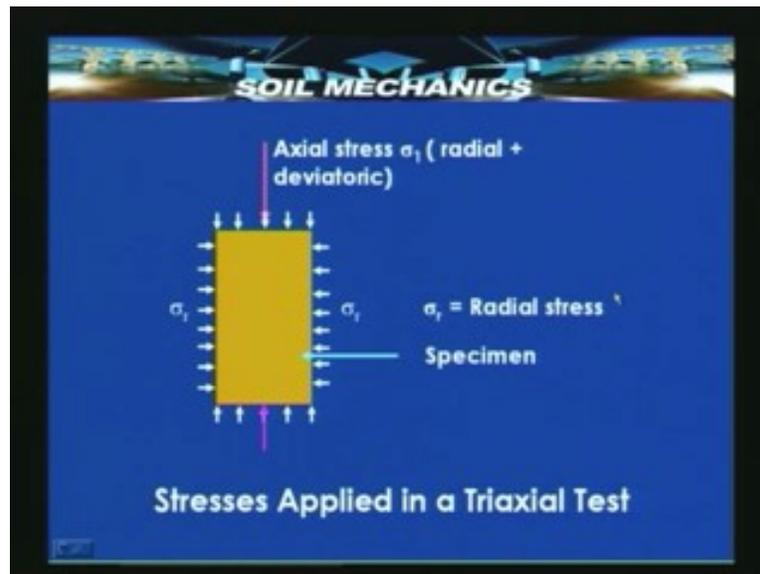
Let us see a little more detailed diagram, a schematic diagram other than this photograph. This is the schematic diagram of a triaxial apparatus. You see here starting from bottom, taking the right hand side this is where we have the pedestal and from the pedestal there is an outlet here rather than an inlet which is attached to the pedestal you can see here through which we apply this cell pressure. That cell pressure gets applied on the specimen inside this cell so this perspex cylinder here thus is so called the cell. On top of this specimen here we have a loading platen and so also at the bottom.

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At the bottom the pedestal there is also a outlet which goes to the pore pressure measuring unit and there is a pipe here which goes into the soil specimen. So the pore pressure inside the soil specimen gets reflected here in the pore pressure measuring unit. Then there is rubber membrane which holds the specimen in place. Then of course we have an air vent to ensure that the water that is stored here through which the lateral pressure is applied is free of air and then we have the proving ring to know the vertical load applied.

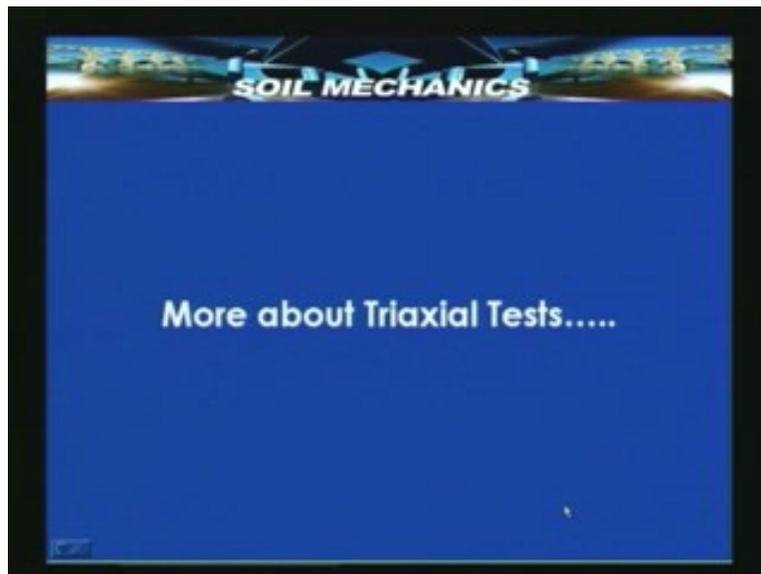
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This shows the stresses that act on the typical triaxial test specimen. The way that the test is conducted as we have seen earlier is first of all a radial all round pressure is applied that's known as the cell pressure  $\sigma_r$  is acting on the circumference of this specimen. And after this the vertical load is increased until the specimen fails and therefore at the end of test, at the point of failure we will have an all-round pressure  $\sigma_r$  which incidentally being watered. It also act at the top and the bottom and then this stress  $\sigma_r$  is increased by an amount known as the deviatoric amount well, so as to get the total axial stress  $\sigma_1$  at the top and bottom at the time of failure.

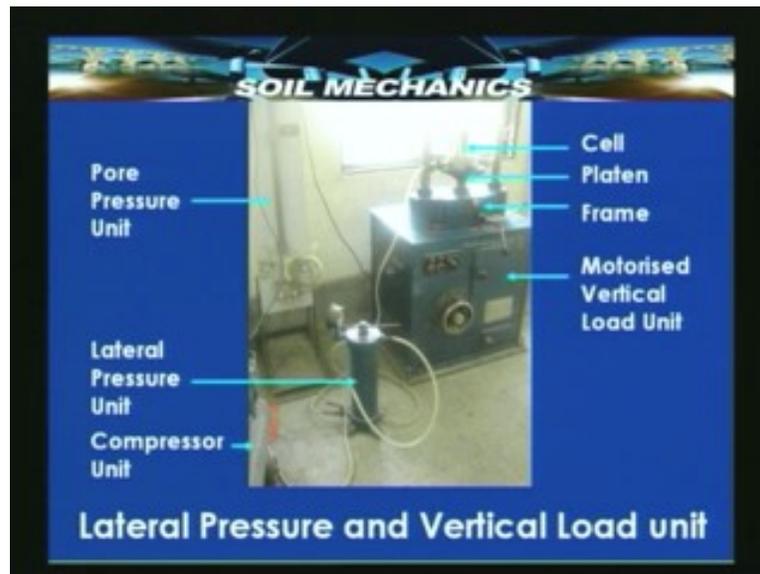
So having seen this quickly let us move on to today's lecture. In today's lecture I plan to go further and talk a little more about details of the triaxial shear test and see a few problems and then move on to the unconfined compression test and finally the vane shear test and everywhere we will understand the principles and the method of utilization of the test through some example. Basically all these test ultimately are intended to give the shear strength parameters C and phi. If they are total strength parameter they are C and phi and if they are effective stress parameter we generally refer to them as C dash and phi dash.

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So we will see a few examples which involve determination of the effective strength parameters or the determination of the total stress parameters. So more about the triaxial tests. Let's see the components of a triaxial apparatus by flitting through a few photographs again. What we see here are the major components of a triaxial apparatus. I have highlighted here the lateral pressure and the vertical load units, here is we have the lateral pressure unit and here is we have the vertical loading unit. So using the same terminology which is used in one of the earlier photograph we have here the cell, we have here the platen, we have here the loading frame and then below that the motorized vertical load application unit.

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And on the left you have a compressor which develops the air pressure that is required to apply the lateral pressure. So this is the lateral pressure unit and then lastly we have the pore pressure measuring unit, to know the pore pressure at any stage of loading or any stage in the test that developed inside the specimen.

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Another photograph, a greater detail of the motorized vertical loading unit. We have here the facility to control the speed of this motor so that the movement in the platen upwards is

controlled in a string controlled manner. So by adjusting the string in the upward direction we can control the load that is applied in the vertical direction.

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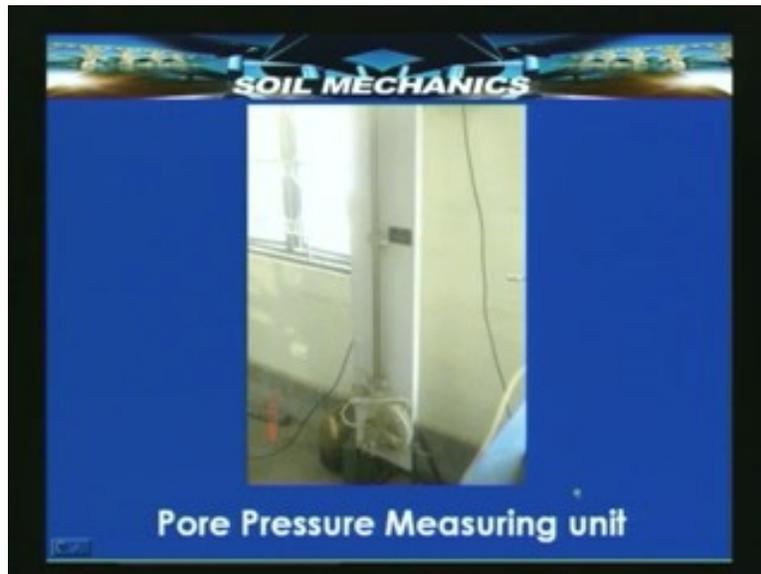
Here is the loading frame, the cell and the motorised unit and the pore pressure unit altogether, one of these views.

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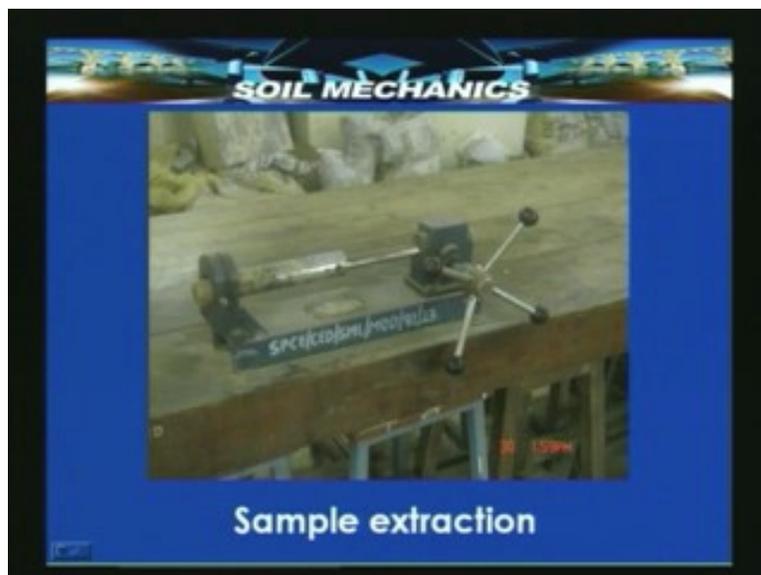
Here we have slightly closer views of the compressor and the lateral pressure unit. Compressor develops the air pressure that is required to drive the water and the pressure into the cell, from here the air pressure comes through this and the pressure is applied to water which goes out through this into the triaxial cell.

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This is the pore pressure measuring unit.

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Now in a case of a triaxial test we need perfectly cylindrical specimens whose ends are quite planar whose sides are perfectly vertical because it has been found from experience that a specimen which does not enjoy these benefits or these characteristics sometimes gives errors in the final results. Sample or specimen preparation is therefore a very important aspect and here is a unit which is meant for extracting a sample from a core box that's brought from the field. We extract a core from the field and then that we take out a sample. From the sample we then cut out a specimen.

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Here is where we have this specimen and this specimen is taken out of the split mould that is if you go back to the earlier slide, when you extract with the help of a plunger the core out of this cylinder then what comes out is collected in a split mould and then because its a split mould, you can open it out and take the sample out and once the sample comes out you can cut it to appropriate sizes and together with this patterns it can be put on the loading machine.

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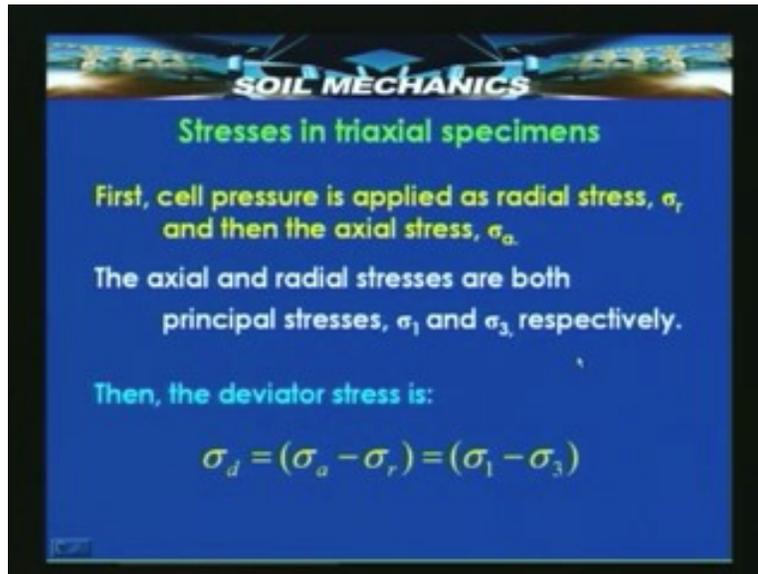
Here we have the closer view of this split mould and the specimen holder in the triaxial apparatus. Now what are the kinds of stresses and strains that develop in the triaxial test when we apply the so called the radial pressure and the axial pressures. So first the self-pressure is applied in the form of a radial pressure and then the pressure is increased in the axial direction until failure take place. So the radial pressure is say  $\sigma_r$  and the axial stress is say  $\sigma_a$ .

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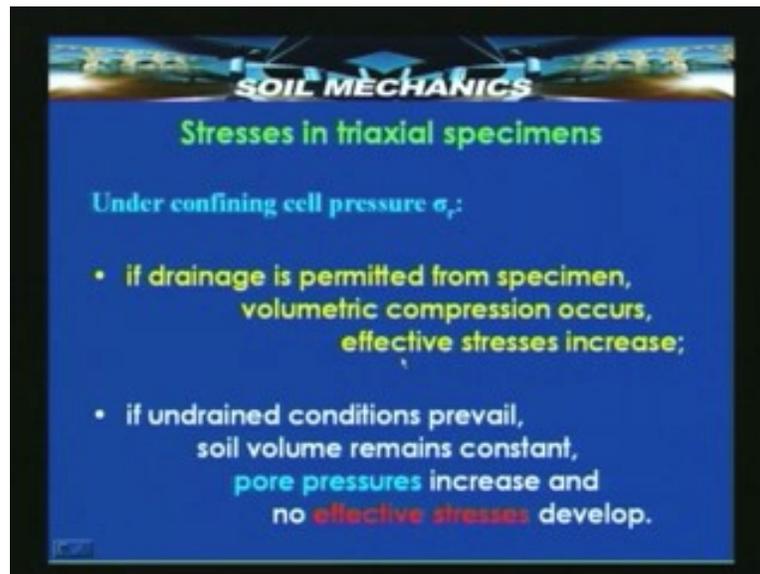
Obviously since no other pressures are acting, the radial pressure is going to be the minor principle stress and the axial stress that is applied is going to be the major principle stress respectively acting on a vertical plane and a horizontal plane.

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The difference between these two  $\sigma_d$  is known as the deviator stress and it is therefore equal to the  $\sigma_a$  minus  $\sigma_r$  and since axial stress is nothing but the major principle stress and the radial minor. We have  $\sigma_d$  taken as  $\sigma_1 - \sigma_3$ . Further the stress  $\sigma_r$  that is applied all around is nothing but a confining pressure that confines the sample.

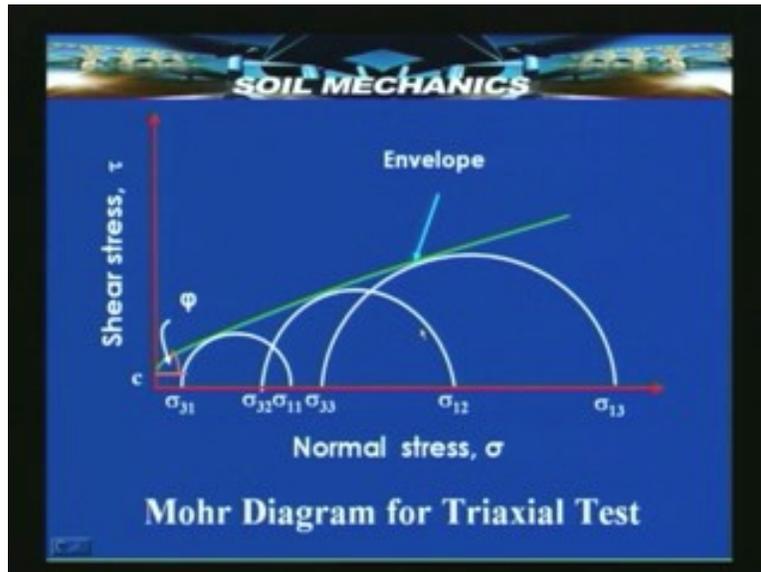
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And the reason why we have applied this is this helps us to simulate the confining pressure that exists at the depth from which the sample has been taken out. Since the behavior of a soil under stress is very much controlled by the stress that it has experienced insitu. We must recreate the insitu stresses in the specimen that we used in the lab and that's the reason why we apply a confining cell pressure  $\sigma_r$ , whose magnitude is equal to the confining pressure that is expected to be existing at that depth from which the specimen has been collected. Now this stress condition inside the specimen which is basically  $\sigma_r$  and  $\sigma_a$  depends on the drainage conditions. If drainage is permitted from this specimen then what happens is as the water expels out there is a compression taking place.

A volumetric compression takes place in the specimen and as the water goes out therefore effective stresses develop in the specimen. So as the test progresses if it's a drained test then the effective stresses in the specimen go on increasing. So if we are applying stresses  $\sigma_1$  and  $\sigma_3$  to start with at the end of the test we will have the effective stresses  $\sigma_1$  dash and the  $\sigma_3$  dash acting on the specimen. If on the other hand we have the undrained conditions then since drainage is not permitted, soil volume does not change it remains constant and therefore the pore pressure get build up in the water inside the specimen and effective stress are not allowed to develop. So in an undrain test purely undrain test, the initially applied  $\sigma_1$  and  $\sigma_3$  remain the total stresses and the total stresses remain the same at the end and effective stresses are not mobilized.

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So this shows the stress conditions through a Mohr diagram which exists in a triaxial test. In a typical triaxial test as we have seen we will be having the minor principle stress and the major principle stress. Generally we conduct at least 3 tests on any particular soil.

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**SOIL MECHANICS**

### Strains in triaxial specimens

By measuring change in height,  $dh$   
and change in volume  $dV$ , of specimen  
we can determine the strains in specimen:

Let the specimen be assumed  
to remain as cylindrical  
even after deformation.

Then, volumetric strain  $\epsilon_v$  is:

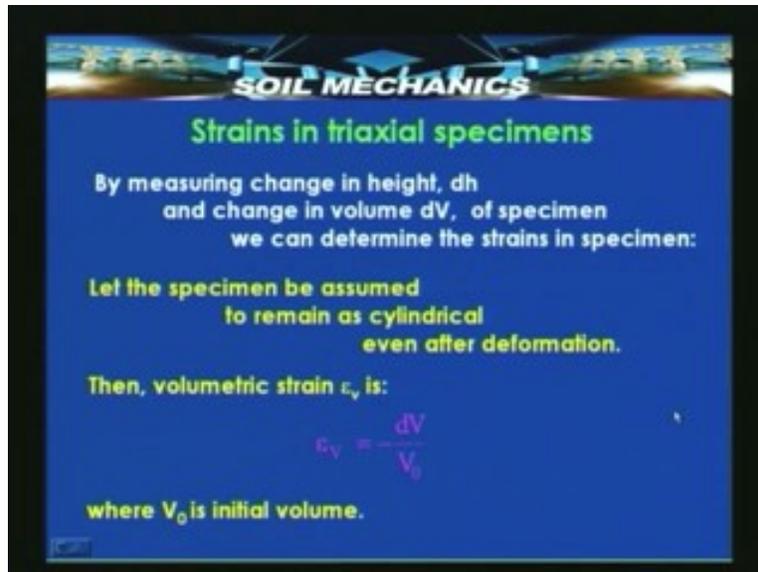
$$\epsilon_v = -\frac{dV}{V_0}$$

where  $V_0$  is initial volume.

So here we have the minor principle stress and the major principle stress and corresponding circle for one pair of these stresses then we can apply another cell pressure  $\sigma_{32}$  get a corresponding failure axial stress  $\sigma_{12}$  so also a third one. So the test is conducted on the same

soil under 3 different confining pressures and 3 different Mohr circles are obtained corresponding to the 3 different failure axial stresses. Now if I take a common tangent to all of these and assume that the Mohr-Coulomb (Refer Slide Time: 19:26 min) failure criteria is valid for this. Then what we will have will be an intercept which represents the cohesion in the soil and an angle here, the angle of internal friction  $\phi$ .

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And the stress state in the triaxial test if that is known then we can plot these graphs and we can calculate the shear strength parameters. That's the reason why need to understand correctly the state of stress in the specimen at any given time. Now what happens to the strain? By measuring the change in height during the axial loading and change in volume during the lateral loading we can determine the strains. If we assume that these cylindrical specimen remain like a cylindrical else specimen even after deformation then we can derive simple expressions for the volumetric and axial strain. It is very obvious that the volumetric strain will be  $\epsilon_v$  equal to minus  $dV$  by  $V_0$ ,  $dV$  is the change in volume. Now the change volume can be easily measured by attaching the pore pressure unit and the original volume  $V_0$  is supposed to be known for the specimen and therefore  $\epsilon_v$  the volumetric strain can be easily computed.

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**SOIL MECHANICS**

**Strains in triaxial specimens**

The axial strain is:  $\epsilon_a = -\frac{dh}{h_0}$

The area of cross-section A of deformed specimen is :

$$A = A_0 \left( \frac{1 + \frac{dV}{V_0}}{1 + \frac{dh}{h_0}} \right) = A_0 \left( \frac{1 - \epsilon_v}{1 - \epsilon_a} \right)$$

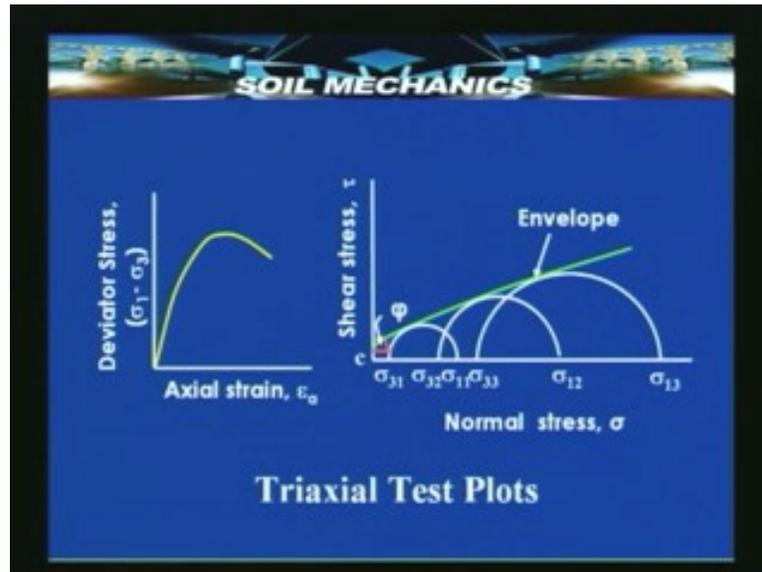
where

$A_0$  is initial area of cross-section of specimen  
 $h_0$  is initial height of specimen.

Then on the other hand the axial strain will be the change in the vertical height of the specimen divided by its original height  $\epsilon_a$  and since on the vertical compression, the area of cross section of the specimen increases as the specimen gets compressed and its diameter increases. The new area of cross section which will tell us the new stress state will be given by this expression in terms of the vertical and volumetric strains and the original area of cross section, we can get the new area of cross section.

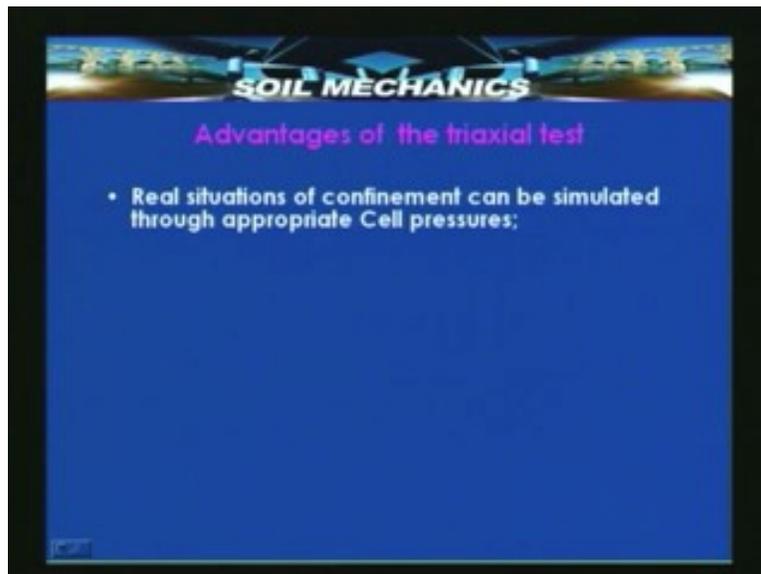
So A is the new area of cross section after deformation,  $A_0$  is the initial area of cross section of the specimen and  $h_0$  is the initial height of the specimen. With this we will be able to tell what will be strains at any given instance in the specimen. So the sum total of all this is during the experiment there will be some stresses and strain developing. The stress state can be expressed in terms of the Mohr circle as shown here whereas the strain state can be expressed in terms of the variation of the deviatoric stress as the function of the axial strain  $\epsilon_a$ .

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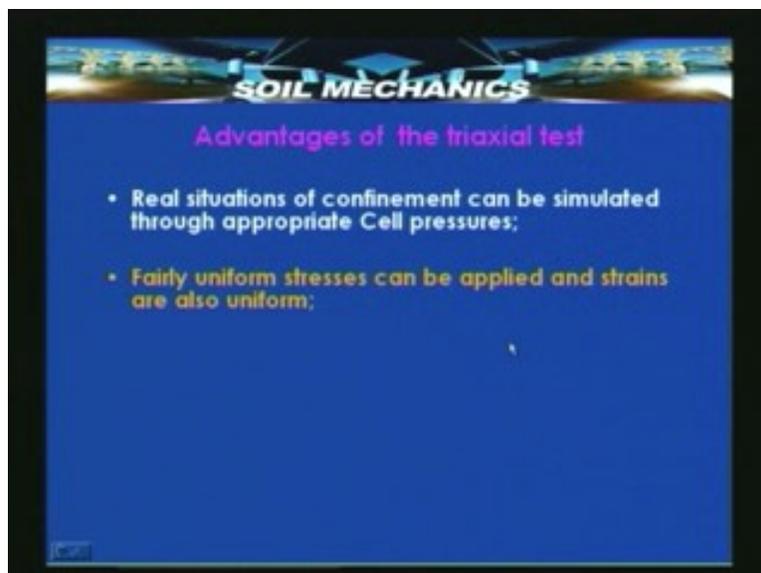
This shows that the deviatoric stress goes on increasing and the axial stress goes on increasing upto a point beyond which the deviatoric stress comes down because the soil would have already reached its failure strain value and the stress cannot be build up anymore. Now why do we need to conduct the triaxial test, why do we need to understand the importance of the distriaxial test? It's because there are several advantages associated with the distriaxial test compared with the direct shear stress. The advantages are one real situations of confinement in the field can be simulated. As I mentioned just a few seconds ago a soil is under certain confining stress at a given depth in the nature, in the field.

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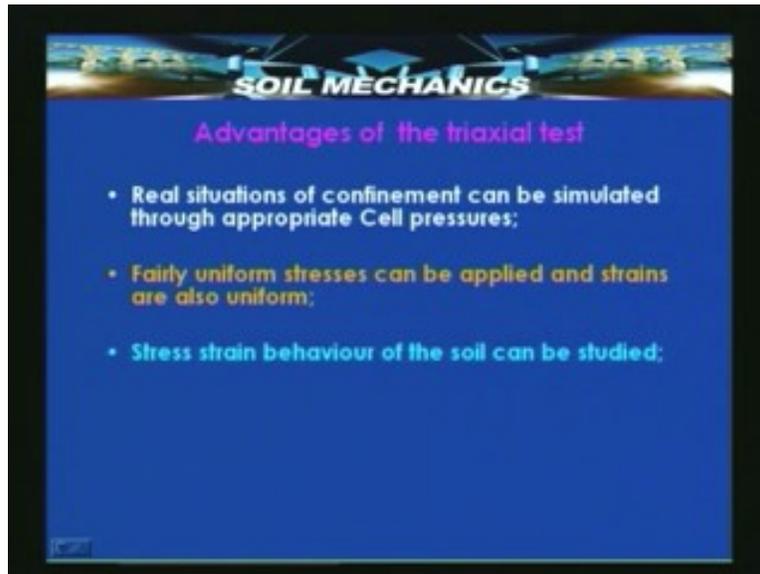
Now when you extract a sample from a certain depth and cut a specimen out of it and test it, obviously in order to simulate the conditions which it experienced in the field, we need to know what was the condition of confinement and reproduce that in the experiment. And the triaxial test helps us to do precisely this because we can apply a cell pressure of any known magnitude which will simulate the confinement that the sample is experiencing in the field at a given depth.

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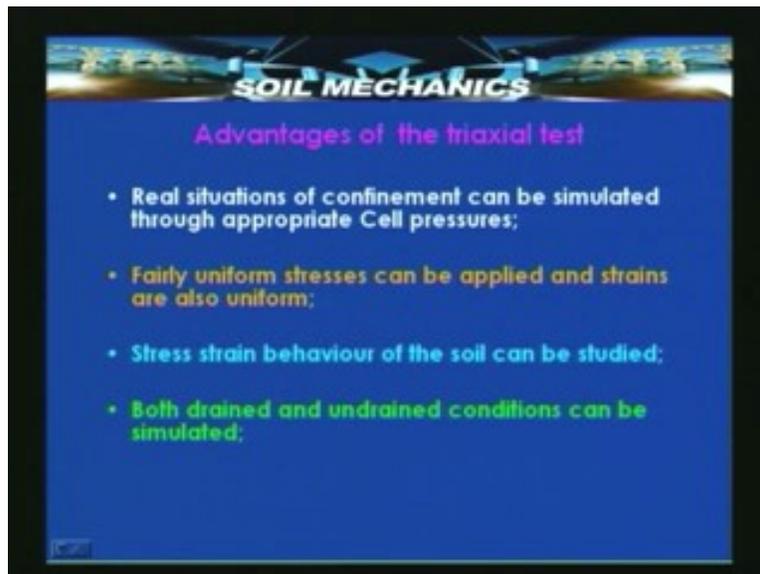
Then the manner in which the pressures are applied gives us the facility to have fairly uniform stresses inside the specimen. The direct shear if you compare for example as the specimen deforms you find that the area over which the normal stress is acting goes on changing.

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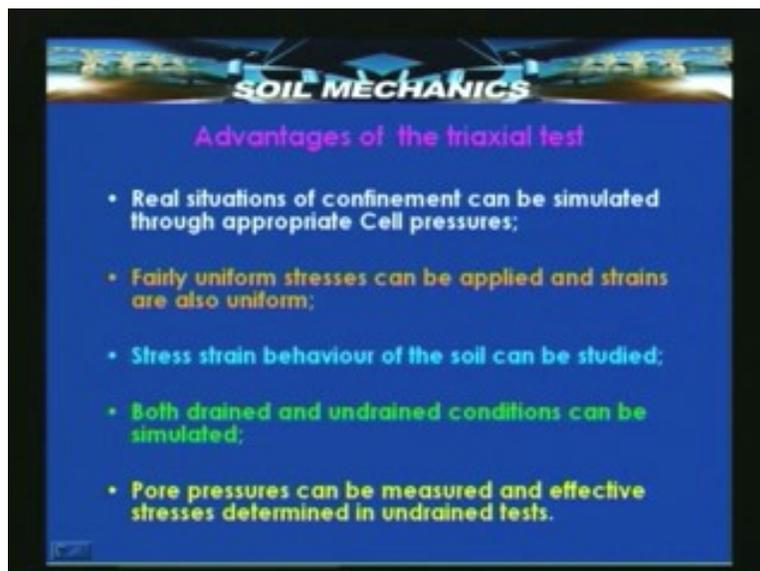
Therefore the stress level goes on changing it doesn't remain uniform whereas on the other hand in the triaxial test we have the advantage of imposing fairly uniform stresses on the sample. Then most important, the shear strain behavior of the soil can be studied in detail. As the test progresses you can measure the strain level, volumetric strain and the axial strain and compute the shear strain.

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Lastly both drained and undrained conditions can be easily simulated in a triaxial test; in the direct shear stress on the other hand we found that undrained conditions can hardly be simulated adequately.

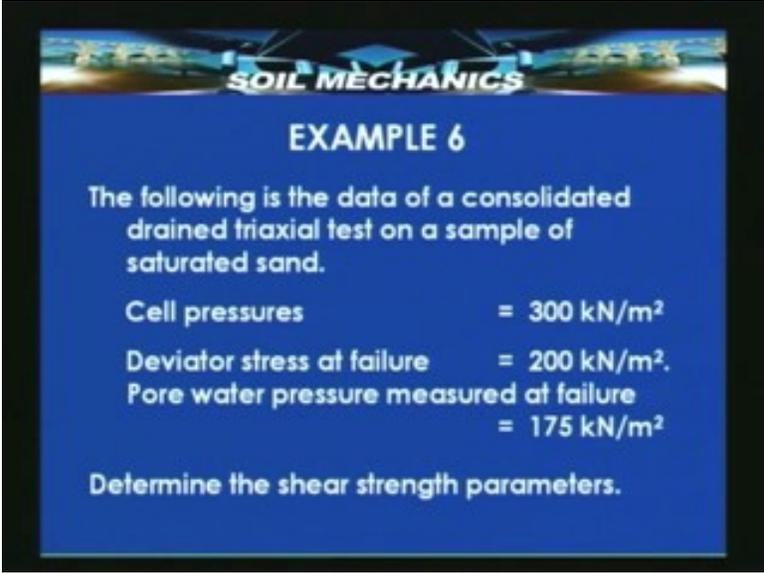
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Then finally pore pressure can be measured and effective stresses can be determined during the undrained test. This is a tremendous advantage because the knowledge of the pore pressure exactly tells us what is the kind of effective stresses that are mobilized during the test and the

shear strength is a direct function of the effective stress and therefore knowledge of an effective stress is very necessary and triaxial equipment can really help you to get that.

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**SOIL MECHANICS**

### EXAMPLE 6

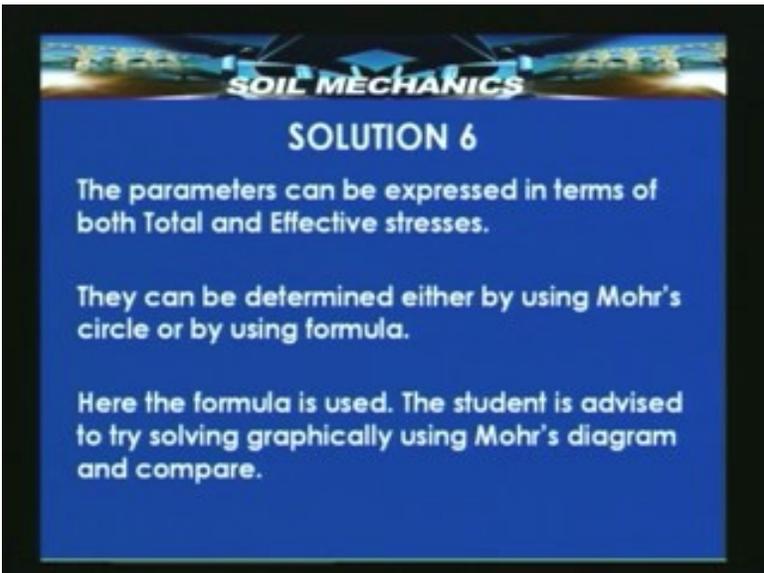
The following is the data of a consolidated drained triaxial test on a sample of saturated sand.

Cell pressures	= 300 kN/m <sup>2</sup>
Deviator stress at failure	= 200 kN/m <sup>2</sup> .
Pore water pressure measured at failure	= 175 kN/m <sup>2</sup>

Determine the shear strength parameters.

Let us quickly take a look at some examples. To begin with let me take the same examples which we saw in the earlier class we have called this class example 6. So let us see example 6 which we had seen in the last lecture, it helps to maintain the continuity. So that example which we saw in the last example was data about the triaxial test on saturated sample of sand.

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**SOIL MECHANICS**

### SOLUTION 6

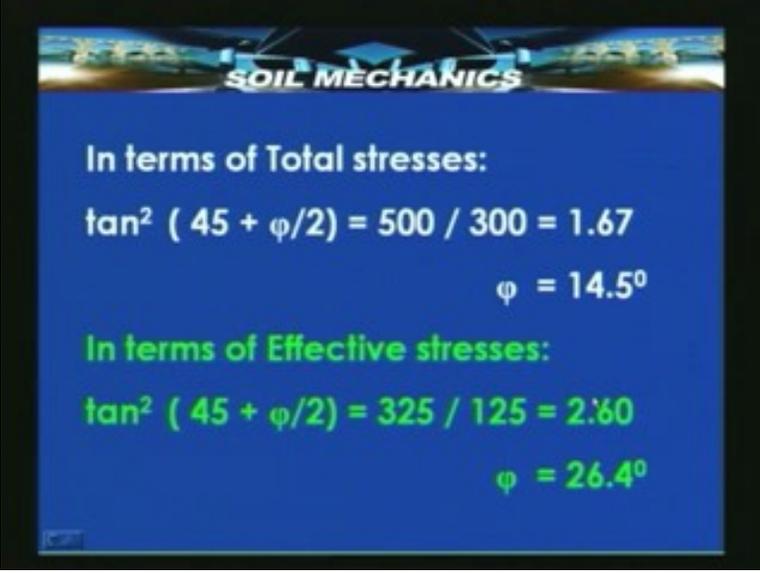
The parameters can be expressed in terms of both Total and Effective stresses.

They can be determined either by using Mohr's circle or by using formula.

Here the formula is used. The student is advised to try solving graphically using Mohr's diagram and compare.



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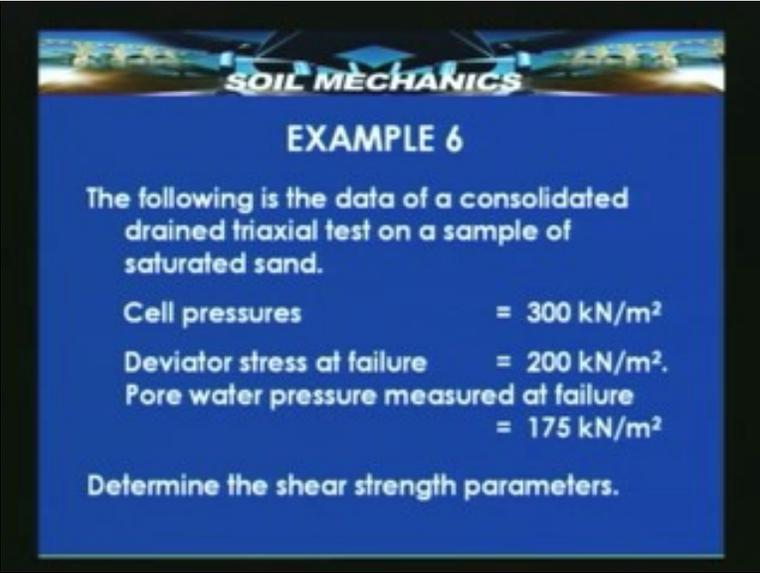
**SOIL MECHANICS**

In terms of Total stresses:  
 $\tan^2 ( 45 + \phi/2) = 500 / 300 = 1.67$   
 $\phi = 14.5^\circ$

In terms of Effective stresses:  
 $\tan^2 ( 45 + \phi/2) = 325 / 125 = 2.60$   
 $\phi = 26.4^\circ$

Here the example which is given makes use of the formula but a Mohr diagram can be easily plotted and the student is advised to try that out himself and compare the results with the formula and find out how right he is. So this is what the typical Mohr circle looks like in a triaxial test. Here we have the point corresponding to the major principle stress and the minor principle stress and this is the failure plane at an angle theta critical. Now in terms of total stresses we can say that tan square 45 plus phi by 2 is equal to 500 by 300. Why 500 by 300?

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**SOIL MECHANICS**

**EXAMPLE 6**

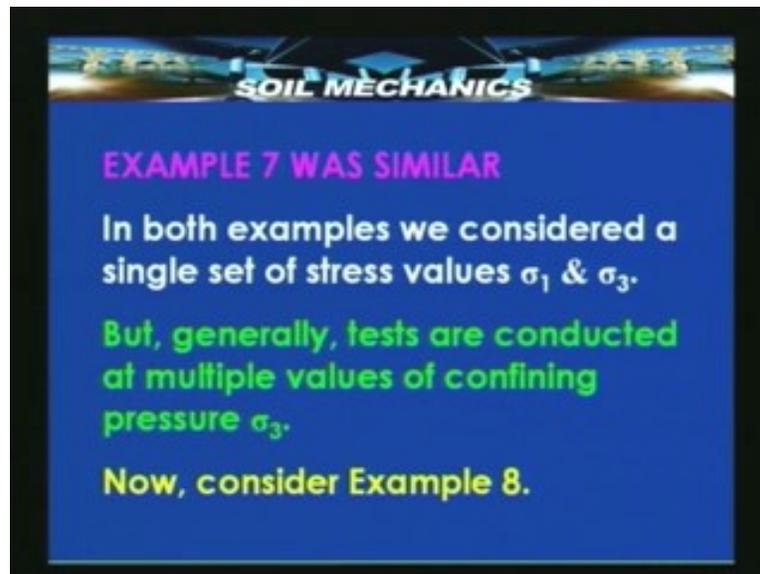
The following is the data of a consolidated drained triaxial test on a sample of saturated sand.

Cell pressures	= 300 kN/m <sup>2</sup>
Deviator stress at failure	= 200 kN/m <sup>2</sup> .
Pore water pressure measured at failure	= 175 kN/m <sup>2</sup>

Determine the shear strength parameters.

It is nothing but you can see here it is cell pressure plus the deviator stress that is  $\sigma_1$  and see here 500 (Refer Slide Time: 27:52) so  $\sigma_1$  upon  $\sigma_3$  that's what we are having here.

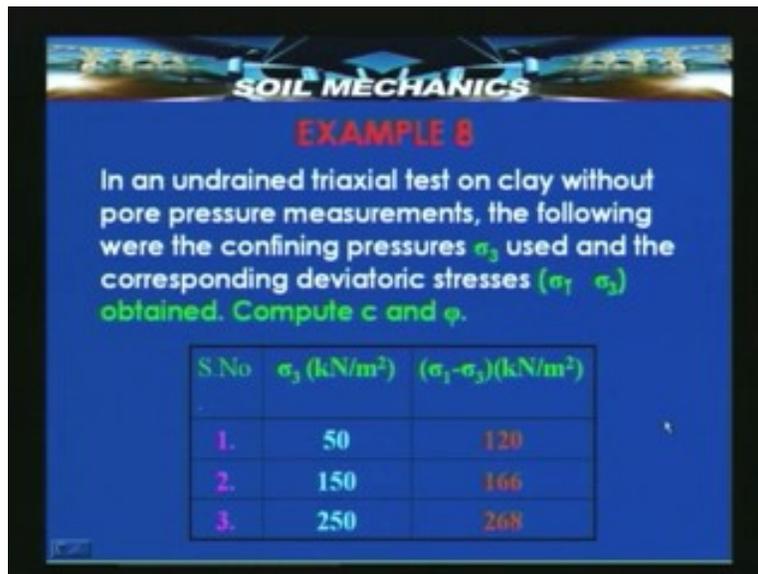
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$\sigma_1$  by  $\sigma_3$  is equal to  $\tan^2 45^\circ + \frac{2c}{\sigma_3 \tan \phi}$ , this is an expression which we had seen in one of the earlier lecture and from this we can get  $\phi$  equal to 14.5 degrees. And if we wish to use the effective stresses, what we need to do is to subtract the pore pressure from the values of  $\sigma_1$  and  $\sigma_3$ . And if you see go backwards, the pore pressure is 175 and coming back if you subtract 175 from here you have 325 and 125 which gives you the effective angular friction  $\phi$  equal to 26.4 degrees. We also saw another example, example number 7 which was very similar.

Now in both these example we considered single test with one  $\sigma_3$  and  $\sigma_1$  but as I mentioned a few minutes back in a typical triaxial test we always make use of several confining pressures and find out the corresponding failure axial stresses and plot Mohr circles one after the other, for each confining stress and find out a common tangent which will give us the failure envelop. So let us see the examples of that, let us go example number 8.

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**SOIL MECHANICS**

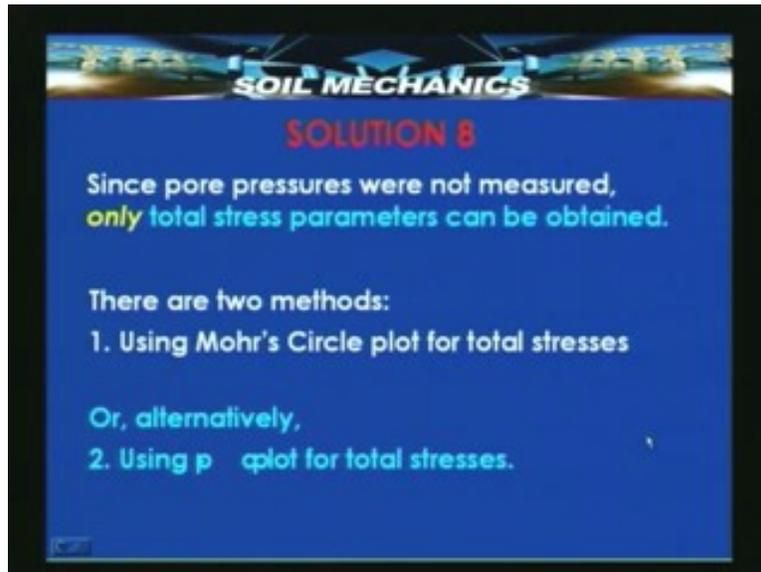
**EXAMPLE 8**

In an undrained triaxial test on clay without pore pressure measurements, the following were the confining pressures  $\sigma_3$  used and the corresponding deviatoric stresses ( $\sigma_1 - \sigma_3$ ) obtained. Compute  $c$  and  $\phi$ .

S.No	$\sigma_3$ (kN/m <sup>2</sup> )	$(\sigma_1 - \sigma_3)$ (kN/m <sup>2</sup> )
1.	50	120
2.	150	166
3.	250	268

In this example we are having data of undrained triaxial test on clay. So undrained triaxial test in drainage is not permitted, obviously it means that the stresses which we have are all total stresses. And incidentally in this example pore pressures were not measured and therefore the stresses which we have are the total stresses and in order to get the effective stresses, if required we need to have the pore pressure measurement. Now the values of the  $\sigma_1$  and  $\sigma_3$  are given here. The  $\sigma_3$  is this, the deviator stress is this. Normally we apply confining pressures of these ranges, of these magnitudes. So if I take these data and go further I find that since pore pressures are not measured, we can only determine the total stress parameters.

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**SOIL MECHANICS**

**SOLUTION 8**

Since pore pressures were not measured,  
**only total stress parameters can be obtained.**

There are two methods:

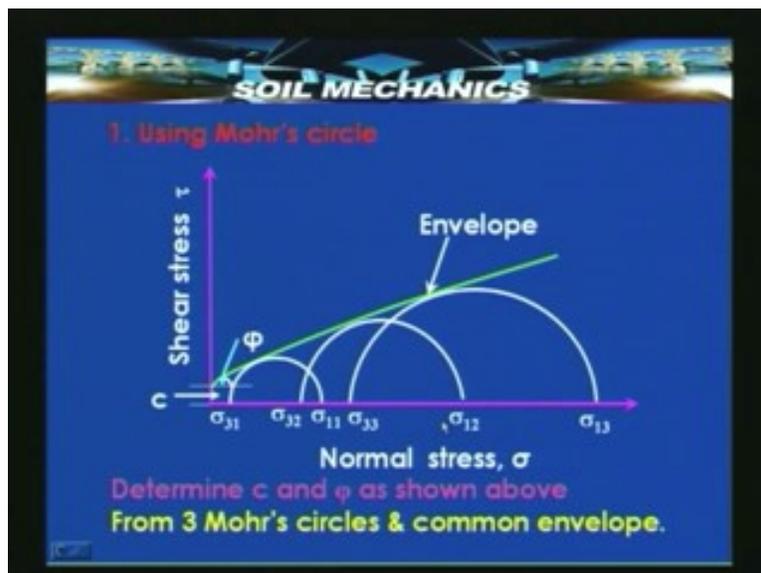
1. Using Mohr's Circle plot for total stresses

Or, alternatively,

2. Using  $p$   $q$  plot for total stresses.

To determine the total stress parameter today we will see two methods, one is of course the Mohr circle plot about which have talked earlier, for a long time. Another is so called  $p$   $q$  plot and both of them can be used for determining the total stress parameters. Here we have the typical Mohr's diagrams that is we have 3 stress values, minor principle stresses corresponding to 50, 150 and 250 and 3 axial stresses. What we need to do is to plot the 3 different Mohr cycle and then plot a common tangent. Now that's not shown here, the Mohr circle plots are not shown here. However the student may attempt drawing these Mohr circles and draw a common tangent.

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**SOIL MECHANICS**

**SOLUTION 8 ... contd.**

Alternatively, consider the already well known equation :

$$\sigma_1' = N_\phi \sigma_3' + 2c' \sqrt{N_\phi}$$

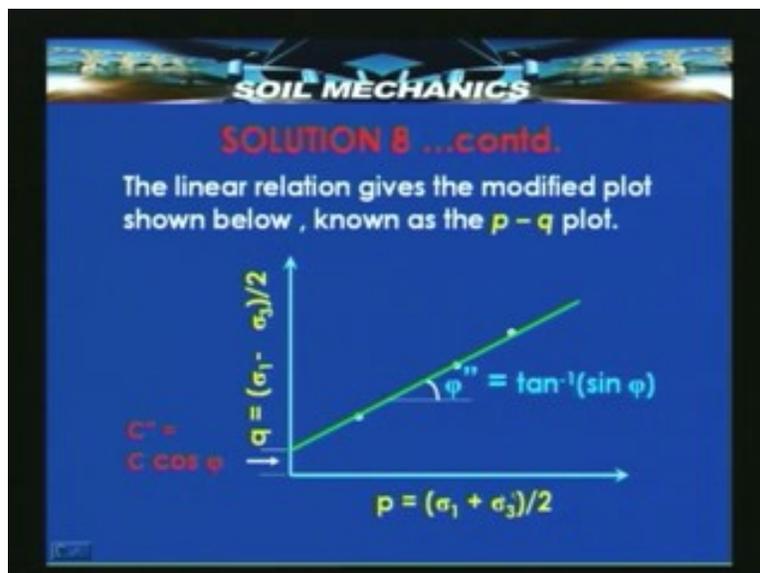
Or, its equivalent :

$$\frac{1}{2}(\sigma_1 - \sigma_3) = c'' + \frac{1}{2}(\sigma_1 + \sigma_3) \tan \phi''$$

Which gives the linear relationship...  
... shown in the next slide .... →

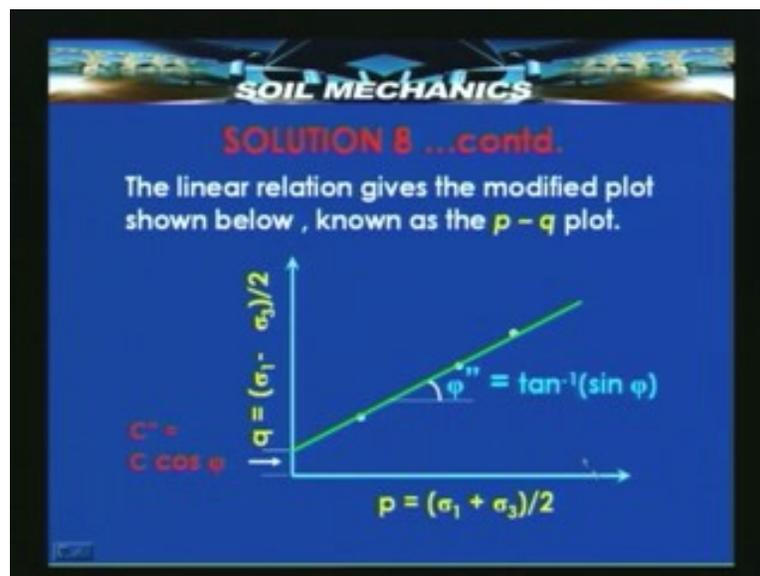
Alternatively we can use the so called p q plot. What is this p q plot? We know already this equation, the relationship between  $\sigma_1'$ ,  $\sigma_3'$ ,  $c'$  and  $\tan^2 45^\circ + \phi$  that is  $N_\phi$  or its equivalent which is this. Now you find this equivalent expression that is the second expression is a linear expression and if you therefore plot half of  $\sigma_1 + \sigma_3$  along the x axis and half of  $\sigma_1 - \sigma_3$  along the y axis we will get a linear relationship which is shown in the next slide.

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Here we have half of  $\sigma_1 + \sigma_3$  and we have half of  $\sigma_1 - \sigma_3$  along the y axis. This is known as  $p$  this is known as  $q$  because it's a linear relationship the  $p$   $q$  plot as it is called will be a straight line. Here I have an example of a  $p$   $q$  plot. What is special about this  $p$   $q$  plot is if you go back to the basic equations. The slope of this will be given by  $\tan \phi'$  and the intercept will be given by  $c'$ . So this intercept  $c'$  is here and the slope of this line is given by the angle of inclination which is  $\phi'$ . Now you can compare this equation with the basic equation which is given here the original one and you can show that  $c'$  is related to  $c$  and  $\phi'$  is related to  $\phi$ .

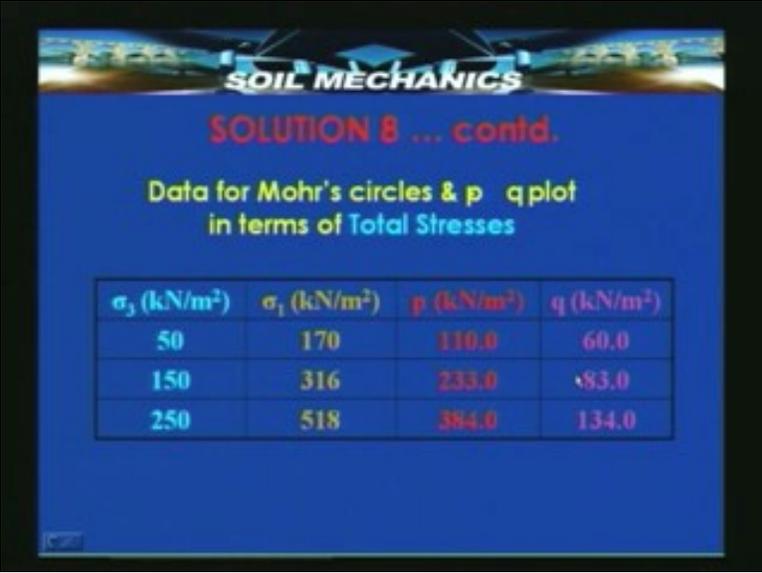
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That relationship is  $\phi'$  is  $\tan$  inverse of  $\sin \phi$  and  $c'$  is  $c \cos \phi$ . So what this means is ultimately we are interested in the shear strength parameters  $c$  and  $\phi$ . By taking the straight line plot we can find out  $\sin \phi$  which is equal to  $\tan \phi'$  and then from  $\sin \phi$  we can find  $\phi$ . Then we have  $c'$  which is equal to  $c \cos \phi$ , knowing  $c'$  which is an intercept here we can always find out  $c$  because  $\phi$  is already determined from this.

So this is what normally done in order to determine the shear strength parameters. So now if I take the total stresses that have been obtained in this experiment, we have  $\sigma_3$  total stress we have  $\sigma_1$  the total stress and corresponding values of  $p$  and  $q$  will be  $\sigma_1 + \sigma_3$  by 2,  $\sigma_1 - \sigma_3$  by 2 and these are the values. These values can be used for drawing the  $p$   $q$  plot.

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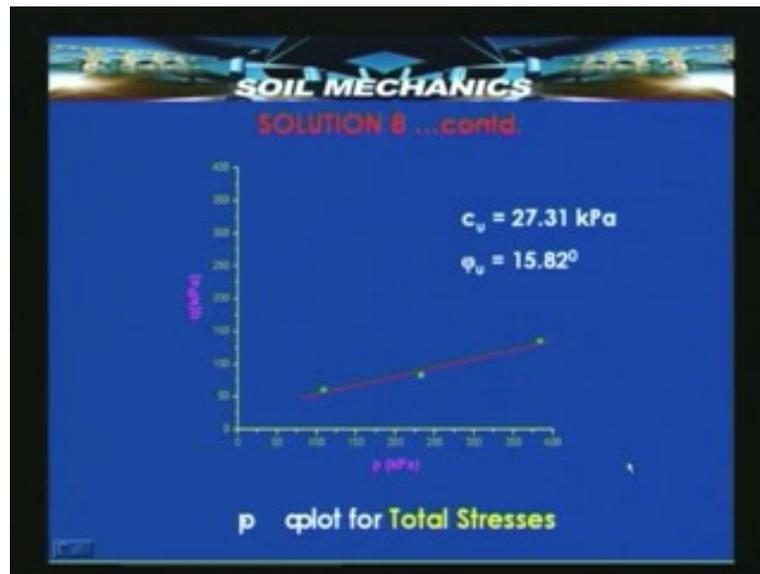


The slide displays a table with the following data:

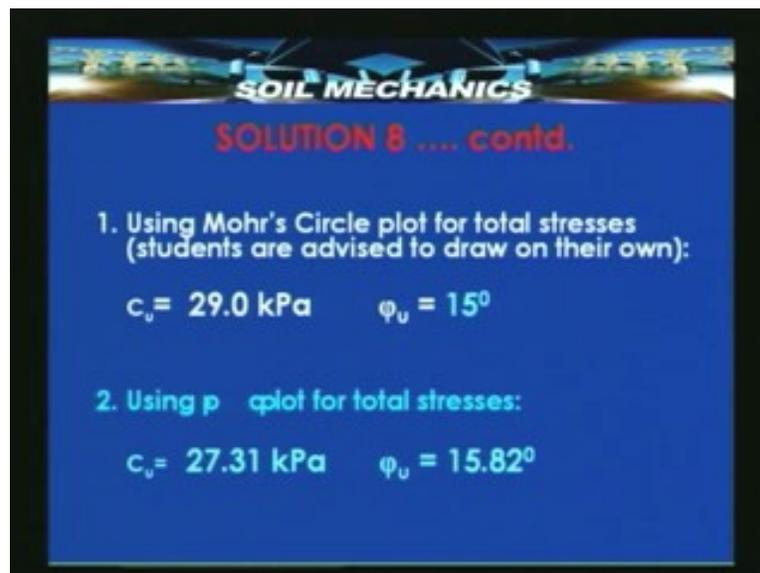
$\sigma_3$ (kN/m <sup>2</sup> )	$\sigma_1$ (kN/m <sup>2</sup> )	p (kN/m <sup>2</sup> )	q (kN/m <sup>2</sup> )
50	170	110.0	60.0
150	316	233.0	83.0
250	518	384.0	134.0

Here is the p q plot, so as we saw the data of the problem is adequate enough to compute p and q values and plot them. So if you plot p and q this is a kind of a plot you will get for the total stresses and this being a straight line you can fit a straight line through all the 3 points and you can find out the intercepts and the angle of inclination from which you can calculate the cohesion and friction parameters corresponding to the total stress conditions that is the undrained conditions which are known as usually  $c_u$  and  $\phi_{iu}$  undrained. So in this particular problem we get 27.31 kilo Pascal's and 15.82 degrees. This is the situation for the total stresses but since in this problem no pore pressure where known, we can only get total stresses and total stress parameters.

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Now let's go further. If you plot the Mohr circle diagram which is not shown here in the slide you will find that you will get slightly different values of  $c_u$  and  $\phi_u$  compared to what we got for the p q plot. In the p q plot we got 27.31 and 15.82 whereas if you plot the Mohr circle diagram and get the values of  $c_u$  and  $\phi_u$  you will find 29.0 and  $\phi_u$  equal to 15 degrees. A slight difference is always permissible because they are both graphical procedures but by in large the p q plot is preferred these days because a straight line can be plotted more reliably than a Mohr circle and a common tangent. And going by that statement, we can say that the values of  $c_u$  and

$\phi_u$  which we have observed from the p q plot may be taken as the appropriate value for the particular soil.

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**SOIL MECHANICS**

**EXAMPLE 9**

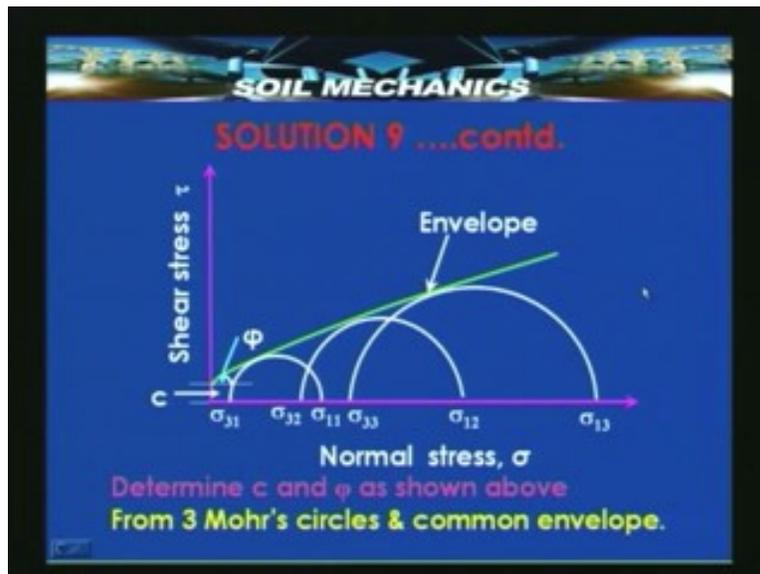
In a triaxial test on undisturbed clay soil specimens, the following were the confining pressures  $\sigma_3$  used and the corresponding deviatoric stresses  $(\sigma_1 - \sigma_3)$  and pore pressures  $u$  obtained at failure- in  $\text{kN/m}^2$ . Compute  $c$ ,  $\phi$ .

S.No	$\sigma_3$ ( $\text{kN/m}^2$ )	$(\sigma_1 - \sigma_3)$ ( $\text{kN/m}^2$ )	$u$ ( $\text{kN/m}^2$ )
1.	50	41	45
2.	150	112	105
3.	250	176	160

One more example here again the triaxial test and undisturbed clay sample and confining pressure used were same 50, 150 and 250 kilo newton per meter square as in the previous example but the deviatoric stress are different 41 and 112 and 176 and the pore pressure were measured. The pore pressure at failure are 45, 105 and 160. So this is the situation in which we are in a position to calculate the effective values of  $\sigma_3$  and  $\sigma_1$  and therefore we can, not only get total stress values  $p$  and  $q$  but we can also get  $p$  dash and  $q$  dash and therefore we can get  $c_u$   $\phi_u$  corresponding to the total stress conditions and  $c$  dash  $\phi$  dash corresponding to the effective stress conditions.

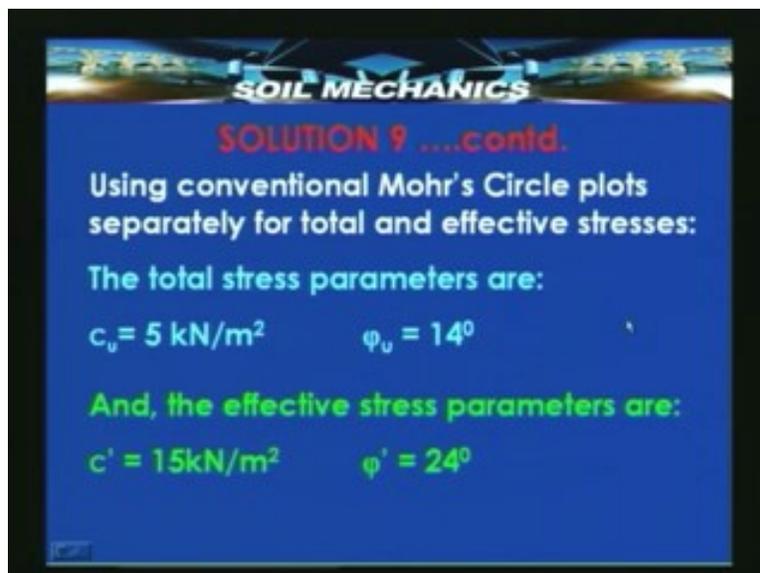
Again we can use either the Mohr circle plot or we can use the p q plot. So let's proceed. If I consider total stresses first I find that, I know  $\sigma_3$  and  $\sigma_1 - \sigma_3$ . It will give raise to three values of  $\sigma_1$  like this. Since we know the pore pressure we can subtract the pore pressure from the given values of  $\sigma_3$  and the computed values of  $\sigma_1$  and we will get  $\sigma_3$  dash and  $\sigma_1$  dash and also  $\sigma_1$  dash -  $\sigma_3$  dash the deviatoric stress. Since the pore pressure  $u$  gets detected from  $\sigma_1$  as well as  $\sigma_3$ , the difference between  $\sigma_1$  and  $\sigma_3$  and  $\sigma_1$  dash,  $\sigma_3$  dash will both be same. So the deviatoric stress either in the total stress or in effective stress manner they are identical.

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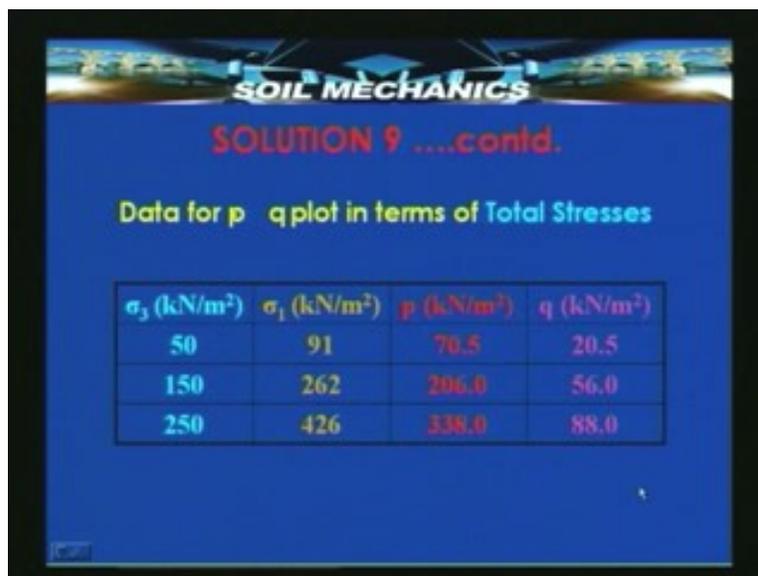
Once again either we get the shear stress parameter using the more envelope like this, plot three Mohr circles draw a common tangent or you can get it from the p q plot. If we use the Mohr circle plots we can draw Mohr circle plots for the total stresses entirely or for effective stresses entirely. If I were to plot the 3 Mohr circles using the total stress value and get the intercept and inclination of the common tangent I will get 5 kilo newton per meter square as the undrained cohesion and 14 degrees as the angular friction.

(Refer Slide Time 39:13)



If on the other hand I use  $\sigma_1$  dash and  $\sigma_3$  dash for each of the 3 tests, plot three Mohr circles draw a common tangent I will get the corresponding effective stress, shear stress parameters intercepts and angle of internal friction as  $c$  dash is equal to 15 kilo Newton per meters and  $\phi$  dash is equal to 24 degrees. You can find that there is a large difference here that's what happens when you don't have drainage and when you have drainage. When you don't have drainage and measure the pore pressure this is what you get. Now suppose you want to determine the same thing using  $p$   $q$  plots then the data for plotting the  $p$   $q$  straight line in terms of total stresses it will be this because we already know the values of  $\sigma_3$  and the  $\sigma_1$ .

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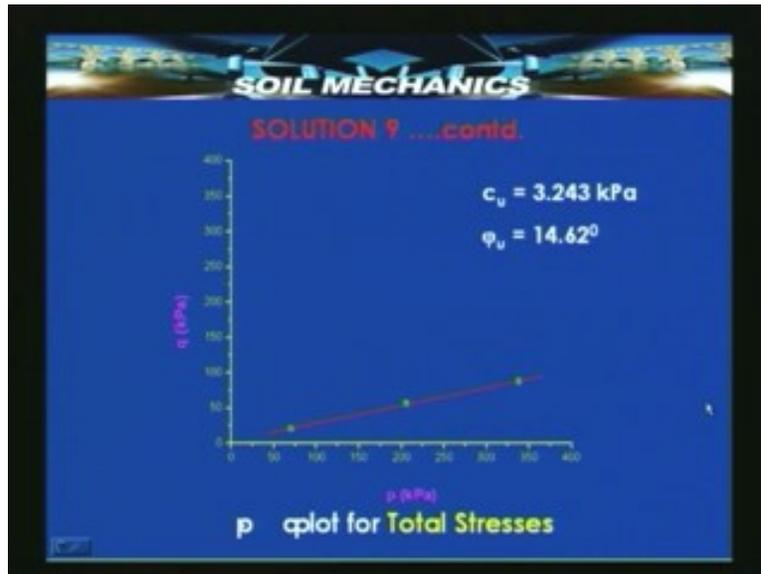
**SOIL MECHANICS**  
**SOLUTION 9 ....conld.**

**Data for p q plot in terms of Total Stresses**

$\sigma_3$ (kN/m <sup>2</sup> )	$\sigma_1$ (kN/m <sup>2</sup> )	$p$ (kN/m <sup>2</sup> )	$q$ (kN/m <sup>2</sup> )
50	91	70.5	20.5
150	262	206.0	56.0
250	426	338.0	88.0

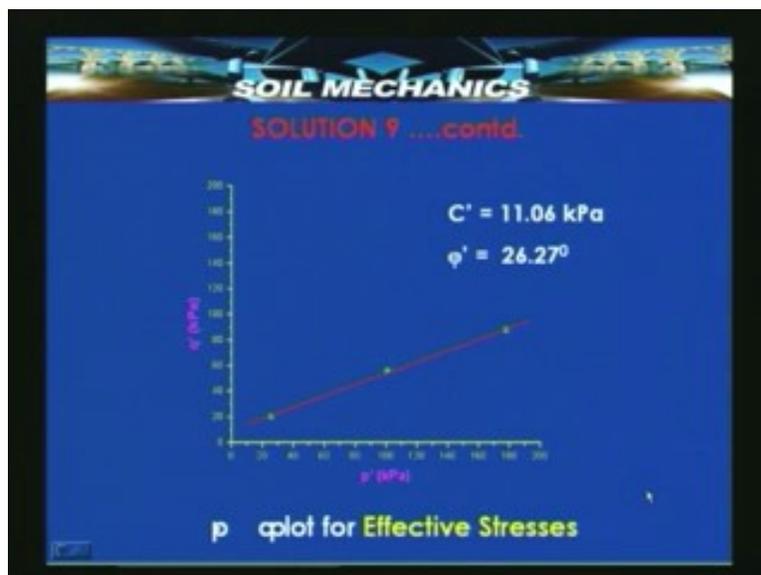
$P$  is nothing but sum of the 2 by 2,  $q$  is nothing but the difference between the 2 divided by 2. So if we plot this value of  $p$  and this value of  $q$  we get the total stress  $p$   $q$  plot. So here is the total stress  $p$   $q$  plot for this test and from this the intercept and the angle of inclination will gives us the undrained cohesion and undrained angular friction as 3 and 14 approximately.

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If we were to use the effective stress parameter then the data that we need for plotting the p q graph is,  $\sigma_3$  dash is this after subtracting the pore pressure,  $\sigma_1$  dash is this after subtracting the pore pressure, p dash will be the addition of 2 divided by 2, q dash is the difference divided by 2 (Refer Slide Time: 41:38).

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So we now have p dash which is quite different from the value of p we had earlier but q dash however remains the same because it is the difference between the stresses and as I pointed out a

little earlier  $\sigma_1 - \sigma_3$  is equal to  $\sigma_1$  dash minus  $\sigma_3$  dash. So we have p dash and q dash if we plot them we get the p q plot for the effective stresses and from here from this intercepts and angle of inclination we can get the effective cohesion and effective angle of friction as 11 and 26 as shown in this graph.

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**SOIL MECHANICS**

**SOLUTION 9 .....contd.**

From modified (p – q) plots separately...  
for Total and Effective stresses:

**1. The total stress parameters are:**

$c_u = 3.243 \text{ kN/m}^2 \quad \phi_u = 14.62^\circ$

→ From Mohr's circle: [5 kN/m<sup>2</sup>] & [14°]

**2. And, the effective stress parameters are:**

$c' = 11.06 \text{ kN/m}^2 \quad \phi' = 26.27^\circ$

→ From Mohr's circle: [15 kN/m<sup>2</sup>] & [24°]

So to summarize we get undrained cohesion and undrained friction under total stress conditions from the p q graph like this. The corresponding Mohr circles gave us 5 and 14 the difference is marginal and this much difference does occur because of the two ways of plotting. And as I said p q plot always preferred because plotting a straight line is always easier and more accurate than plotting a circle and a common tangent. Similarly under effective stress condition c dash and phi dash are like this for p q plot whereas the corresponding Mohr circles values which we got a little earlier on 15 and 24 degrees. So this is what a typical triaxial test is all about we conducted the test, measuring the pore pressure usually so at the end of the test we not only have the total stresses that we have applied but also the pore pressure and therefore the effective stresses. So we can calculate both the total strength parameters and the effective strength parameters.

The utility of these parameters we shall be seeing in the next lecture. When we are going to look at shear strength in a different way that is from point of view of the effective of drainage undrained and drained conditions and their effects on shear strength. Let us move further, take another example. Example 10 is a test on rock.

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**SOIL MECHANICS**

**EXAMPLE 10**

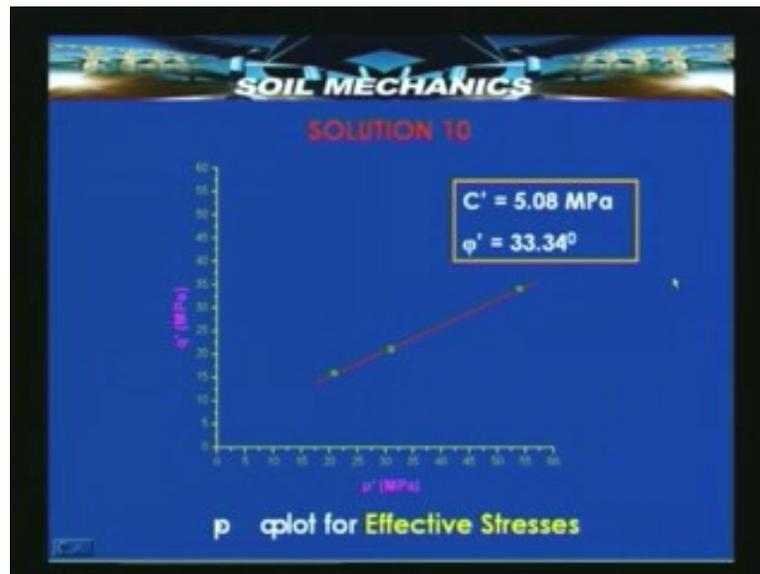
In a drained triaxial test on Volcanic Breccia, the following were the confining pressures  $\sigma_3$  used and the corresponding deviatoric stresses ( $\sigma_1 - \sigma_3$ ). Compute  $c$  and  $\phi$ .

S.No	$\sigma'_3$ (MPa)	$(\sigma'_1 - \sigma'_3)$ (MPa)	$\sigma'_1$ (MPa)
1.	5	31.8	36.8
2.	10	42.2	52.2
3.	20	68.0	80.0

Using  $p - q$  plot we get →

The triaxial test can also be conducted on rocks but what important to notice is the kind of stress that are required to make a rock sample fail under shear in triaxial conditions are much higher than what we require for soils. So in a drained triaxial test on volcanic breccia we get values of  $\sigma_3$  and deviatoric stresses  $\sigma_1$  and  $\sigma_3$  which are much higher than what are usually obtained in the case of soils. So if  $\sigma_3$  is 5, 10 and 20 mega Pascal's, deviatoric stresses are 31.8, 42.2 and 68 mega Pascal's. Then  $\sigma_1$  will be 36.8, 52.2 and 80 mega Pascal's. Now from this data we can calculate  $p$  and  $q$  and get the  $p - q$  plot. Since this is a rock sample we cannot measure pore pressure in a rock sample because it's virtually impossible and we get a  $p - q$  plot therefore in terms of the effective stresses.

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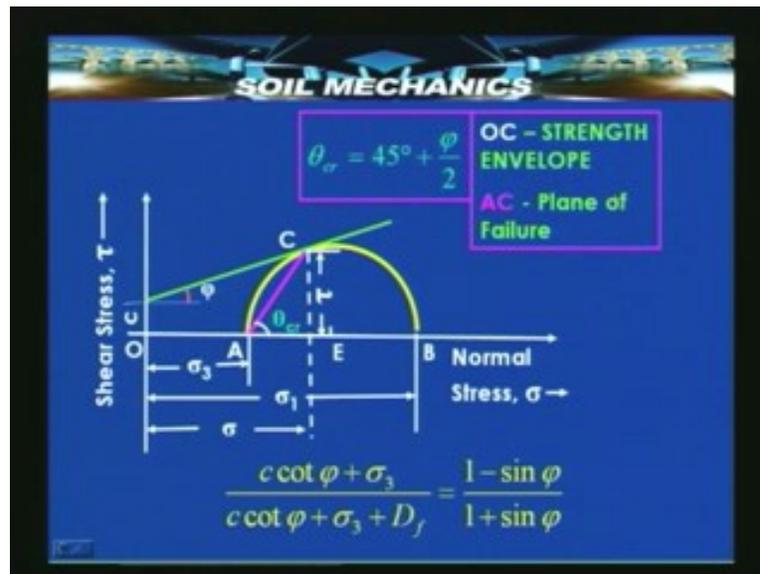
The figure is a slide titled "SOIL MECHANICS EXAMPLE 11". It contains the following text:  
A clay specimen is tested under fully drained condition in a triaxial test. The cell pressure used for consolidation was  $80 \text{ kN/m}^2$ .  
The shear strength parameters with respect to effective stresses are  $c' = 10 \text{ kN/m}^2$  and  $\phi' = 20^\circ$ .  
Determine the compressive strength.  
From Lecture No. 3 we know that, in a Mohr's circle at failure

A blue arrow points to the right at the end of the last line of text.

The p q plot in terms of the effective stresses for this rock gives us a c dash of 5 mega Pascal's and phi dash of 33.34 degrees. So this is the utility of the p q plot, there is a lot of advantage in drawing a straight line through the test points than in plotting Mohr circles and drawing a common tangent. Now let us see one more example of triaxial testing. A clay specimen here is tested under a condition of full drainage; the cell pressure used is 80. Here these shear stress parameters are given c dash and phi dash the effective stress parameters. What's required is the compressive strength of the specimen that is now from shear strength we want to determine the

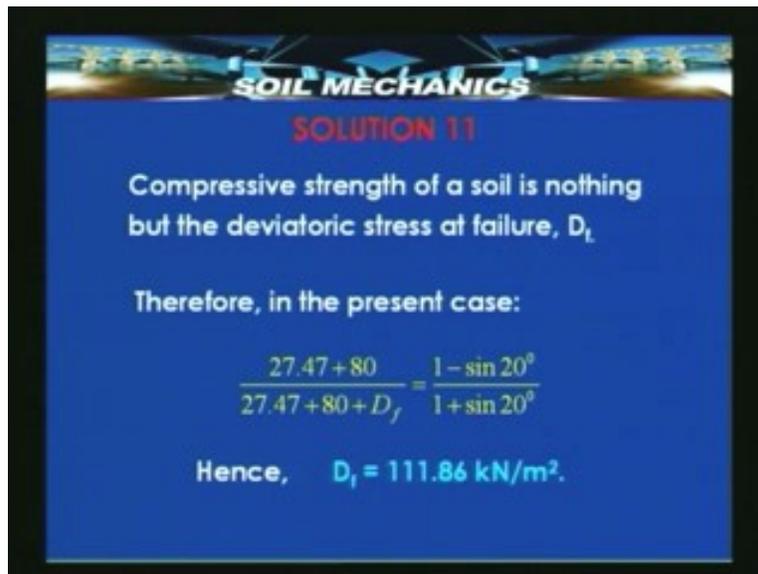
compressive strength of the soil. It's possible from the triaxial shear test data. From lecture number 3 we know that there is a certain relationship that holds good in a Mohr circle at failure. Let's see what is the relationship in the next slide.

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This is the Mohr circle and from geometry we can easily derive this relationship here which holds good for a Mohr circle at failure,  $c \cot \phi$  that's what you will get here on the x axis when you produce the envelop and make it cut the x axis here. This is  $c$  the angle which will be made will be  $\phi$  and therefore the distance here will be  $c \cot \phi$ ,  $c \cot \phi + \sigma_3$  is the distance from the shifted origin up to the point A and  $c \cot \phi + \sigma_3 + \sigma D$  the deviatoric stress or  $D_f$  at failure is the total length here. So one divided by the other can be shown to be equal to  $1 - \sin \phi$  by  $1 + \sin \phi$ . This is simply a modified form of writing an equation which we saw a little earlier in terms of  $\tan^2 45 + \phi$  by 2. So from this equation it's very easy to calculate  $c$  and  $\phi$  and that's what we will do.

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**SOIL MECHANICS**

**SOLUTION 11**

Compressive strength of a soil is nothing but the deviatoric stress at failure,  $D_f$

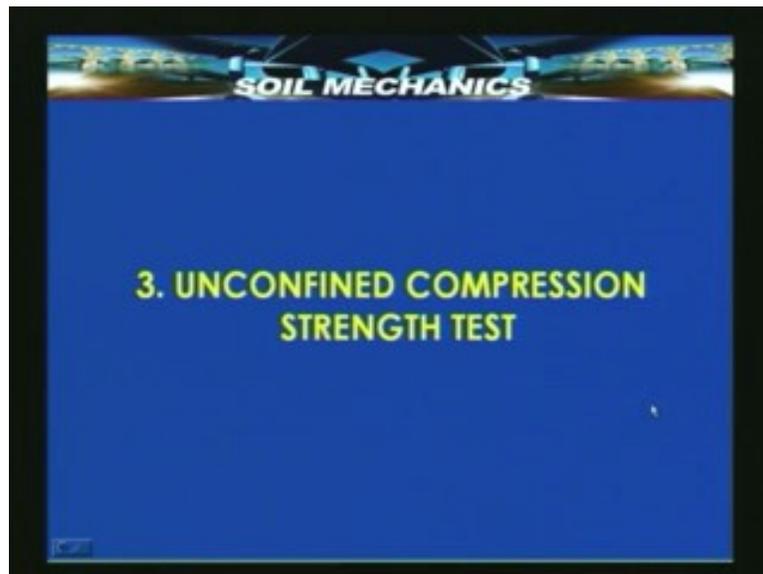
Therefore, in the present case:

$$\frac{27.47 + 80}{27.47 + 80 + D_f} = \frac{1 - \sin 20^\circ}{1 + \sin 20^\circ}$$

Hence,  $D_f = 111.86 \text{ kN/m}^2$ .

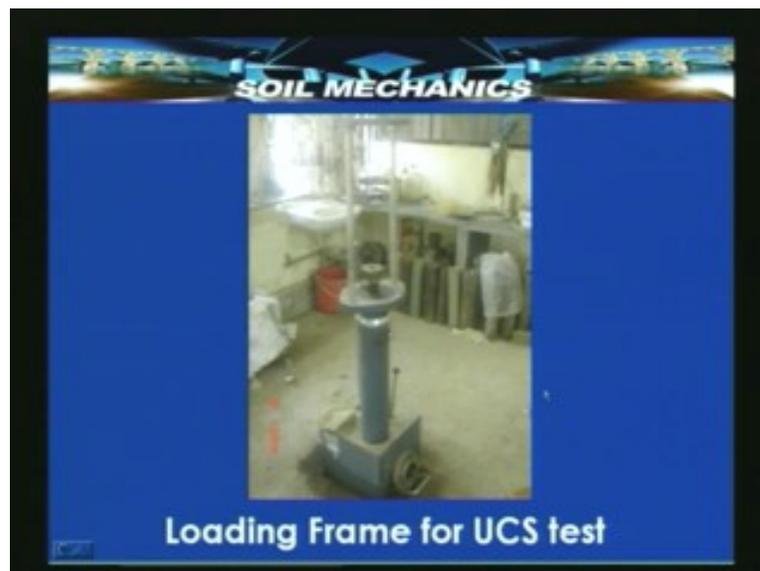
Compressive strength of a soil is nothing but the deviatoric stress at failure  $D_f$ . So straight away from this equation the unknown  $D_f$  can be calculated because  $\sigma_3$  is known,  $\phi$  is known and  $c$  is known. So if you substitute the known values, you will find that  $c \cot \phi$  is nothing but 27.47. This is  $\sigma_3$ ,  $c \cot \phi$  plus  $\sigma_3$  plus  $D_f$ . So this is equal to  $1 - \sin 20$  by  $1 + \sin 20$  where 20 is the angle of internal friction which is known. This gives us  $D_f$  equal to 111.86 kilo newton per meter square that's the compressive strength of the soil sample. Now there are two other test, the unconfined compression strength test and the vane shear test which we will see now. These two tests are primarily meant for a clay soils, they can be done in the laboratory as well as in the field.

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So if I take unconfined compressive strength test this is what the equipment looks like. It's a simple loading frame with a facility to keep a soil specimen and load it and test it to failure without applying any confining pressure. So this is known as the UCS test.

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**SOIL MECHANICS**

**Unconfined Compression Test**  
**is a special case of Triaxial Test**  
**where there is no confinement,**  
**i.e.  $\sigma_3 = 0$**

Hence, usually employed to test cohesive soils.  
Hence, Axial load is applied rapidly to failure;  
Therefore, undrained conditions prevail,  $\phi_u = 0$ ;

An unconfined compression test is a special case of a triaxial test because the only difference between the two is in unconfined compressive test  $\sigma_3$  is zero. Now this is usually employed to test cohesive soils, hence an axial load is applied rapidly to failure. There is no confining pressure applied at all therefore undrained condition will prevail obviously because the loading is rapid which means the angle of internal friction will not get mobilized at all. That means  $\phi_u$  is zero therefore the corresponding Mohr circle for this will be a single circle which will pass through the origin.

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**SOIL MECHANICS**

Since there is only one confining pressure ( $= 0$ ),  
The Mohr's circle for UCS Test is as given below:

$\sigma_3 = 0$        $\sigma_1 = q_u$

Normal stress,  $\sigma$

Shear stress  $\tau$

Envelope

$\phi_u = 0$

$c_u$

$\sigma_1 =$  Axial stress at failure  
= Unconfined Compressive Strength,  $q_u$   
 $c_u = q_u/2$

This semi-circle is a Mohr circle,  $\sigma_3$  being zero this point will coincide with the origin and  $\sigma_1$  is the stress axial stress at failure. If you draw a tangent to this obviously this tangent here will give you the undrained cohesion and  $\sigma_1 - \sigma_3$  is nothing but the compressive strength and therefore what you get here at  $\sigma_1$  is nothing but the unconfined compressive strength the specimen that is  $q_u$  and it is from this that the name UCS test is derived. So once you have this kind of a Mohr circles you just measure the axial stress  $\sigma_1$  that's the confined compressive strength.

Then what happens to the cohesions? The cohesion is nothing the radius that's equal to half the diameter that is it is equal to  $q_u$  by 2. So unconfined compressive strength  $q_u$  divided by 2 gives you the undrained cohesions. So this is a very simple and very effective convenient method of determining the shear strength parameter and the compressive strength of a clay soil can be done both in the lab and in the field. Let's take an example.

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**SOIL MECHANICS**

UCS is computed as:

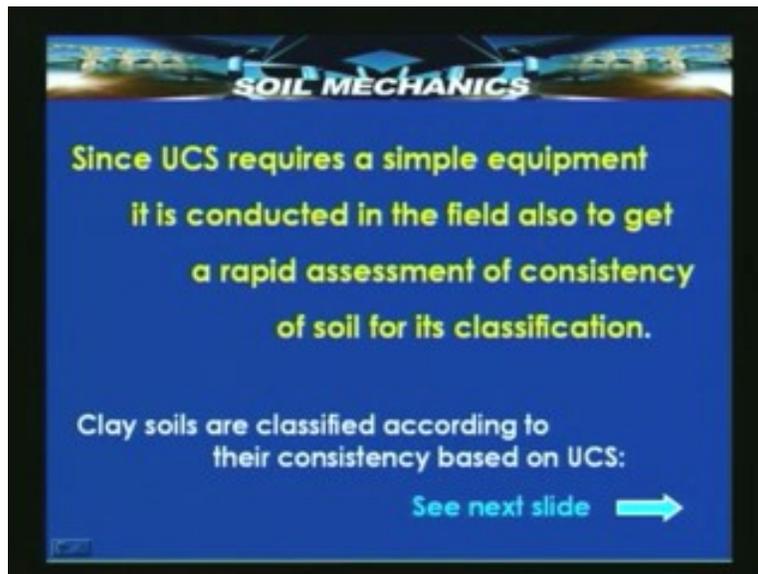
$$q_u = \sigma_1 / A_f \text{ where } A_f = A_0 / (1 - \epsilon)$$

Here,

- $A_0$  = Initial area of cross-section of specimen  
 $= \pi d^2 / 4 = 11.34 \text{ cm}^2$
- $d$  = initial diameter of specimen ( 38 mm )
- $A_f$  = Area of cross-section at failure
- $\epsilon$  = axial strain in specimen =  $\Delta h / h$
- $\Delta h$  = change in height of specimen
- $h$  = original height of specimen ( 76 mm )

The example would involve a number of parameters which are described here and their relationships are also given here.  $A_0$  is the initial cross section; there is a vertical axial strain that the sample undergoes. So if you take that into account and calculate the modified area of cross section of the specimen you will get  $A_f$ . So from this you can always find out what will be the final stress that acts on the specimen which is what required in order to plot the Mohr circles.

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Now UCS requires simple equipment. It is conducted in the field and it's a method for rapid assessment of consistency of soil for its classification. Clay soils are classified according to their consistency based on the value of unconfined compressive strength as shown in this slide. Depending upon the value of the undrained cohesion or the compressive strength  $q_u$  the consistency of a sample is defined like this.

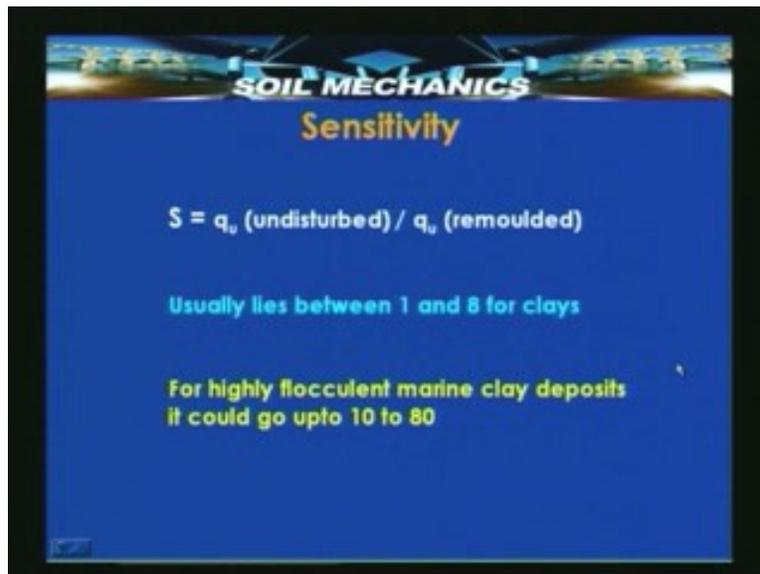
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**SOIL MECHANICS**

**General Relationship Between  
UCS and Consistency for Clays**

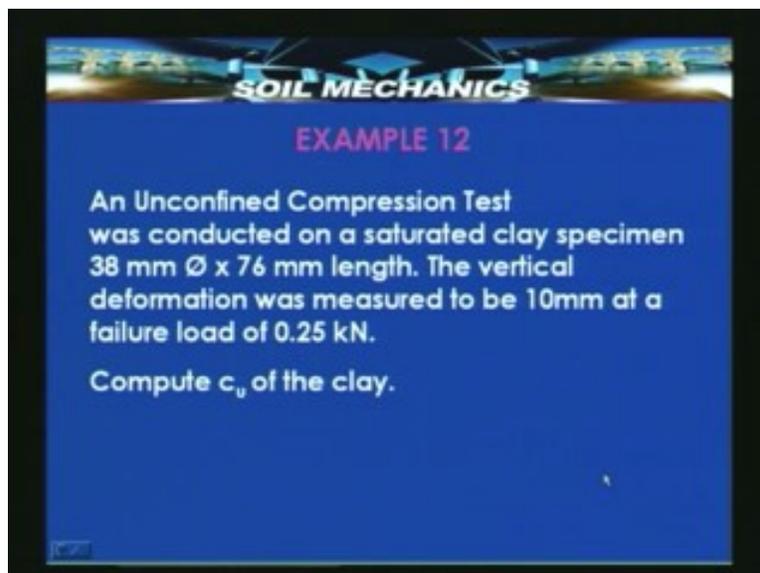
Consistency	$q_u$ (kPa)
Very Soft	0 to 24
Soft	24 to 48
Medium	48 to 96
Stiff	96 to 152
Very Stiff	192 to 383
Hard	> 383

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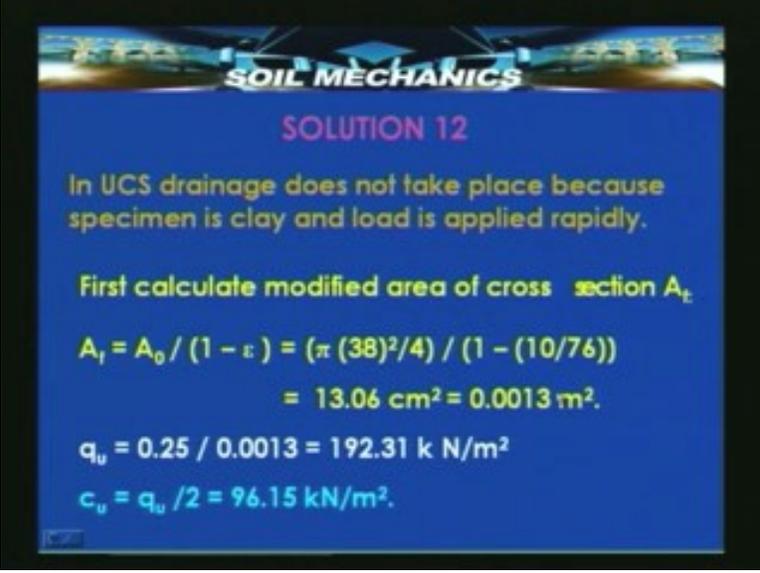


Now further the unconfined compressive strength of a soil depends upon the nature of disturbance that the soil is undergone. In the undisturbed state if it has a compressive strength  $q_u$  undisturbed and after remoulding if it has got an undrained compressive strength of  $q_u$ , the ratio of this is known as sensitivity that shows how sensitive the soil is to disturbance because there is a reduction in strength due to disturbance. This value usually lies in between 1 and 8 for clays and for flocculent marine clays it can even go from 10 to 80.

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**SOIL MECHANICS**

**SOLUTION 12**

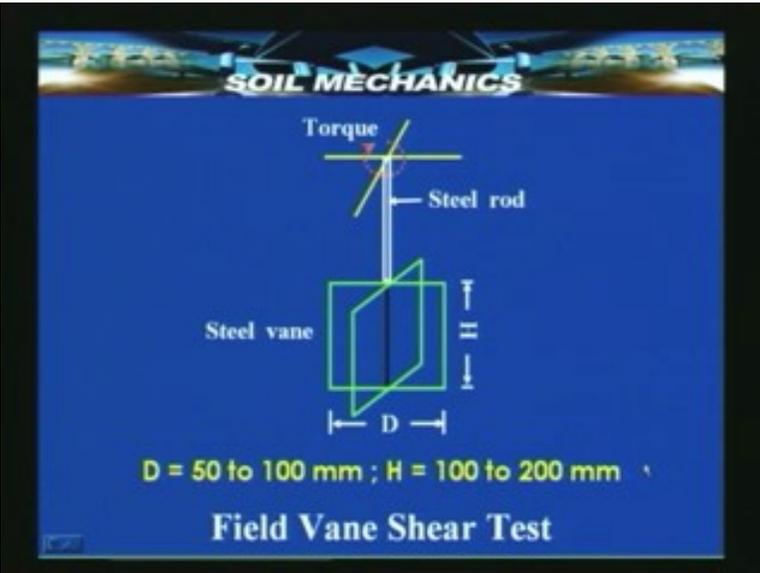
In UCS drainage does not take place because specimen is clay and load is applied rapidly.

First calculate modified area of cross section  $A_t$

$$A_t = A_0 / (1 - \epsilon) = (\pi (38)^2 / 4) / (1 - (10/76))$$
$$= 13.06 \text{ cm}^2 = 0.0013 \text{ m}^2.$$
$$q_u = 0.25 / 0.0013 = 192.31 \text{ k N/m}^2$$
$$c_u = q_u / 2 = 96.15 \text{ kN/m}^2.$$

Let us take one quick look at an example. The clay specimen which was tested in UCS has these dimensions 38 mm and 76 mm. The vertical deformation was measured to be 10 mm at failure and the load at failure was 0.25 kilo newton. In order to compute  $c_u$  of this clay all that we need to know is what is the final area of cross section corresponding to the strain vertical strain and then the load divided by the final area of cross section gives you the unconfined compressive strength of 192.31 kilo newton per meter square and the corresponding cohesion of 96.15. Now we will probably take a look at the vane shear stress in a subsequent lecture.

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**SOIL MECHANICS**

Torque

Steel rod

Steel vane

D

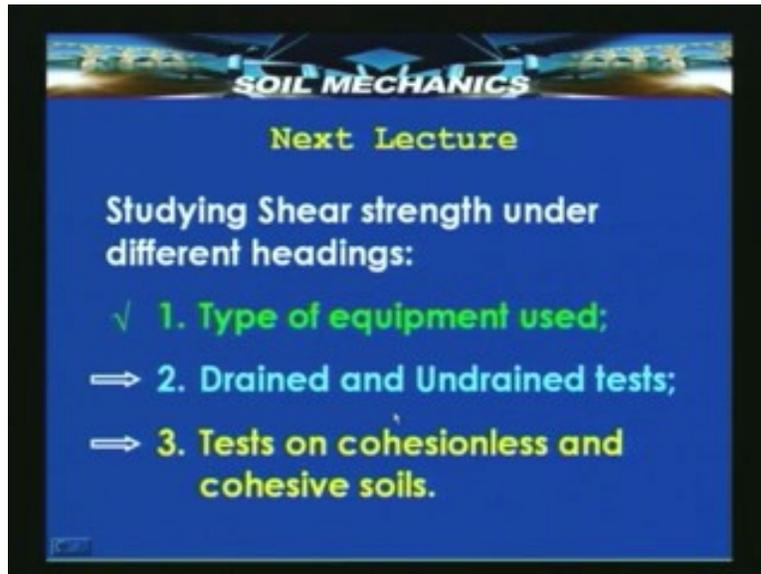
H

$D = 50 \text{ to } 100 \text{ mm} ; H = 100 \text{ to } 200 \text{ mm}$

**Field Vane Shear Test**

So let us now summarize what we have seen today. In this lecture today according to the classification of shear strength based on the type of equipment used we have completed seeing direct shear test, triaxial shear test and unconfined compression test and a few examples.

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In the next lecture we will continue with the type of equipment used and see the vane shear stress and proceed further with the undrained and drained conditions of testing.  
Thank you.