Fluid mechanics

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Lecture – 40
Pipe flow systems

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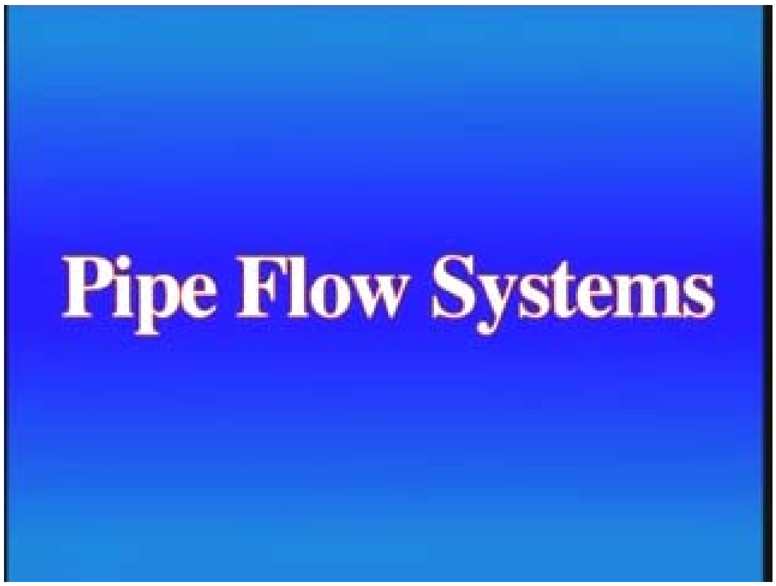

Welcome back to the video course on fluid mechanics. In the last lecturer we have discussed about the pipe flow systems. We have seen the pipes in parallel, pipes in series like that, we can have other system like branched pipe in systems and pipe network systems. So we have seen that, most of the time we will be solving this pipe flow systems either in parallel series or branched type of systems.

We will be solving mainly, by using the energy equations between various sections, and then we will be utilizing the continuity equations to get system situations and then, we will be solving for the unknowns. Whether it can be the unknowns can be the discharge at various locations at, or it can be at various pipes or it can be the head particle locations.

We will be writing all the equations by using the energy equation and then, or is it continuity equation and then we will be trying to solve the system. So in the last lecturer we have discussed about the pipes in series system and pipes in parallel system, now today we will discuss the branched pipe systems or branching pipe systems.

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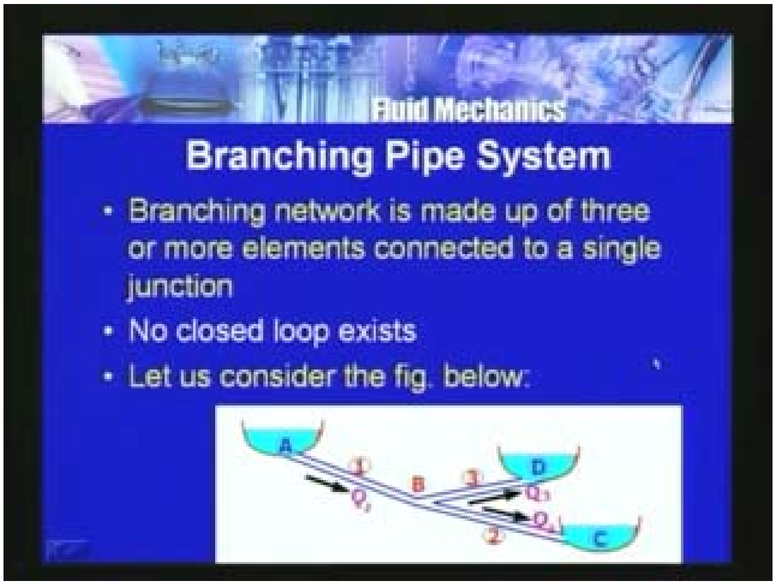

Here, the branching network is made up of either three or more elements connected to a single junction. Here say number of, may be connecting number of reservoir as number of tanks and then number of piping system is will be there between them.

So, there exists there is no looping or say networking of the pipes but we are connecting simply connecting various reservoir or various tanks together to form the branching pipe systems. So here there is no closed loop exists as such, so say for example, if we consider a system like this, here we have three reservoirs here reservoir A, reservoir C and reservoir D and we connect this three using three pipe line.

So here pipe 1, pipe 2 and pipe 3, there will be junction at B. We will first discuss how we can solve such a pipe kind of problem where, branching of pipe takes place. Here the non values may be say the head values at our elevation head reservoirs A, C and D and unknowns may be the discharge throw each pipe 1, 2, 3 and then, what is that head at location B?

First, we discuss how we can solve such a typical kind of these kinds of branching pipe systems, and then we will discuss about the pipe networks. Now as we discussed earlier, so here also, we will be using the basic energy equations between, say between towards otherwise between two points and then also the using the continuity equation say where ever junction comes.

So as we discuss earlier this kind of problem, in the analysis direction of flow we do not necessary. You can see that depending upon the elevation here A, C and D the flow can be in this direction like as shown in this arrow mark or depending on the elevation of D and C. It can be other way also, so we exactly do not know what the direction of the flow is.

So we will assume the direction of flow and then, we will work it out and see, what we assumed is right or not. Then as I mention earlier the energy equation for each element is written using an equivalent length account for the minor losses. So as we discuss in the case of pipe is series and pipe in parallel system.

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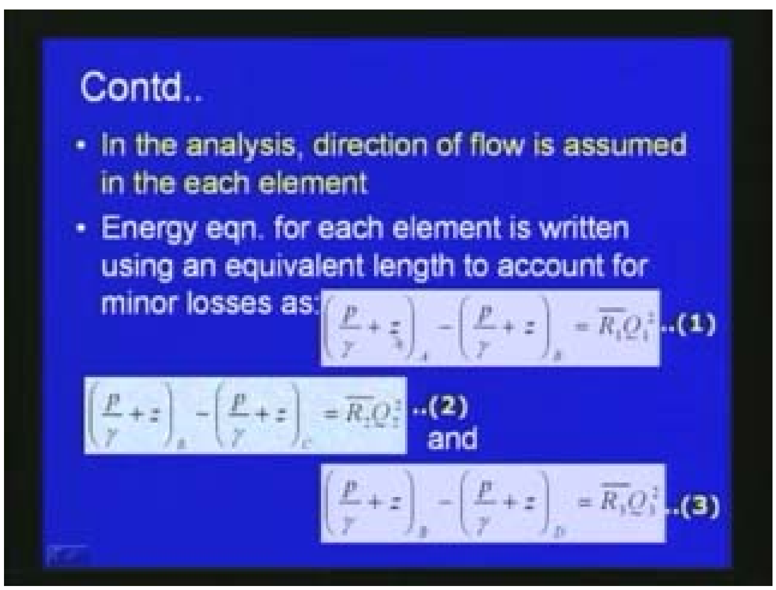

So the branching pipes say for example, if we consider we will consider now this between section A and B then B and D and B and C. So if we consider section between A and B, we can right the energy equation for each pipe element like as given here so between A and D. P by gamma plus z at A minus p by gamma plus z at B is equal to R_1 bar Q_1 square, where z is the determination at corresponding location say corresponding to A and B and then, where R_1 bar is the equivalent resistant coefficient for as we discuss in our in the earlier lecturer.

When we discuss about the pipe in parallel system for pipe in series system, so similar way we can equation (1) is return for between either energy equation between A and D. So now if you right the energy equation between this B and C, here we can write, P by gamma plus z at B minus P by gamma plus z at C is equal to R_2 bar Q_2 square, and similarly, we can right between B and D this between this point B and D as P by gamma plus z at B minus P by gamma plus z at D that is equal to R_3 bar Q_3 square.

So where this Q_1 is the discharge throw pipe 1, Q_2 is the discharge throw pipe 2, and Q_3 is discharge throw pipe 3. So now these 3 are the energy equation which we written between A and B and B and C and B and D.

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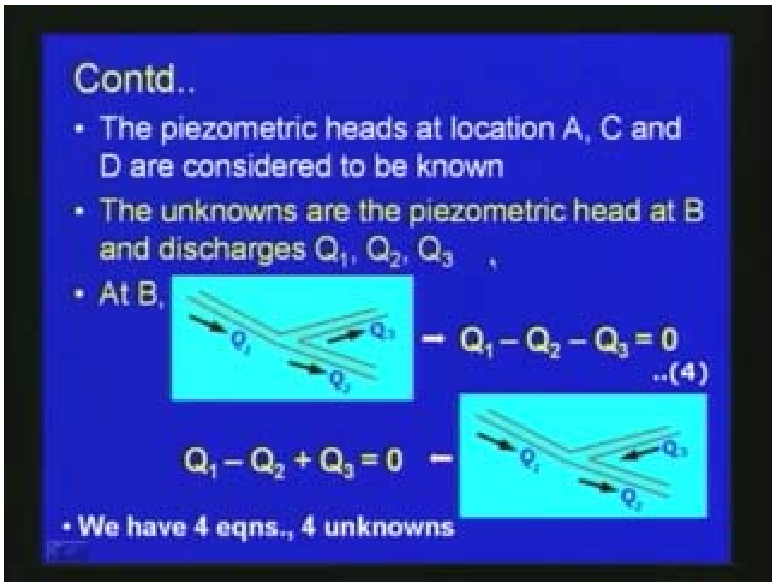

So now next say the piezometric heads at say location at A, C and D are considered to be known as I mentioned, so the reservoirs which are open or which we considered that head is not we assume that this here the piezometric heads at A, C and D are known then the unknown are the piezometric head at D and the discharges Q_1, Q_2, Q_3.

So by considering the typical problem, here we assume that the piezometric head is known at A, D and C and but this charges Q_1, Q_2, Q_3 are not known and also the head at B. So, there are four unknowns so this Q_1, Q_2, Q_3 and the head at B. So here at junction B is considered here, we can assume that, if we assume that flow is taking place in this direction Q_1 to the junction and Q_2 from the junction and Q_3 from the junction.

So that we can write the continuity equation with respect this given minus Q_1 minus Q_2 minus Q3 equal to ~~zero~~ 0, and as in equation number (4). But if we assume the other way Q_1 in this direction, Q_2 in this direction, and Q_3 towards the junction, then we will be writing Q_1 minus Q_2 plus Q_3 is equal to 0.

So we can assume the direction initially, and then we will do the calculations will find out the discharge and then accordingly make later, we can adjust the direction and depending upon our assumption. Now here you have three energy equation as written one, two three here between A and B between B and C and between B and D, and then either one of this, we are having one continuity equation either four or five. So we let us use this equation here assuming that, Q_1 is going in this direction, Q_2 from the junction and Q_3 also from the junction, Q_1 towards the junction, so we use this equation number (4).

We have ~~4~~ four equations and ~~4~~ four unknowns so that, we can easily solve this system the unknowns here are Q_1, Q_2, Q_3 and the head at junction B and then the equations have at junction B. We have written the continuity equation one equation, and then we have written three equations with respect to the energy equation between A and B, B and C and B and D. So these ~~4~~ four equations we can solve to find out these ~~4~~ four unknowns, so here in the slide the basic steps for such kind of solutions are given here. So what we assume here we start with step number one, assume a discharge of Q_1 in element 1 with or without pump if there is pump also in this particular system.

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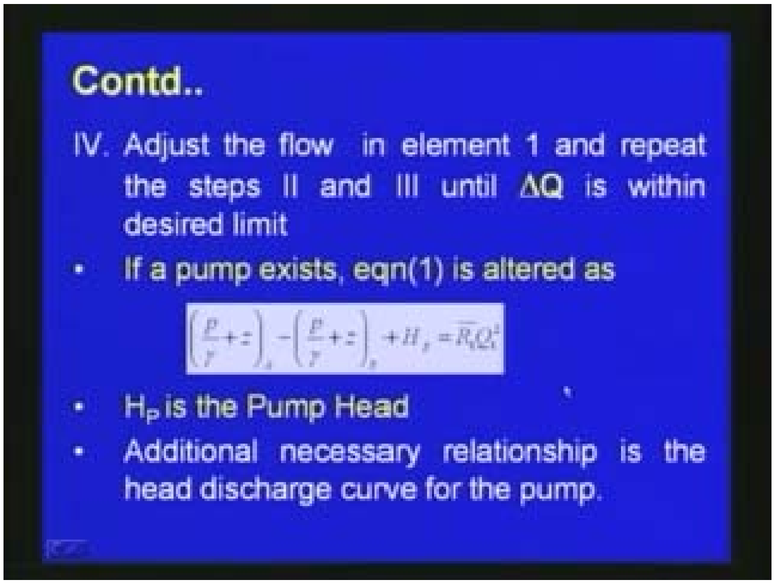

We have to consider that pump separately, so we assume a discharge Q_1 in element 1, and then establish the piezometric head, H at the junction by solving equation number (1), and then we compute the discharge in the remaining using equations (2) and (3), and we substitute Q_1 equation 4 to check for the continuity balance, flow imbalance will be delta Q is equal to Q_1 minus Q_2 minus Q_3.

So basically, we assume that say, we start with some assumption of the flow throw one of the pipe and then accordingly, then we solve equation 1and equation 2 and 3 which are the energy equations, and then we put back the continuity equations and see that weather imbalance is there, and then if there is imbalance we will again iterate.

So it is iterating procedure, the solution is iterative so then the fourth step is, we adjust the flow in element 1 and repeat the steps 2 and 3 until delta Q is within the desired limit. So for delta Q so when we do the iterate procedure, we can set the prior value this will be 0.005 or 0.0005 and like that, we can put some value and then, when the computer does the iteration when it reaches this particular limit, iteration is stopped and we finally get the value.

So that is the iterate procedure will we adopt for this kind of problem. So if a pump exists equation say for example say for there is pump say with respect A and B then we have to consider that pump also, that can be the equation can be return as P by gamma plus z at A minus P by gamma plus z at B plus H_p is equal to R_1 bar Q_1 square. So the equation is one is modified by adding the pump head H_p.

So where H_p is the pump head and then additional necessary relationship is the head discharge curve for our pump. So the pump which are using, we the manufacturer would given a pump head discharge equation for the pump, so the discharge equation curve we can utilize to get a solutions here. So if the pump exist in say for example between pipe A and B we will be writing P by gamma plus x at A minus P by gamma plus at B plus H_p is equal to R_1 bar Q_1 square, where Q_1 is the discharge throw the pipe, R_1 bar the resistance coefficients and H_p is the pump here.

So now, we have seen with respect to branching pipe systems typical system, with three reservoirs. We have seen with three pipes and then there is a junction, so what we generally do, we write the energy equations between various points, and then also we use the continuity balance equations at junction and then, we solve for the unknown and we have seen iterative procedure. So here we use an iterative procedure to solve for the unknowns and we keep tolerance limits. So that, when the iteration reaches that tolerance we stop the iteration, and then we get the solution. Here now we will discuss an example, so here the example problem is for the figure given here below.

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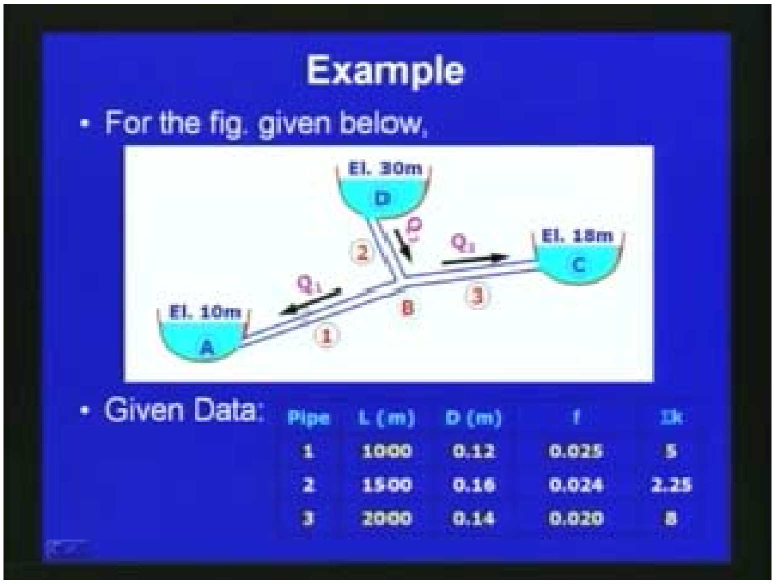

So here three reservoirs are there, ~~so~~ here at A the elevation is 10 meter here that means the corresponding to piezometric head is given as 10 meter, and here elevation is 30 meter, and here elevation is 18 meter and these three reservoirs are connected by using three pipes and there is junction at B. So here the pipe 1 is between B and A, and pipe 2 is between B and D and pipe 3 is between B and C.

So now the given piped data regarding the pipes are, for pipe 1 the length is thousand meter, and the diameter is 0.12 meter, and friction factors is given as 0.025, and then the minor loss a coefficient sigma k is given as ~~five~~5. And similarly, for pipe 2 the length of the pipe is given as ~~fifteen hundred~~1500 meter between D and B, and then also the diameter of the pipe is given as 0.16 meter, and friction factor f is given as 0.024, and sigma k is given as 2.25 between B and D.

Similarly for pipe 3, the length is given as between beyond C, the pipe length is given as ~~two thousands~~2000 meter, and the diameter of the pipe is ~~fourteen~~ 14 centimeter or 0.14 meter, and friction factor is 0.020, and sigma k and minor loss coefficient is given as ~~eight~~8.

So for this system we have to determine the flow rates Q_i. We have to determine the flow through each pipe, and piezometric head H at the junction. So for this junction we want to find out Q_1, Q_2, Q_3 and we have to find out the piezometric head H at this location. We want to find out the piezometric head, so the elevations at A, C and D are given and also various pipe factors or pipe elements details are given like length, diameter, friction factors and sigma k. So we have to find out Q_1, Q_2, Q_3 and the piezometric head at H, also assume the constant friction factor for the given pipe. When you consider each pipe, the constant friction factors can be considered.

So now, to solve such a system we have already discussed is very similar system, now the various numerical values are given for such system. So to solve such a system we will be writing the equation energy equation and the continuity equation as we discuss. So before that, corresponding to the minor losses we will be finding the equivalent lengths so the equivalent lengths as we discussed in the last lecturer.

We can see that the equivalent lengths corresponding to the minor losses can be written as, this sigma k into D by f. So that is the equation corresponding to the equivalent length see D into sigma k by f so say for pipe number 1 the diameters 0.12, and sigma k is ~~five~~ 5 and friction factor 0.025. We will get the equivalent length 24 meters and for pipe 2 corresponding diameters 0.16, and then sigma k is 2.25, and then the friction factors 0.024.

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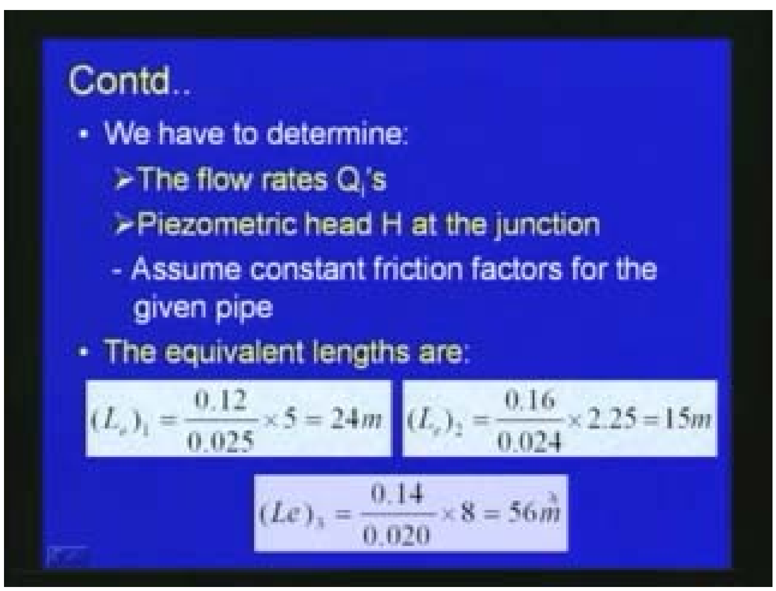

So we get the equivalent length ~~fifteen~~ 15 meters. Similarly, for pipe~~s~~ 3 the equivalent length will be 0.14 into ~~eight~~ 8 divide by 0.02, so that will give ~~fifty six~~56 meters. So initially, we found all equivalent lengths corresponding to the minor losses. Now the resistance coefficients as we have already seen the basic equations earlier.

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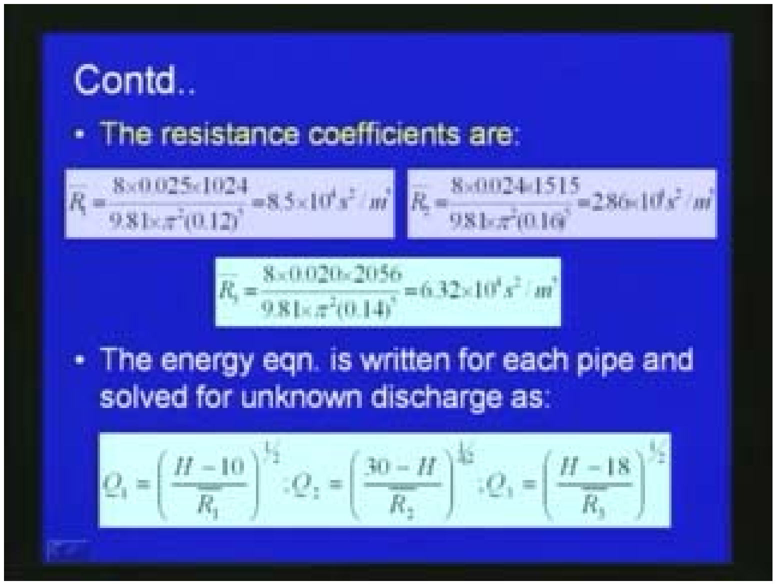

So now we will be finding the resistance coefficients for pipe 1, pipe 2 and pipe 3. So the resistance coefficients as we discussed earlier, we can write the resistance coefficient equation as R_1 bar is equal to~~ eight into so~~, ~~f eight~~ 8 into f into the length divide by g into pi~~e~~ square into d t~~ho~~ to the power ~~five~~5.

So this equation for R_1 bar will be ~~eight~~ 8 into so for pipe ~~one~~ 1, f is 0.025, and equivalent length is now ~~thousands~~ 1000 plus ~~twenty four~~24 so ~~thousands twenty four~~1024 into divide d by g is 9.81 into pi~~e~~ square into d is 0.12, so d to the power ~~five~~5, 0.12 to the power ~~five~~5. So we get the resistance coefficients for pipe 1 as 8.5 into ~~ten~~ 10 to the power ~~four~~4.

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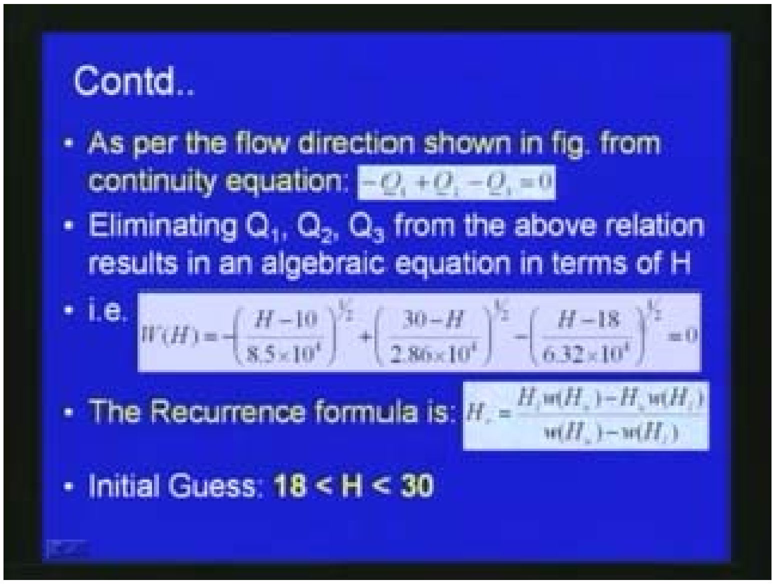

So similarly, we can find out resistant coefficient for pipe 2, R_2 bar that will be equal to 2.86 into ~~ten~~ 10 to the power ~~four~~4, and R_3 bar will be 6.32 into ~~ten~~ 10 to the power of ~~four~~ 4, so now we found R_1 bar R_2 bar R_3 bar resistance coefficient. And now the energy equation we can write for each pipe, and then we can solve for the unknown discharge. So as we have seen earlier, so with respect to the resistance coefficient and then from the energy equation we can write, Q_1 is equal to say if for this first pipe H minus ~~ten~~ 10 divided by R_1 bar to the power ~~one~~ 1 by ~~two~~2, and then Q_2 is equal to ~~thirty~~ 30 minus H divide by R_2 bar the power ~~one~~ 1 by ~~two~~2, and Q_3 is equal to H minus ~~eighteen~~ 18 divided by R_3 bar to the power ~~one~~ 1 by ~~two~~2.

So here in this problem we assume that, the flow direction is this Q_1 this towards A, and Q_2 towards B, and Q_3 B to C so this direction. We assume this direction and then based upon this equation, so here the basic equations we have already seen this equation, we can now write the final equation, with respect to discharge, so Q_1 we get as H minus ~~ten~~ 10 divided by R_1 bar to power ~~one~~ 1 by ~~two~~ 2 or square root of H minus ~~ten~~ 10 by R_1 bar, and Q_2 is equal to square root of ~~thirty~~ 30 minus H by R_2 bar, and Q_3 is equal to square root of H minus ~~eighteen~~ 18 by R_3 bar.

So now as for the flow direction shown in figure, we can write the flow direction is shown here, one is here this towards the junction and other to here from the junction. So towards the junction it is positive, and away from the junction it is negative, so minus Q_1 plus Q_2 minus Q_3 is equal to ~~zero~~ 0.

So this is the continuity equation so we can now eliminate Q_1, Q_2, Q_3 from the above relation. Here we have relationship for Q_1, Q_2, Q_3 so we can eliminate Q_1, Q_2, Q_3 and we can write in terms of H as follows, so here we get a function in terms of H, so W(H) is equal to minus H minus ten divide by 8.5 into ten to the power four to the power one by two plus thirty minus H divide by 2.86 into ten to the power four to the power one by two minus H minus eighteen divide by 6.32 into ten to the power four to the power one by two minus H minus eighteen divide by 6.32 into ten to the power four to the power one by two.

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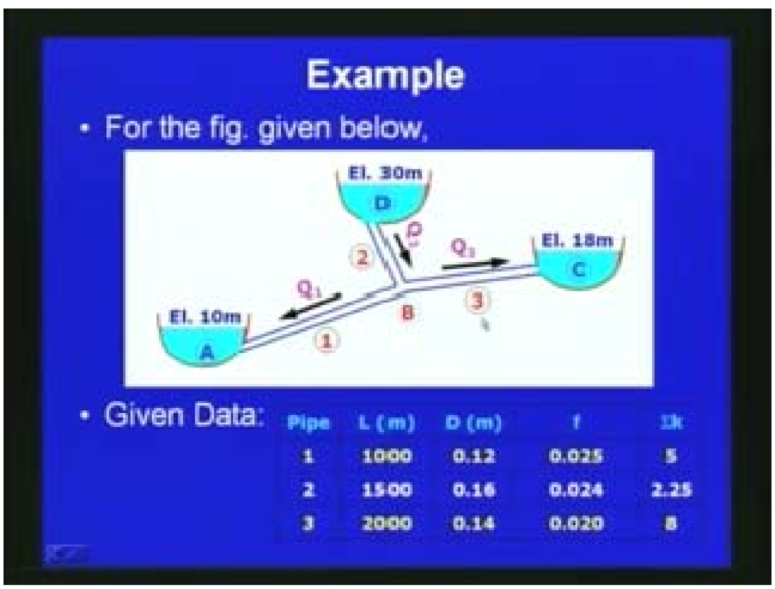

So that is equal to ~~zero~~ 0 by using the continuity equation. Now here as we discussed we will be using the iterate procedure. To do the iteration we will assume a range of values between, so now here we got an expression for H, to get H we will assume certain value an upper limit and lower limit and then, we will write recurrence formula and then we will check whether we reached the tolerance which we set.

So here the recurrence formula, if you want to find out the head between an upper limit and over limit the recurrence formula is H_r is equal to H_l into w (H_u), w function of H minus H_u and w (H_l) it is function H_l divide by w (H_u) minus w (H_l), where H_l is the lower limit to be assume, and H is the upper limit to be assume and then w(H_u)is corresponding the upper limit if we put here in this equation, so that we will be getting received.

So that will be w (H_u) similarly w (H_l) when we put the lower limit we get received that will be the w (H_l) and then we find out different, and then that iteration we find out the head value H_r. So for this typical problem you can see that here the maximum elevation is thirty meter, and minimum elevation is a ten meter ,and here the other one is eighteen meter so we will assume a value between eighteen meter and thirty meter. So we will put an initial head value between eighteen and thirty, and we will then do the iteration. The iterative procedure is given in this table, so here we set a convergence by limit of 0.005. So here we assume that, H value is between eighteen and thirty.

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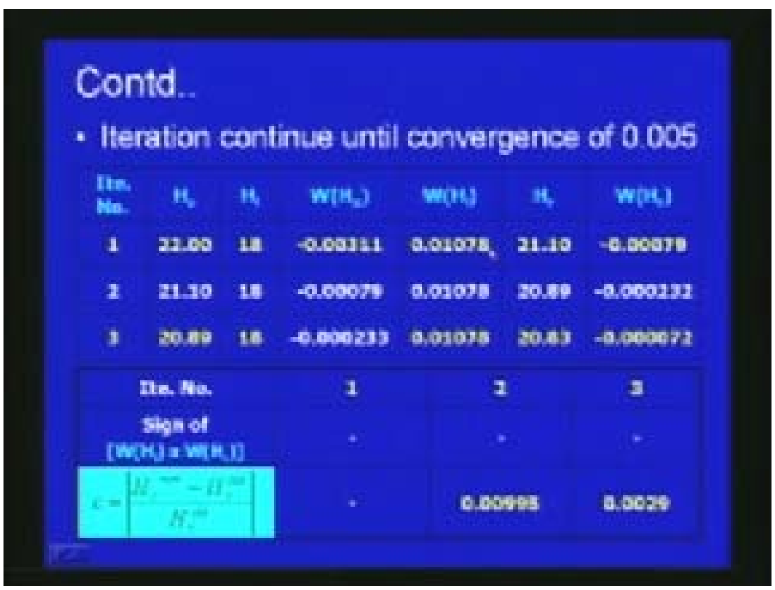

So we will start the iteration with value of H as twenty two meter, and then we will use the recurrence formula given here. So in this table we can see first column is iteration number one, two, three and here the upper value which we use head value H_u, and H_l

lower head value, and then w (H_u) this assumed for the corresponding upper value the w function which is given by this equation.

And then w (H_l) corresponding to lower value function, and then we get the corresponding H_r from this equation using this equation we get H_r and then corresponding w (H_r) again we will find, what is the residue we get here corresponding to this equation, and then we will see that what is sign of this w (H_l) and H_r and then in this column here.

This direction and then we will see their residue epsilon is the modulus of H_r new minus H_r old divided by H_r old. So we start with value of ~~twenty~~ 22 ~~two~~ and lower limit is always set as ~~eighteen~~18, so now after substituting twenty two here we will get w (H_u) as minus 0.00311 and correspondingly by putting H_l as ~~eighteen~~18, we get w (H_l) as 0.01078 and then correspondingly by solving this equation we will get H_r as 21.1 and w (H_l) we the minus 0.00079 and then a since w (H_l) is positive and w (H_r) is negative.

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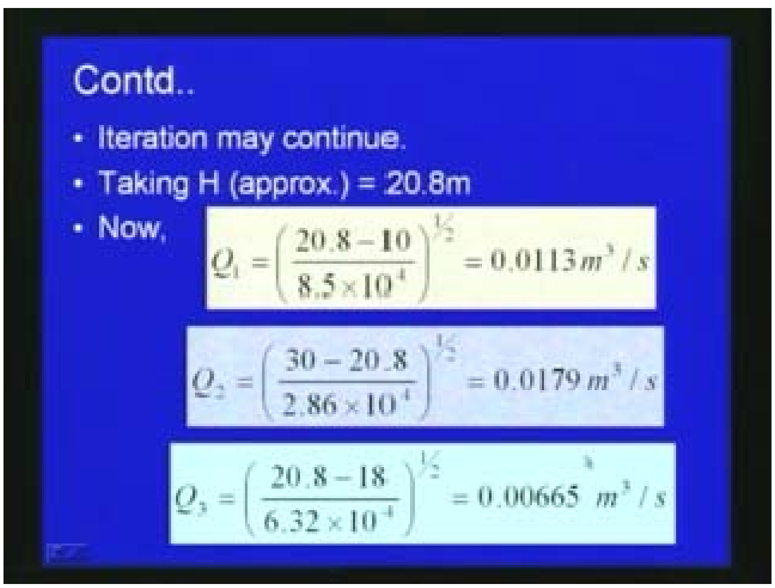

So we get this as negative, and then say corresponding to iteration number ~~one~~ 1 and then the epsilon anyway here we are not calculating since, the first iteration and then the second iteration we will use this now this H_r 21.1 as our upper limit and lower limit already assume ~~eighteen~~18, and then again the found out double H_u so that would be obtain this is same value and 0.00079, and then w (H_l) is constant it will be same 0.01078 since the lower limit is same and then.

If you calculate again H_r back we will get H_r as, 20.89 and then the corresponding residue will be residue will be minus 0.000232, so then again this sign will be minus here as shown, and then if we find out the epsilon, the error H_r new minus H_r old by H_r old is modulus we will get 0.00995. So now we have improved this solution into 20.89 but you can see our expected error is convergence is 0.005. We have to continue one more iteration, so now the next upper values 20.89 and then lower is same, and then we again calculate w (H_u), w (H_l) so w (H) is minus 0.000233 and w (H_l) is 0.01078 and then H_r we can find out here we get as 20.83.

So then we can see that the residue will be minus 0.000072. You can see that now here again it is minus sign is minus but we can seem here the epsilon, we calculate it is 0.0029 which is smaller than the convergence limit as 0.005. So here we can stop the calculation even though some error is there, we assume that the values is almost near to 20.83. After this if you want further accurate value we can continue the through a number of iteration. We get almost very near by value then, here say let as assume that approximately the value is equal to, even though we got here is 20.83 as so we will assume it is almost equal to 20.8 meter and then.

We can find out the discharge Q_1 is equal to 20.8 minus ~~ten~~ 10 divided by R_1 8.5 into ~~ten~~ 10 to the power ~~four~~ 4 its square root, we will get 0.013 meter cube per seconds, and Q_2 will be ~~thirty~~ 30 minus 20.8 divide by 2.86 into ~~ten~~ 10 to the power of ~~four~~ 4 its square root. We will get 0.0179 cubic meters per seconds similarly, Q_3 equal to get as 0.00665 meter cube per second. So like this branching pipe system as we discussed, we can right the energy equation we can use the continuity equation, and then we can solve this equation to find out the unknowns, like flow through each pipe, and also we can solve for the piezosmetric head say for example, here we found the piezometric head at the junction points.

So like that we can solve different kinds of problems, so here we assumed a simple system. The complex system there can be wolves there can be pumps like that number of compounds will be there. So the branching pipe system we can solve in a very similar way.

Next topic is analysis of pipe networks. So far we have seen the pipe in series, pipes in parallel, and branching of pipe so that is what we had now discussed. Now we will discuss the pipe network so we can see that, most of the say water supply lines or most of the system where the a number of pipes, number of uses are there just like in the water supply system.

We will be using the pipe networks for the distribution. So a simple pipe network like we can see here in the slide, can see number of pipes are connected through with respect to reservoir or there will be number of outlets, and then can see that through there will be number of junction points, and then with respect to junction points various junction points. We can have also loops like this, so a simple pipe network consists of pipe elements fitted with bends, valves, joints and number of pipes, and then also correspondingly we can say where ever pipes are joining that junction which is called node. We can have the number of nodes likes this so this is one node, second node like that number of nodes will be there, and then we can also have a loop so like this is typical loop.

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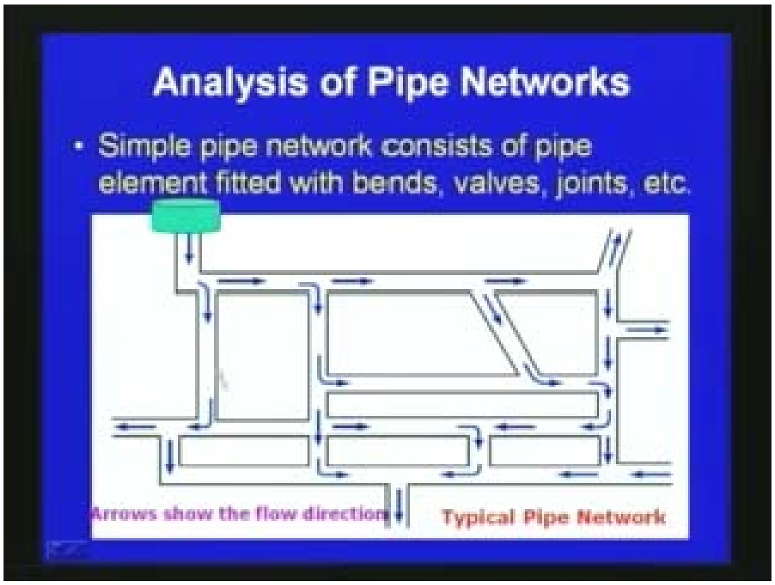

So like that another loop we can, have number of loops also, this figure shows a typical pipe network the arrows show the flow direction. Here most of the time we can see that we may not be knowing the flow direction also the flow direction may change with respect to time also depending upon the pressure, depending upon the water going from junction the even the direction can also change.

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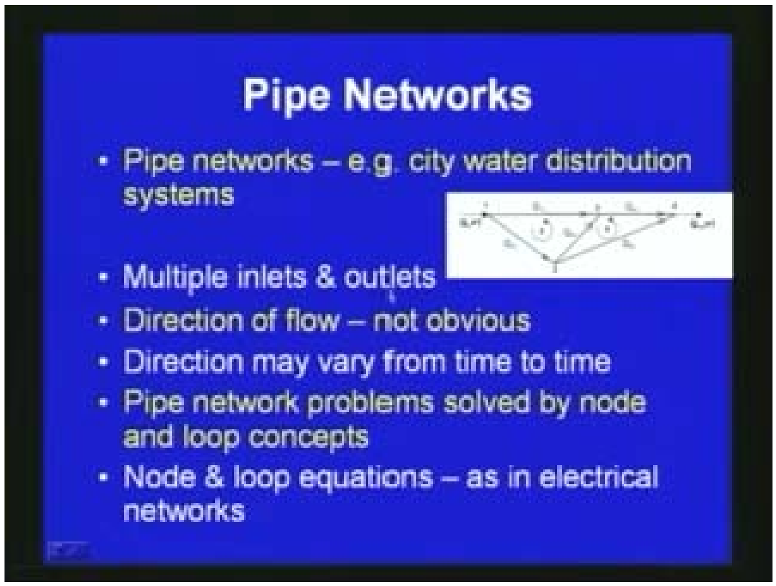

So that is why we call it as a network or pipe network. As I mention the pipe network and say can be typically in the case of say water distribution system, say where a number of say systems number of pipes are included and number of connections are there number of junction are there so as we have seen. There can be multiple inlets and multiple outlets, and mention the direction of flow is not obvious, and then direction may vary from time to time, and then pipe network say here as I mention there can be say, where ever the junctions are there, these are called nodes, and then also there are loops between that.

So here for this simple pipe network system, this is one loop and this another loop so there will be loops so generally, say we will be considering the nodes and loops and so we will write equations with respect to nodes and loops, and we can just compare this kind of pipe flow systems are pipe network system with respect to electrical network.

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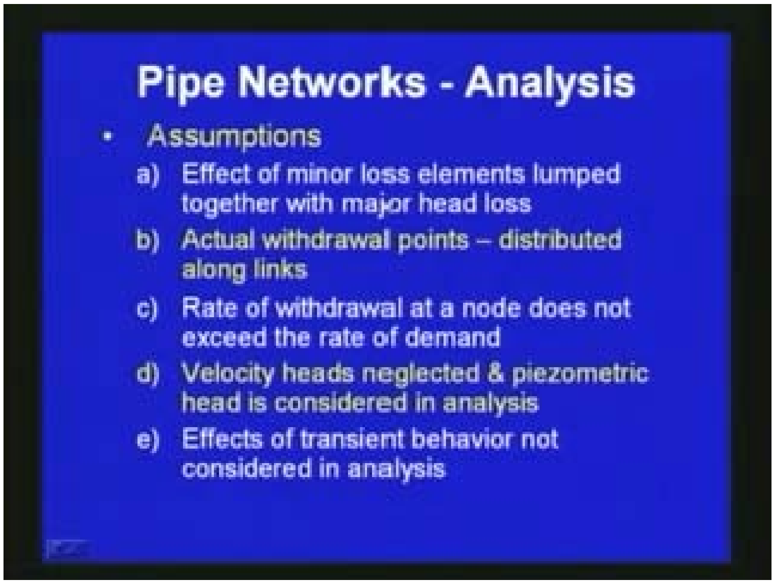

So you can see that electrical system is considered, so there will be number of say number of wires to be connected together to form a network, electrical network so pipe flow system is also, we can consider as a typical very similar to electrical network. So this as we have seen the major challenges here are, we do not know exactly the direction of flow, and even from time to time the direction may change, and also since the number of pipes are there, number of fittings would be there, so all this losses we have to consider, and then also we have to see the continuity equation, and the energy equation in satisfied to nodes with respect to that.

We have to consider the equation with respect to loops also, so these are some of the major challenges when we solve such kind of pipe network. So to make it simple while solving these kinds of pipe network problems, we put some assumption so here the important assumptions are listed here.

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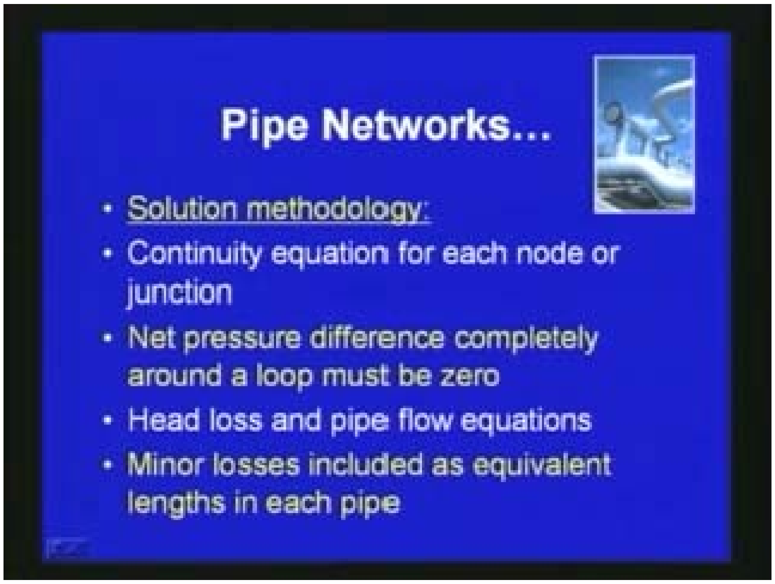

So the assumption here are first one is, the effect of minor loss elements generally as we have done earlier and lumped together with major head loss depending upon the system, so that is assumption number 1, and assumption number 2 the actual withdrawal points the distributed along the links. Here we can see along the links from one to another the actual withdrawal points here considered, and then the right of withdrawal at a node does not exceed the rate of demand.

So each node is considered there will be certain demand, so with respect to demand only rate of withdrawal what we considered. And then fourth one is, this the velocity head generally neglected, and say since the system is so complex, that we consider all the thing are very difficult to solve, so velocity head are neglected and piezomteric head is generally considered in analysis.

So as we have seen in the system earlier, we will considering the piezometric head instead of the say velocity head is neglected and the piezometric head is considered, and effect of transient behavior and not generally consider in this kinds of analysis, since in this analysis is so complex ,you may not considered the transient effect.

So based upon this assumptions we will try to solve this kinds of complex pipe networks. Generally, the solution methodology so we have already seen in the branching the pipe system also, we consider the continuity equation, and we consider the energy equation so very similar way also here we approach the pipe networks. So the solution methodology roots continuity equation will be written for each node or junction say, we have seen that say for such a system there can be number of junction one, two, three like that number of junction so first, we will consider the continuity equation for each node for junction, and then net pressure difference say completely around a loop must be ~~zero~~ 0.

So here, we can see that number of loops here like this, like this number of loops will be there. We consider the net pressure difference completely around a loop must be ~~zero~~ 0, and then third point is the head loss and pipe flow equation is use as we have seen earlier, we will be using the head loss equation and the pipe flow equations based upon the energy equations, or the Darcy weisbach equations we will be using, and then the minor losses are included as equivalent lengths in each pipe.

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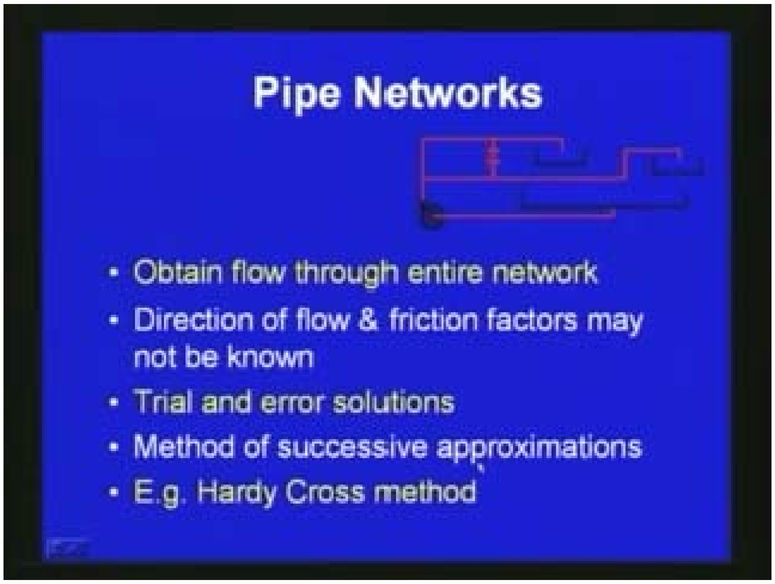

So as we assume the minor losses, we include as with the major losses, and then we obtain the flow through the entire network. So depending upon the problem there will be number of networks, so number of loops, and number of junctions. So we obtain the flow through the entire network, and then the direction of flow and friction factors the some of the major challenges include may not know the flow direction, and also if the friction factors are known it will be more difficult. So to solve this kind of problems we go for trial and error solutions.

So say system is so complex, we may assume the direction of flow, and we may start with some of the, by writing the basic equations we start and then, we assume the flow directions or we assume some of the flow values, and then we go for a trial and error solution. So generally to solve these kinds of pipe network problems we go for successive approximations. So, one of the commonly used successive approximation is hardy cross method which we will be discussing in detail later. Now, we have seen the major challenges to solve this pipe network complex pipe networks, and then we have also seen the assumptions which we used to solve the system. You have also seen the solution methodology is starting from the basic equation like energy equation and the continuity equation we try to solve the system.

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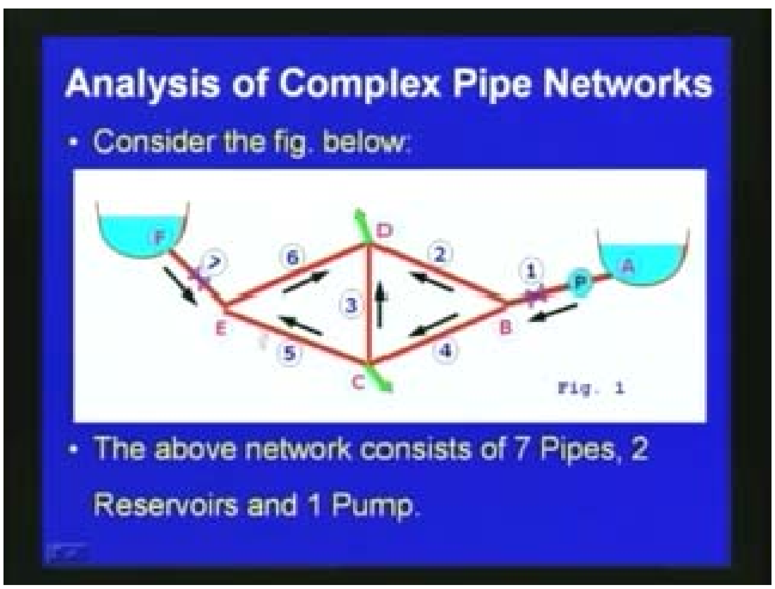

Now, we will consider typical system, and then we will write the equations which we can utilize to solve such a systems. Here in the slide we consider a pipe network system here it includes two reservoirs at A and F and then, we have seven pipes connected like this, and we have here four junctions here E, C, D and B and the reservoirs are located at A and F and there is a pump.

The pump at this location P so here we have seven pipe elements, so the above network consists of seven pipes, two reservoirs and one pump, and then we have four junctions and four nodes. We will now see how to write the system equation for such a system typical system, and then for any kind of system we can just extend in a very similar way the analysis considered, and then the solution part we will discuss in detail later.

So now here between this A and F say, here we have two reservoirs, of this two reservoirs A and F are assume the values the hydraulic grade lines at A and F are assumed to be known. So the piezometric heads are known at A and F and these nodes here at A and F is called fixed grade nodes.

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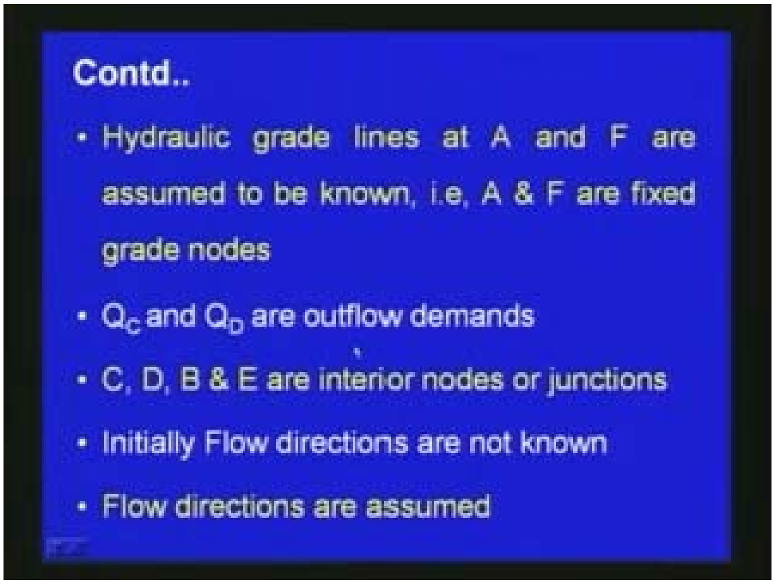

So this A and F consider the values are known, and we call them as fixed grade nodes. And then here this system is considered from C and D there is the flow is going out so, the discharge Q_C and Q_D these are outflow demands, and C, D, B and E are interior nodes or junctions.

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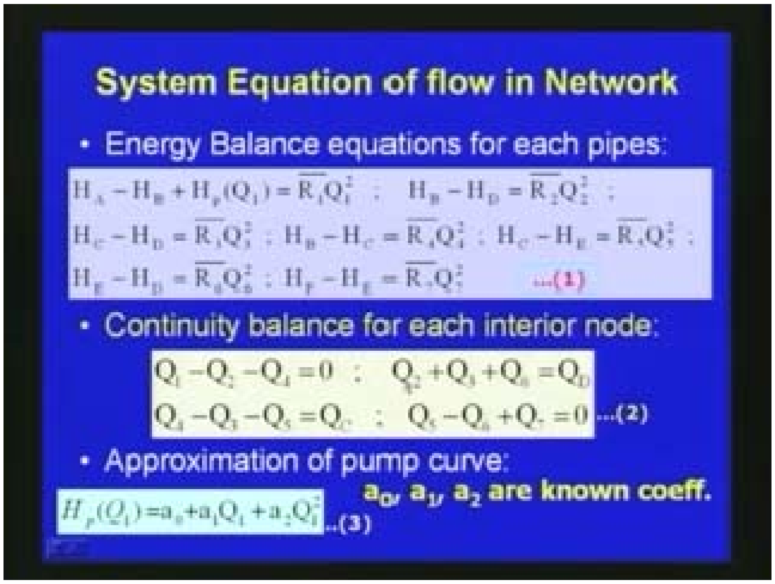

So here we have C, D, B, E so we have four interior nodes or junctions, and initially as we discuss flow direction are not known, so we can assume the flow direction say for example this system is considered we can assume that, flow is going this direction A to B, this direction and B to D, this direction B to C, and then C to D flowing, and E to D flowing, C to E and F to E these are the flow directions, and here out flow from D and C. So now we assume the flow directions, now between for each junction each node we can right the energy balance equations for each pipes we can write. So if we consider A and B so that means this points A to B so, between A to B we can write H_A minus H_B plus H_P (Q_1) is equal to R_1 bar Q_1 square so that is the energy balance between A and B. So similar way between B and D so this is from B to D we can write H_B minus H_D is equal to R_2 bar Q_2 square, and similarly between C and D, H_C minus H_D is equal to R_3 bar Q_3 square, and between B and C it is H_B minus H_C is equal to R_4 bar Q_4 square, and between C and E is H_C minus H_E is equal to R_5 bar Q_5 square, and between E and D, H_E minus H_D is equal to R_6 bar Q_6 square, and between F and E it is H_F minus H_E is equal to R_7 bar Q_7 square.

So these are the equations with respect to the energy balance, and now we have four junctions of four nodes. So we can write the continuity balance for each interior node, here we have four junction so for each junction or each node we write the continuity as,

Q_1 minus Q_2 minus Q_4 is equal to ~~zero~~ 0, and this is coming with respect to we assume the flow direction with respect to the assumed the flow direction we are writing the continuity balance for each interior node.

So Q_1 minus Q_2 minus Q_4 is equal to ~~zero~~ 0, and then Q_2 plus Q_3 plus Q_6 is equal to Q_D since there is an outflow and D that is equal to Q_D, and Q_4 minus Q_3 minus Q_5 is equal to Q_C as there is an outflow at C, and then Q_5 minus Q_6 plus Q_7 is equal to ~~zero~~ 0.

So these are with respect each node, we have written four equations like this, and then as far as pump is considered, so here we have got a pump at this location, so for pump we can write equation like this $H_p (Q_1)$ is equal to a_0 plus a_1Q1 plus a_2Q_1 square where a_0, a_1 and a_2 are known coefficients.

So this equation corresponds to the pump curve. So now all the equations are written here with respect to the energy balance equations, and then continuity balance and the pump equations. Here we can see that, say the unknowns generally say here this system is considered as unknowns are the discharge through each pipe.

So we are go to seven pipe so Q_1 to Q_7. Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, so seven unknowns and then the unknowns will be the heads at each interior nodes.

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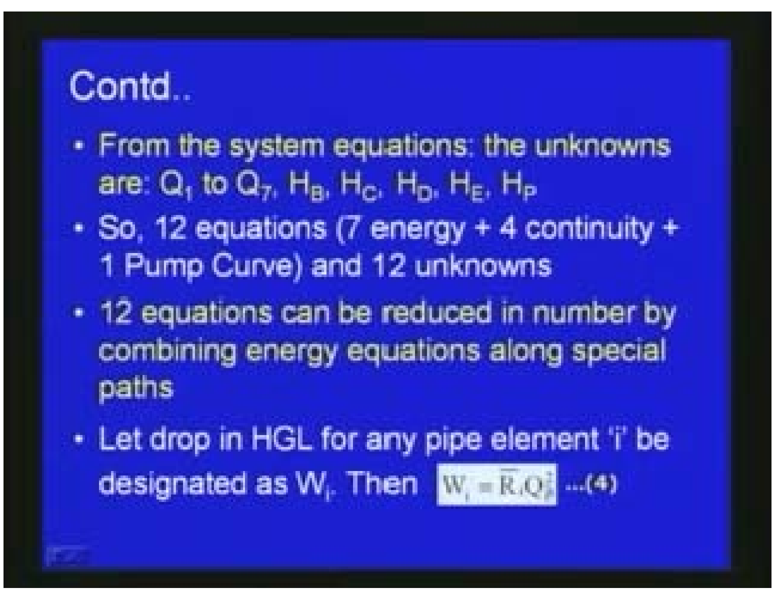

H_B, H_C and H_D and H_E and then corresponding to the pump to the discharge is not known. So H_P will be another unknown. We have got here twelve equations, so here we can see here we have seven equations, and here also we have four equations, and then we have one equation corresponding to this pump curve. So we have ~~12~~ twelve equations, ~~7~~ seven energy equations, ~~4~~ four continuity equations and ~~1~~ one pump equation and we have ~~12~~ twelve unknowns.

So this way the solution is ok, since we can easily get solution ~~12~~ twelve equations and ~~12~~ twelve unknowns. Now since you can see that equations combined with respect to the discharge and heads, so these equations we will be trying to reduce number by combining the energy equations along special paths.

So here we can see that you can have different paths can be written like this or like this. Along different paths we will combine the energy equations such that, we can reduce the number of equations and correspondingly number of unknowns also. So we will assume let the drop in hydraulic radiant line for any pipe line that is 'i' be designated as W_i.

So that as we as seen this drop in hydraulic radiant line can be W_i is equal to R_i bar Q_i square. This is relationship we have seen earlier, so this is the drop in HGL for any pipe element 'i' is W_i is equal to R_i bar Q_i square as in equation number (4).

So now for the system you can see that, we have possibility of two loops one is this loop, and another one is this loop. So there is possibility of two loops here for this typical system. Two closed paths or interior loops can be identified.

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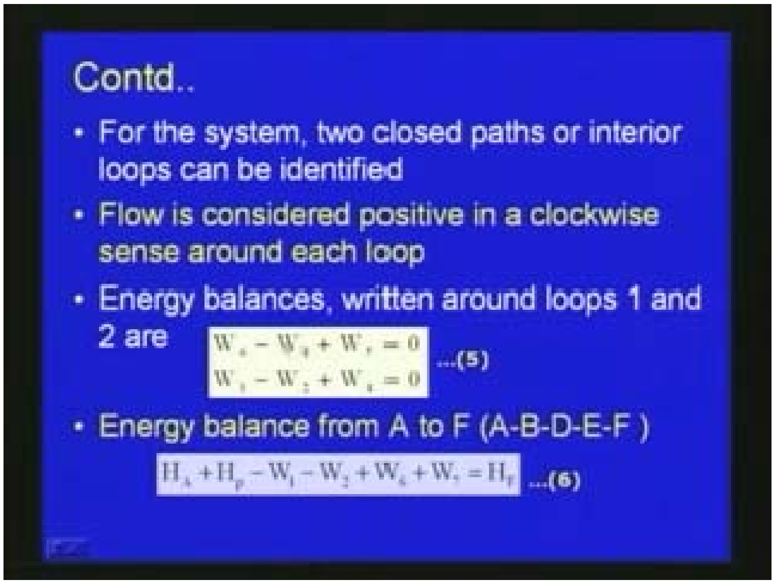

Flow is considered for positive in a clockwise sense around each loop. Here we consider the flow is to be positive clock wise, and then the anticlockwise they consider as negative. So now we can write energy balance we can write around loops 1 and 2, so if we consider here this figure you can see that this loop is considered, this loop say this flow is going this is same direction and this is the opposite direction.

So this is clock wise, this is anti clock wise and this way clock wise and this is the anti clock wise so correspondingly, between these two loops we can right W_6 minus W_3 plus W_5 is equal to ~~zero~~ 0, and second equation is W_3 minus W_2 plus W_4 is equal to ~~zero~~ 0 as in we given here. So now with respect to the energy balance, with respect to two loops we got two equations. Now the energy balance from A to F so if we consider here this figure as say from this reservoir A to reservoir F, if we consider here we can write the equation from A to F by concerning A-B-D-E-F this A-B-D-E-F here A-B-D-E-F.

So if we consider this we can write H_A plus H_P the pump head minus W_1 minus W_2 plus W_6 plus W_7 that is equal to H_F, so we can write the energy balance between A to F in the direction of A, B, D, E and F like in equation number (6). So now the path energy equation (6) connects two fixed grade nodes, so depending upon the network we can see where are the fixed nodes, so between this fixed grade nodes we can write the path energy equation as written here in this case equation number (6). So such a path is timeless pseudo-loop so here this is called pseudo-loop because an imaginary pipe with infinite resistance or no flow can be considered to connect the two reservoirs.

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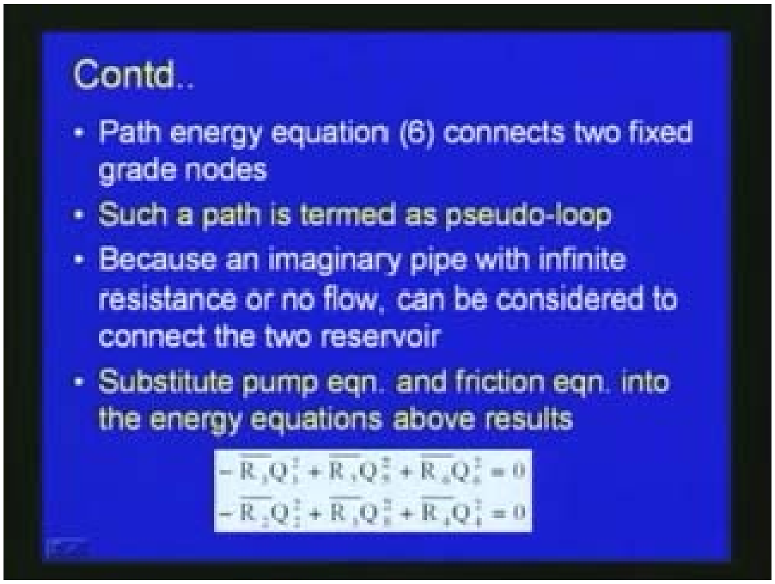

So that is why it is called as pseudo-loop. For the pseudo-loop which we considered here this equation is to be written, and now we substitute pump equation and friction equation into the energy equation above this equations, we will get per height all the corresponding equations here and then this equation equations number (5).

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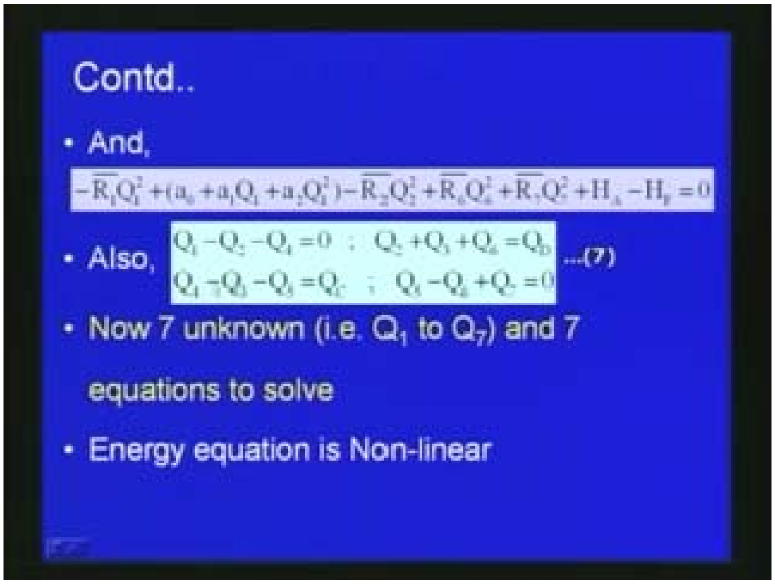

We will get minus R_3 bar Q_3 square plus R_5 bar Q_5 square plus R_6 bar Q_6 square is equal to ~~zero~~ 0, and second equation will be R_2 bar Q_2 square plus R_3 bar Q_3 square plus R_4 bar Q_4 square is equal to ~~zero~~ 0.

So then if you write corresponding to equation number (6) when we substitute all the values it will be correspondingly for the pseudo-loop it will be minus R_1 bar Q_1 square plus corresponding the pump equation a_0 plus a_1Q_1 plus a_2 Q_1 square minus R_2 bar Q_2 square plus R_6 bar Q_6 square plus R_7 bar Q_7 square plus H_A minus H_F is equal to ~~zero~~ 0.

So similarly, with respect to this we have already seen this equation here the continuity balance that also we can write here Q_1 minus Q_2 minus Q_4 is equal to ~~zero~~ 0, and Q_2 plus Q_3 plus Q_6 is equal to Q_D, and Q_4 minus Q_3 minus Q_5 is equal to Q_C ,and Q_5 minus Q_6 plus Q_7 is equal to ~~zero~~ 0.

So now we have written all the equations with respect to the discharge since here in this equations H_A and H_F already known values, so now we have seven unknown Q_1 to Q_7 all the discharge values, and here we have in have this two equations, and this third equations three plus this four equations is ~~7~~ seven equations and ~~7~~ seven unknown in terms of the discharges Q_1 to Q_7.

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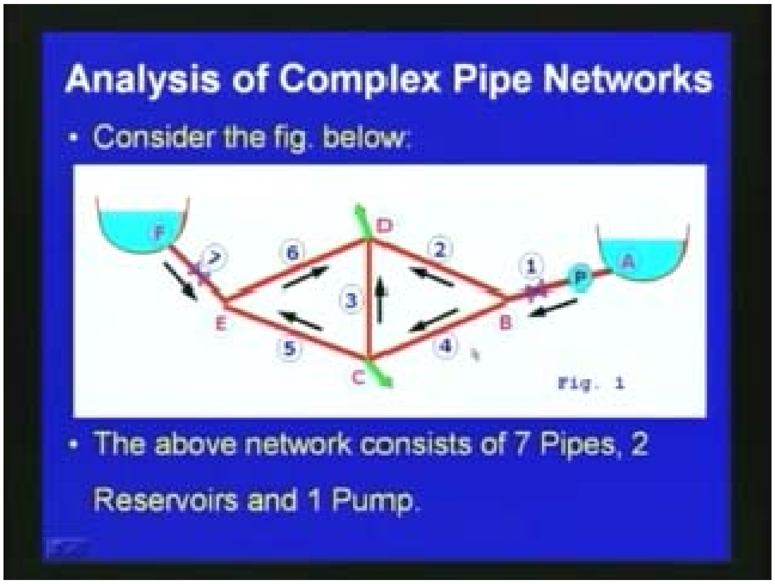

So here now we can now solve the seven equations first to find out the discharges Q_1 to Q_7, and then we can solve for the head values, but here we can see that equations like here the energy equation. We can see these equations are non-linear so we have to either linearize these equations, or we have to go for an iterative solution. So that is the general for this typical network which we considered we have seen how to write this equations how to say see the various equations for with respect to continuity or with respect to the energy, and then with respect to pump equation we have seen.

The various system equations we have seen so this we consider for a typical system like, this is the typical system which we considered, so now based upon this equation which we want to generalize so for a general system, what will be the equations.

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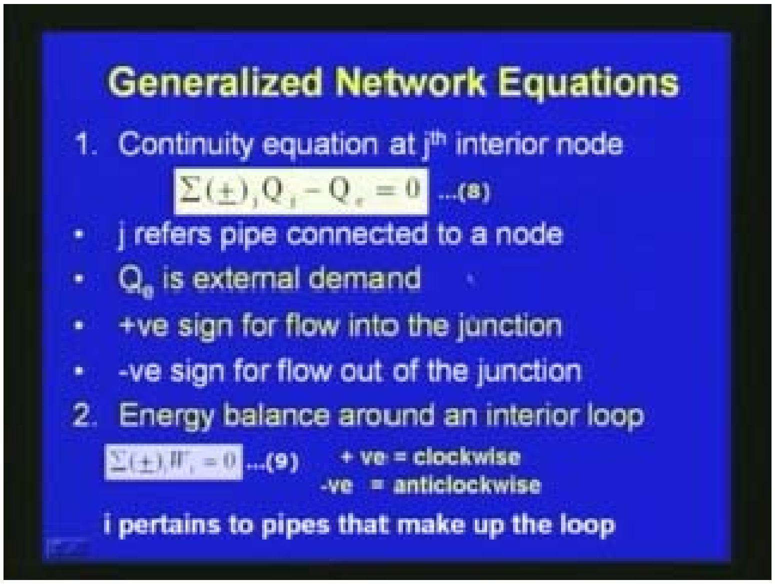

So here for generalize network the equations can be written, so first one is the Continuity equation at the jth interior node we can write, sigma plus or minus $_j$ Q_j minus Q_e is equal to ~~zero~~ 0 as in this equation number (8). Where j refers the pipe connected to particular node, which node we connect, so that is for this j and Q is the external demand. So if there is in clock going out from the particular node that is called external demand, and then positive sign for flow into the junction, so we are seen various junction so when ever flow is going to the junction, we consider it as positive, and when the flow is out of the junction we consider the flow it as negative.

So in this equation number (8) we are sigma plus or minus$_j$ Q_j minus Q_e where Q_e is external demand, and Q_j is corresponding to the particular pipe j and negative is for flow out of the junction, and positive is flow into the junction. And then the next equation generalize equations is, energy balance around an interior loop so we have seen as in the earlier case are where two loops.

So depending upon the problem number of loops may be there, so for each loop we can write the equation. So the energy balance around interior loop we can write as sigma plus or minus$_i$ W_i is equal to ~~zero~~ 0 as in equation number (9) so here we consider positive as clockwise and we consider negative as anticlockwise.

So here i pertains to the pipe that make up the loop, so this is the second equation the generalize equation in a first one is continuity equation, and second one is the energy balance around an interior loop. Now next lecturer we will be discussing how to solve this kinds of system and then get solution whether we go for iteration method, so we will discussing in details about the solution techniques in the next lecturer for the pipe networks.