

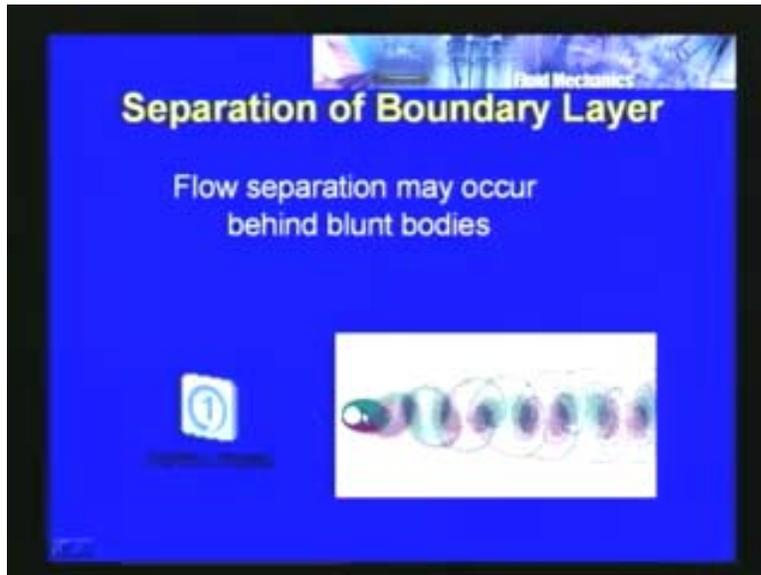
Fluid Mechanics
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Lecture - 32
Boundary Layer Theory And Applications

Welcome back to the video course on fluid mechanics. The last lecture we were discussing about the boundary layer and its formation and various theories to analyse the boundary layer for various parameters like velocity, variation shear stress and pressure variation. Starting from the Prandtl's boundary layer equations we have seen the various solutions with respect to the blasius solution, then the moment integral equations and then we have derived various equations which we can utilize to analyse the boundary layer especially for flow over immersed bodies, flow over a flat plate and also flow through pipe with respect to the boundary layer formation for the external flows as well as internal flows.

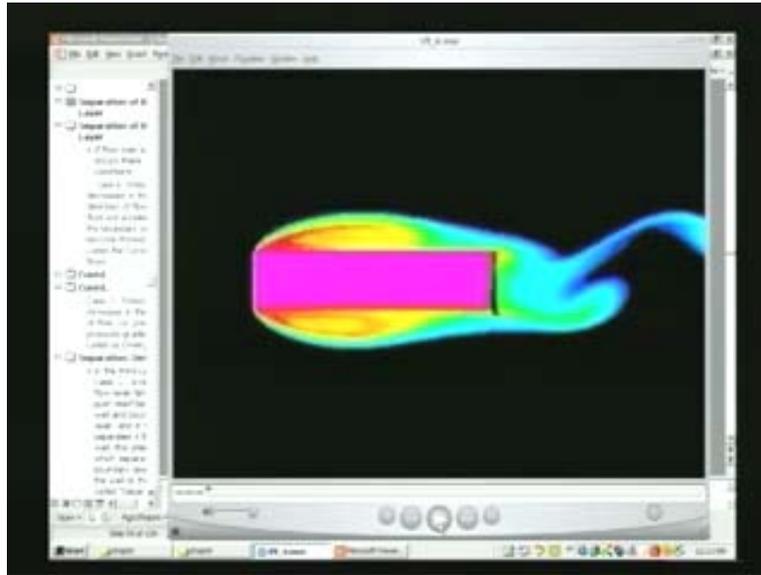
Now, as far as boundary layer is concerned one of the important aspects is the separation of the boundary layer. We have seen that the boundary layer when it forms with respect to the flow over the external bodies depending upon the shape of the body, depending upon the fluid properties the boundary layer thickness can vary; boundary layer thickness can change with respect to the various parameters. Most of the blend bodies like a circular cylinder or depending upon the sphere or depending upon the body shape the boundary layer can separate.

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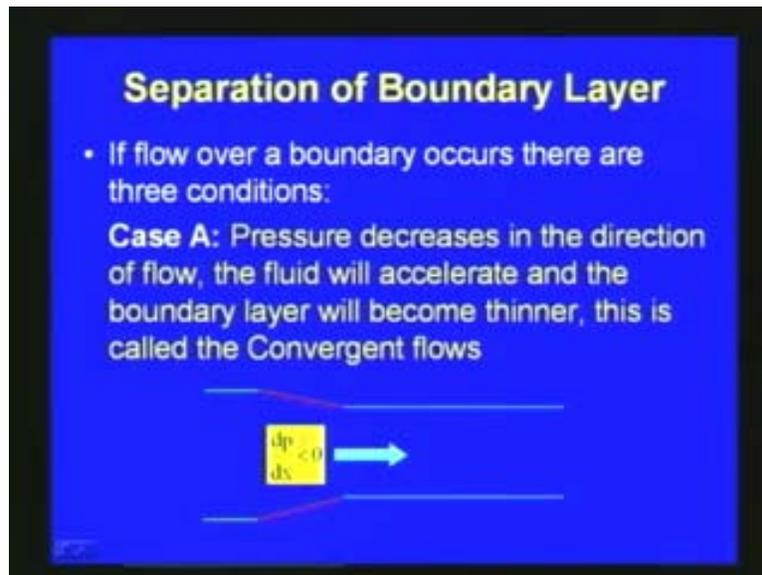
Today, we will discuss the separation of boundary layer so depending upon the case the boundary layer separate depending upon the shape of the body, depending upon the fluid flow parameters the boundary layer can separate at various locations with respect to the flow. Here, we can see the flow over a cylinder is animated; we can see that the flow is coming here and then we can see that it is separated at some locations. Like this depending upon the problem here I will show another figure. This video is taken from the book of fluid mechanics by Munson.

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We can see similar beam, the flow over a cube or flow over a rectangular block. You can see that it is coming here and then separating the boundary layer after sometimes. Depending upon the case, the flow separation takes place and then the flow at some particular location the proportion takes place and then we have to find out in the boundary layer and shear the flow separation starts and what are the various parameters as far as various properties with respect to the boundary layer separations. When we consider the separation boundary layer if flow over a boundary occurs there are typically three conditions.

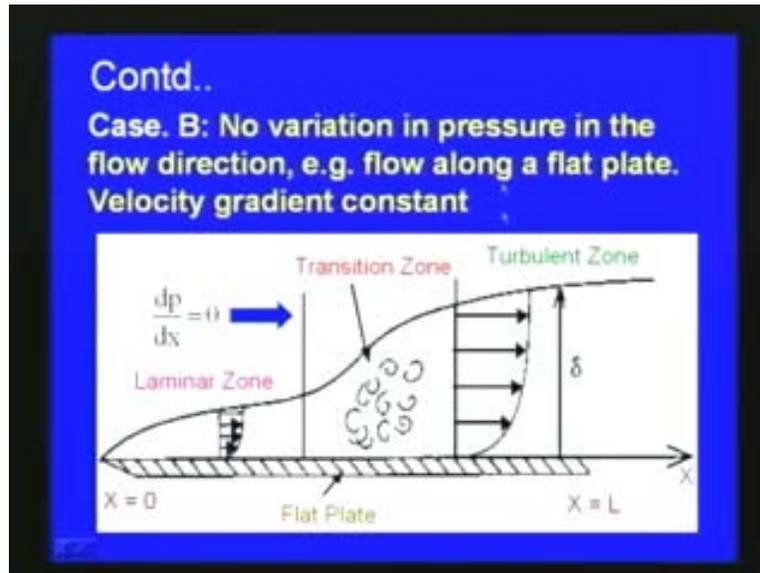
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First condition is case A, where pressure decreases in the direction of flow; the fluid will accelerate and the boundary layer will become thinner, this is called the convergent type of flow. We can see that here the flow comes and then there is a convergent as far as the flow is concerned. Then, from here onwards the dp by dx is less than 0 so that pressure decreases and the fluid will accelerate and the boundary layer will become thinner in this case.

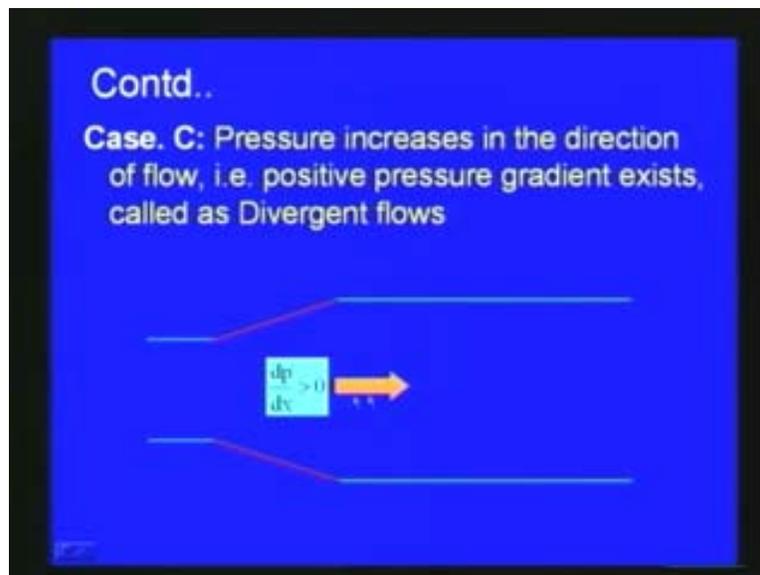
The second case is case B, where if you consider the flow over a flat plate when we can see that here there is no variation in pressure in the flow direction. Example, flow along a flat plate velocity gradient is constant. You can see that here the free stream velocity coming and then the boundary layer is developed and initially laminar and transition turbulent boundary layer.

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Here is the second case; there is no pressure variation in the flow direction.

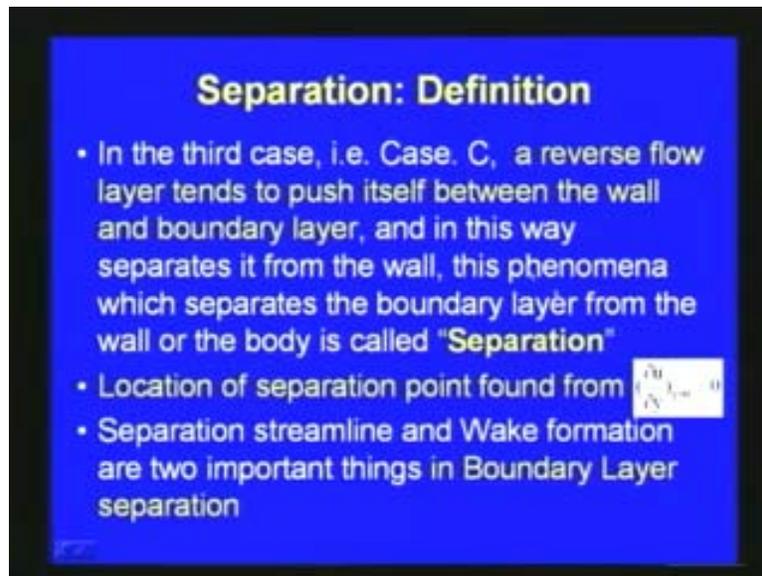
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The third case is here the pressure increases in the direction of flow, that means, here you can see that here the flow diverges; the dp by dx is greater than 0. You can see that positive pressure gradient exists and this kind of flow is called divergent flow. So depending upon case A or case B or case C whether convergent flow or flow over a flat

plate which we consider or the divergent flow, you can see that separation of the boundary layer properties changes.

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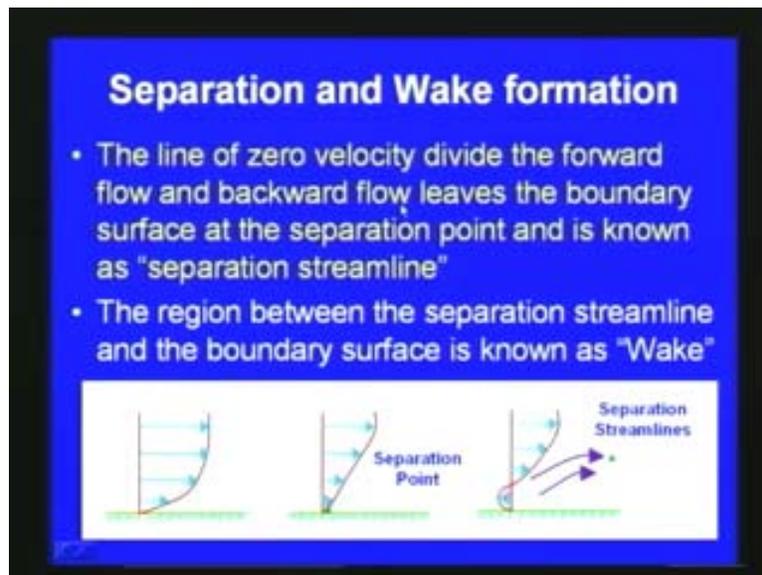
Especially, in the third case, the last case where we consider the divergent flow you can see that reverse flow layer tends to push itself between the wall and boundary layer and in this way separates it from the wall, this phenomenon which separates the boundary layer from the wall or the body is called separation.

So these three cases, I have presented here to show how the variation takes place and which the most obvious case is where the separation is immediately visible. In the case c for the divergent flow you can see that a reverse flow layer tends to push itself between the wall and boundary layer and then separation starts. This process where the boundary layer separates from the wall or the body that phenomenon is called boundary layer separation. Generally, this separation, the location of the boundary layer separation we can find out the velocity gradient $\frac{\partial u}{\partial y}$ in the y direction is equal to 0. If you equate this to 0 then we can find out the location of the separation points

You can see here in the case of flat plate what is happening and then divergent flows what is happening we can see. With respect to which we have already analysed with respect to case C, the divergent flow is very easily visible; how the separation of the

phenomena separation takes place and then we can find out by equating $\frac{du}{dy}$ is equal to 0 the location of the separation point. Then, we can draw the separation streamline and then a wake formation also that means due to the pressure difference here the pressure changes with respect to boundary layer formation and then a wake is formed. This separation streamlines and wakeup are two important things in the boundary layer separation which we have to consider while analyzing the boundary layer separation we have to find out the separation streamline and then we have to see that whether any now pressure or the wake formation takes place behind the body or after this separation takes place.

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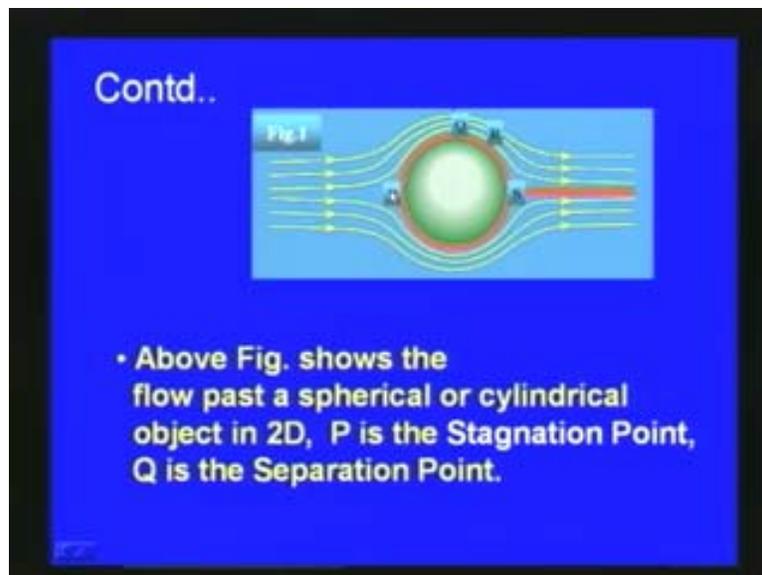


This separation and wake formation here this slide shows how the separation point where two starts and then how the process taking place. The line of zero velocity divide the forward flow and backward flow leaves the boundary surface at the separation point and is known as separation streamline. We can just find out the line of zero velocity which divide the forward flow and backward flow which leaves the boundary surface at the separation point and we can find out the streamline. The region between the separation, streamline and the boundary surface is known as wake. Here, we can see that before any wherever the boundary layer formation takes place and then in the separation starts, if you plot the velocity it will be in this form and wherever the separation points starts we

can see that it will be going to this level. You can see that after the separation streamline forms. You can see that the direction of the velocity reverse and you can see that it will be of this from so that we can find out the separation streamline; we can plot the separation streamline.

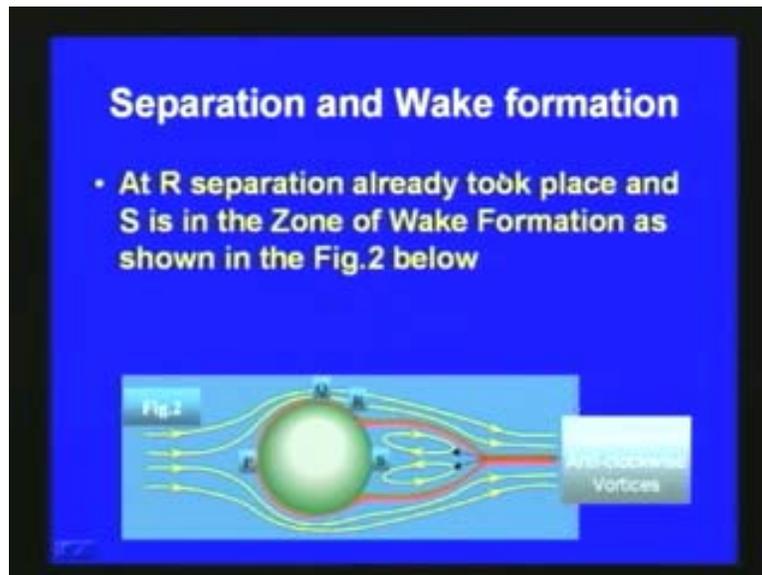
Separation streamlines and wake formation, we have to analyse when we analyse the boundary layer separation we have to discuss with respect to the separation streamlines and the wake formation.

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If you consider a cylinder like this we can see that here the flow past a spherical or cylindrical object in 2D. We can see that free stream is coming like this and then here there will be location where the flow velocity is 0 so that is called stagnation points; here P is the stagnation point. Then you can see that the separation flip flops, the boundary layer we can see that the about this location at Q the boundary layer separation starts; we can say that this is the separation point. We can see that here between the two separating between the separate streamline we can see that wake will be formed. Here, in this figure, P is the stagnation point, Q is the separation point and then S is where it is located in the wake as far as the flow over a spherical or cylindrical object in two dimensions is concerned.

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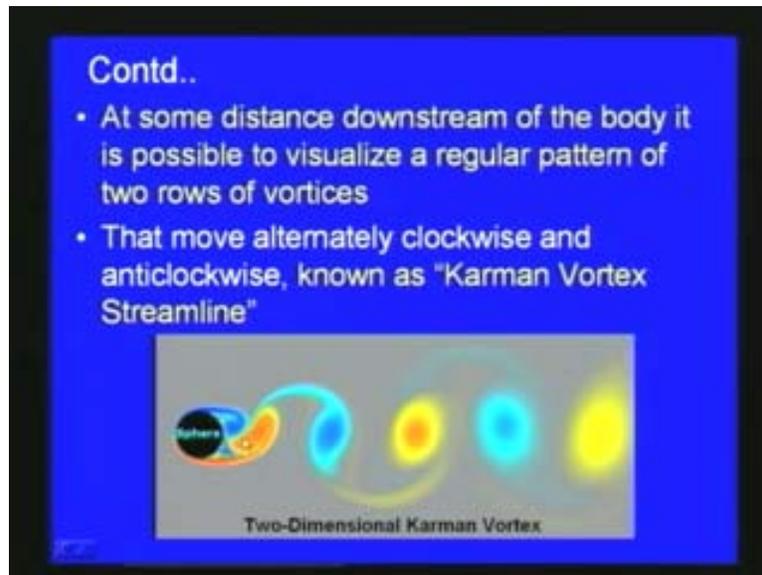


Now, as for as separation wake formation is concerned again we can see depending upon various parameter whether the cylinder is smooth or whether the cylinder is rough; we can see that the boundary layer formation as well as the boundary layer separation changes. So, with respect to the cylindrical flow over a sphere or cylinder in two dimensions which we have seen in the previous slide, at R separation already took place.

As I mentioned earlier Q is the separation point and then where onwards the separation already took place and then we can see that the wake is formula and this shows the wake. We can see that after this within this wake there will be clockwise and anticlockwise vertices will be formed and then this region is called the wake and this process is called wake formation as shown in this figure. Here, the free stream is coming like this and then here is a stagnation point and Q is a separation point and at R the separation already takes place. Then, we can see that the separating streamline is like this and then in this region a wake is formed and then S is this point; S is in the zone of wake formation. Depending upon the case, depending upon whether we are considering circular cylinder or whether depending upon the smoothness of the cylinder and also the fluid properties we can see that clockwise and anticlockwise vertices may be formed. Depending upon the case you can see that this is clockwise and anticlockwise vertices are formed within this way. So this is what we can generally observe. We can easily visualize experimenting how this

separation and then the wake formation take place. Now, as I mentioned after the separations starts and the separating streamline also identified then we can see that some distance downstream of the body we can visualize a regular pattern of two rows of vortices.

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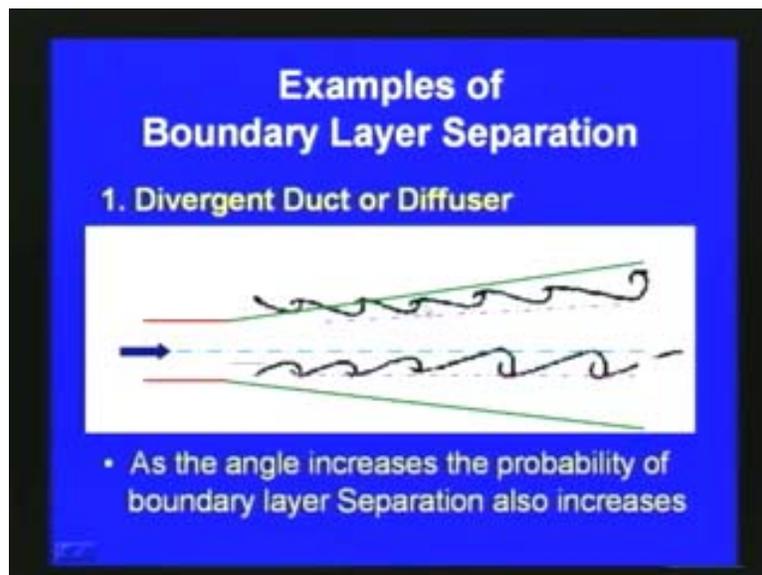


With respect to this slide it is very much clear; a sphere is there and then flow takes place. Then, we can see that at some distance downstream of the body we can visualize a regular pattern of two rows of vortices and these two rows of vortices move alternative clockwise and anticlockwise. This is called Karman vortex streamline. We can see that this is two dimensions Karman vortex. So, depending upon this it is plotted for this sphere or we can also plot for the circular. Depending upon the various flow parameters as well as the nature of the sphere or the cylinder integral smoothness then we can see that a Karman vortex will be formed.

We can see that in regular pattern two rows of vortices which is as we can see that here it starts and then Karman vortex will be formed with respect to the sphere or cylinder which we consider. So, this is the after effect of the boundary layer separation and then within the wake, this Karman vortex is formed. Like this boundary layer separation is very important in the design of many of the automobiles like car, truck, bus and also this very

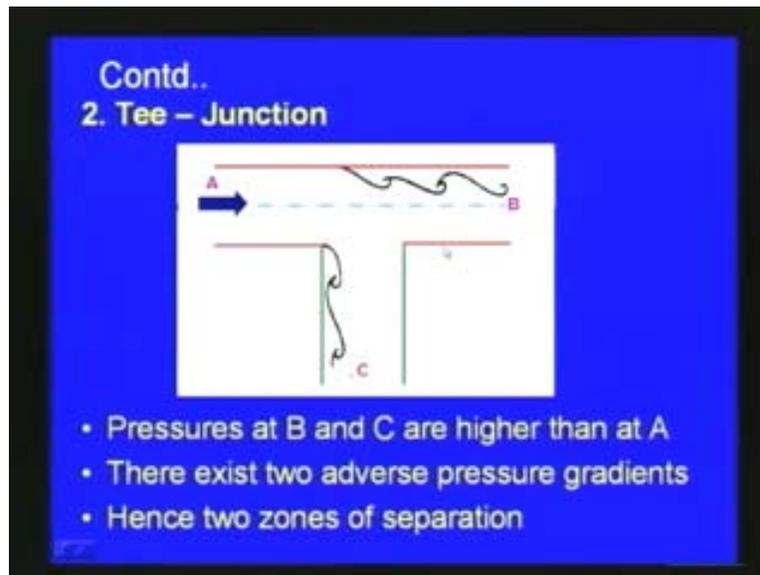
important in the case of aero plane, missile, space shuttle or the rocket designer depending upon the shape. The separation may start or earlier or separation will be at the end. Accordingly, various fluid flow parameters the velocity, the pressure etc., will be changing. It is very important to analyse where the boundary layer separation starts and how the wake formation takes place and its properties. Now, we will just discuss some of the examples.

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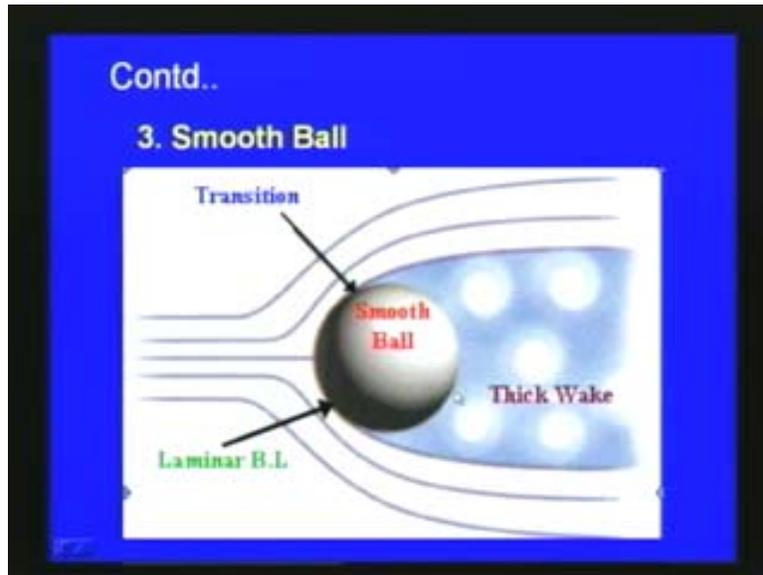
Boundary layer separation: First case is divergent duct or diffuser. As we have seen earlier, the flow is coming like this and then there is a divergent duct; it is divergent like this or as in the case of diffuser you can see that here the boundary layer is formed and then the separation starts; then, as the angle increases the probability of boundary layer separation also increases depending upon the case. First example is here: you can see the divergent duct or diffuser; the boundary layer depending upon the angle, if the angle is more then we can see the probability of boundary separation also increases. This is the divergent duct or diffuser where boundary layer separation takes place.

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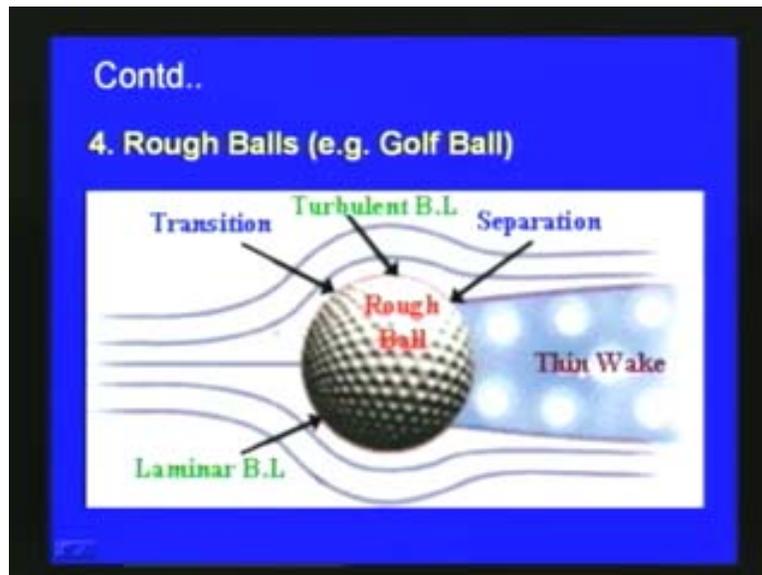
The Second case is a tee junction, this is another example. You can see that flow come this direction and then it will be going through the direction of B as well as in the direction of C. Here, the pressures at B and C are higher than at A depending upon the case and there exist two adverse pressure gradients with respect to A. You can see that in this case there is a possibility of two sounds of separation as far as the boundary layer is concerned. You can see that this boundary layer will be formed here and then the separation and here the boundary layer formation and the separation. There are possibilities of two sounds of separation so this is the case of a tee junction.

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As we have already seen in the third case the flow is over a cylinder or a sphere. We consider a ball. The free stream flow over a ball you can see that depending upon the property this smoothness of the ball we can have a thick wake or a thin wake. In the case of smooth ball, we can see that this separation starts much earlier. You can see the free stream is coming like this and then here the laminar boundary layer will be somewhere and then transition. Then, we can see that the boundary layer separation starts much earlier. So that at this location the boundary layer separation starts and the separating streamline is this one and then we can see there will be a thick wake. The reason is that here the ball which we consider is smooth. We will be having a thick wake as drawn in this slide.

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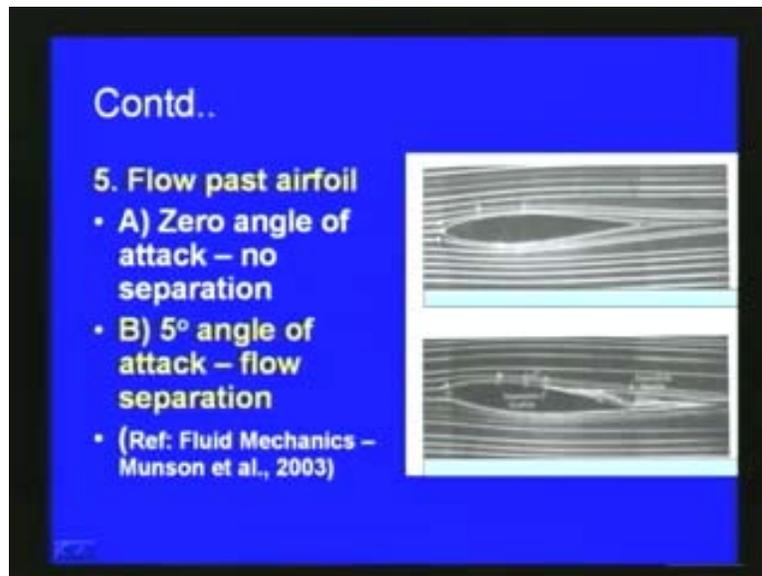
As I mentioned this depends upon the separation as well as the wake formation depends upon the smoothness of this surface, also over which the flow takes place the same ball which we consider in the last slide.

If the ball is rough or the sphere is rough then we can see that the boundary layer separation starts later stage and then there will be a thin wake. So, this shows here the free stream velocity and then laminar boundary layer and then transition. You can see that there will be a turbulent boundary layer and here due to the roughness of the ball I for example, in the case of a golf ball or various other kinds of ball where the surface is rough you can see that separation is much after. Here, in this location the separation starts and then we can see that this is the separating streamline. In this case, the separation is much. Here, in this smooth ball case where the separation is much earlier and here the boundary layer separation is much lighter so that we will be having a thin wake.

That is why I mentioned, we have to see that not only the flow properties and the fluid properties but depending upon the case whether the surface is smooth or surface is rough again the boundary layer characteristics will be different. Then this separation also starts earlier or later. Also the wake formation will can be thin or thick depending upon the

case. So this is the fourth case where the flow over a rough ball and then the separation with respect to the rough ball.

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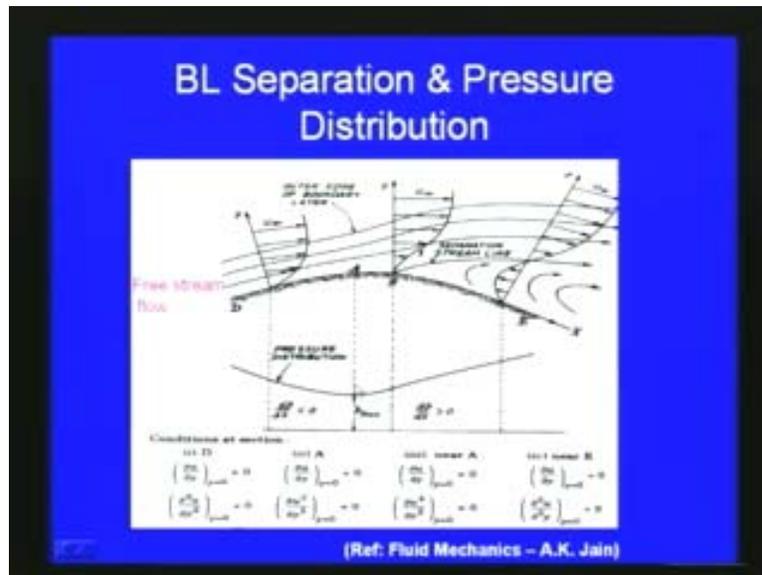


Then, the fifth case is the flow past airfoil. This airfoil we can use to see how the separations take place by placing different angles. Here, we can see that in this slide the flow past an airfoil. If you place it, the free stream is coming like this and if this is the airfoil and then you can see that the angle of attack is 0. You can see that if the airfoil is perfectly made in such that we can have a structure where there is no separation takes place. You can see that the flow is going like this so there is no boundary layer separation. This is the case for zero angle of attack but the same airfoil if you just slightly tilt so that some angle is formed. Then, we can see that for example, 5 degree of angle of attack, then you can see that the boundary layer will be totally of different nature and then the separation flow separation also takes place.

Here, in the second figure we can see that these figures are taken from fluid mechanics by Munson. You can see that in second figure this separation point is here and then you can see that even there is a possibility of small make formation even for this airfoil equation. So, the same airfoil if you put it with zero angle of attack then we can see that no separation take place but if the angle of attack is 5 degree then they will be separation and

so depending upon the angle of attack again the location of separation changes and also the boundary layer characteristics change. So, this case shows how the flow separation and then this separation of stream line behave with respect to the placement of the body. So, this is the fifth case.

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Now, we have seen that the flow separation, the boundary layer separation takes place and it is very important to analyse from where the separation starts and what are the various characteristics. With respect to the fluid flow parameters it is very important in the analysis and the design of various bodies wherever this boundary layer is forming and then separation can take place. So, before discussing further the various applications of the boundary layer separation. Here, we can see that a figure with respect to for which here I have shown the variation of the velocity and the pressure. This figure is taken from the book of fluid mechanics by A.K. Jain. Let us consider the free stream flow over a curved surface like this. So, this is the curved surface and the free stream is coming like this. You can see that with respect to the surface a boundary layer will be formed. Initially, at this location approximately near to this point d consider if you plot the velocity will be going like this and then we can see that this region of the where the boundary layer is formed; a special distribution, if you plot you we see that dp by dx will be negative of dp by dx will be less than 0.

At this location, if you plot the velocity gradient $\frac{du}{dy}$ at y is equal to 0 we can see that gradient will be greater than 0 and if you take the second derivative of the velocity you can see that $\frac{d^2u}{dy^2}$ will be less than 0 at this location. Then, you can see that due to the curvature of the surface here a separation occur at this location S. So before that for example at A we can identify a section where the pressure will be minimum, appear the pressure p is minimum. Then, you can see that at this location A if you take the velocity gradient you can see that the velocity will be deforming like this which is keep on reducing like this case at A. At this location, you can see that the gradient of the velocity $\frac{du}{dy}$ will be greater than 0 and $\frac{d^2u}{dy^2}$ will be almost 0 since this pressure is p minimum. Here, at this location it will be $\frac{d^2u}{dy^2}$ will be 0. So, that is at this section at A.

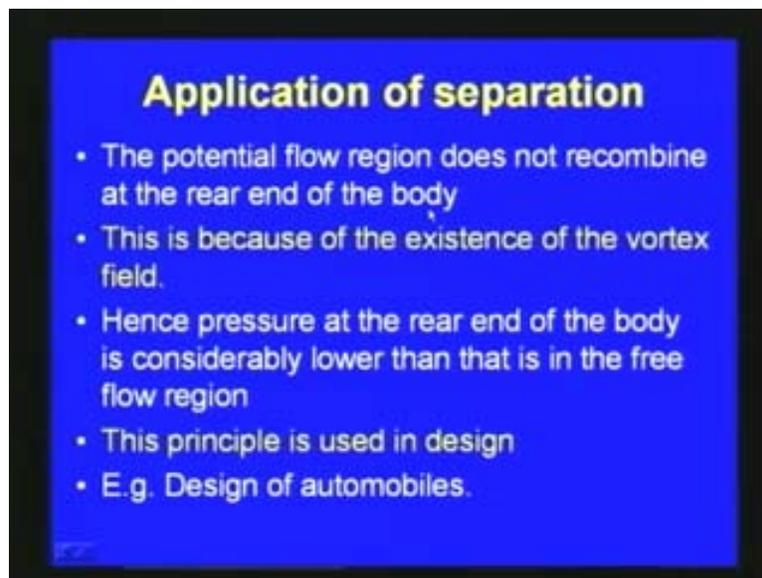
Now, if you consider a section at S where the separation already starts so you can see that the separation stream line is here; the velocity plot is here. The velocity will be changing like this and if you plot the velocity gradient you can see that $\frac{du}{dy}$ at y is equal to 0 will be 0 near A. So this location near A or at S will be the velocity gradient which will be zero at this location and $\frac{d^2u}{dy^2}$ will be also 0 at this location. Then the flow separation takes place and you can see that the pressure $\frac{dp}{dx}$ is from here onwards after this stream minimum the pressure minimum, the pressure gradient increases $\frac{dp}{dx}$ will be greater than 0 like this. Then, near E wherever after the separation takes place we can see that $\frac{du}{dy}$ at y is equal to 0 will be less than 0 and $\frac{d^2u}{dy^2}$ will be greater than 0.

You can see that the velocity if you plot you can see that reversal velocity takes place after the separation of the stream line or the boundary layer separation we can see that the velocity will be of the state. Then, here a wake will be formed. So, here with respect to this figure we have analysed with respect to the flow over a curved surface how the velocity changes with respect to the boundary layer separation; from starting and after the separation takes place. Initially, with respect to the boundary layer the velocity pattern is different and where the stream separation starts, the boundary layer separation start the velocity parameter the velocity behavior is different with respect to section. After the separation, you can see that reversal of the flow direction will change in the direction of

the velocity and then that is why we say wake is formed. Then we have also seen how the pressure is changing. Initially, the pressure gradient is less than 0 and then there is a location where the pressure is minimum and then the pressure gradient is $\frac{dp}{dx}$ is greater than 0 after the separations starts. So, this slide shows how the boundary layer separations with respect to pressure distribution how the parameters like velocity in the gradient of velocity and the second derivative of the velocity $\frac{\partial^2 u}{\partial y^2}$ is changing.

So this way we can analyse various structures of various bran bodies surfaces, how the boundary layer separation takes place and how the pressure changes and how the velocity gradient changes. Now, before going to the other topics like drag let us see what the application of these separations.

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We have seen with respect to the shape of the body, with respect to whether it is blend body or with respect to surplus cylinder or whether it is a sphere or a curve surface the boundary layer can separate at various locations and then wake formation takes place. So, I already mentioned there is large number of applications of this boundary layer separation especially, in the design of aero planes and then space shuttle and also the design of automobiles.

So, some of the applications of separation are listed here. The potential flow regions with respect to the boundary layer separation you can see that the potential flow region does not recombine at the rear end of the body. As we have already seen various cases, this is because of the existence of the vortex field. We have seen that after the separation takes place there is possibility within the wake, there is a possibility of vortex field so that the potential flow which does not recombine at the rear end of the body. Hence, pressure at the rear end of the body is considerably lower than that is in the free flow region. The pressure change at the rear end of the body is considerably lower than that is in the free flow region. After the boundary layer separation and then is the wake formation or due to the existence of the vortex field what happens after the separation take place. This principle we can utilize in the design of automobiles and also in the design of various flying machines like aero planes and other equipments. This is some of the important applications with respect to the separation of the boundary layer. Here, now in this slide as I mentioned there are number of applications of this boundary layer formation and its separations. So you can see that.

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The boundary layer separation after its identification of its boundary layer formation separation we can see that this is very much used especially in the design of vehicles in automobile engineering. Hence, see that the shape of the cars also change in last few

decades since we can see that wherever the separation for example, in this case here this car is concerned; the shape of car flow with respect to the boundary layer formation and the separation is plotted here. So, you can see that at this location the separation starts but here you can see this car is considered we can see at the rear end separation starts. As we have already seen in the case of airfoil, if the airfoil is placed in such that angle of attack is 0 so that there can be a condition where there is no separation so that the flow is much smoother.

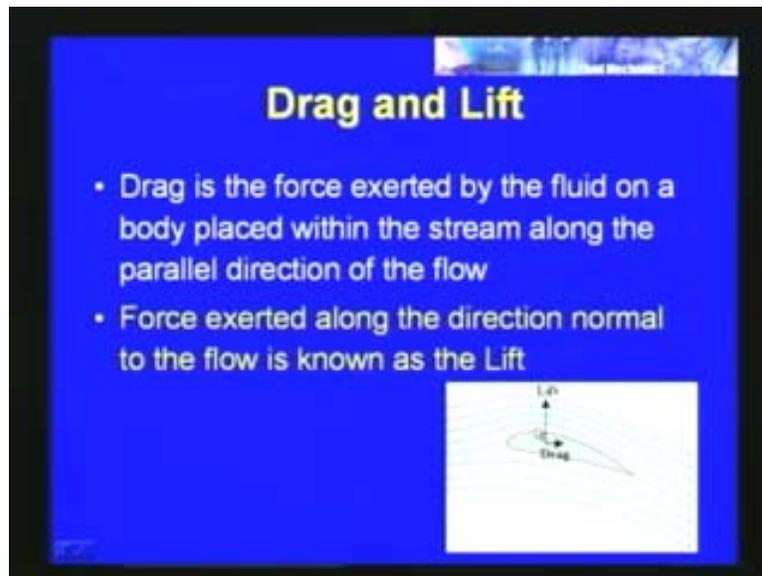
Like this in the automobile industry also we can utilize this principle. So that there will be less resistance by driving the car or while driving the vehicle there will be of course, wind effect will be there on the vehicle. So, the design of the surface of the vehicle is such that the separation is taking at the rear end and then the flow resistance will be much lesser so that we can design, we can see that the shape of the car, how the shape of the cars are evolved say, starting earlier we can see that in 1980's and 1950's the design was like this and now you can see the latest design of the vehicles you can see here like this. There will be especially for the case of sports car you can see that the shape is designed such a way that the separation takes place at the rear end. So that there will be very less resistance to the moment with respect to the wind effect on the vehicle.

Similarly, the design of buses, trucks and all these automobiles we can do in such a way that in the resistance with respect to the wind effect is much lesser on the vehicle which we consider and then separation we can; if it is at the rear end it will be much better compared to the case if you say with respect to the vehicle. Here, the study of the boundary layer separation and its analysis helps in the automobile designing the shape so that there will be lesser assistance with respect to the wind effect on the automobile.

Like this in a very similar way, when we design the shape of the aero plane its wings and its various components we can utilize this boundary layer separation theories so that we can design the aero plane wings and other component such a way that there will be minimum resistance with respect to the wing on the aero plane. Like this, there are number of applications we can find out this boundary layer formation and the separation of the boundary layer. So, further we have seen how the boundary layer forms and then

what are the govern equations and then we have also seen the boundary layer separation and its applications

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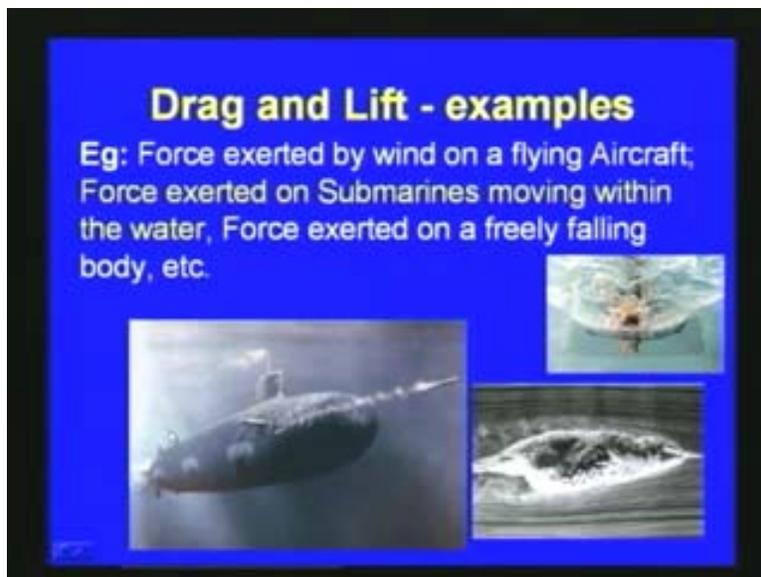
Now, another important thing with respect to the external flows are concerned, say flow over a body or flow over a surface two important parameters are drag and lift. So, next topic which we will discuss is drag and lift effect. You can see that if you consider the flow over an airfoil like this we were discussing earlier. You can see the wind of the free stream velocity is coming and then you can see that there will be two important effects with respect to the flow over the body which is immersed in a fluid flow or the body which is moving through a fluid. There are mainly two effects: one is called drag effect and second one is called lift effect.

With respect to this figure, if we can see that drag is in this direction of the flow and lift is normal to that. So, drag is the force exerted by the fluid on a body placed within the stream along the parallel direction of the flow. So, drag is defined as the force exacted as I mentioned here we are consigned with the external flows where the body is immersed and fluid flow or the body is moving within the fluid. So, drag is defined as the force exacted by the fluid on a body placed within the stream along the parallel direction of the flow; the flow is taking place this direction so the body is the moving body or the body

placed stationary, the body is with respect to the direction of flow what is the force acting on the body? So that force is called the drag.

This is very important aspect when we consider many design of aero plane, design of some submarines and many other types of equipment. The force exerted along the direction normal to the flow is known as lift. So, here the lift is indicated like this. What is the force acting along the direction normal to the flow is known as the lift. Two important aspects: one is the drag and the second one is the lift. As far as the external flow **circlets**(32:45) or flow over a surface is concerned or the moment of a body within a fluid is concerned, two important aspects which we have to consider are drag and lift. So it is all defined here the drag and lift with respect to the slide here

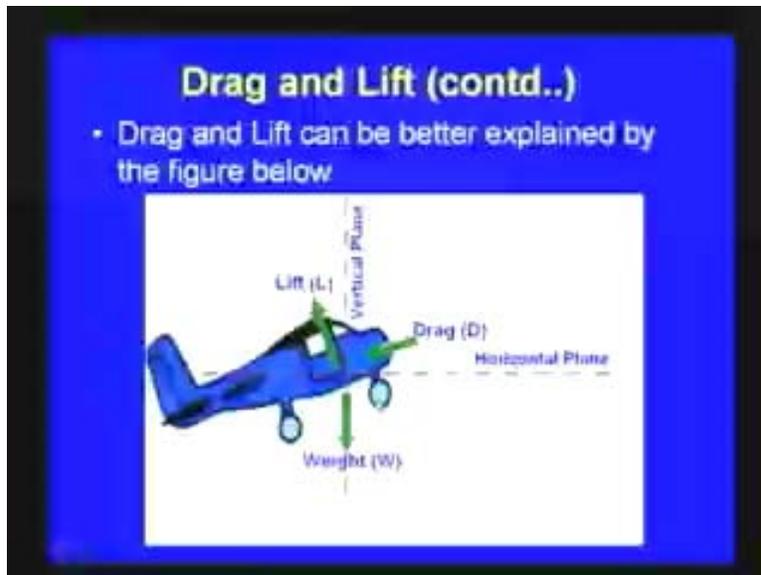
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We will be discussing in details about this drag and lift effect. Now, some of the examples as I mentioned in this case the submarines are moving through the sea. We can see that submarines are moving then it is moving through the sea water. You can see there will be of course with respect to movement of the submarines there will be drag effect which we have already defined and also there will be lift effect. A sea bird moving through a fluid you can see that how it is already plot the stream line and how the moving boundary layer we can see how it forms and then there will be definitely a drag effect on

the body surface of the bird; also, there will be a lift effect. We can visualize this person swimming in the swimming pool you can see that with respect to the direction of the swimming there will be drag force and then also and there will be lift force. We can have a number of examples with respect to the force exerted by wind on a flying aircraft. As I mentioned there will be the drag and lift are to important force which we have to consider and we have to critically analyse with respect to the flying aircraft. How much drag will be there? How much will be the lift? These are two very important parameters which we have to consider and also in the design of submarine and also many other also the force exerted on a free falling body like in the sedimentation process in server we have to consider the drag and lift effect. Some of the examples are shown here with respect to this slide.

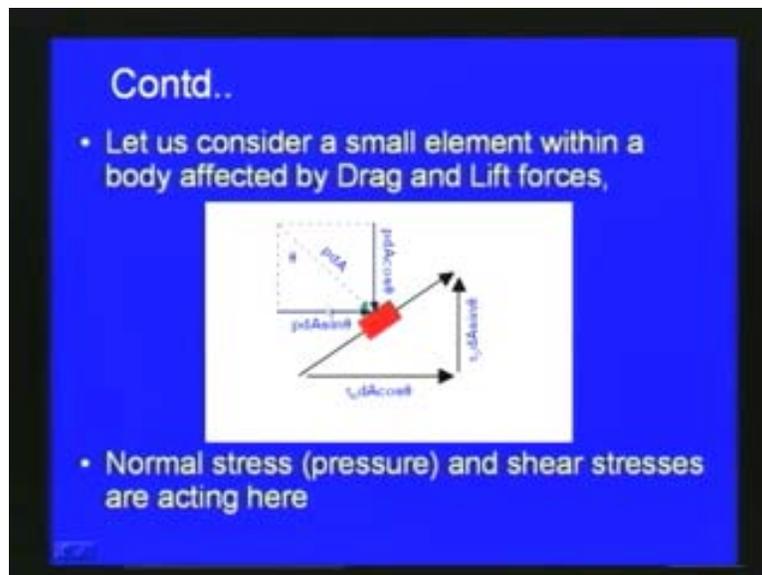
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Now, further to explain this drag and lift, let us consider an aircraft like this. We can see that here in this slide this aircraft is moving this direction. So you can see there will be drag and lift effect on this aircraft; so horizontal plane is like this and vertical plane is here. With the direction of the aircraft, the aircraft is moving this direction so with respect to that wind will be coming on the draft. Then, you can see their drag will be in this direction and then lift will be normal to that; lift will be in this direction and then of course the weight of the aircraft will also be there which we have to consider. So, weight

of the aircraft w and then drag force d and then due to force in this direction. So, these are the forces which we have to consider with respect to the flying of this aircraft. It is the drag and lift effects are marked here in this figure. Now, we have seen the definition of drag and lift and then we have seen various examples. We have to critically analyse this drag effect and lift effect of the bodies so especially when we consider the external flows. We have already defined the drag in the direction of the fluid flow or the direction of the moment of the body or the vehicle or the body which we consider lift is normal to that. With respect to this slide here we can identify the drag and the forces.

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You can see that generally the drag and lift forces mainly due to the pressure, the normal stress variation of the pressure variation and then with respect to the shear stress acting on the surface. If you consider a small element within a body affected by drag and lift force as shown in this slide, we can see that this is the body which we consider and then the pressure is p and if you consider an element variant da then you can see that the pressure will be p into da and with respect to this angle is θ . We can see that this direction will be the pressure force p into $da \sin \theta$ and normal direction it will be p into $da \cos \theta$

Similarly if you consider as a shear stress, with respect to viscous fluid a shear stress will be there on the body. If you consider the shear stress, you can see with respect to this angle theta this direction shear force will be $\tau_0 dA \cos \theta$ and normal to that a flow direction the shear force will be $\tau_0 dA \sin \theta$, where τ_0 is the shear stress acting on this elemental area. With respect to this the pressure or the normal stress and the shear stress we can derive the expression for the drag and lift.

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- Referring to the fig. we have,

$$dF_L = \tau_0 dA \sin \theta - p dA \cos \theta \quad \dots(1)$$

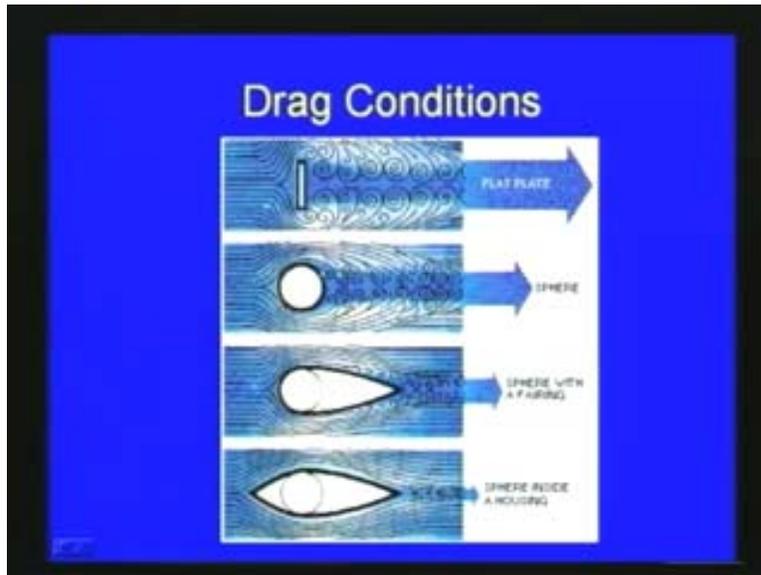
$$dF_D = p dA \sin \theta + \tau_0 dA \cos \theta \quad \dots(2)$$
- Hence the Drag force, $F_D = \int p dA \sin \theta + \int \tau_0 dA \cos \theta$
- The Lift force, $F_L = \int \tau_0 dA \sin \theta - \int p dA \cos \theta \quad \dots(3)$
- The shear stress component of the lift force may be neglected as it is small compared to pressure, hence $F_L = - \int p dA \cos \theta$

With respect to this figure here we define on that element area which we consider the lift force is equal to $d F_L$ is equal to $\tau_0 dA \sin \theta$ minus $p dA \cos \theta$ so this $p dA \cos \theta$ in this direction and $\tau_0 dA \sin \theta$ in the opposite direction. That is why we are considering with negative sign here. The lift force on the element layer is equal to $\tau_0 dA \sin \theta$ minus $p dA \cos \theta$ is written in this equation number 1. And then the next the drag force is concerned. You can see that both this force the $p dA \sin \theta$ and the with respect to the shear stress both are in the same direction. We can write dF_D - the drag force on the elemental area, dF_D is equal to $p dA \sin \theta$ plus $\tau_0 dA \cos \theta$ as in equation number 2. So these are the equations which we consider for the lift force and the drag force for which we consider an elementary area

Then, we consider the normal stress or the pressure force and then the shear stress or the shear force. Then, we have the form with respect to the direction. With respect to the angle θ , we go to equations for the lift force on the elemental area and for the drag force on the element area as shown in equation number 1 and 2. To find out the total drag force on the area which we consider, we can just integrate with respect to the area for the two dimension case which we consider. Hence, the drag force is consigned. We can just integrate. So, this equation number 2 we can integrate. In this case, drag force is f_D is equal to $\int p dA \sin \theta$ plus $\int \tau_0 dA \cos \theta$ with respect to area A . So, this expression gives the equation for the drag force for the body which we consider. Similar way the lift force is consigned; we can now say the element area dF_L we have seen with forces on the element area and equation number 1.

We can integrate this expression equation number 1. So that we get a lift forces F_L is equal to $\int \tau_0 dA \sin \theta$ minus $\int p dA \cos \theta$ over the area A as in equation number 4. This equation gives the lift force for the body which we consider. Equation number 3 and 4 gives the equations for the drag force and then lift force for the case which we consider. Now, regarding the lift force is concerned you can see that generally for viscous fluid flow shear stress component of the lift force maybe neglected and as it is small compared to the pressure force depending upon the case the exact equation is given by equation number 3 and 4 for drag force and lift force. Depending up on the case, say, maybe in some cases, the shear stress component we can neglect and some cases we can neglect the pressure force depending upon the problem. Generally, if you neglect the shear stress component as for lift force is consigned then we get this lift force F_L is equal to $\int p dA \cos \theta$ is minus since we consider this equation here so this is the lift force.

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As I mentioned how we place the body this drag force and lift force effect can be different. For example, if I consider this as a body, let me consider the plate like this. Now, I am placing this plate here so this is the plate and then you can see that the fluid flow is coming like this. Then, we can see as far as the drag force is concerned you can see that the shear stress effect will be much higher than the pressure force.

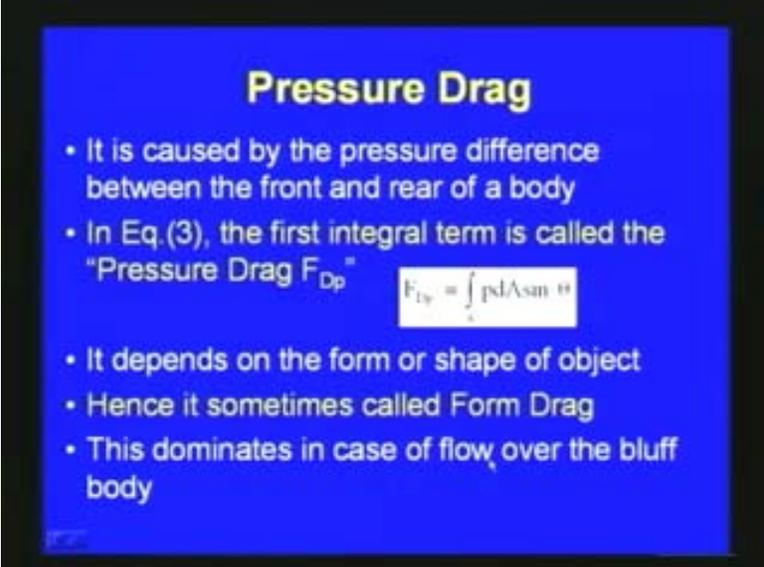
In this case, if the board is placed like this we have to consider the shear stress component will be much larger. So, the drag force effect on this will be mainly with respect to the shearing effect. If I place the same body in the other direction we can see that there will be a pressure difference between this and other side and then you can see that we have to consider the major component in this will be the pressure force. So, accordingly the equation which we consider, we have to see that whether the pressure drag is much higher than the drag to the shear effect is much higher.

So this drag condition with respect to the placement of the body with respect to the body shape is some of the effects are plotted here. If you consider this flat plate which we consider like this if you put in this location with respect to free stream then you can see that large wake will be formed and here the pressure effect will be much higher. This is in the case of a flat plate and then as far as sphere is concerned here the boundary layer

formation and wake formation are also shown and then correspondingly the drag effect will be vary.

So if the sales sphere is put in **with fairing**(42:33) like this so that we can see that union boundary layer separation changes. Then, instead of the pressure effect it will be more, the shear effect will be much higher and then we have to consider the equation correspondingly and the same sphere is inside housing like this we can see that here then we can see the boundary layer expression is at the end. The drag effect will be mainly with respect to the shearing effect. Like this the drag effect we have to depend upon how you orient the body or how the body is moving. With respect to that we have to consider whether the pressure effect in this equation number 3 whether it is major component this pressure effect of the sheer force or shear stress component accordingly. Similarly, in the case due to force also we have to consider. Accordingly, now this drag is concerned. We have already seen the major components are due to the pressure and the pressure variation and then with respect to shear stress or the viscous effect.

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Pressure Drag

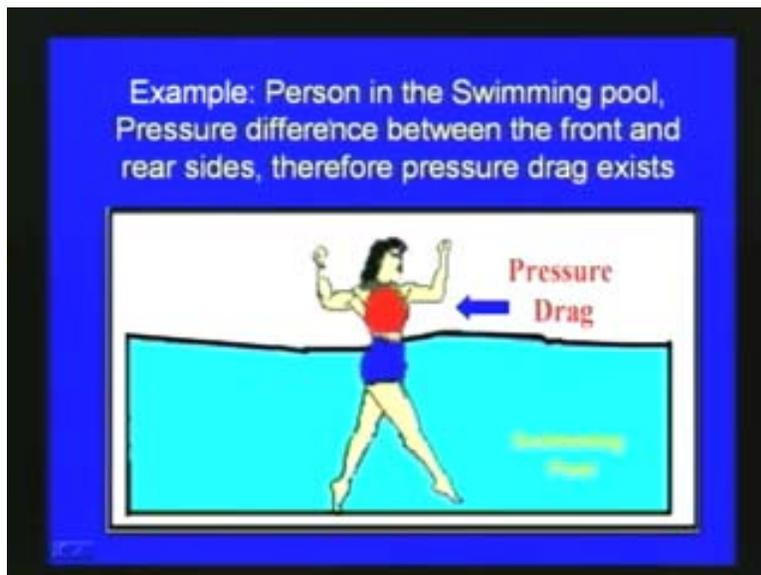
- It is caused by the pressure difference between the front and rear of a body
- In Eq. (3), the first integral term is called the "Pressure Drag F_{Dp} "
$$F_{Dp} = \int_C p dA \sin \theta$$
- It depends on the form or shape of object
- Hence it sometimes called Form Drag
- This dominates in case of flow over the bluff body

Accordingly, we can classify the drag: first one is the pressure drag. The pressure drag is caused by the pressure difference between the front and rear of a body. As I mentioned if the body is placed across the free stream velocity then a pressure drag is created and the

pressure drag is caused by the pressure difference between the front and rear of the body. Here, you can see a pressure drag is at the rear end is created. In equation 3, accordingly, the first integral term is called the pressure drag. So, F_{DP} is equal to $\int p \, dA \sin \theta$. This depends on the form or shape of the object as we are already seen here.

Hence, sometimes we call this form drag. Depending upon the shape of the body the drag effect is changing. According to the shape this can be varying. So, that is what is obvious, according the shape the drag effect will be defined. This is some literature; this special drag is also called as form drag; this dominates in case of flow over the bluff body. As we can see here this is the bluff body; pressure drag is dominating. In the case of this here this and here also the pressure drag will be much but when it is putting this way pressure drag reduces and the shear effect will be much larger. So, the pressure drag dominates in case of flow over a bluff body.

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Another example is placed here. A person is in the swimming pool. Pressure difference between the front and rear sides, therefore pressure drag exists. So between this side and the rear side, a pressure drag we have to consider.

Then, the second one is the friction drag. As I mentioned the drag can be mainly due to pressure drag and friction effect or the viscous effect. So, this friction drag exists due to

the viscous effects of the fluid and second integral term is called the friction drag or the shear drag.

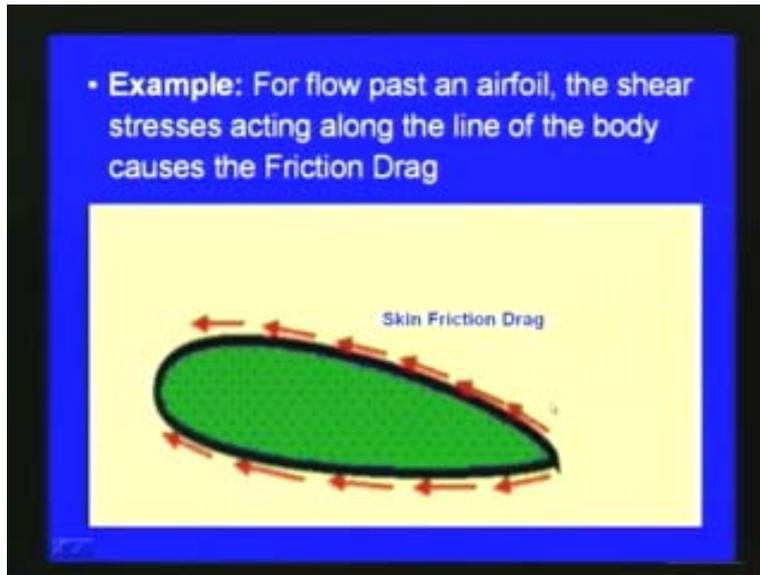
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Friction Drag

- It exists due to the viscous effect of the fluid
- Second integral term is called the "Friction Drag or Shear Drag F_{DF} "
$$F_{DF} = \int_A \tau_w dA \cos \theta$$
- It depends on the extent and character of boundary layer
- It sometimes called as Viscous drag
- The entire friction drag is created within the Boundary layer

So, F_{DF} is equal to integral $\tau_w dA \cos \theta$ over the area A. This depends on the extent and character of the boundary layer as we have seen here in the last figure. Here, the friction drag is much higher; it depends on the extent and character of the boundary layer. It is sometimes called as viscous drag. The entire friction drag is created within the boundary layer. Like this we can have the pressure drag and also we can have the friction drag.

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So, here this is an example. For flow past an airfoil, the shear stress acting along the line of the body causes the friction drag. This shows the skin friction drag. Here, the friction effect will be or the viscous effect will be much larger and then we have the skin friction drag. Like this, depending upon the cases we have to consider the pressure drag or the friction drag.

The next lecture, we will be discussing more about the drag and its effects and then with respect to the laminar boundary layer or with respect to the turbulent case, how the drag will be? How we will calculate the drag and how various parameters we have to consider with respect to the drag on the boundary will be discussed in the next lecture.