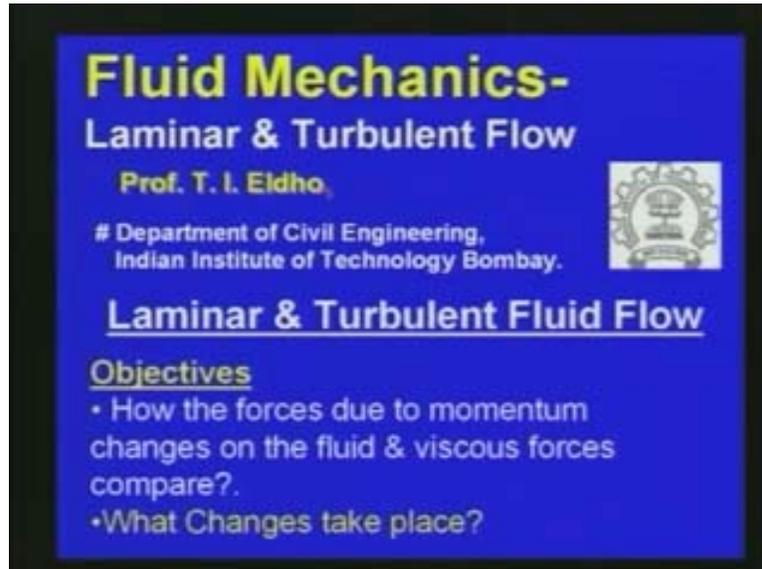


Fluid Mechanics
Prof. T.I. Eldho
Department of Civil Engineering
Indian Institute of Technology, Bombay

Lecture - 17
Laminar and Turbulent flows

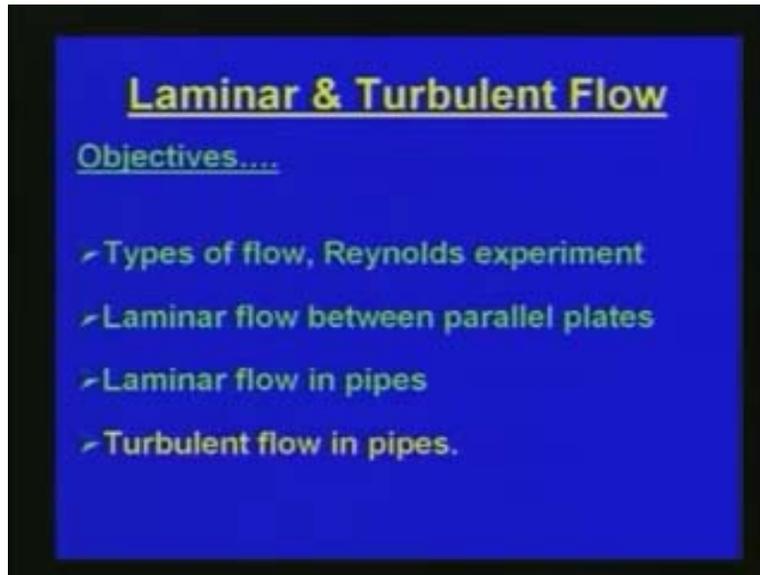
Welcome back to the video course on fluid mechanics. In this lecture, today we will discuss one of the important topics on laminar and turbulent flow.

(Refer Slide Time: 01:54)



The main objectives of what we are going to study in this chapter: we will be checking how the forces due to momentum changes on the fluid and viscous forces and how to compare? The main effect of viscosity will be how the flow parameter is changing? What are the changes that take place? How the fluid will be behaving with respect to the viscosity? How the system will be the laminar or turbulent? We will be checking that.

(Refer Slide Time: 02:14)



We will be looking into various types of flow. We will be checking one of the important aspects called Reynolds experiments. Where Reynold was born? Where did he utilize these experiments to classify the flow as laminar turbulent and the various cases? Then, we will be discussing the laminar flow between parallel plates; we will be discussing laminar flow in pipes. Also, we will be discussing the turbulent flow in pipes. So, these are the main objectives which we will be studying or which will be discussing with respect to this particular chapter on laminar and turbulent flow.

If you see the nature, we can see that the flow can be most the time either laminar in nature or it can be turbulent in nature.

(Refer Slide Time: 03:05)



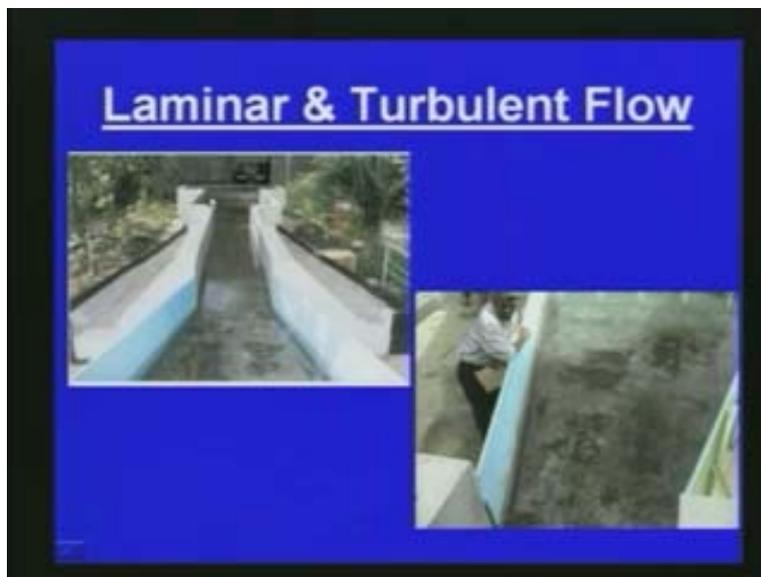
So, here we can see this. If we consider a river we can see that some parts of the river, the flow is called laminar; it is flowing in a very slow phase and we can see that the flow is in laminar type or it is in flow in layers but you can see some parts where we can see some gates are there with respect to this bridge here. We can see that some disturbances are created and then we can see that the laminar type of flow in this river is transforming to turbulent flow or we can see whenever a boat is moving just like here, that is, just behind the boat the flow is totally turbulent or totally disturbed. That means here the same river at some parts the flow may be laminar and in some parts the flow may be turbulent. Similarly, this is the case as far as open channel is concerned or river is concerned; you can see the same in a pipe flow also. So in this photographic we can see that whenever we open the pipe very slowly, when the valve is opened slowly we can see that flow will be coming just like in layer type or laminar type and when you are opening more, when the discharge is more, when the velocity is more we can see that lot of mixing takes place and the flow become turbulent. We will see this again how the flow can be laminar or turbulent in an experiment which we conducted in our hydraulics lab

So here we can see one of the experiments which we did in our hydraulics lab where some pump test is done (Refer Slide Time: 04:39). Here, we can see one open channel which is coming. The flow is initially laminar; then, there are some pumps and due to the

pumping effect the flow velocity of the discharge variation takes place and the velocity is increased. You can see that the laminar flow at some part of the channel becomes turbulent. You can see that here, the flow becomes very much disrupt. Then the laminar flow is transformed in to turbulent flow. Like this we can see many places where initially the flow may be laminar; sometimes it may be turbulent.

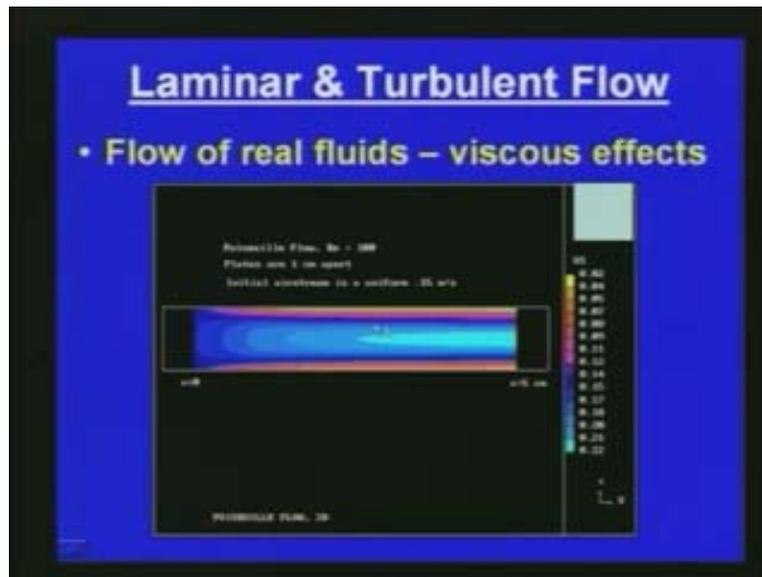
With respect to this, we will be discussing further the important parameters which decide the flow is laminar or whether the flow is turbulent. How we can classify whether the flow is laminar or turbulent and then we will be discussing further theories related to the laminar and turbulent flow. Now, the same experiment which we saw in the video show we can see here

(Refer Slide Time: 05:50)



Here, this is laminar flow and here due to the pumping effect each second becoming turbulent. Here, some places is laminar, some place it is in transitional stage. You can see the flow becomes turbulent. Now, with respect to the various theories on laminar and turbulent flow we can see that most of the times we are dealing with real fluids. Real fluids are concerned (06:00) varies of course viscosity of the fluid is very important parameter and viscous effects play a major role.

(Refer Slide Time: 06:18)



You can see that due to the viscous effect, as shown in this slide here, we can see there will be no slip condition on the boundaries and then we can see there the flow before becoming fully developed due to the boundary layer effects and viscosity effects. Some parts initially the flow can be laminar and then after sometimes depending upon the flow condition it can become turbulent. So, viscous effect is very important in fluid mechanics in the case of real fluids.

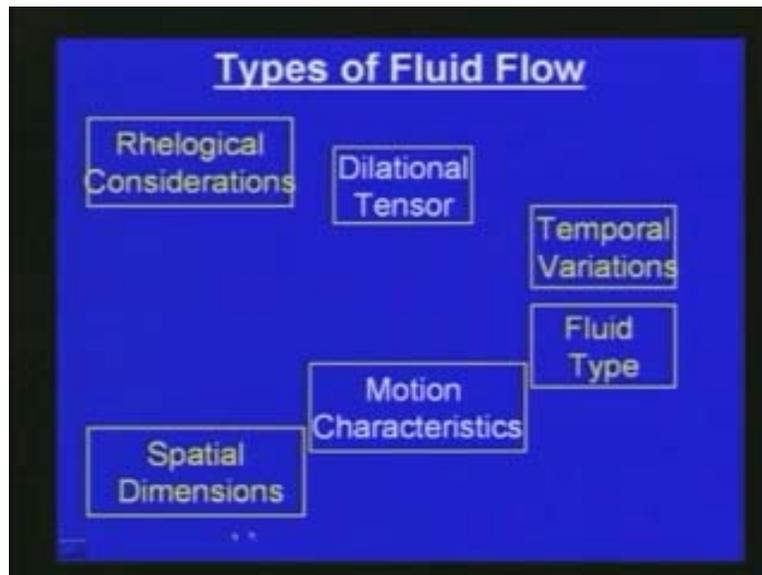
With respect to this we can see that due to the viscosity or the viscous effects the fluid tends to stick to the solid surface. In the previous slide, you can see due to the viscous effects or viscosity the fluids tending to stick to this solid surface in the both sides.

(Refer Slide Time: 07:20)

The slide has a blue background with white text. The title is 'Laminar & Turbulent Flow' in white, underlined. Below the title are three bullet points in white: '• Tend to “stick” to solid surface', '• Stresses within their body', and '• Newton’s law of viscosity'. To the right of the second and third bullet points are two white boxes containing mathematical equations. The top box contains $\tau \propto \frac{du}{dy}$ and the bottom box contains $\tau = \mu \frac{du}{dy}$.

Due to this sticking effect or no slip condition the boundary layer is created. The flow pattern can change whenever flowing fluid is interacting with the solid surface. Similarly, we can see the stresses also plays major role as far as various flow is concerned. When we are classifying the flow into laminar or turbulent the stresses are also very important; shear stress, normal stress and within the fluid how it will play is also very important. While discussing this laminar turbulent flow, another important aspect is the Newton’s law of viscosity. We have already seen in our introductory lectures that Newton’s law of viscosity says that the shear stress is proportional to the velocity gradient. So, τ is proportional to du by dy . With respect to the dynamic coefficient of viscosity we can write τ is equal to μ du by dy . This is the Newton’s law of viscosity; the shear stress is equal to dynamic coefficient viscosity multiplied by the velocity gradient.

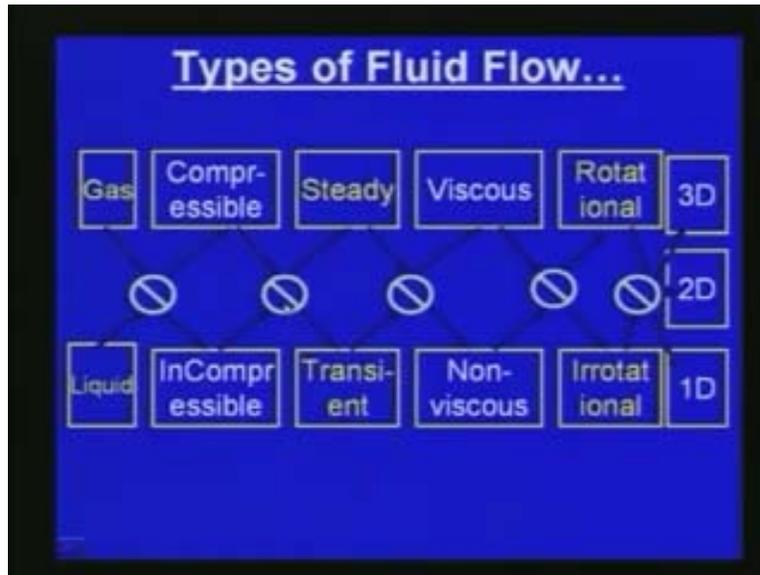
(Refer Slide Time: 08:50)



This Newton's law of viscosity is also very importance when we discuss the laminar and turbulent flow. Now, we are discussing the laminar turbulent flow. With respect to this it will be interesting to see the earlier classification which we have discussed in the introductory lectures, say, the types of fluid flow.

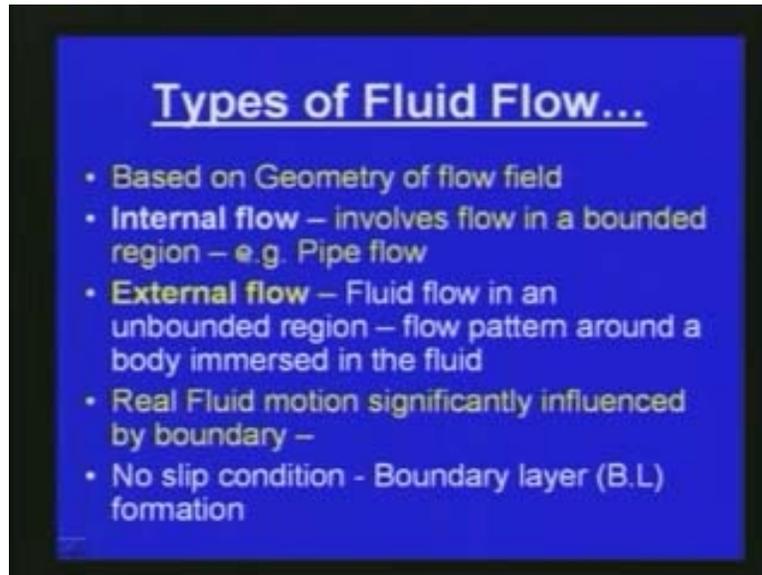
We discussed in earlier lectures the fluid flow can be classified with respect to the Rheological consideration like liquids and gases; Dilational consideration like which is compressible or incompressible; then temporal variations with respect to time, how the fluid is whether it is transient or steady state; and, spatial dimensions whether one dimensions or two dimensions, that is, whether the flow which we are considering is one dimensions, two dimensions or three dimensions. Also, the motion characteristics we have seen that whether it is a laminar or turbulent. These types of fluid flow we have discussed earlier. With respect to this, we are going to discuss in detail about the laminar and turbulent flow; the various aspects as what kind of fluid we are dealing; what kind of the motion characteristics or whether the spatial dimensions which we are considering or whether the temporal variations. All these are very importance when we discuss whether the fluid is laminar or turbulent.

(Refer Slide Time: 10:13)



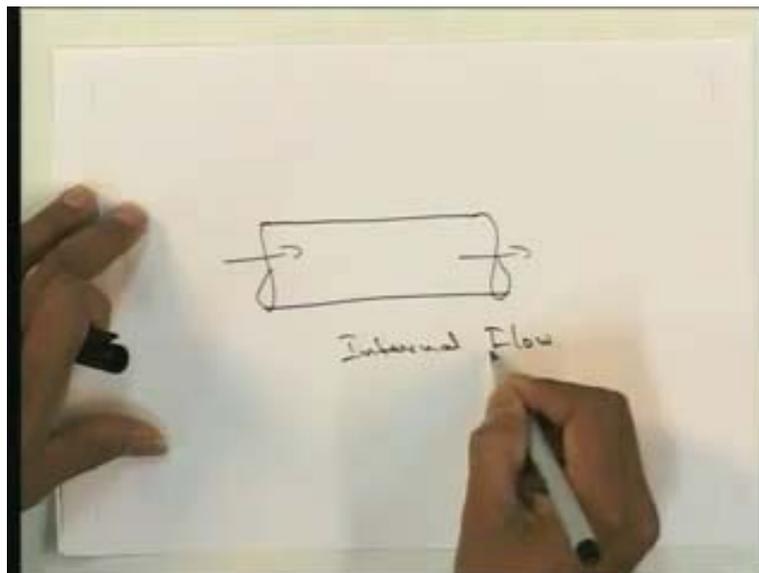
Accordingly, this chart we have seen earlier in our introductory lecture. We have seen that fluid flow can be classified as whether gas or liquid; with respect to the dilatational characteristics whether the fluid flow is compressible or incompressible; or with respect to temporal variations whether it is transient or steady stage; with respect to the viscous effect whether it is viscosity considerable or in viscous or non-viscous fluid which we are considering and also the rotational flow or irrotational flow. Especially, in turbulent flow rotational effect is very important and then also the dimensions which we consider whether one dimension, two dimensions or three dimensions. This type of fluid flow with respect to what we are discussing now laminar or turbulent flow, many of these parameters are very important. Before going in more details, the laminar and turbulent flow types of fluid flow also can be classified according to the geometry of flow field.

(Refer Slide Time: 11:22)



Most of the flow with respect to the geometry, we can classify whether the fluid has the internal flow or external flow. Now, internal flow means it involves flow in a boundary region so just like in a pipe flow.

(Refer Slide Time: 11:56)

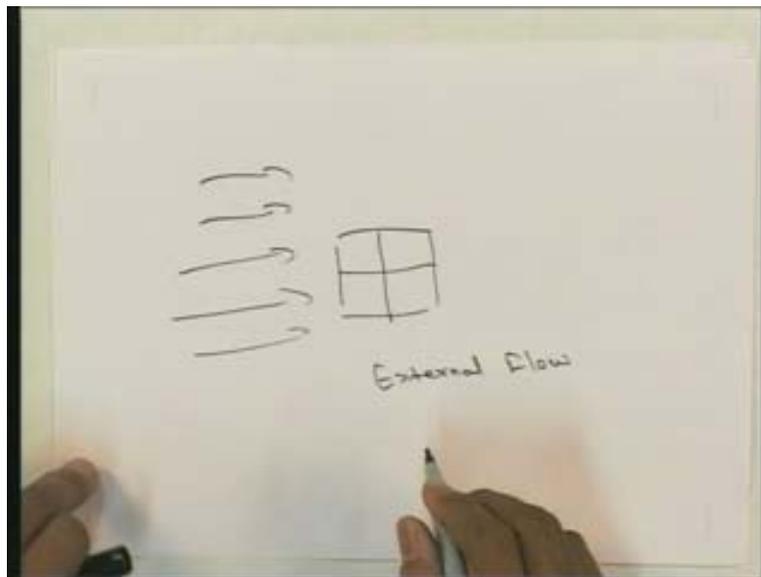


We can see there is a pipe flow; the flow is taking place in this way. We can see here the internal flow is in a pipe is the reason is we can see that the flow is confined within this boundary of the pipe. That is why we say that it is internal flow.

The approach which we will be analyzing in this kind of flow will be different just what we are discussing so called external flow. The pipe flow is an example of the internal flow. Now, other classification based upon the geometry is the external flow. So, here the fluids flow in an unbounded region.

We are considering the fluid flow in an unbounded region so it can be just like in a flow pattern around a body immersed in the fluid.

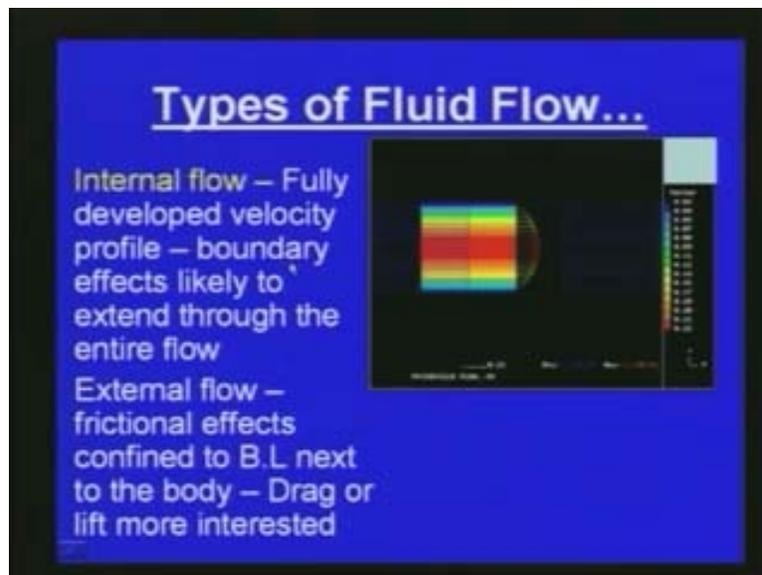
(Refer Slide Time: 12:40)



For example, a building is concerned and if this is the building which we consider, with respect to this if the wind comes what we can see with respect to surrounding the building. This is very important for a many engineering design. These kinds of flow where the flow happen with respect to the surrounding of a particular body or particular structure or this kind of flow where the fluid flow is in an unbounded region. That is called external flow. This is also very important when we are discussing the laminar and turbulent flow. The real fluid motion significantly influenced by the boundary. Either it can be an internal flow or it can be an external flow. As in the case a pipe flow we can

see that the solid surface where the fluid is having a no slip conditions, the boundary is very important. So the no slip condition and the boundary layer formation are very important as far as flow is concerned. Whether it is internal flow or the external flow, where surrounding a building or whenever a boat is moving in a river we can say that with respect to that surrounding the boat or surrounding the ship what is happening that is very important. The formation of the boundary layer is also very important. With respect to this laminar and turbulent flow which we will be discussing today whether it is external flow or internal flow, that is also very important.

(Refer Slide Time: 14:30)

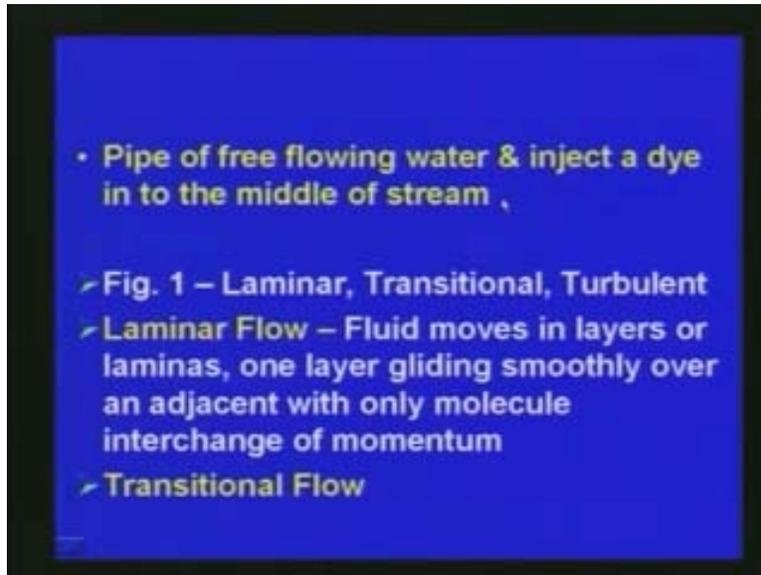


Now, as far as internal flow which we discussed is concerned, we can see that here is a pipe flow. The boundary effects are likely to extend throughout the entire flow as we can see here in this slide. So, we say here a fully developed velocity profile is formed and then after some time the boundary effects is still there but still it has overcome. Then, the boundary effects are likely to extend through the entire flow. This is again the internal flow.

As far as external flow is concerned we can see that the frictional effects are confined to the boundary layer next to the body; they drag or lift is more important. Here, when we will discussed, for example, whenever a balloon is just lifting through the air you can see

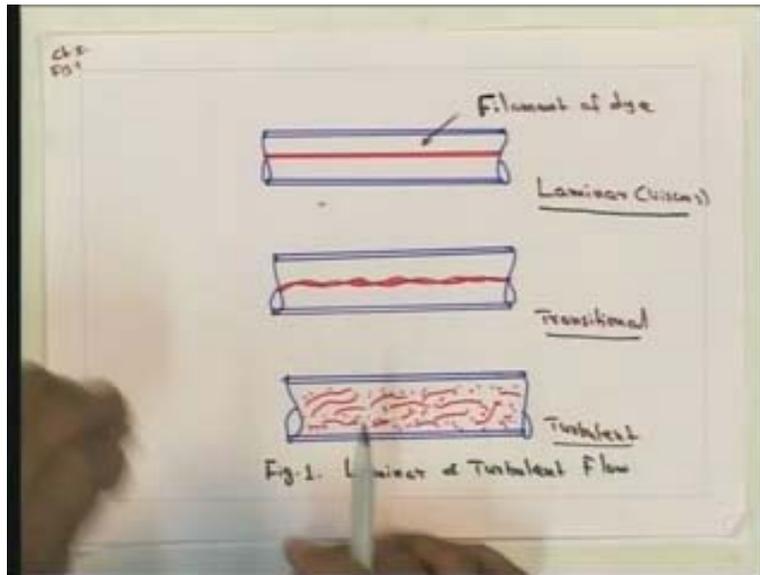
that with respect to lift effect or the drag effect what happens. That is also very important as far as the type of fluid flow is concerned. In the case of an external flow as far as the laminar and turbulent flow is concerned we have to see that whenever we discuss these kinds of problems, we have to see whether the flow is external flow or internal flow.

(Refer Slide Time: 16:00)



Now, for the pipe of free flowing water if we inject a dye into the middle of a stream, you can see just if we inject for example, a dye to the middle of the pipe flow, you can see that with respect to the velocity of the flow in the pipe so initially we can see that the fluid velocity is small.

(Refer Slide Time: 16:08)



Then you can see that the dye which we are injecting it will be just moving like a filament of dye in layers. We can say now this is laminar flow where the flow is in layers and then there is no disturbance takes place. We can see that the velocity of the flow increases then we can see that after some time here in the second figure, we can see that there is slight disturbance starts. Earlier, the flow was in layers and then slight disturbance started and finally we can see when the velocity increase we can see that the dye is totally mixing within the fluid. Then due to a lot of disturbance we can see the fluids are intermixing and then we can see that they will be diluted totally mixing with the fluid inside the pipe.

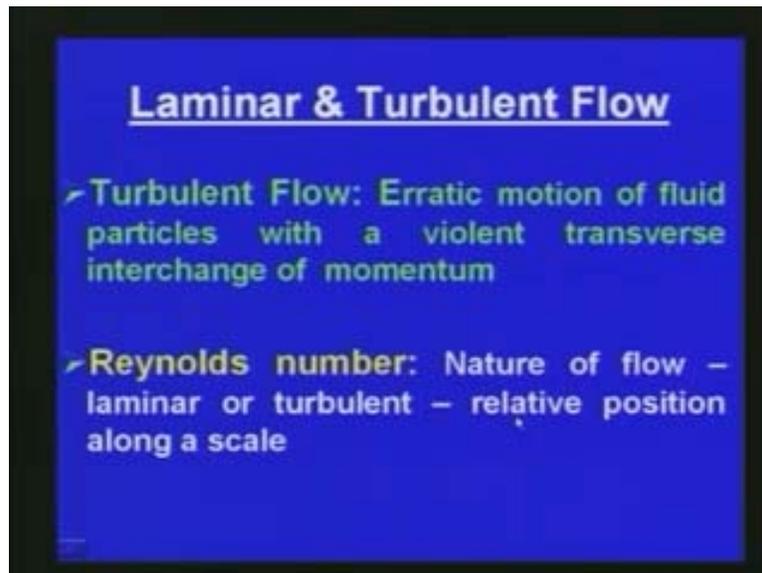
We can see initially when the velocity is small we are injecting small mass of filament of dye. We can see it is laminar and when the velocity is increased we can see it becomes transitional flow and then finally lot of mixing takes place and we can see that it becomes turbulent flow.

In a similar we can see for the flow sometime can be laminar but in the flow parameters like velocity changes then it becomes transitional. Further, we can see that it will become turbulent. This is the way depending upon various flow parameters the fluid flow changes from laminar to turbulent. With respect to what we discussed we can say what is a

laminar flow? The laminar flow (Refer Slide Time: 18:15) we can see that the fluid moves in layers or laminas. We can define the laminar flow as fluid moves in layers or laminas, one layer gliding smoothly over an adjacent with only molecule interchange of momentum. With respect to the first figure we can see the fluid moves in layers or laminas; one layer is gliding smoothly over an adjacent with only molecule interchange of momentum takes place.

This is the definition of a laminar flow but then we can see that as I mentioned when the velocity or other flow parameter changes then small kinds of mixing takes place instead of smooth gliding. We can see that there is mixing starts and then the flow is transitional.

(Refer Slide Time: 19:17)



As far as turbulent flow is concerned, we can see here there are erratic motion fluid particles with a violent transverse interchange of momentum. The turbulent flow we can define as erratic motion of fluid particles with a violent transverse interchange of momentum. So, here we can see whenever we injected the dye, when the velocity is increased or other flow parameters changed, we can see that there is total mixing of the dye takes place within the fluid and then the flow become total turbulent.

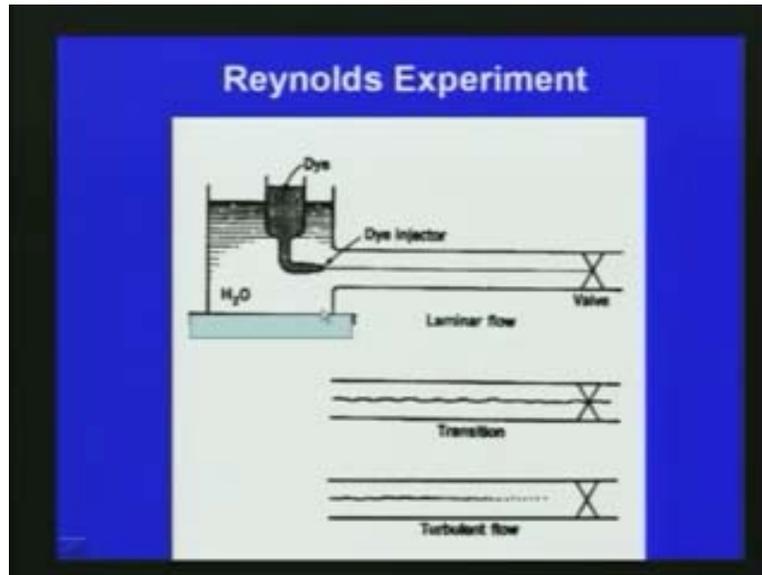
We can define the turbulent flow as wherever there is erratic motion of fluid particles with a violent transverse interchange of momentum. Now, the question here is if a fluid

flow is there how we can define whether it is at laminar stage or whether it is at transitional stage or whether it is in turbulent stage? This is a very important question. Most of the time we will be dealing with pipe flow or we will be dealing with open channel flow or river flow or many kinds of flow. It is very important to see that whether the fluid flow is laminar or whether the flow is transitional stage or it is turbulent. The reason is that most of theory whatever we are applying to laminar flow may not be applicable to turbulent flow. So, many flow parameters have to be changed. The theory will be changing when the flow is changing from laminar to turbulent.

In all the cases, it is in most of the fluid flow cases it is very important to define whether the existing flow is in laminar stage or whether it is just in the transitional stage or it is turbulent stage. A large number of scientists and engineers worked on this topic and then in the 19th century, say, in 80s one famous scientist was born. Reynolds defined the number called Reynolds number. After this, Reynolds introduced this dimensional number called Reynolds number. With respect to this Reynolds number we can define whether the concern flow is laminar or turbulent. This Reynolds number is to define whether the flow is laminar turbulent. It shows the nature of flow, whether the flow is laminar turbulent and then it says relative position along a scale. With respect to the dimensionless number so called Reynolds number, defined by Osborne Reynolds, we can put scale whether with respect to this scale: if the Reynolds number is 500 or 1000 or 2000, then it is laminar as in the case of pipe flow; when it exceeds above 1000 it become turbulent. For example, in open channel flow it is less than or equal to 500 whether it is laminar; when it exceeds 600 or 1000 it become turbulent.

So, this Reynolds number is a scale relative which gives the relative position whether the flow is laminar or turbulent. Now, Osborne Reynolds defines this Reynolds number based upon the large number of experiments with respect to fluid movement. He conducted experiment called Reynolds experiment and he conduct this experiment in 1880's, end of 19th century and then he define this so called a Reynolds number.

(Refer Slide Time: 22:55)



This Reynolds experiment we can see here. What Osborne Reynolds did is there is a small tank where water is filled and then a pipe is connected with respect to that tank; there is a valve attached at the end of the pipe; he introduces a small tank inside this large tank where some dye has been put and then through a small nozzle he introduced the dye to the pipe where the pipe starts like in this figure. And then what he did? With respect to valve opening, he started slowly to open this valve and you can see as shown here. The flow will be with respect to the dye and then the fluid movement here we can see that the fluid movement is now with respect to dye; we can see it is laminar in nature.

With respect to what we have seen here, with respect to this color we can identify. It is initially whenever the valve opening is small the velocity is small and discharge is also small. Then, we can see the flow is with respect to dye. We can visualize the fluid is moving in layers; they are in laminas.

When the valve is opened more we can see that this fluid which is moving in laminas or in layers some disturbance starts. We can see that there is some oscillation for the fluid and further when the valve is opened we can see that the velocity increases and then with respect to dye movement we can visualize that there is mixing taking place; interchange of masses takes place; there is erratic motions taking place. Finally, as we have seen here

the flow becomes turbulent. So, that is what Osborne Reynolds did in 1880s. Then, with respect to a large number of experiments as mentioned here he defined so called Reynolds number which we have seen.

(Refer Slide Time: 25:00)

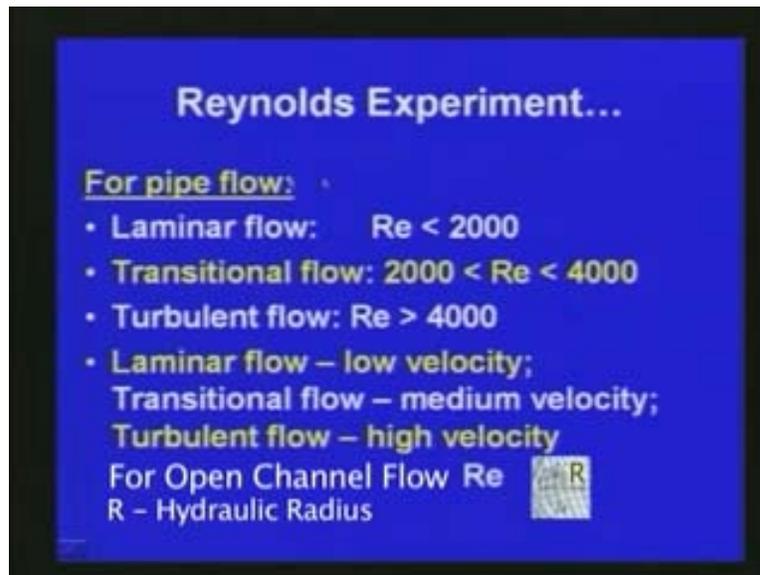
Reynolds Experiment...

- After many experiments, Reynolds showed that $\frac{\rho V D}{\mu}$ help to predict change in flow type (where V – mean velocity, D – diameter of pipe)
- Reynolds defined a dimensionless number (Re):
- Reynolds number = $\frac{\text{InertiaForce}}{\text{ViscousForce}} = \frac{\rho V D}{\mu}$

After many experiments Reynolds showed that a particular ratio $\rho V D$ by μ , where ρ is the mass density of the fluid, V is the average velocity, D is the diameter of the pipe and μ is the dynamic coefficient of viscosity. So, this ratio $\rho V D$ by μ helps to predict the change in flow type. He defined this number as Reynolds number; this number is dimensionless number. So, Osborne Reynolds defined this number as Reynolds number and actually this number is the ratio of inertial forces to viscous forces. He puts this Reynolds number is equal to inertial force divided by viscous force. For pipe flow, we can define as $\rho V D$ by μ , where ρ is the mass density of the fluid, V is the average of velocity, D is the diameter of the pipe and μ is the coefficient of dynamic viscosity.

Similar way we can define this Reynolds number for open channel also. With respect to his experiments he put this Reynolds number as a scale to define whether the concern fluid flow is laminar or whether it is in the case of transitional stage or whether it is in the case of turbulent stage.

(Refer Slide Time: 26:25)



Now, with respect to this Reynolds number we can say for example if we consider pipe flow. As i mentioned whether the pipe flow can be pipe flow or open channel flow or river flow like that, if we consider for pipe flow we can show that generally with respect to various conditions we can say that when the Reynolds number is less than or equal to 2000 in this range of 2000 or less than 2000, we can see that the flow is laminar. A range of between 2000 and 4000, we can see that the mixing starts and then the disturbance start and then a transitional state occurs for pipe flow. Beyond 4000 when the Reynolds number is greater than 4000, we can see that the flow become turbulent. With respect to for pipe flow and with respect to Reynolds number we can say, whenever the Reynolds number is less than 2000 the flow is laminar; between 2000 to 4000 we can say the flow is at a transitional stage and beyond 4000, the flow is turbulent.

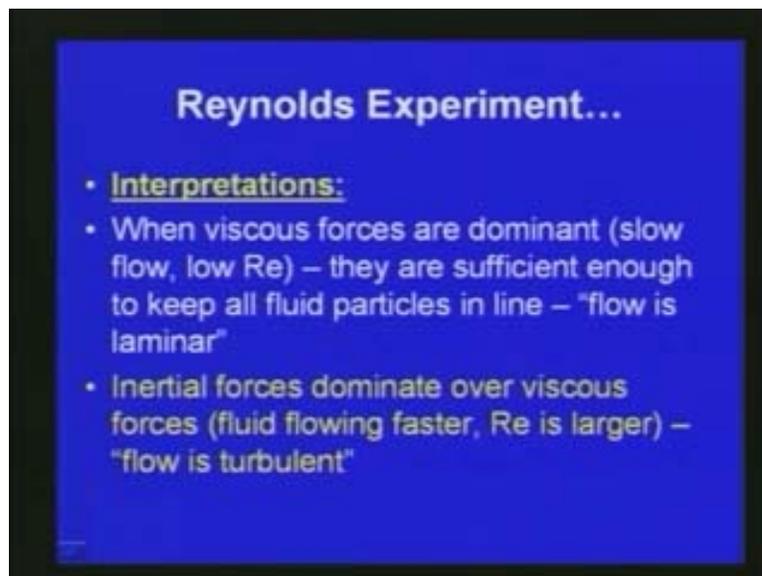
Through experiments we can show when the flow is laminar with respect to the Reynolds number we can see that whenever the flow is laminar the velocity will be low which will be low velocity; in transitional flow we can see the medium velocity and when the flow become turbulent the velocity is high or high velocity flow. This is the case of pipe flow.

Similar way if we do experiments in open channel with respect to various conditions generally we can say that the flow will be laminar when the Reynolds number is less than

or equal to 500; from 500 to 2000 it may be at a transitional stage but this can vary slightly and also beyond 2000 we can see for open channel flow it can be turbulent in nature.

The exact range depends on various other flow conditions exist in the flow regime, various other boundary conditions or the initial conditions. Depending upon this kind of various conditions this range can vary. But, approximately this is the range put forward by various scientists and engineers for open channel flow and pipe flow. With respect to the Reynolds experiments which we have seen we can put forward certain interpretations.

(Refer Slide Time: 28:55)

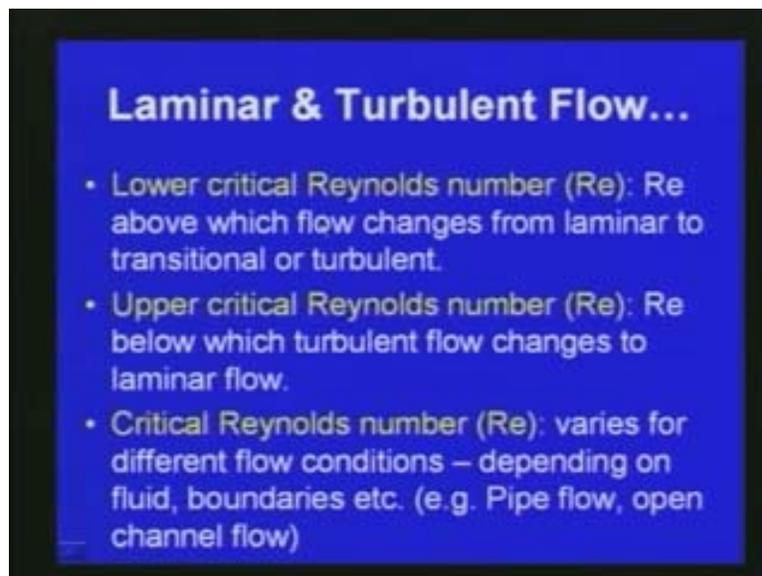


First one is when viscous forces are dominant. (Refer Slide Time: 29:06) We have seen that the Reynolds number is the ratio of inertial force to viscous force. With respect to this, we can interpret when the viscous force are dominant the flow is slow or the Reynolds number is low and then they are sufficient enough to keep all fluid particles in line or flow is laminar. Even though there is any tendency for any disturbances then here for this particular kind of flow, wherever velocity is small or the slow flow and low Reynolds number we can see that the disturbance will be dampened. Here, the viscous force is dominant and flow is laminar. Whenever the fluid flow is faster or the velocity is

larger the Reynolds number is larger then we can see that inertial forces dominant over viscous forces and then finally the flow is becoming turbulent.

This is what is happening. Whether with respect to the fluid flow velocity we can see that when the Reynolds number is lower or higher and whether the viscous force is dominant or the inertial force is dominant. So, accordingly, we can classify whether the concern flow which we are dealing is laminar in nature or turbulent in nature. Also, while conducting these experiments even though we have seen that in certain range the flow is laminar and beyond that it become transitional or turbulent.

(Refer Slide Time: 31:05)



When we conduct large number of experiments we can define a lower critical Reynolds number where the Reynolds number above which flow changes from laminar to transitions or turbulent. This is so called lower critical Reynolds number and then we can define, for example the velocity of flow with respect to the valve movement with the velocity is reduced. In a similar way when the flow is transforming from laminar to turbulent in the opposite direction, when we reduce the valve opening then you can see that flow velocity will be slowly and coming down and then there will be a transitional from turbulent to transitional or turbulent to laminar.

There we can define a Reynolds number called upper critical Reynolds number. Here, the Reynolds number below which turbulent flow changes to laminar flow. So this critical Reynolds number we can have two ranges: one is lower critical Reynolds number and another one is the upper critical Reynolds number. This critical Reynolds number which we have seen is based upon which we classify the flow as laminar or turbulent. This varies depends upon various flow conditions like what kind of fluid and then what are the boundaries just as we have seen whether it is pipe flow or whether it is open channel.

It is difficult to put an exact range; when exact, the flow is transforming from laminar to turbulent but this depends upon various other different flow conditions. Generally, we prescribe a lower critical Reynolds number where the flow changes from laminar to turbulent or transitional or and then upper critical Reynolds number where they are turbulent flow changes to laminar flow.

(Refer Slide Time: 32:50)



Now also we can see whenever we conduct these kinds of experiments in the laboratory we can see that this laminar flow which is we have seen is flowing in layers and then in some stages we can see instability of laminar flow. Here, the inertial forces associated with fluid mass try to amplify disturbances in the flow. Initially, the flow is laminar but then there can be different kinds of disturbances like boundary friction or the shearing

effect or the other kinds of effect then we can see that the inertial forces with respect to the fluid which is flowing. So, inertial forces associated with fluid mass try to amplify disturbances in the flow.

Then there is instability of laminar flow. Viscous forces try to damp this disturbance but the various forces which are again trying to amplify the disturbances are dominating above the viscous forces. Then we can see that the laminar flow become unstable; instability of laminar flow takes place and depending on the flow fluid characteristics like mass, density, viscosity, velocity gradient proximity to boundary etc., the disturbances are dampened or amplified.

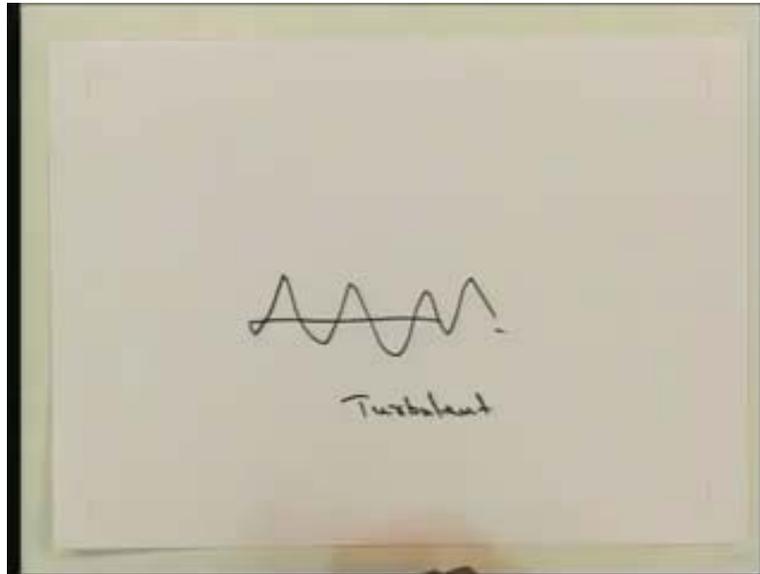
For the given conditions, if the disturbances are dampened again the flow will be continuing as laminar flow but if the disturbances are amplified further due to various forces like shear force or various frictional forces then it may change from laminar to turbulent flow or that the laminar flow become unstable or instability of laminar flow takes place. So if the disturbances are dampened the flow is laminar; otherwise, that flow becomes turbulent. Also, with respect to what we have discussed, we see that as far as turbulent flow is concerned various important characteristics which we can observe in nature or observe in the laboratory includes irregularity. (Refer Slide Time: 35:22). That means here when flow become turbulent we can see that when the flow is laminar we can see it is very regular; one layer is sliding over the other one; it is very much regular.

But you can see when the flow become turbulent, some of the important flow characteristics of turbulent flow are: first one is irregular in nature, the flow is irregular; and the second one is diffusivity, as far as diffusivity is concerned we can see the case of laminar flow we that the molecular diffusivity is more important but as for as turbulent flow is concerned the turbulent diffusivity is much more than the molecular diffusivity and then the flow is changing from laminar to turbulent.

Some of the other important characteristics are higher Reynolds number as we have seen and then in many cases we can see that the flow may be rotational in nature. In most of the laminar flow the rotational effective is very minimal or it can be even rotational in nature. But in most of the turbulent flow we can see that the motion will be rotational

nature and due to this most of the time we have to consider the fluid flow in three dimensions. Since in one dimension or two dimensions it will be very difficult to see what is realistically happening and how the disturbance takes place? Which direction it takes place. Most of the time as far as turbulent flow is concerned we have to do three dimensional analysis.

(Refer Slide Time: 37:11)

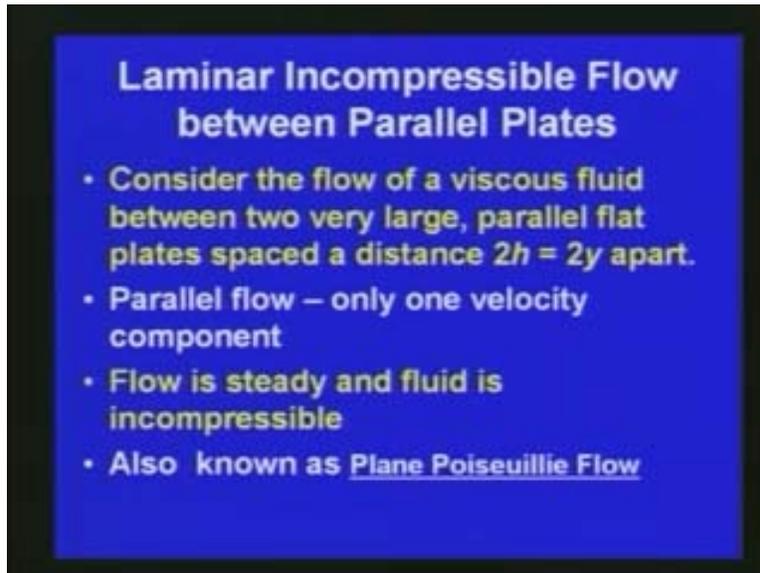


As far as turbulent flow is concerned motion can be rotational and three dimensions and then the fluctuations we can see here; the fluctuations can be wide spectrum. If you just consider for example, if the flow is laminar but as far as turbulent flow is concerned we can see that the whenever the flow becomes turbulent the fluctuations that takes place will be wide spectrum .

Another important flow characteristic in turbulent flow motion is dissipative. Various energies like heat energy and all will be dissipated with respect to the turbulent flow. With respect to the discussion so far we can say that the turbulent in a flow is feature of flow but it is not a property. It is very difficult to say that turbulent is not a fluid property but we can define a turbulent is a feature of the flow.

Whenever a flow is there we have to see that if it is if we turbulent flow it is the future of the flow but it is not a fluid property. Further, we will be discussing various aspects of the laminar flow and turbulent flow in detail in this chapter.

(Refer Slide Time: 39:11)



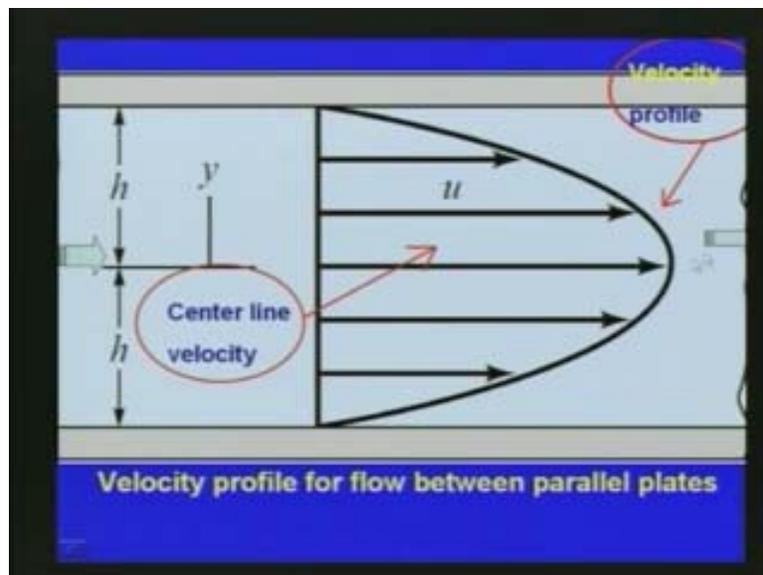
So initially we will be discussing the laminar flow and then we will be discussing the turbulent flow. Now, we will discuss laminar flow in detail. Initially, we will see the various flow characteristics as far as laminar flow between parallel plates and then will be discussing the laminar flow in pipes. Further, we will be discussing the turbulent flow.

Now first one is laminar in compressible flow between parallel plates. Here we will be deriving various relationships like the velocity variation and then the shear stress variation etc., as far as laminar incompressible flow between parallel plates. Let us consider the flow of a viscous fluid between two very large parallel flat plates spaced at a distance of $2h$ or equal to $2y$ apart. We consider here the laminar flow so the laminar flow what we are considering here is we are considering two very large parallel plates like this so now if we consider this sheet of paper as the parallel plates what we are doing here is in between two parallel plates are placed like this and then two parallel flat plates. In between if fluid flow is there laminar flow how the flow parameter varies so that we are going to discuss here; the incompressible flow between parallel plates.

Since it is parallel flow we assume the flow is only one velocity component and then we also assume the flow is steady state and fluid is incompressible. This kind of flow wherever the flow between two large parallel plates this kind of flow is general, you called a plane poiseuille flow.

The flow between two parallel plates which is laminar in nature at the parallel plates is placed at distance. This kind of flow between these parallel plates is known as plane poiseuille flow.

(Refer Slide Time: 40:24)



Now we will discuss in detail various flow parameters or the velocity variation takes place. So here we can see the velocity profile for flow between parallel plates: one parallel plate is here and another one just placed $2h$ above at distance $2h$. Here the flow is between these parallel plates. If we plot the velocity profile we can see that we are discussing the real fluid.

Due to the viscosity of the fluid we can see the no slip condition. Here the velocity on this side on the upper plate where the contact between fluids takes place the velocity will be 0; here the velocity will be 0 and then we can see since the fluid is between two parallel plates. Generally, the velocity will be maximum at the center and then you can see if you brought the velocity then you can see it will be parabolic in nature. So starting

was 0 both sides and then it will be maximum at the center. If you bring the velocity we can see that it will parabolic in nature 0 on the both sides of the parallel plates and then maximum at the center.

Now, we want to get the velocity variation we want to find out how the velocity variation and once the velocity variation is known we can find out the shear stress or other fluid flow parameters.

(Refer Slide Time: 41:44)

Laminar Incompressible Flow between Parallel Plates...

- For two-dimensional flow, Continuity Eqn:
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \dots (1)$$
- Since the streamlines curve straight and parallel to x – direction, $v = 0$
$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = 0; \therefore \frac{\partial u}{\partial x} = 0$$
- Velocity doesn't change with x velocity field $v = 0, u = u(y)$

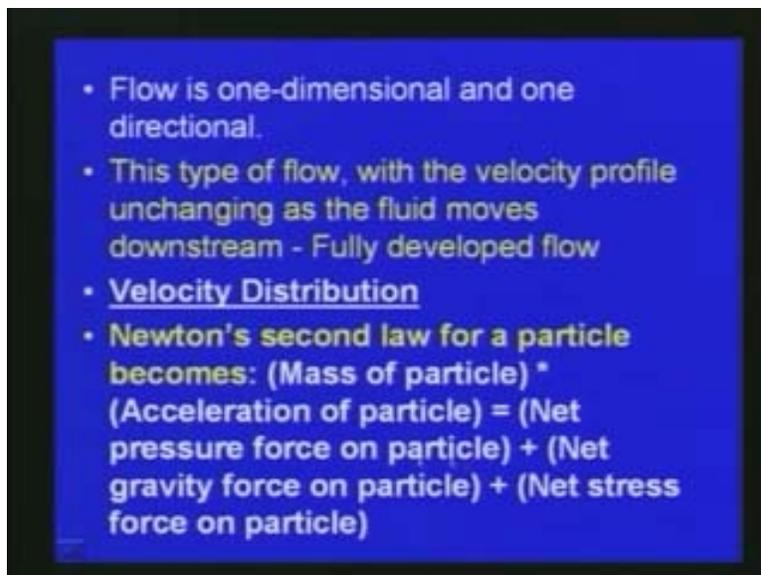
Now, we are considering the flow which we consider it has two dimensions in nature. For two dimensional flow we have already seen that the continuity equation we can write $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ so if the velocity in x direction is u velocity in y direction is v then we can write from the continuity equation as $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$. So this is the continuity equation and now we can see that since the streamlines curve is straight here is the stream if you bring the streamlines, the stream lines are parallel so that we can say that it is parallel to the x direction. You can see that here the velocity component v is equal to 0.

Finally, what we can see for this particular case there will be only one velocity component in x direction the other velocity component v is equal to 0. From this

continuity equation we can write $\frac{dv}{dx}$ is equal to $\frac{dv}{dy}$ since v is equal to 0 that $\frac{dv}{dx}$ is equal to $\frac{dv}{dy}$ is equal to 0.

If we put into this continuity equation we can write $\frac{du}{dx}$ is equal to 0. That means, for this particular case laminar incompressible flow between parallel plates velocity does not change with x ; velocity field, v is equal to 0 and the velocity u is changing with respect to y only as we can see the flat here we can see the velocities varying with respect to y only. So u is a function of y as written here.

(Refer Slide Time: 43:22)



Finally, with respect to this simplification we can see that the flow is one dimensional and unique directional or one directional. This type of flow with the velocity profile unchanging as the fluid moves downstream is so called fully developed flow. This type of flow with the velocity profile unchanging we can see the velocity profile here which is unchanging with respect to the x as the fluid moves down downstream so we this fully developed flow. We have already seen the velocity variation here and now we want to find out the velocity distinguishes; we want to get an expression for the velocity variation so that we can draw the velocity. Here we will derive the expression for velocity by using the first principle or the Newton's second law we can apply here. We have already discussed about Newton's second law in this application earlier.

By using Newton's second law for a particle if we can say particular particle in the control volume which we are dealing, we can write the mass of particle multiplied by the acceleration of particle is equal to net pressure force on the particle plus net gravity force on particle plus net stress force on particle. So, we can see from the Newton's second law for the particle which we consider the control volume we can write mass of the particle multiplied by acceleration of particle that is equal to net pressure force on particle plus net gravity force on particle plus net stress force on the particle. So here we use the Newton's second law to derive the equation. For the x direction, from the Newton's second law we can write mass into acceleration is equal to pressure force plus gravity force plus shear force.

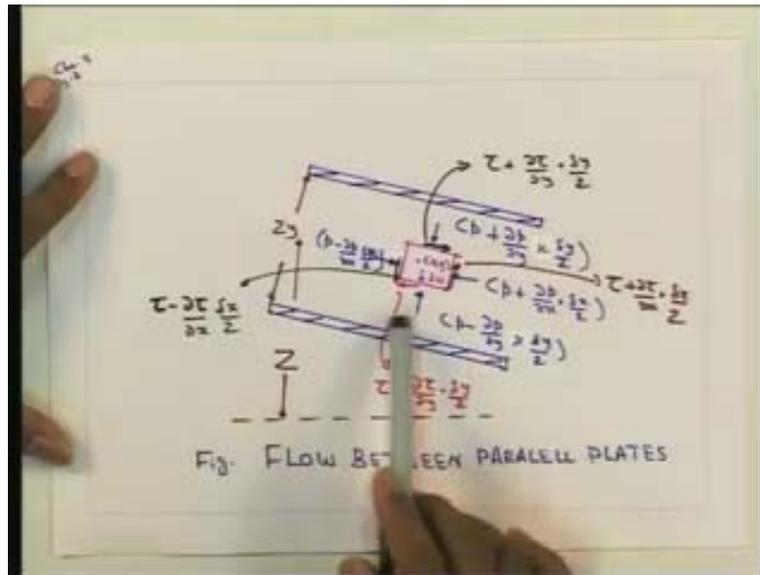
(Refer Slide Time: 45:10)

• For x-direction, Newton's second law is

- $$(\delta M_p) a_x = \delta F_{x, pressure} + \delta F_{x, gravity} + \delta F_{x, shear}$$
- $$\delta F_{x, pressure} = -\frac{\partial p}{\partial x} (\delta x)(\delta y)(1)$$
- $$\delta F_{x, gravity} = \rho g_x (\delta x)(\delta y)(1)$$
- $$\text{Shearforce } \delta F_{x, shear} = \left\{ \tau + \frac{\partial \tau}{\partial y} \left(\frac{\delta y}{2} \right) \right\} \delta x(1)$$
- $$- \left\{ \tau - \frac{\partial \tau}{\partial y} \left(\frac{\delta y}{2} \right) \right\} \delta x(1)$$
- $$= \frac{\partial \tau}{\partial y} (\delta x)(\delta y)(1)$$

If we consider a small fluid element for which the mass is delta Mp and acceleration is ax in x direction so that is equal to delta Fx, pressure plus delta Fx, gravity plus delta Fx shear. Here, if we consider, the delta Fx is concerned so here we can write down the expression for this by using here.

(Refer Slide Time: 45:47)



We will consider a control volume that means here the fluid from between parallel plates is here so we consider a small fluid element and then we can write the pressure force we write with respect to this fluid element on left side we can write p minus $\frac{\Delta p}{2}$ by Δx into Δy by 2 Δx is the size of the fluid in x direction; Δy is the size of the fluid in y direction and the plate is placed at distance of $2y$; here the pressure force is p minus $\frac{\Delta p}{2}$ by Δx into Δy by 2.

And other side of the fluid element p plus $\frac{\Delta p}{2}$ by Δx into Δy by 2 and then other side is concerned, we can write p plus $\frac{\Delta p}{2}$ by Δy into Δx by 2 and the opposite side it will be p minus $\frac{\Delta p}{2}$ by Δy into Δx by 2. Similarly, the shear stress is concerned which is another important force shear force. So we can write on this side τ plus $\frac{\Delta \tau}{2}$ by Δy into Δx by 2. On this side, it is τ plus $\frac{\Delta \tau}{2}$ by Δx into Δy by 2 and this side of the fluid element it will be τ minus $\frac{\Delta \tau}{2}$ by Δy into Δx by 2 and this side it will be τ minus $\frac{\Delta \tau}{2}$ by Δx into Δy by 2.

Now we are considering this control volume and we are considering fluid element; all the pressure force and shear force is concerned. Finally, we can write with respect to this the ΔF_x pressure we can write $\frac{\Delta p}{2}$ by Δx into Δy into, if we consider the unit thickness, it is multiplied by 1.

Similarly, we can write with the gravity force is concerned ΔF_x which is acting on the fluid element will be $\rho g x \Delta x \Delta y \Delta z$. This gives the gravity force and as the shear force is concerned we can write with respect to the earlier figure shown we can write $\tau \Delta y \Delta z$.

Now by using Newton's second law, we have written for the fluid particle of the fluid element is concerned the mass of the fluid element or fluid particle multiplied by acceleration that is equal to the pressure force on the fluid particle plus gravity force on the fluid particle plus shear force on the fluid particle. So we have derived the expression for the pressure force on the fluid element $p \Delta x \Delta y \Delta z$ and gravity force, we got $\rho g x \Delta x \Delta y \Delta z$ and shear force we got $\tau \Delta y \Delta z$.

Now, the acceleration of the particle as we have already seen earlier we can write a_x is equal to $\frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy}$. This gives the local acceleration plus convective acceleration. So a_x is equal to $\frac{du}{dt} + u \frac{du}{dx} + v \frac{du}{dy}$.

You can see here we are considering the fluid steady state, this is 0 and $\frac{du}{dx}$ is 0 and then v is 0. Finally, a_x is equal to 0. The Newton's law reduces to the force is equal to the pressure force plus gravity force plus shear force on the fluid is equal to 0. Finally, we get the equation as $-\frac{dp}{dx} + \rho g x + \tau \Delta y \Delta z = 0$.

(Refer Slide Time: 48:19)

• Acceleration of the particle is

$$a_x = \frac{\partial u}{\partial t} + u\left(\frac{\partial u}{\partial x}\right) + v\left(\frac{\partial u}{\partial y}\right) = 0 + u(0) + 0\left(\frac{\partial u}{\partial y}\right) = 0$$

• Newton's second law reduces to

$$0 = \left(-\frac{\partial p}{\partial x} + \rho g_z + \frac{\partial \tau}{\partial y}\right) (\delta x) (\delta y) (1)$$

$$-\frac{\partial p}{\partial x} + \rho g_z + \frac{\partial \tau}{\partial y} = 0 \dots \dots \dots (2)$$

This is the expression which we get for the fluid element which we consider. Now, in the earlier equation if you introduce here this rho g x is concerned, introduce z coordinate pointing upward opposite the gravity vector. We can write minus del p by del x minus rho g del z by del x plus del tau by del y is equal to 0 and here you can see that if the pressure p and the acceleration gravity g are constant. We can combined these two terms and write del by del x p plus gamma z plus del tau by del y is equal to 0. We can finally get the expression as del by minus del by del x of p plus gamma z plus del tau by del y is equal to 0.

Similar way what we have seen in the earlier equation is the x direction. So, similar way we can write the y direction: minus del by del y of p plus gamma z plus del tau by del x is equal to 0. Here you can see since the velocity profile do not change in x direction we can write del tau by del x is equal to 0 and hence we can write del by del y of p plus gamma z is equal to 0 with respect to this earlier equation.

So that we can say the pressure distribution in the y direction is hydrostatic and the term p plus gamma z is a function of x only. Now this earlier equation number 3, which is written the x direction we can finally write as minus d by dx of p plus gamma z plus del tau by del y is equal to 0 equation number 5.

(Refer Slide Time: 50:37)

• ∴ Eqn (3) ->

$$-\frac{d}{dx}(p + \gamma z) + \frac{\partial \tau}{\partial y} = 0 \dots \dots \dots (5)$$

• From Newton's law of viscosity, $\tau = \mu \frac{\partial u}{\partial y}$

• Substituting in (5),

$$-\frac{d}{dx}(p + \gamma z) + \mu \frac{\partial^2 u}{\partial y^2} = 0$$

u function of y only, ∴ $\frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dy^2}$

Earlier, we have seen the Newton's law of viscosity as τ is equal to μ into $\frac{\partial u}{\partial y}$. If you substitute for this τ here finally we get minus $\frac{d}{dx}$ of $p + \gamma z$ plus μ into $\frac{\partial^2 u}{\partial y^2}$ is equal to 0. This is the final expression which we get connecting the pressure variation and the velocity variation. Here, you can see u is only the function of y . So that we can write $\frac{\partial^2 u}{\partial y^2}$ can be written as $\frac{d^2 u}{dy^2}$.

Therefore, finally we can write the expression as μ into $\frac{d^2 u}{dy^2}$ is equal to $\frac{d}{dx}$ into $p + \gamma z$ and if you put this $\frac{d}{dx}$ is equal to $\frac{d}{dx}$ of $p + \gamma z$, finally we can write $\frac{d^2 u}{dy^2}$ is equal to $\frac{d}{dx}$ of $p + \gamma z$ by μ , where μ is the coefficient dynamic viscosity and now our aim is to get an expression for velocity. So this expression we can integrate two times and that write u is equal to $\frac{d}{dx}$ of $p + \gamma z$ by $2\mu y^2$ plus $C_1 y$ plus C_2 .

(Refer Slide Time: 51:19)

• Therefore,

$$\mu \frac{d^2 u}{dy^2} = \frac{d}{dx}(p + \gamma z) \dots \dots \dots (6)$$
$$\text{Let, } \tilde{p} = \frac{d}{dx}(p + \gamma z), \quad \therefore \frac{d^2 u}{dy^2} = \frac{\tilde{p}}{\mu} \dots \dots \dots (7)$$

• Integrating twice,

$$u = \frac{\tilde{p}}{2\mu} (y^2) + C_1 y + C_2$$
$$\text{At, } y = \pm Y, u = 0 \Rightarrow C_1 = 0, \quad \therefore C_2 = \frac{-\tilde{p} Y^2}{2\mu}$$
$$\therefore u = \frac{-\tilde{p} Y^2}{2\mu} \left[1 - \left(\frac{y}{Y} \right)^2 \right]$$

Now, we will introduce the boundary condition. Here we can see the boundary conditions on the plate. Due to the no slip condition we can write at y is equal to plus or minus y the velocity is 0 since due to no slip conditions the velocity is 0 from which we will get the constant of integration C_1 is equal to 0 and finally we will get C_2 is equal to minus p delta y squared by 2μ . So we get C_2 is equal to minus p y squared by 2μ and C_1 is equal to 0. Finally, we get an expression $4u$ - the velocity variation is equal to minus p delta y squared by 2μ into 1 minus y by Y whole squared, where as we have seen the distance between the plate is $2y$.

(Refer Slide Time: 52:42)

• Maximum velocity $u_{max} = \frac{-\bar{p}Y^2}{2\mu}$ at $y=0$

• Average velocity,

$$\bar{v} = \frac{1}{2Y} \int_{-Y}^Y u dy = -\frac{2}{3} \frac{\bar{p}Y^2}{2\mu} = \frac{2}{3} u_{max}$$

Introduce g, \bar{p} .

$$u = -\frac{Y^2}{2\mu} \frac{d(p+\gamma Z)}{dx} \left[1 - \left(\frac{y}{Y} \right)^2 \right]$$

$$u_{max} = -\frac{Y^2}{2\mu} \frac{d(p+\gamma Z)}{dx}$$

$$\bar{v} = \frac{2}{3} u_{max} = -\frac{Y^2}{3\mu} \frac{d(p+\gamma Z)}{dx}$$

Finally, we get an expression for the velocity variation and then we can see that with respect to velocity variation maximum at the center. So y is equal to 0, so that u_{max} is equal to minus $p \Delta y$ squared by 2μ and then also we can find out the average velocity \bar{v} is equal to $\frac{1}{2Y}$. We can integrate between minus y to plus y $u \, dy$. That would give the average velocity as minus $\frac{2}{3} p \Delta y$ squared by 2μ that is equal to $\frac{2}{3} u_{max}$.

And then if we introduce the term g and \bar{p} which we have already seen with respect to that the final expression for velocity can be obtained as u is equal to minus y squared by 2μ $d(p + \gamma z)$ by dx into $1 - (y/Y)^2$ or u_{max} also can be written as y squared by 2μ $d(p + \gamma z)$ by dx and the average velocity we can show that it will be two third of the maximum velocity that is equal to minus y squared by 3μ into $d(p + \gamma z)$ by dx .

So, like this we can derive the relationship for velocity and from the velocity we can go for the other parameters like shear stress, variation or pressure distribution. Further we will be discussing on this laminar flow in the next lecture.