

Fluid Mechanics
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Lecture – 11
Kinematics of Fluid Flow

Welcome back to the video course on fluid mechanics. So in the last lecture, we were discussing about the potential flow of theories and then we were discussing superposition of elementary flows.

So in the last lecture we discussed about the various kinds of superposition like direct method, inverse method and then in superposition process we have seen how we are superposing uniform flow and then a source how we can superpose to get a half body that we have seen in the last lecture and also we have seen how to superpose a two uniform flow with source and sink of equal strength with uniform flow a combination of source and sink of equal strength with uniform flow to get rankine ovals.

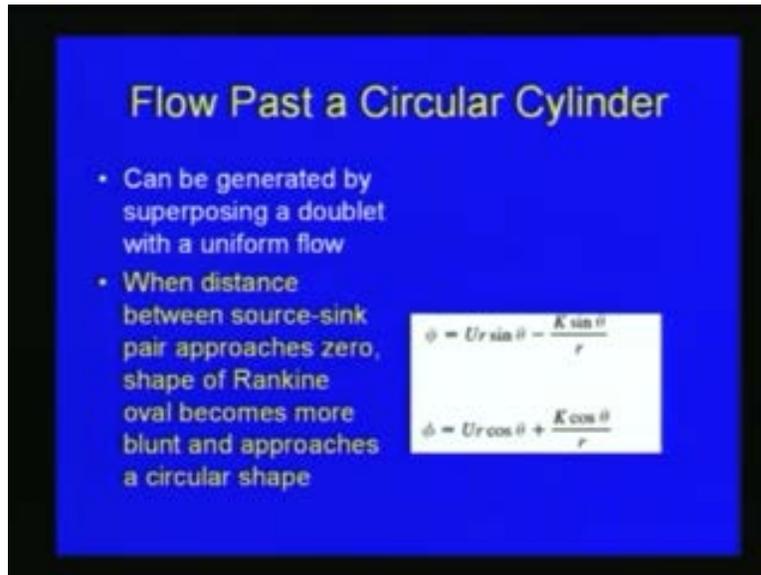
This also we have seen, from the uniform flow equations and then the source and sink equations how we are deriving the potential expression for potential and then stream function and then, how we are getting the velocities and various other flow parameters.

Then we have seen after the rankine ovals, we have also seen the flow past circular cylinders. Flow past circular cylinder we can generate by superposition of a doublet with a uniform flow which we have seen earlier.

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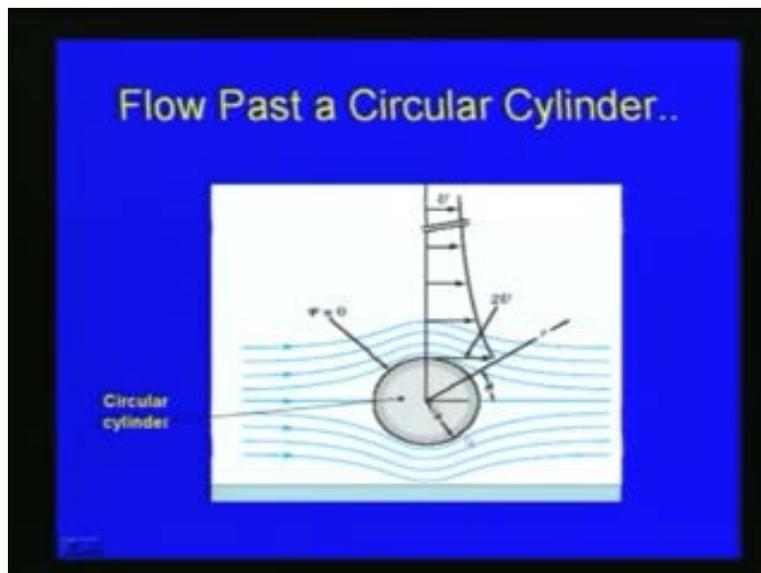
Flow Past a Circular Cylinder

- Can be generated by superposing a doublet with a uniform flow
- When distance between source-sink pair approaches zero, shape of Rankine oval becomes more blunt and approaches a circular shape

$$\psi = U r \sin \theta - \frac{K \sin \theta}{r}$$
$$\phi = U r \cos \theta + \frac{K \cos \theta}{r}$$


A doublet is superposed in a uniform flow. Then, with respect to superposition of a doublet and uniform flow, we have seen that the stream function can be represented by ψ is equal to $U r \sin \theta - \frac{K \sin \theta}{r}$ and velocity potential ϕ is equal to $U r \cos \theta + \frac{K \cos \theta}{r}$ and then the flow past circular cylinders can be seen in this slide.

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This is the cylinder and then, this is the uniform flow and then with respect to the doublet and uniform flow, we are analyzing flow past through circular cylinder and then we got the various parameters here for the expressions for ψ and ϕ with respect to the potential flow and then the uniform flow and then the doublet.

So, up to that we have seen how to determine the pressure with respect to the Bernoulli's equation.

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Flow Past a Circular Cylinder...

- Therefore stream function and potential function can be written as:

$$\psi = U r \left(1 - \frac{a^2}{r^2}\right) \sin \theta$$

$$\phi = U r \left(1 + \frac{a^2}{r^2}\right) \cos \theta$$
- Now velocity components can be obtained as:

$$v_r = \frac{\partial \psi}{\partial x} - \frac{1}{2} \frac{\partial \psi}{\partial x} - U \left(1 - \frac{a^2}{r^2}\right) \cos \theta$$

$$v_\theta = \frac{1}{2} \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x} - U \left(1 + \frac{a^2}{r^2}\right) \sin \theta$$

So with respect to the previous figures here in the slide, say here as is the radius of the cylinder and then say u is the velocity of uniform flow. Then we can use to find the pressure distribution, here since the maximum velocity on surface of cylinder at r equal to a that means θ is equal to plus or minus $\pi/2$ by 2 V_r is equal to 0 and the tangential velocity on surface V_θ is equal to minus $2 U \sin \theta$ and pressure distribution is obtained from the Bernoulli's equation $p_0 + \frac{1}{2} \rho u^2 = p_s + \frac{1}{2} \rho v_\theta^2$ where p_0 is the pressure far from cylinder and U is the velocity far from the cylinder and p_s is the surface pressure.

So surface pressure we can obtain as p_s is equal to $p_0 + \frac{1}{2} \rho U^2 (1 - 4 \sin^2 \theta)$ with reference to this figure here.

So this you have seen in the last lecture and now say as far as since the cylinder is immersed in a fluid that means flow past circular cylinder only we are analyzing.

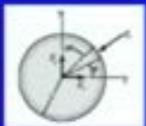
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Flow Past a Circular Cylinder...

- Drag force parallel to direction of uniform flow:
- Lift force perpendicular to direction of uniform flow
- Substitution of p_s indicates that $F_x = 0$ and $F_y = 0$

$$F_x = - \int_0^{2\pi} p_s \cos \theta \, d\theta$$

$$F_y = - \int_0^{2\pi} p_s \sin \theta \, d\theta$$



So definitely there will be a drag force and lift force acting upon the cylinder, if you have analyzed the real fluid flow cases. So then if you analyze a flow past circular cylinder with respect to the superposition theories, which you have seen now the drag force parallel to the direction of uniform flow generally we can obtain as F_x is equal minus integral 0 to 2 pi $p_s \cos \theta \, d\theta$ with reference to this previous figure here.

A is the radius of the cylinder and then θ is mentioned here in this shown here. So with reference to this we can get the drag forces which is parallel to the direction of uniform flow is obtained as f_x is equal to minus integral 0 to 2 pi $p_s \cos \theta \, d\theta$ and the lift force perpendicular to direction of uniform flow is obtained as F_y is equal to minus integral 0 to 2 pi $p_s \sin \theta \, d\theta$ so now the substitution of p_s from the previous expression.

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Flow Past a Circular Cylinder...

- Max. velocity occurs on surface of cylinder at $r=a$ ($\theta=\pm\pi/2$):
- $V_r=0$; $V_\theta = -2U \sin \theta$
- Pressure distribution from Bernoulli's equation: (p_0 – pressure far from cylinder, U – velocity far from cylinder, p_s – surface pressure)

$$p_0 + \frac{1}{2}\rho U^2 = p_s + \frac{1}{2}\rho v_s^2$$
$$p_s = p_0 + \frac{1}{2}\rho U^2 (1 - 4 \sin^2 \theta)$$

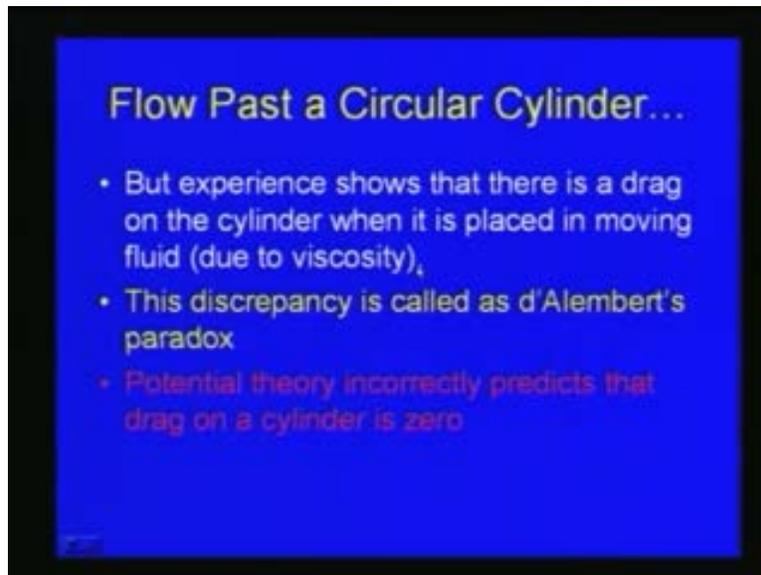
p_s is equal to p_0 plus half rho U square into $1 - 4 \sin^2 \theta$ if you substitute here for p_s what you will get is this F_x is equal to 0 and F_y is equal to 0. So with respect to this figure here, F_x is in this direction x direction and which is the drag force F_y is the lift force in the normal vertical direction y and then p_s is the surface pressure which is acted on the surface like this. So if you substitute these values with respect to our earlier analysis you get the drag force F_x is equal to 0 and the lift force F_y is equal to 0 with respect to the superposition of this doublet and uniform flow.

But, we can see that in the real case in the real fluid flow case, this is impossible. Since definitely there is lift drag force and also possibility of lift force with respect to that there is a flow and surrounding the cylinder or the cylinder is immersed in a flow. So there is definitely drag force and lift force, So the experience that there is a drag on the cylinder when it is placed in moving fluid due to the viscosity, so this discrepancy, so when we analyze these kinds of problem with respect to the potential flow theory or the superposition of the elementary flows with which we have seen so far, then some parameters are not obtained properly under predicted like drag force is 0.

Here lift force is 0, which is not realistic when we consider real fluid flow where the viscosities are also considered.

So this discrepancy is called the d'Alembert's law paradox that means when they are using the potential flow theory for the real fluid flow say like, the flow surrounding a cylinder or which we have seen here so the drag force becomes 0 or the lift force is 0 which is not realistic as per the real fluid flow is concerned.

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So this discrepancy is called the d'Alembert's paradox so that means the potential theory we can see here, it incorrectly predicts the drag on a cylinder is 0 which is not realistic.

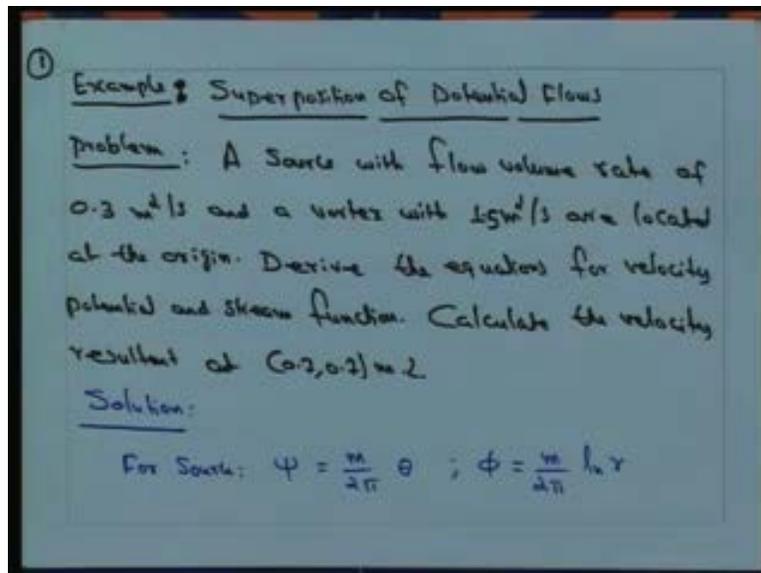
So that is what is called d'Alembert's paradox. So here the reason is obvious that here we are considering as potential flow and then viscosity is not considered then effectively, when we superpose the uniform flow and then the doublet in this particular case just like flow past circular cylinder it will be all the parameters will not be predicted properly, but as we can see the velocities and then the pressure can be predicted with respect to this superposition of the uniform flow and doublet as far flow past a circular cylinder is considered.

So that means when we use this potential flow theory for many of the real flow problems we should be very careful which parameter is obtained getting proper accurate which parameter is not predicted properly. So we should be very careful while using the potential theory in the case of real fluid flow problems, so that is very important.

So now with respect to this superposition of potential flows before completing or before closing this chapter we discussed numerical examples here. We can see we will first see superposition of potential flows.

First example is superposition of potential flow. So the problem is a source with a flow volume rate of 0.3 meter square per second and vortex with 1.5 meter square per second are located at the origin. Derive the equations for velocity potential and stream function; calculate the velocity resultant at 0.7 meter.

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So here, we have got a source of strength flow rate of 0.3 meter per square seconds and then vortex. So a source and vortex combined superposition superpose.

So vortex strength is 1.5 meter square per second, so we have to derive the equation of velocity and potential and stream function and calculate the resultant velocity. So for source which we have already analyzed earlier as far as the source is considered concerned we can derive we obtained the stream functions ψ is equal to m by two θ .

Where, m is the flow volume rate which is given as 0.3 meter square per second and then we have also seen earlier the velocity potential can be expressed before say source as ϕ

is equal to m by 2π natural log r , where r is the distance between the points which we are considering. So here, for ψ and ϕ source is obtained as ψ is equal to m by 2π theta and ϕ is equal to m by 2π natural log r and then also we have seen for vertex is concerned um earlier you have seen from the vertex ψ this stream function can be defined as ψ is equal to minus Γ by 2π natural log r and ϕ is equal to Γ by 2π theta.

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② For Vortex: $\psi = -\frac{\Gamma}{2\pi} \ln r$; $\phi = \frac{\Gamma}{2\pi} \theta$
 $m = 0.3 \text{ m}^2/\text{s}$; $\Gamma = 1.5 \text{ m}^2/\text{s}$
 Combined: $\psi = \frac{0.3}{2\pi} \theta - \frac{1.5}{2\pi} \ln r = \frac{1}{\pi} [0.15\theta - 0.75 \ln r]$
 $\phi = \frac{0.3}{2\pi} \ln r + \frac{1.5}{2\pi} \theta = \frac{1}{\pi} [0.15 \ln r + 0.75 \theta]$
 Now $u_r = \frac{\partial \psi}{\partial r} = -\frac{0.75}{\pi r}$
 $u_\theta = \frac{1}{r} \frac{\partial \psi}{\partial \theta} = \frac{0.15}{\pi r}$

So here, this is Γ this is vortex strength 1.5 meter square per second this is given. So now for vortex we have seen ψ function and ϕ function and m is already given as 0.3 meter square per second and Γ is equal to 1.5 meter square per second. So now as per our superposition theory, since now we are dealing here with source and then vortex, so we can use the combined equation or the combined equation for the stream function will be ψ is equal to so now which we have already seen here m by 2π theta is for ψ function so m is equal to point 3 by 2π into theta then we superpose the for the vortex ψ function which is minus Γ by 2π natural log r .

So that means ψ is equal to point 3 by 2π theta, minus 1.52 π natural log r which can be simplified as 1 by π 0.15 theta minus 0.75 natural log r .

So now this is the expression for the stream function for the superposed the source and vertex which we can say in this particular problem and now the velocity potential is concerned ϕ is equal to $m \ln r$ for this source so m is equal to 0.3.

So ϕ is equal to $0.3 \ln r$ and then say for vertex is concerned ϕ is equal $\frac{\gamma}{2} \theta$. So, γ is 1.5 plus $1.5 \ln r$. So that is equal to $1.1 \ln r + 0.75 \theta$.

So thus, we get the expression for stream function and potential function after the superposition of the source and the vertex equation.

So now we want to find the velocity the resultant velocity. So for that we will just calculate the radial velocity, the radial velocity expression is V_r is equal to $\frac{\partial \phi}{\partial r}$ so here if you differentiate this ϕ function which is obtained here one differentiation we will get V_r is equal to $\frac{\partial \phi}{\partial r}$ is equal to $0.15 \ln r$.

So similarly we can obtain the expression for the tangential velocity V_θ is equal to $\frac{1}{r} \frac{\partial \phi}{\partial \theta}$. So the tangential velocity is equal to $\frac{1}{r} \frac{\partial \phi}{\partial \theta}$. So if you differentiate this function for ϕ you will get this is equal to V_θ is equal to $0.75 \ln r$. So this gives the expression for the tangential velocity. So thus, we got the expression for the radial velocity and expression for the tangential velocity.

So now the next part of the problem is, we want to find the resultant velocity at say 0.7 meter square so x is equal to 0.7 and y is equal to 0.7. So r with respect to the x and y we can write the position r is equal to square root of $0.7^2 + 0.7^2$ that is equal to 0.89 meter.

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③

$$x = 0.2 \text{ m}, \quad y = 0.2 \text{ m}$$
$$r = \sqrt{0.2^2 + 0.2^2} = 0.2833 \text{ m}$$
$$u_r(0.2, 0.2) = \frac{0.15}{\pi \times 0.2833} = 0.0482 \text{ m/s}$$
$$u_\theta(0.2, 0.2) = \frac{0.75}{\pi \times 0.2833} = 0.241 \text{ m/s}$$
$$u_{\text{resultant}} = \sqrt{u_r^2 + u_\theta^2} = \sqrt{0.0482^2 + 0.241^2} = 0.2459 \text{ m/s}$$

So that now we can obtain the value of the radial velocity at 0.7 as V_r is equal to 0.7 is equal to since the expression for radial velocity we obtained V_r is equal to 0.15 by ϕr . So this is equal to 0.15 by ϕ into 0.9899 which r is obtained. So this is equal to V_r is equal to 0.0482 meter per second and then similarly we can find the expression for V_θ we obtain the value for the tangential velocity at 0.7.

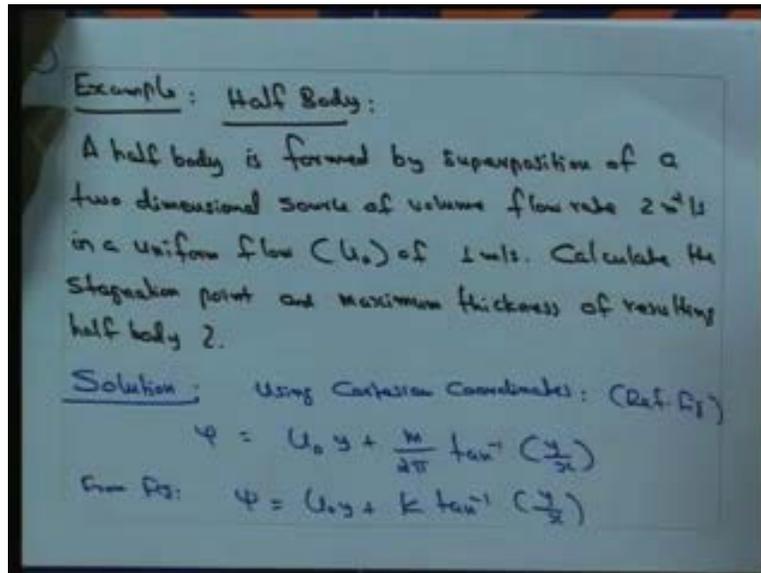
So the expression for V_θ as obtained is 0.75 by ϕr so 0.75 by ϕ into r is 0.9899. So this is equal to 0.241., So then once we obtained the radial velocity and tangential velocity we can obtain the resultant velocity is equal to $V_{\text{resultant}}$ is equal to square root of V_r square plus V_θ square so that V is here, root of 0.0482 square plus 0.241 square.

So if you take square root of these, you will get the resultant velocity is equal to point two four five nine meter per second. So like this here in this particular problem, we have superposed a source and then a vortex we superposed and then we have found the velocity for the expression for the potential velocity potential and then the expression for the stream function and then finally, we found the radial velocity and tangential velocity and we got the resultant velocity at a particular point from the radial velocity and

tangential velocity. So this is one example with respect to superposition of a source and then vertex.

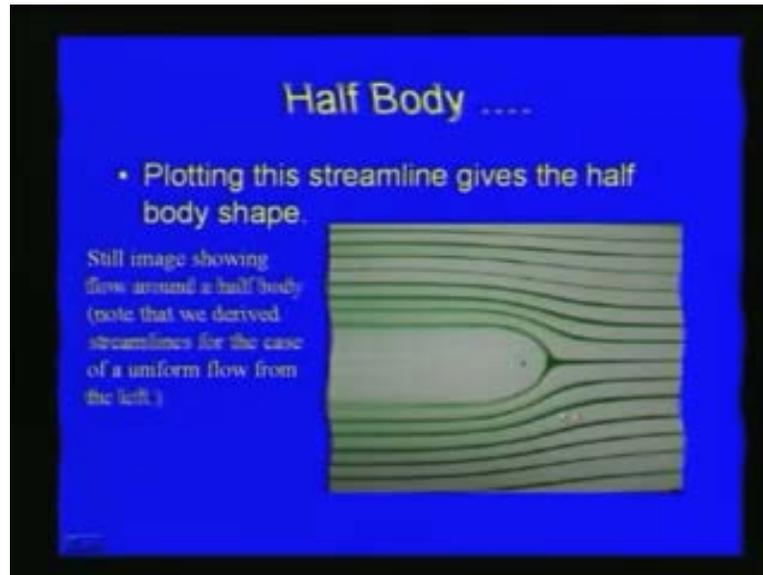
Now as a second numeric example here we consider a half body.

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So as we have seen the half body which is a superposition of source and then the uniform flow so, here the problem is a half body is formed by superposition of a two dimensional source of volume flow rate two meter square per second in a uniform flow of velocity one meter per second. Calculate the stagnation point and maximum thickness of resulting half body? So this is the problem.

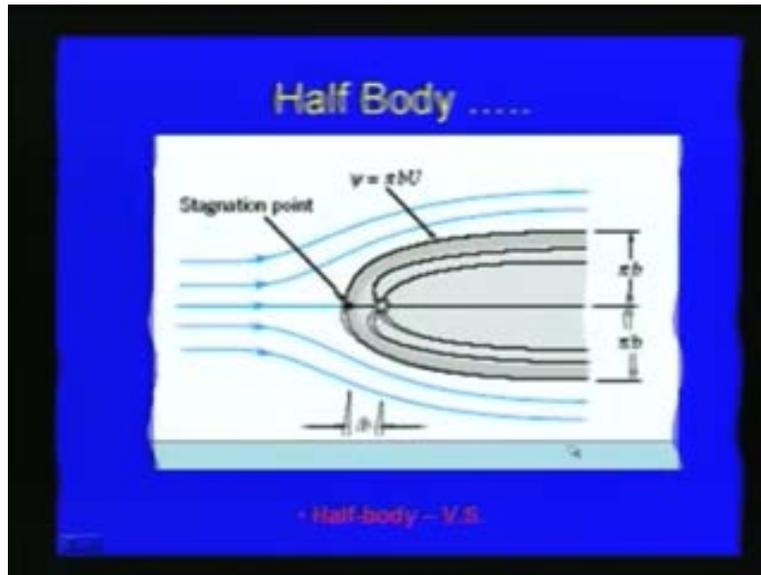
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So here half body is as we have seen in the seen earlier half body is concerned here we get a half body by superposition of a source and then the uniform flow in this slide here. So here we have superposed here the uniform flow velocities of one meter per second then the volume flow rate is for the source is concerned it is two meter square two meter square per second.

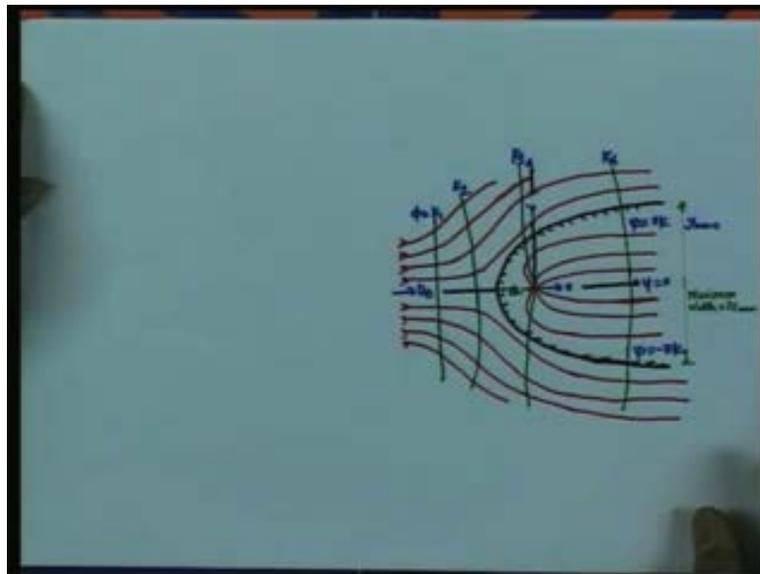
So we want to find the stagnation point and maximum thickness of the resulting half body. So as far as the half body is concerned various parameters, these things we also discussed the stagnation point and then the thickness and all other parameters so with respect to the figure here.

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So we have diagnosed in this problem. So now you use the Cartesian coordinate system with respect to this figure.

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This figure shows this is the origin x axis is this direction and here this is the y axis here, there is uniform flow of velocity 1 meter per second and here we have a source of 2 meter square per second.

So now with respect to this figure, we can obtain in Cartesian coordinate system, this stream function. So there is uniform flow and then the source is there, so with respect to this ψ is equal to stream function can be written as ψ is equal to $U_0 y$ plus $\frac{m}{2\pi} \tan^{-1} \frac{y}{x}$. So here y and x is already defined y is this direction x is this direction and m is the strength of the source and U_0 is the velocity uniform flow velocity.

So now this stream function is obtained by superposing the stream function for the uniform flow and the stream function for the source and then this we can write as ψ is equal to $U_0 y$ plus, if $\frac{m}{2\pi}$ is represented as K then plus $K \tan^{-1} \frac{y}{x}$. So this is we can write in this way with respect to this figure. So this is the velocity potential velocity potential $K_1 K_2 K_3$ etc. and then we have the stream lines and the potential line is drawn here for various values.

So now the velocity, the expression for velocity is U the velocity in x direction velocity in x direction is u is equal to minus $\frac{\partial \psi}{\partial y}$ u is equal to $\frac{\partial \psi}{\partial y}$, so we can differentiate this expression for the expression for ψ is $U_0 y$ plus $K \tan^{-1} \frac{y}{x}$.

So if you differentiate u is equal to $\frac{\partial \psi}{\partial y}$ is equal to U_0 plus $K \frac{x}{x^2 + y^2}$ plus y square so we get u is equal to $\frac{\partial \psi}{\partial y}$ is equal to U_0 plus $K \frac{x}{x^2 + y^2}$ plus y square.

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Handwritten mathematical derivation on a piece of paper:

Now velocity $u = \frac{\partial \psi}{\partial y} = U_0 + K \frac{x}{x^2 + y^2}$
 $v = -\frac{\partial \psi}{\partial x} = -K \frac{y}{x^2 + y^2}$
 At stagnation point, $u=0, v=0$, demand from eqn. (1) & (2)
 $\therefore x = -\frac{m}{2\pi U_0} = -\frac{2}{\sqrt{\pi} + 1} = -\frac{1}{\pi}$
 Coordinates of stagnation points are $(-a, 0)$ or $(-\frac{1}{\pi}, 0)$
 ψ at stagnation point $= 0 + \frac{m}{2\pi} \tan^{-1} 0 = \frac{2}{\sqrt{\pi}} = 1.107$

So similarly the velocity in y direction v is equal to minus $\frac{\partial \psi}{\partial x}$ so that is equal to $\frac{K}{y^2}$. So thus, we got the expression for velocity in x direction and velocity in y direction u and v from the obtained expression of stream function with respect to the superposition of the uniform flow and source for the half body which we considered here.

Now, as per our definition of the stagnation points this U_0 that is the velocity should be 0 at stagnation point as defined here, the stagnation point is shown here.

So for stagnation point u is equal to 0, v is equal to 0. Now if you use this equation now with respect to this if you substitute for u is equal to 0, v is equal to 0 you will get x is equal to minus $\frac{m}{2\phi U_0}$ where K is taken as m by 2ϕ . So x is equal to $-\frac{m}{2\phi U_0}$, so that is equal to $-m$ is already given as 2 meters per second, so this is equal to $-\frac{2}{2\phi U}$ is equal to the velocity is equal to 1, So $-\frac{2}{2\phi}$ into 1 so this is equal to $-\frac{1}{\phi}$.

So now we want to determine the coordinates of the stagnation point. So from this you will get the coordinates of the stagnation points as $a, 0$ or it will be $\frac{1}{\phi}, 0$, so the coordinates of the stagnation point is $a, 0$ or $\frac{1}{\phi}, 0$ and this stream function at stagnation point with respect to this now y is equal to here say 0 here.

So we will get ψ stream function at stagnation point is 0 plus $\frac{m}{2\phi} \tan^{-1} \frac{y}{0}$. So $\tan^{-1} 0$. So this is equal to $\frac{m}{2\phi}$ is 2, so $\frac{2}{2\phi}$ into ϕ so this is equal to 1. You obtained ψ stream function at stagnation point is equal to 1.

So now the half body is described by dividing stream lines. So as you can see this is the half body with respect to this figure here or this figure here, this is the half body. So the half body is described by the dividing stream line. So that we can write ψ is equal to $\frac{m}{2}$ so ψ is equal to $\frac{m}{2}$ that is equal to ϕ into $\frac{m}{2\phi}$ so that we will get ψ is equal to ϕK .

So that is with respect to this figure (Refer Slide Time on 18:55) it is obvious ψ is equal to ϕK which we have already seen when we derived all the equation so we can write

$U_0 y + \frac{m}{2\pi} \tan^{-1} \frac{y}{x}$ is equal to $\frac{m}{2}$ with respect to the half body which is shown here.

So then we can write $U_0 y + \frac{m}{2\pi} \theta$ with respect to this figure this is equal to $\frac{m}{2}$ or you will get y is equal to $\frac{m}{2U_0} (1 - \frac{\theta}{\pi})$ and at θ is equal to 0 we get y_{max} is equal to $\frac{m}{2U_0}$ which is the maximum ordinate and at θ is equal to $\frac{\pi}{2}$. We get y is equal to $\frac{m}{4U_0}$ that is the upper ordinate at origin and θ is equal to π y is equal to 0 which is the stagnation point.

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③ Half body is described by dividing streamlines.

$$\psi = \frac{m}{2\pi} = \pi \cdot \frac{m}{2\pi} = \pi K.$$

$$\text{or } U_0 y + \frac{m}{2\pi} \tan^{-1} \frac{y}{x} = \frac{m}{2}$$

$$U_0 y + \frac{m \cdot 0}{2\pi} = \frac{m}{2} \quad \text{or } y = \frac{m}{2U_0} \left(1 - \frac{\theta}{\pi}\right)$$

At $\theta = 0$, $y_{max} = \frac{m}{2U_0}$ → max. ordinate

At $\theta = \frac{\pi}{2}$, $y = \frac{m}{4U_0}$ → upper ordinate at origin

$\theta = \pi$, $y = 0$. Stagnation point.

So here with respect to this figure, everything is clear this is stagnation point and y_{max} this is y_{max} . So, all these parameters for this particular problems are clear with respect to the figure here. This figure you can see that the y_{max} is defined here and then ψ is equal to πK here and ψ is equal to πK here and all other parameters are already defined so maximum width is equal to $2 y_{max}$. So a θ is equal to $\frac{3\pi}{2}$ y is equal to $-\frac{m}{4U_0}$ which is the lower ordinate at origin so here lower ordinate at origin y is equal to $-\frac{m}{4U_0}$.

So now the equation of half body becomes $U_0 y + \frac{m}{2\pi} \tan^{-1} \frac{y}{x}$ is equal to $\frac{m}{2}$ and then maximum thickness occurs as x tends to infinity is the

maximum thickness that means here the maximum thickness with respect to figure is maximum this two times y_{max} . So that is equal to 1 into y since U_0 is 1 into y plus 1 by ϕ m is to 1 by $\phi \tan^{-1} 0$ that is equal to 1.

So we get y is equal to 0.5 meter so maximum thickness with respect to this particular problem is 1 meter. Here since it is two times y_{max} y_{max} is 0.5 so two times 0.5 is equal to 1 meter.

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$AL \theta = \frac{3\pi}{2}, y = -\frac{m}{4l_0} \rightarrow$ (outer circle change)
 Equation of half body becomes,

$$l_0 y + \frac{m}{2\pi} \tan^{-1}\left(\frac{y}{x}\right) = \frac{m}{2}$$
 Max. thickness occurs as $x \rightarrow 0$

$$l_0 y + \frac{1}{\pi} \tan^{-1}(\infty) = 1$$

$$\therefore y = 0.5m$$
 Max thickness = $2y_{max} = \underline{\underline{1m}}$

So, with respect to figure this figure (Refer Slide Time on 18:55). So now we have obtained this particular problem the superpose the half body is obtained by superposition of a source and then a uniform flow and then we calculated the stagnation points and also the maximum thickness of the resulting half body is calculated. So like this we have say in two examples for a as far as one is concerned in super position of vertex and source and then another one is superposition of a uniform flow and then the one source is superposed, so that we go the half body.

So like this we can superpose various flows various elementary flows of uniform flow or we can superpose the source sink doublet or the vertex, so these are the elementary flows and then we can obtain the complex flow phenomena like which we have seen half body or the flow past circular cylinder.

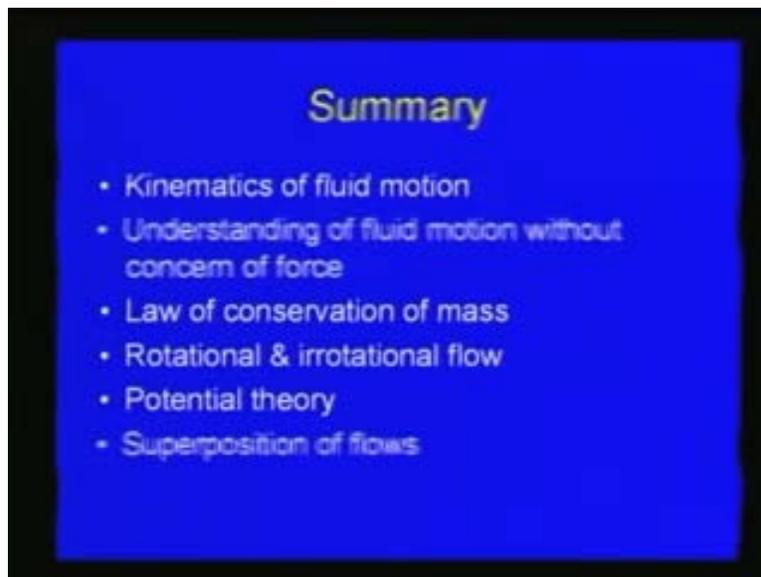
So like that or rankine oval like that we can obtain various complex flow condition by superposition of the elementary flows and then this can be applied with for various practical cases just like say in a homogenous isotropic media like ground water flow is concerned. Somehow the particular cases like we can apply this superposition theories to solve some of the complex problem.

But as I mentioned we should be careful that here we are assuming the potential flow theory. So we should be very careful in the use of this superposition techniques to express determine its certain parameters like drag lift etc. may not predict properly.

So we should be careful in the usage of this potential theory and then this kinds of superposition.

So now this chapter on kinematics fluid flow, now we will summarise what we have done so far in this particular chapter we have discussed the kinematics of fluid motion so as we have seen here we are here say not considering as for the particular force is considered.

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But we are considering various other parameters like velocity field acceleration without giving much attention to the force which is causing the flow. So that the analysis becomes much simple and then we have also derived with respect to various theories.

We derived the law of conservation of mass, the continuity equation in differential form and integration form. We have derived the equations and then we have seen how we can apply for various kinds of practical problems as far as law of conservation of mass is concerned.

Then in this chapter the kinematics of fluid flow, we discussed the rotational and irrotational flow and then as far as irrotational flow is concerned which is potential is concerned various applications we have seen potential theory we have seen the Laplace equation how we can utilize with respect to stream function and potential function and then we have also discussed the elementary potential flow uniform flow source sink doublet and the vortex and finally we have seen the how we can superpose various this elementary flows to obtain complex flows and then how we can utilize to solve various complex problem we have discussed.

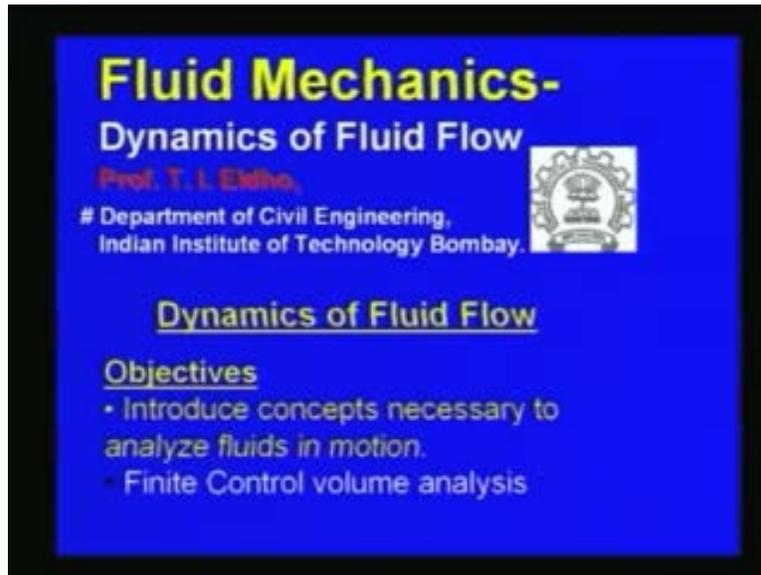
So this is chapter on kinematics of fluid flow further the next chapter we will be discussing dynamics of fluid flow so far we have not given much attention to the force causing the flow but the dynamics of fluid flow.

Consider in next chapter we will be giving much attention to the force causing the flow.

So that we can have the problem will become much more complex and then we will be analyzing it with various fundamental principal of the physics and the fluid mechanics.

So on the in the video course on fluid mechanics now we will discuss the dynamics of fluid flow.

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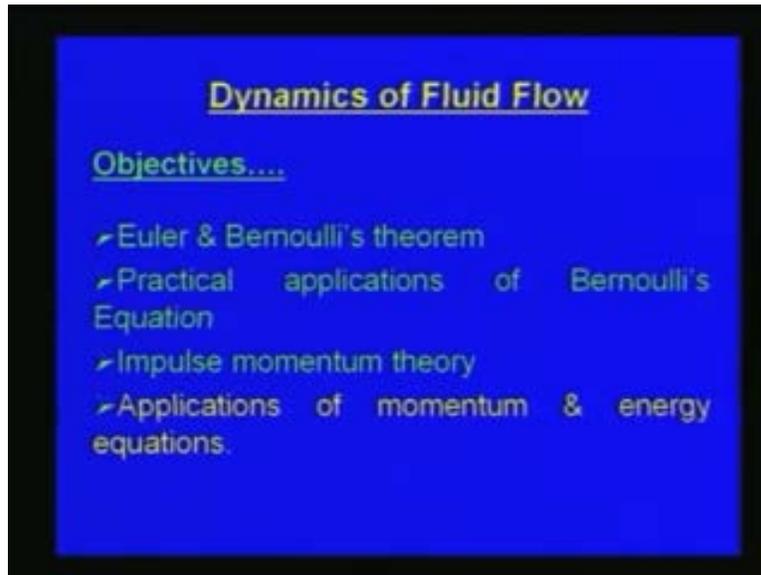
So the last chapter we have seen the kinematics of fluid flow where we did not give much attention to the force causing the flow, but with respect to potential flow and then with respect to various other approximations only we have considered.

So now we will see the dynamics of fluid flow. So as I have mentioned the dynamics of fluid flow is concerned since we are considering, we have to consider all the forces acting upon it and then on the flow which is causing the fluid flow. So now in dynamics of fluid flow the main objectives which we have said here, this particular video course is concerned.

First we will introduce the concepts necessary to analyze fluids in motion. So as far as dynamics of fluid flow is concerned and then secondly we discussed the finite control volume analysis with respect to which we will be deriving the momentum equation energy equation and various other fundamentals equations and then we will be discussing the Euler and Bernoulli's theorem.

So with respect to this control volume analysis and then we will be discussing the practical applications of Bernoulli's equations and then the inverse momentum theory and finally we will be discussing more about the momentum and energy equation's and its applications with respect to the dynamics of fluid flow.

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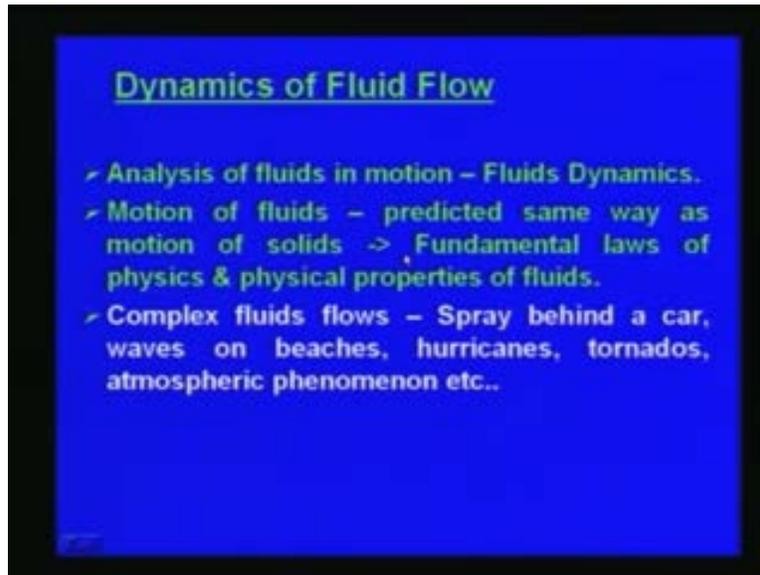


So now as I mentioned say the dynamics of fluid flow is concerned say here we are considering the forces causing the flow and then we are say as far as the fluid dynamics is concerned it is the analysis of the fluid motion.

So as we have seen the fluids statics earlier there the motion is not considered the fluid in is considered at rest so here the fluid is in motion. So we are trying to analyze with respect to say various forces acting due to which the flow fluid movement or flow takes place. So we are trying to analyze these fluids in motion with respect to various fundamental principles fundamental theories and then we are trying to obtain the various fluid flow properties like velocity pressure depth and other parameters with respect to when the particular section is considered or particular point is considered with respect to fluid movement.

So the motion of fluids here predicted same way as motion of solids as I mentioned earlier so the fluid mechanics is most of the theories which we use in engineering mechanics or mechanics of solids or very much valued for fluid mechanics also.

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So just like we deal with the solids in motion or the moving solids, so like that how we predict the various parameters like velocity and other parameters like that only as far as motion of fluid is concerned we are trying to predict the various parameters very similar to the motion of solid and then the various fundamental laws of physics and physical properties of fluids.

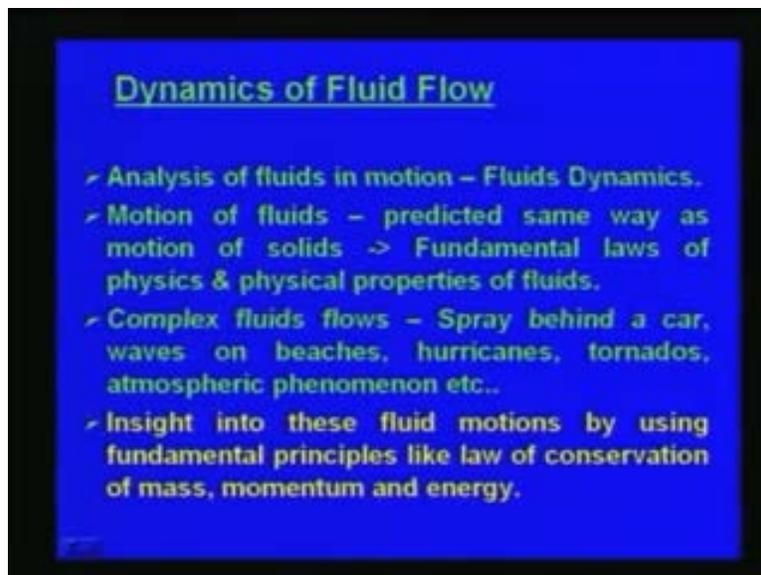
We can directly utilize just like the fundamental laws which we use as far as motion of fluid is concerned those laws are also very much valued as far as fluid mechanics or fluid dynamics of fluids is concerned then other than this fundamental laws of physics, we will be using various physical properties of the fluid which say particular case which we are concerned and then the physical properties also will be considered so that we would be able to solve the particular problem or we will be able to predict the various fluid flow properties that are considered.

So now as I mentioned the fluid flow analysis is complex just like complex flow if you consider the spray behind a car or wave on beaches hurricanes or tornados or various atmospheric phenomena like the movement of clouds or rain fall. All these fluid flows are concerned all these system is very complex to the movement with respect to.

We are trying to analyze with respect to the fluid flow or fluid motion the analysis becomes much more complex. So now for this all these particular problems are concerned say we want insight into this fluid motion.

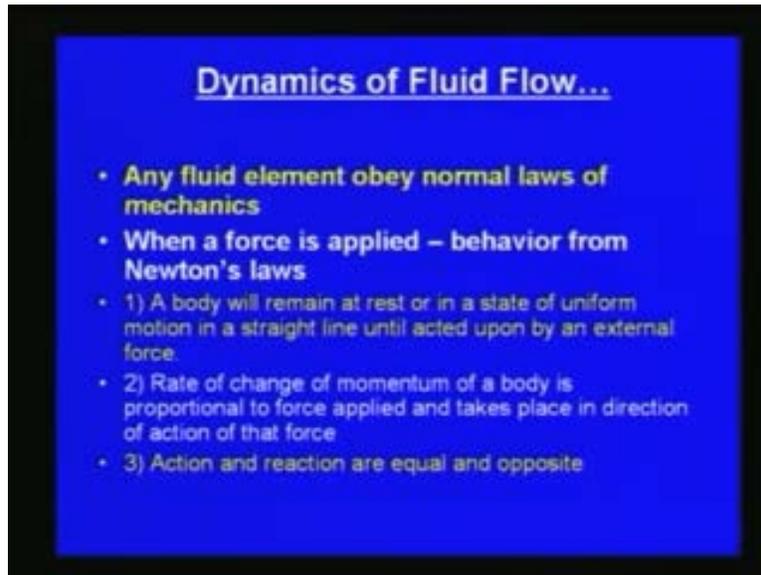
Our aim here is we want insight into this fluid motion by using fundamental principles like law of conservation of mass conservation of momentum, conservation of energy and other fundamental principles like Newton's laws. So that is the way which we are going to approach the dynamics of fluid flow.

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So as far as dynamics of fluid flow is concerned any fluid element obey normal laws of mechanics as I mentioned, the laws of mechanics which we are using for solid mechanics is considered most of the laws are valid for fluid flow also concerned. So since most of the time compared to the solid is concerned we have a definite shape and that definite shape only we will be considering in the analysis. But as far as fluid is concerned we have to, it is continuously flowing we have to consider instead of considering the total flow we may be considering particular element or particular control volume with respect to that only what happens we will be analyzing. So that is the difference between the solid analysis of motion of solids and the analysis of motion of fluids.

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So now when a force is applied say we can obtain the behavior with respect to the application of forces using Newton's three fundamental laws. This Newton's laws, which are applicable to solids movement or motion of solid, these are also valid for fluid also. So as far as the force is applied the behavior we can obtain from Newton's law three laws.

First law is a body will remain at rest or in a state of uniform motion in a straight line until acted upon by an external force. So this is one of the fundamental principles which we use always in physics and also solid mechanics and now this we will be also using fluid mechanics theories of fluid mechanics principles the development of fluid mechanics principles are concerned.

So the first law is body will remain at rest or in a state of uniform motion in a straight line until acted upon by an external force. So if there is only external force acting upon the body. So here the body means here the fluid then only if there is no external force the body will remain at rest or in a uniform flow in a state of uniform motion in a straight line there are no external force it will be a rest or it will be keep on moving

And second Newton second law is rate of change of momentum of body is proportional to the force applied and takes place in direction of action of that force. So this is the Newton's second law.

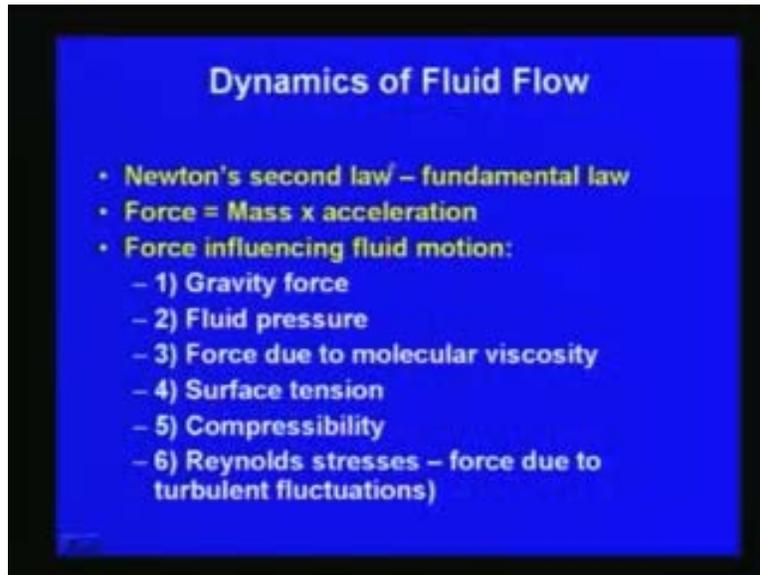
So here, the rate of change of momentum of a body is always say when a force is acting upon the momentum will be we have to consider the momentum of the body. So the rate of change of momentum is proportional to the force applied and then takes place in the direction of that force which is acting upon the body. So this is Newton's second law.

So this Newton's second law is very much used in fluid mechanics principles fluid mechanics theories most of the fluid mechanics theories we have derived based upon this Newton's second law, based upon which we can obtain force is equal to mass into acceleration and other related theories which we will be discussing further and third Newton's law is action and reaction are equal and opposite so these also we can use as far as dynamics of fluid flow is concerned.

So the basic fundamental principles just like Newton's laws are we can definitely utilize as far as dynamics of fluid flow is concerned we will be using these principles to derive various fundamentals equations as far as dynamics of fluid flow is concerned.

So now we will be looking into what are the different forces acting as far as fluid flow is concerned. So as I mentioned say this Newton's second law is one of the fundamental law which we will be utilizing to derive most of the equations concerned to dynamics of fluid flow.

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So as I mentioned force is equal to mass into acceleration. So this we will be utilizing to derive like conservation of mass conservation of energy and all these principles so here the various forces say as far as fluid.

So when the fluid motion is concerned the various forces which have influenced the fluid motion are: first one is the gravity force. Gravity force has got a definite impact upon the fluid motion. So first one which we will be generally considering is gravity force and second one is fluid pressure, based upon which the fluid is moving from one position to another. So second force influence the fluid motion is the fluid pressure and third force is due to the molecular viscosity within the fluid itself and then fourth one is the surface tension say which is acted upon fluid surface, so surface tension is important and then if the fluid is compressible or if it is incompressible so the compressibility of the fluid that affects compressible force that we have to consider as far as the fluid motion is concerned.

And then lastly the Reynolds stresses force. Forces due to turbulent fluctuations the flow can be either laminal or turbinal we will be discussing further.

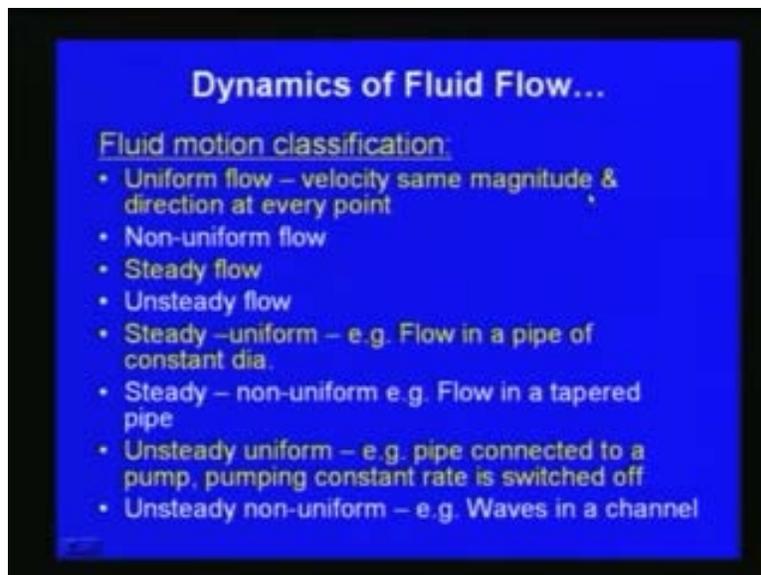
So with respect to turbulence there will be Reynolds stresses will be produced these are some of the important forces which influence the fluid motions just like gravity force,

fluid pressure, then force due to molecular viscosity, then surface tension compressibility Reynolds stresses. So all these forces we have to consider as far as dynamics of fluid flow.

So depending upon the type of problem which we are considering or type of equation which we are going to derive some of the forces will be which will be more important for that particular case and then others can be the effect will be negligible and that negligible forces can be neglected as far as the particular derivation is concerned or the particular problem is concerned important forces will be considering as far as the derivation of the equation or the particular problem is concerned.

Now, based upon these as since, we are now dealing with dynamics of fluid flow. So the fluid motion we can classify depending upon the how the fluid properties are changing with respect to motion, we can classify the fluid motion like uniform flow, non uniform flow, steady flow, or unsteady flow.

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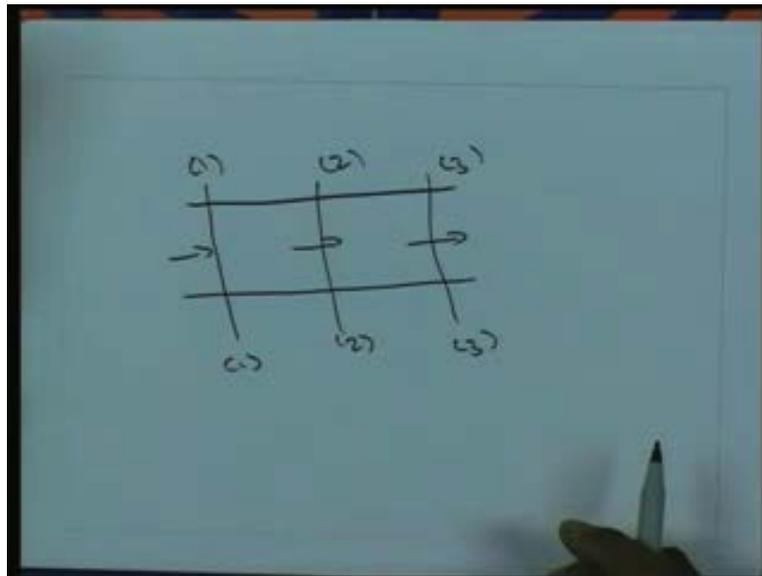


So this we have already seen earlier. Uniform flow as we discussed earlier means the velocity is same magnitude and direction at a very point. So if you consider for example, a channel flow say that if it is uniform flow if you consider with respect to various

sections say here section one, section two, section three, here the flow is in this direction say this piece of channel which we consider here.

For uniform flow means we will say that flow is uniform for this particular case when the velocity remains same velocity and other fluid parameters like depth remains same in magnitude and direction at every point of the flow is concerned. So this is what is called uniform flow and that means the.

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Variation with respect to space if there is spatial variation is not there that means not there is a flow parameters remains same in magnitude and direction. So that is the uniform flow. But as far as non uniform flow is concerned we can see a flow is said to be non-uniform.

Whenever we go from section one to section two the flow parameters like velocity pressure and all these parameters will be changing with respect to space. So the flow with respect to fluid motion when we consider a spatial wise so the parameters the velocity pressure and other depth parameters are changing that is what is called non uniform flow. The third category is concerned steady flow.

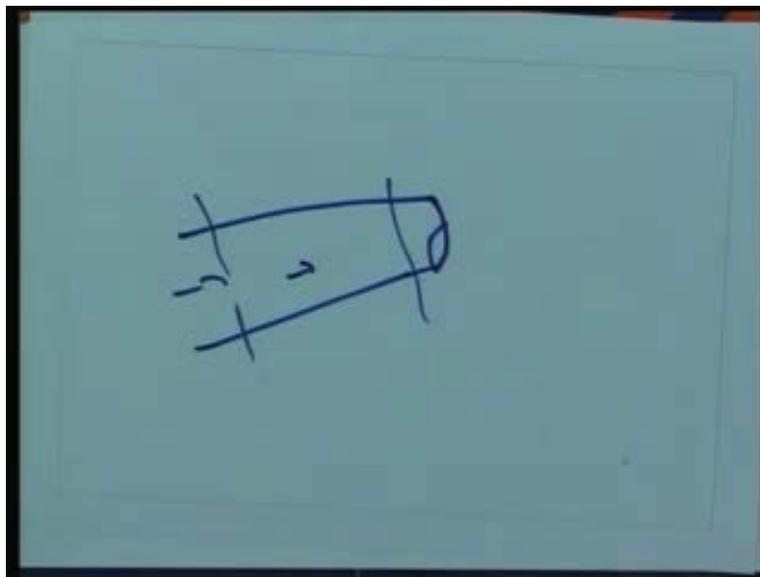
So if we say that the flow parameters are not changing with respect time means only the spatial variation is there in that case, we say that the flow is steady that means with respect to time there is no change for the particular fluid flow is concerned and whenever there is change with respect to the velocity is changing with respect to time, if it is one d one dimensional flow with respect to space and time or with respect to x y z and time then we say that it is unsteady flow that means or we call it as transient flow.

So with respect to this uniform flow, non uniform flow, steady flow, we can again classify the fluid motion to various characteristics like steady uniform, so we say that there is no change with respect to space and time that means it is steady and uniform.

So for example for a pipe of constant diameter it is steady uniform it can be considered as a steady uniform flow and then we can also say classify the fluid motion.

As steady non uniform so that means if you consider the flow in name tapered pipe, so if you consider the flow with respect to a tapered pipe like this, so here you can see the diameter is changing from one section to another, so it is tapered pipe. So in this case we can say that the flow is steady non uniform flow with the definition of the steady flow and non uniform flow.

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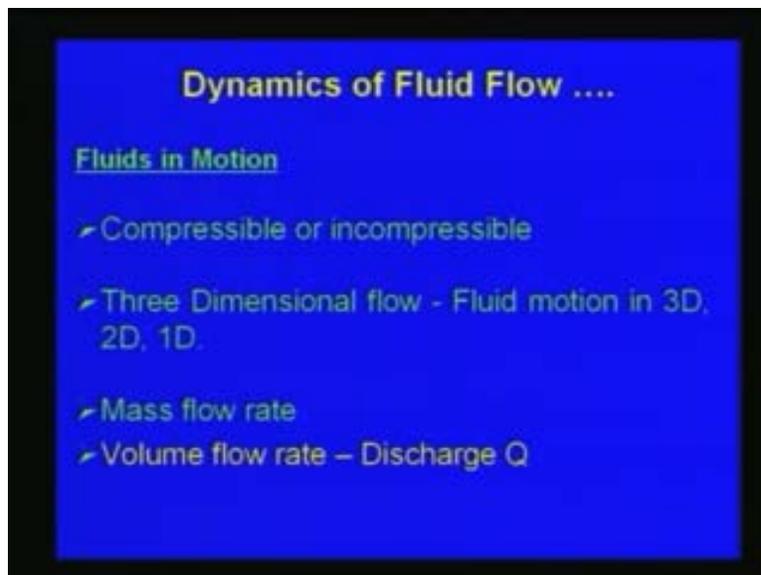


And then we can also classify the unsteady flow as unsteady uniform flow that means for example, a pipe connected to a pump pumping constant rate is 0.

So this is a case of unsteady uniform flow or we can have an unsteady uniform flow. For example, waves in a channel (Refer Slide Time on 43:32). So these are some of the typical fluid motion classification like with respect to how the changes takes place with respect to space x y z axes the space and then with respect to time how the changes takes place. So the dynamics of fluid flow or the fluid motion we can classify accordingly.

So now as far as fluid motion is concerned as I mentioned there can also be the effect of compressing if any compressible force is acting then, the fluid is compressible then definitely the motion ,the fluid motion is affected accordingly, we can say that the fluids in motion, we have to consider the rate is compressible or incompressible depending upon the property of the fluid and then if it is compressible then with respect to various forces acting then properties will change and then we have to consider.

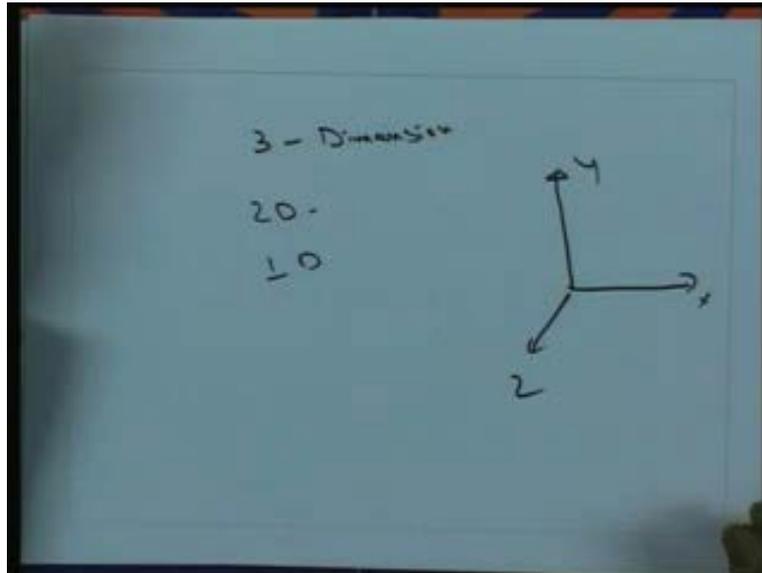
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Whether the flow is compressible or incompressible and then obviously we have to also consider with respect to the dynamics fluid flow dynamics or dynamics of fluid flow, we have to consider the flow is whether it is 3 dimension as I mentioned earlier most of the time the fluid flow is varying with respect to x y and z but that means 3 dimension.

So most of times we can say approximate the flow instead of if you consider the flow as three dimension then it will be analysis especially since the fluid motion is we are dealing with so the analysis becomes much complex.

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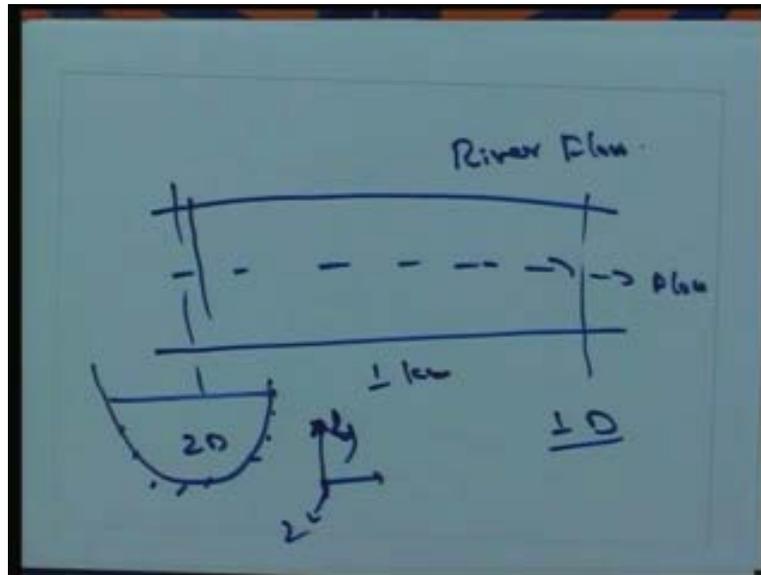
So we can just simplify the 3 dimension flow to 2 dimension or 1 dimension depending upon the case. so we have to also see as far as dynamics of fluid flow is concerned whether we are dealing with 3 dimension flow or whether we are dealing with 2 dimension flow or 1 dimensions flow.

So as we have seen earlier for example, if you consider a river flow is concerned say a large river is concerned. So now if you consider the 2 sections say at distance of 1 kilometer so this particular case is concerned if you are dealing if you are interested only what happens on the direction of the flow if the direction of flow is in this direction, what happens in this direction? Even though this river flow is concerned it is 3 dimension flow depending upon the particular problem, we are solving we can analyze as 1 dimension flow that means in this direction of flow we can consider so that using 1 dimension how the variation of 1 dimension parameter.

Since it is more important for this particular problem is concerned so we can analyze it 1 dimension or when we consider a particular section of the river like these, here say what

happens in this particular say section then we can take this as 2 D that means it will be say with respect to vertical and with respect to the lateral direction, so what may be z and y so that will be the y and z will be considered here or say if you want a complete analysis you can go for a 3 dimension analysis.

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As far as dynamics of fluid flow is concerned depending upon the problem which we are dealing we will be considering 3 dimensional flow or 2 dimensional flow or 1 dimensional flow whenever we need to say approximate or we want to simplify the problem we will be considering the 3 D flow as 2 D flow or 1 D flow.

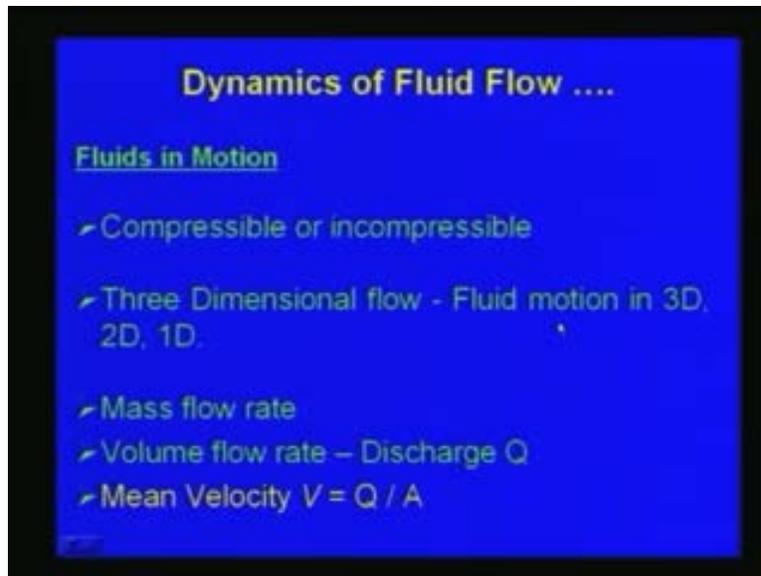
And then some of the other important parameters as far as dynamics of fluid flow is concerned we will be considering since fluid is flowing, fluid we are considering fluid motion. So the mass flow rate how much mass is flowing that is an important parameter so that we may have to consider fluids or fluids in motion and then volume flow rate that means discharge, discharge cube that means how much discharging is passing at a particular section or between two sections, how much is the change in discharge.

So another important parameter is so called volume flow rate or the discharge. If you know the cross sectional area we can say, most of the time we will be interested in the

mean velocity. So mean velocity can be expressed as V is equal to Q by A which is the volume flow rate divided by area of cross section.

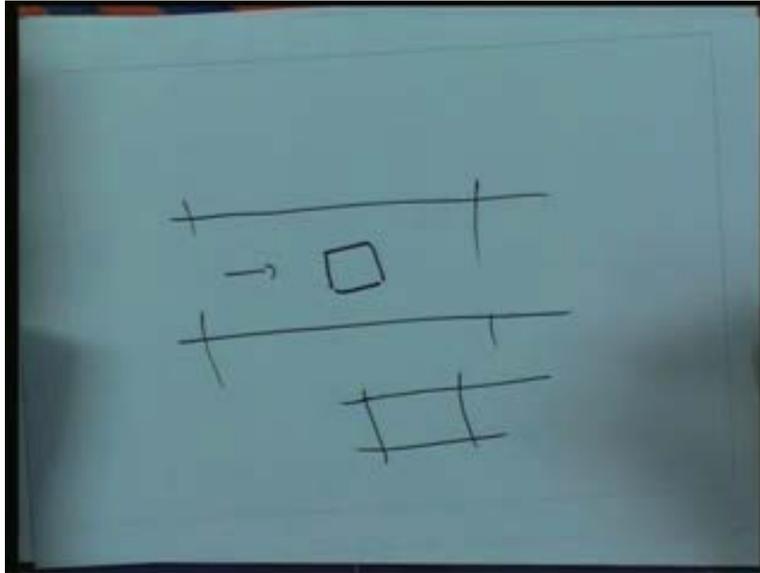
So now further we will discuss about the analysis of fluid that means with respect to motion.

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So as I mentioned here, when we deal with the fluid flow compared to the solid movement, here fluid is continuously moving say, if you consider river flow or a pipe flow or any kind of this flow you can see that continuous movement is endless stream of fluid is there. So it is very important that we have to see what part of this stream that we will be analyzing or the stream starts constitute the system to analyze. So generally we are dealing with system so we have to see which part of that which one we are analyzing.

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So as far as solid motion is concerned particular solid how it is moving we will be dealing, but as far as fluid motion is concerned we have to see, since we are dealing with an endless stream of fluid we have to see what part of that will be considering the analysis.

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Dynamics of Fluid Flow

Analyzing Fluid Flow

Endless stream of fluid – what part of this stream shall constitute the system to analyze? – Two alternatives

- 1) Behavior of a specific element of fluid of fixed mass – closed system
- 2) Define system to be studied as a fixed region in space – known as control volume – through which fluid flows – open system

So we have two alternatives generally - first one is we can see the behavior of a specific element of fluid. So here is you consider this particular element what happens? That is one approach. So behavior of a specific element of fluid of fixed mass that means generally we will be considering a closed system just like a pipe this particular system what happens? So that is one way approach. So within that fixed mass what happens with respect to fluid motion and second one is define the system to be studied as a fixed region in space known as control volume through which fluid flows.

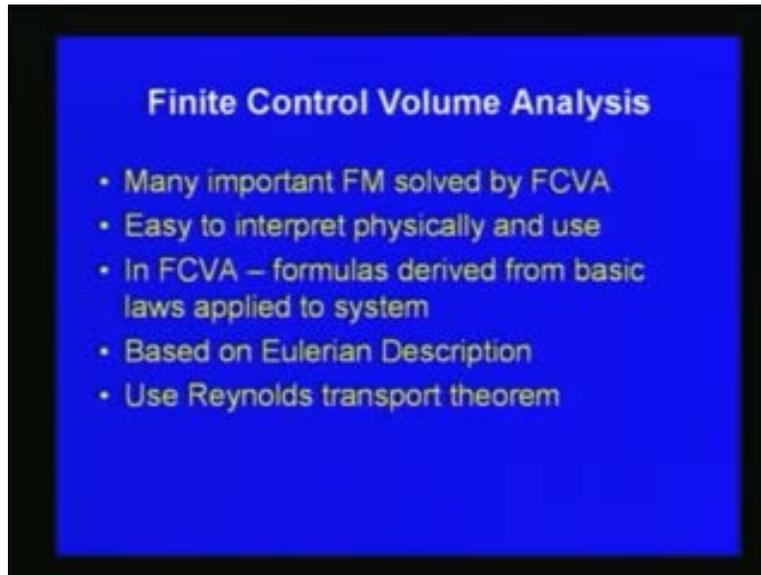
So generally, this will be an open system so this here we can see a channel flow or river flow. So if you consider a control volume like this between section one and section two what happens? So that is we can define a system of fixed region in space and then what happens in that is so called control volume and then this is this is an open system.

So many of the important fluid mechanics problems, we can solve using this finite control volume. So this is we are just defining a control volume and then what happens to this control volume that is what we are analyzing. So many of the fluid dynamics problem we can analyze using finite control volume analysis.

Here this finite control volume analysis is much easy to interpret physically and the usage is also vary we can use this concept is simple to interpret also the formula derived from basic laws we can apply directly to the system.

So since we are considering a finite control volume with respect to the fluid motion of dynamics of flow, it is the formula derived we can easily apply to the system which we are dealing with.

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Then as we have seen the flow description is concerned we can either use the Eulerian description or Lagrangian description, but here as far as the finite control volume analysis is concerned we will be dealing with the Eulerian description and then we will be using the Reynolds' transport theorem which we have seen earlier.

So now with respect to this finite control volume approach we will be deriving the conservation of momentum equation earlier. We have already discussed how to derive the conservation of mass equation the continuity equation with respect to the differential approach and integral approach that we have already discussed earlier in the kinematics of fluid flow.

So now in this dynamics of fluid flow first we will be discussing how to derive the conservation of linear the conservation of momentum equation and then we will be discussing about the conservation of energy equation and further we will be going for the with respect to the dynamics of fluid flow of the fluid motion is concerned, we will be analyzing various problems with respect to fundamental principles and its basic applications.