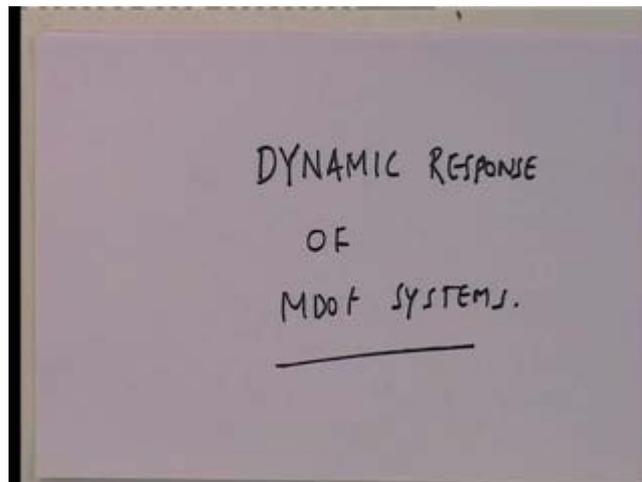


Structural Dynamics
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Lecture - 28
Dynamic Response of Multi Degree of Freedom Systems

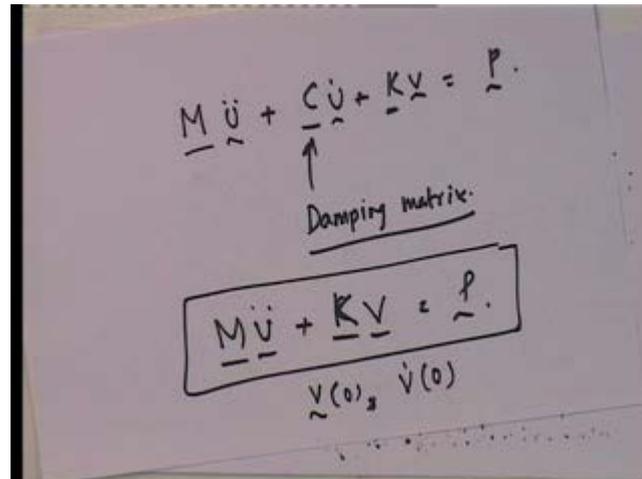
Hello there. In the last lecture, we looked at practical free vibration analysis and now we are done with free vibration analysis this thing. Now, what we are going to do is that you are going to be looking at lab situation and in the lab you will find, you will be taken through how to do a multi degree of freedom system and to find out its frequencies and mode shapes. So, I am going to stop here as for free vibration is concerned, and I am going to move on to the dynamic analysis of single, I mean sorry multi degree of freedom system problems.

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So, now the final one as far as multi degree of freedom system problems is dynamic response, we will be looking at multi degree of freedom dynamic response. Well, let us start.

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$$\underline{M} \ddot{\underline{U}} + \underline{C} \dot{\underline{U}} + \underline{K} \underline{V} = \underline{P}$$

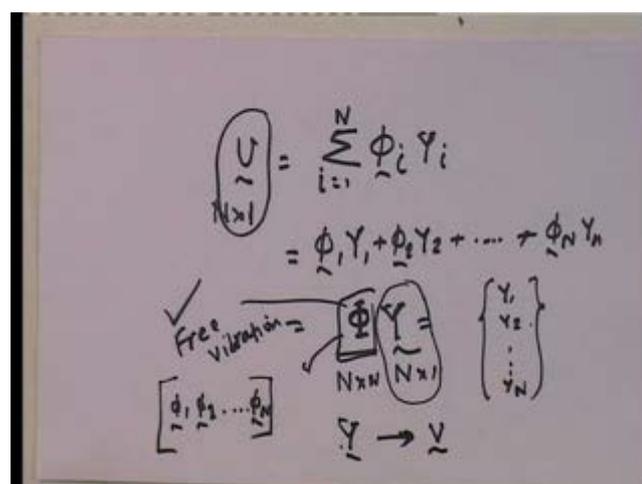
↑
Damping matrix:

$$\underline{M} \ddot{\underline{U}} + \underline{K} \underline{V} = \underline{P}$$

$\underline{U}(0), \dot{\underline{U}}(0)$

Remember last time I did say that you know you could remember, I said we spent a little bit of time on damping matrix. Now, let us not worry about damping. Let us look at the undamped system response. So, undamped system response is essentially this subject to initial conditions given by these two. So, this is the problem that we are looking at undamped system. So, we have MV double dot plus KV is equal to P , where P is a load vector, M is a mass matrix, K is the stiffness matrix corresponding to the degrees of freedom vector given by V and subjected to initial conditions V_0 and V dot 0. Note that all of these are pro force vector because it is an end degree of freedom structure.

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$$\underline{U} = \sum_{i=1}^N \underline{\phi}_i Y_i$$
$$= \underline{\phi}_1 Y_1 + \underline{\phi}_2 Y_2 + \dots + \underline{\phi}_N Y_N$$

✓ Free vibration ⇒

$$\begin{bmatrix} \underline{\phi}_1 & \underline{\phi}_2 & \dots & \underline{\phi}_N \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \underline{U}$$

$\underline{Y} \rightarrow \underline{V}$

So, now how do we solve this problem? Well, let us start from the beginning again. You know I just showed you in the last lecture that well, we can indeed write the displacement as a sum of i going from 1 to N $\phi_i Y_i$. So, if you look at this one, then in

essence becomes the following that $\phi_1 Y_1$ plus $\phi_2 Y_2$ blah blah blah plus $\phi_N Y_N$, and I can write this in metrics form in this format. This is n by 1 , this is N by 1 , and this is N by N . What is this vector? This vector is vector of the modal amplitudes. What is this? This is a matrix whose row vectors are, so each row of this vector is this anyway.

So, now you know we also saw that. So, that means, if I know the modal amplitudes, if I know this, I can find this because this is known from free vibration. So, the way of going from Y to V is relatively easy. Now, how do I find out Y given V ? Well, we did that and we saw that using the orthogonality condition, right Y_N .

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$$Y_n = \frac{\phi_n^T M V}{\phi_n^T M \phi_n}$$

$$V \rightarrow Y$$

$$Y \rightarrow V \quad V = \Phi Y$$

I am just repeating, you know repeating something that I have already derived if equal to $\phi_n^T M V$ upon $\phi_n^T M \phi_n$. So, using the orthogonality condition, I can always find this out and so from given V to find out Y , this is the way of finding it out. Given V , finding out Y and then you got Y . So, V to Y is through this and Y to V is through simple V to Y , this Y to V .

So, now given this I can write my equation of motion. Let us look at this equation. This is $M \ddot{V}$ plus $K V$ is equal to P . So, this is how this is written. Now, let me write I know I want to write down V in terms of Y . What will it be? It will be M into this is ϕ into Y plus K into ϕ into Y is equal to P . All I have done is substitute the fact that V is equal to ϕY and obviously, V double dot ϕ is not a function of T . So, the only thing that the function of T is Y , and so that is what comes, right and that is all I have done.

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The whiteboard shows the following equations and matrix definitions:

$$v = \underline{\Phi} \underline{y} \quad \dot{v} = \underline{\Phi} \dot{\underline{y}}$$

$$\underline{M} \ddot{\underline{v}} + \underline{K} \underline{v} = \underline{P}$$

$$\underline{\Phi}^T = \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_N^T \end{bmatrix}$$

$$\underline{M} \underline{\Phi} \ddot{\underline{y}} + \underline{K} \underline{\Phi} \underline{y} = \underline{P}$$

$\begin{matrix} N \times N & N \times N & N \times 1 \\ N \times N & N \times N & N \times 1 \end{matrix}$

$$\underline{\Phi}^T \underline{M} \underline{\Phi} \ddot{\underline{y}} + \underline{\Phi}^T \underline{K} \underline{\Phi} \underline{y} = \underline{\Phi}^T \underline{P}$$

$\begin{matrix} N \times N & N \times N & N \times N \\ N \times N & N \times N & N \times 1 \end{matrix}$

Now, you know what I am going to do? I am going to pre multiply. I mean as long as I pre multiply all terms on the left hand side and right hand side by anything, it should be valid. Now, what I am going to pre multiply? Now, note here that this is N by N, N by N, this is N by 1, this is N by N, N by N, N by 1. So, this is actually a set of N differential equations coupled because even if M is lumped mass, K is still coupled. So, we have a problem. So, now, let us see I am going to pre multiply this entire thing by phi transpose which is N by N, where what is phi transpose matrix equal to.

Now, this matrix is equal to, its row vectors are equal to phi 1 transpose phi N transpose. This is what this N by N matrix is. So, phi N transpose is that into M into phi, this is N by N, this is N by N and plus phi transpose K phi, sorry this is Y double dot. This is Y equal to P. Again all of these are N by N. So, this matrix multi by the way, sorry, phi transpose. Now, this is N multiplied by N by 1. It is still N by 1. So, what we have is, we have continued taking N coupled linear differential equations, transform it into still N coupled little differential equations and this is also N differential equations, but let us see something very interesting.

Let us see what this phi transpose M phi is. Now, this is N by N which is nothing but phi 1 transpose phi 2 transpose blah blah blah up to phi N transpose into M into, what is this is phi 1 phi 1 blah blah blah till phi N. So, every row here is a mode shaped vector, and every column I mean, sorry every column over here is a mode shaped vector, and every row here is a transpose of the mode shape vector, ok.

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$$\begin{aligned} \underbrace{\Phi^T}_{N \times N} \underbrace{M}_{N \times N} \underbrace{\Phi}_{N \times N} &= \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_N^T \end{bmatrix} \underbrace{M}_{N \times N} \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_N \end{bmatrix} \\ &= \begin{bmatrix} \phi_1^T M \phi_1 & 0 & \dots & 0 \\ 0 & \phi_2^T M \phi_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \phi_N^T M \phi_N \end{bmatrix} = \begin{bmatrix} M_1 & & & 0 \\ & M_2 & & \\ & & \ddots & \\ 0 & & & M_N \end{bmatrix} \end{aligned}$$

So, now let us do this. So, this is N, sorry on E by N, this is 1 by N. All these are 1 by N and so 1 by N's such give you N by N. This is N by N and these are N by 1, N by 1, each one is a row of N by 1 vectors to make N by N. So, let us see this entire thing what does it look like. What does this matrix look like? Well, I can easily see this is a row, and a row is multiplied with a column. So, if you look at it, the first one if you look at it, what does it represent? It represents 1 by N. Let me see what this represents. This is 1 by N, this is N by N, this is N by 1. What is that? N into N into N by 1 is N by 1, 1 by N into N is a scalar. So, this is in this matrix. The first one is this.

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$$\begin{aligned} \underbrace{\Phi^T}_{N \times N} \underbrace{K}_{N \times N} \underbrace{\Phi}_{N \times N} &= \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_N^T \end{bmatrix} \underbrace{K}_{N \times N} \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_N \end{bmatrix} \\ &= \begin{bmatrix} \phi_1^T K \phi_1 & 0 & \dots & 0 \\ 0 & \phi_2^T K \phi_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \phi_N^T K \phi_N \end{bmatrix} \end{aligned}$$

What is the second one? Phi 1 transpose M. Now, let us see phi 1 transpose M phi 2. What is that? I mean all of these, it goes up to phi 1 transpose M phi N. What are these? By definition this is M1 and these are all 0 orthogonality property. So, if you look at this

matrix, the next one is $\phi^T M \phi$ $\phi^T M \phi$. So, if we look at this one, this matrix looks like this $M_1 M_2 0 0$. It is just a diagonal. This is a diagonal matrix. Let us now look at $\phi^T K \phi$.

Well, let us see what $\phi^T K \phi$ turns out to be? It is equal to same thing, $\phi^T M \phi$ $\phi^T M \phi$ $\phi^T M \phi$, and then is equal to what is it become. It becomes nothing but $\phi^T M \phi$ and all other terms are 0. Again, only the diagonal terms exist because all the up diagonal terms are 0. It is because of what? It is because of orthogonality property. Note that since these are all 1 by N's and these are N by 1's, each one even though it is a matrix multiplication is actually a scalar, ok.

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$$\underbrace{\Phi^T M \ddot{\Phi}}_{\text{diag.}} + \underbrace{\Phi^T K \Phi \dot{\Phi}}_{\text{diag.}} = \Phi^T P$$

$$M_n \ddot{Y}_n + \phi_n^T K \phi_n Y_n = \phi_n^T P$$

N - uncoupled ODE

Modal space — Normal space

So, therefore, very interesting thing what happens now once I have put this $M \phi$ transpose into \ddot{Y} plus $\phi^T K \phi$ into \dot{Y} is equal to $\phi^T P$. What is this? It is a diagonal matrix. So, what do we get if you look at this? Now, we just get note that by this procedure I have transferred, I have transformed a problem which was in the original space, N coupled linear differential equations. I have transformed it into the normal space which the modal spaces called the normal space, and the modal space is also called the normal space by transforming it from the original space to the modal space. What have we done? We have taken N coupled linear differential equations, and made them into N uncoupled linear ODE's. So, therefore, you know you look at this does not have any other Y, it does not have any other Y, ok.

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$$\begin{aligned} \phi_n^T K \phi_n &= \phi_n^T \omega_n^2 M \phi_n \\ &= \omega_n^2 \phi_n^T M \phi_n \\ &= \omega_n^2 M_n \end{aligned}$$

$$M_n \ddot{Y}_n + \omega_n^2 M_n Y_n = P_n$$

Identical SDOF - system ODE

So, if I look at this problem and now, look at something else. Let me just look at something else and that is, what is phi n transpose K phi n. Let us look at it, phi n transpose K phi n. What is K phi n equal to? K phi n is equal to omega n squared M phi n. The omega n squared goes to outside. Omega n squared is outside because it is a scalar. So, what you have is phi n transpose M which is omega n squared Mn. So, if you look at this problem, this problem is this. If you look at it because you look at this, what was M into V plus K into V is equal to P. What is K? K is equal to omega squared M because K upon M is equal to omega squared. So, you know I could substitute in this way. If you look at this is identical, this is identical to a single degree of freedom system.

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$$V = \Phi Y$$

$$M \ddot{V} + K V = P$$

N coupled ODE (ω_n^2)

$$M_n \ddot{Y}_n + K_n Y_n = P_n$$

$\omega_n^2 M_n$

N uncoupled SDOF ODE
 $Y_n(0), \dot{Y}_n(0)$

So, what we have done through this procedure is by incorporating the fact that V is equal to we have taken it from original space to normal space because these are the amplitudes

in the normal space. This thing by doing this, I have taken coupled equations, taken them N coupled ODE's to which is equal to $\omega_n^2 M_n Y_n$ is equal to P_n uncoupled single degree of freedom ODE. So, you know you have made a multi degree of freedom problem into M degree of freedom, multi degree of freedom problem into N single degree of freedoms.

So, you have taken the entire problem and made into the single degree of problem of course. Note that this one, how do I evaluate these? Well, M_n is equal to $\phi_n^T M \phi_n$. So, you still have to do this, and ϕ_n is equal to ϕ_n^T the load vector. You still have to do these and ω_n^2 comes from the free vibration analysis. So, once you do these, you get this. You got N uncoupled shift differential equations is much easier to solve than N coupled differential equations, and once you solved for these. Now, these will be solved for what? This is the general kind of problem, single degree of freedom problem. Well, how do I find out Y and 0? It is very simple. Why cannot I find Y and 0?

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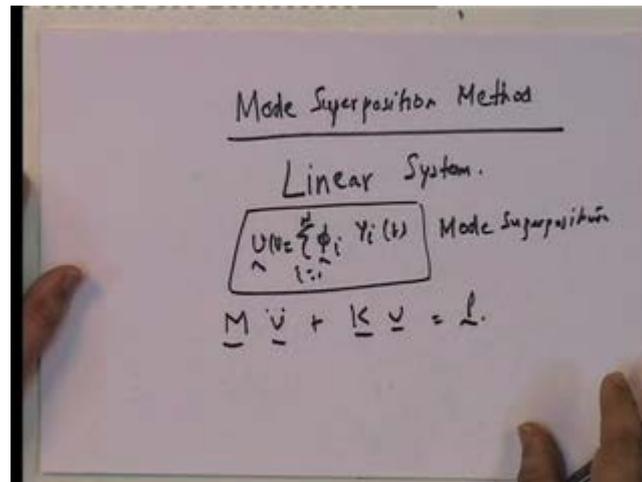
$$Y_n(0) = \frac{\phi_n^T M \underline{v}(0)}{\phi_n^T M \phi_n}$$

$$\dot{Y}_n(0) = \frac{\phi_n^T M \dot{\underline{v}}(0)}{\phi_n^T M \phi_n}$$

I have already done it $Y_n(0)$ is equal to $\phi_n^T M \underline{v}(0)$, which is \underline{v} at time equal to 0 upon $\phi_n^T M \phi_n$ and therefore, $\dot{Y}_n(0)$ is $\phi_n^T M \dot{\underline{v}}(0)$. Note that none of these other terms are dependent on time excepting for these two. So, automatically this comes in. I can find out the initial conditions through this procedure using the same thing. Remember that in one, you go from \underline{v} to Y . This is through going from \underline{v} to Y and the other one is going from $\dot{\underline{v}}$ to \dot{Y} .

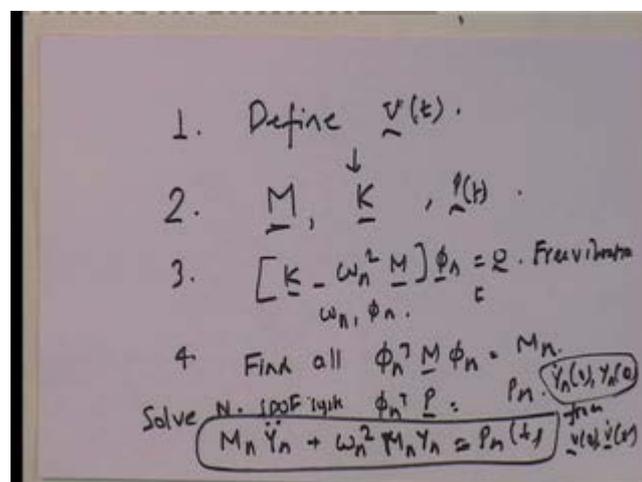
So, once you solved for these Y_n , you plug it back in here and you have got your displacement time history. So, this is a fairly easy way of doing things, and this approach of solving these problems is known as the mode super position method. As all super position methods, this is only valid when you have a linear system.

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So, this is what is the mode super position method. The mode super position method is based on the fact that any displacement quantity time is given by ϕ_i going from 1 to n. This is your mode super position, where ϕ_i is the mode shape which is time independent and the modal amplitude is time dependent. So, based on this and using the procedure, once this is the basic mode super position that is what we are saying and once we have this, so the problem then becomes this. Let me write it down in an algorithm fashion as to how you are going to use the mode super position method to solve.

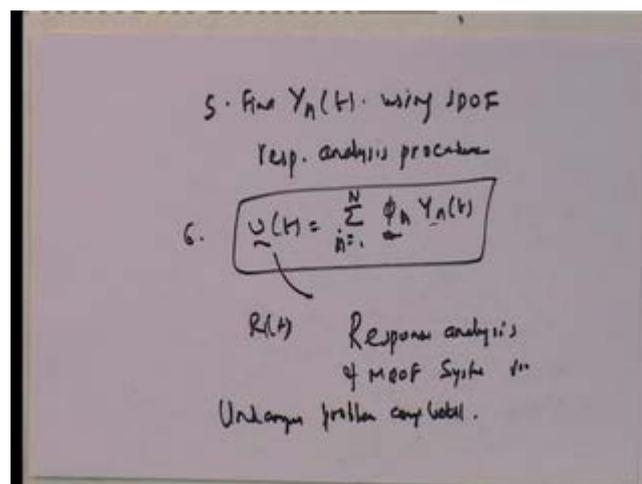
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Multi degree of freedom problem start. 1. Define degrees of freedom that will give you this, right. So, once you do that, then corresponding to this, find M, K and P vectors. So, mass stiffness and find that. Then solve this free vibration Eigen value problem for ω_n and ϕ_n . 4. Find all $\phi_n^T M \phi_n$, that is $M_n \phi_n^T P$ which is P_n and then solve n single degree of freedom system problems, where $M_n \ddot{Y}_n + P_n Y_n = F_n(t)$. Well, this is single degree of freedom. We already have solved many single degrees of problems given.

So, find this and also, find out $Y_n(t)$ from V_0 and \dot{V}_0 , so that basically you can solve this differential equation given initial conditions. So, once you have done that, now this one gives you what you solve for it and you get what? Think back to single degree freedom.

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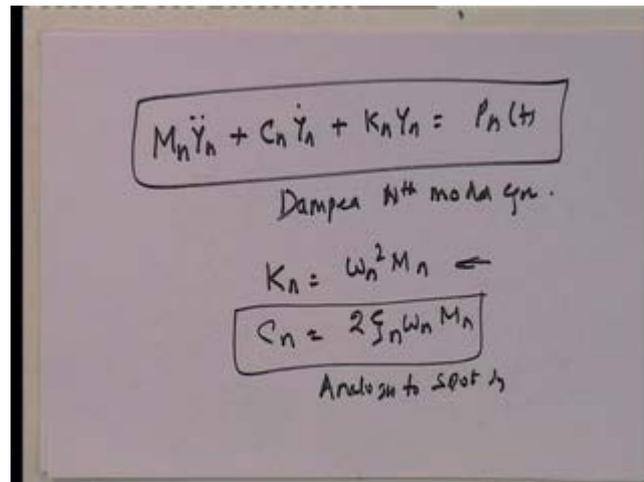


Single degree of freedom gives you Y_n of t . Find Y_n of t using single degree of freedom response analysis procedures. So, find Y_n of t . Once you find Y_n of t , V of t is equal to summation i going from 1 to n ϕ_n , sorry this is ϕ_n . So, $\phi_n Y_n$ of t that gives you your displacement. In fact, we can show later on, we will see that you can find out any displacement, any response can be found out in terms of these modal amplitude and response analysis because this is nothing but response analysis of multi degree of freedom system problem. So, we have solved the undamped problem completely.

So, we have solved the undamped problem. Now, let us look at the damped problem and today, I shall only introduce you to the concept of the damping and remember, I talked about proportional damping. Well, we will see if you look at this. See we are getting by

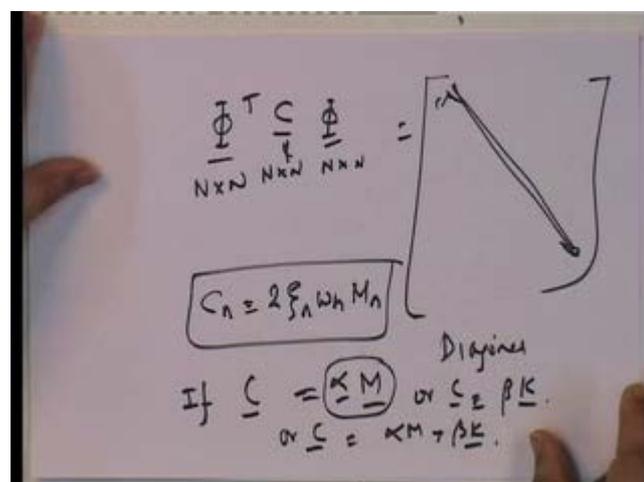
using mode super position, we are getting the equation which looks $M_n \ddot{Y}_n + C_n \dot{Y}_n + K_n Y_n = P_n(t)$. This is a single degree of freedom. It is undamped single degree freedom. So, now, if I want to take this and make it into include damping at the single degree of freedom level, how should it look?

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It should look like this. This is how the damped you know nth eighth modal equation look like, right. This is how it should look and note that by definition K_n is $\omega_n^2 M_n$, this is known. So, therefore, we should say that C_n is equal to $2 \zeta_n \omega_n M_n$ analogy to single degree of freedom system because this is still the single degree of freedom. So, all that we say is that if C_n could be this, so what is that mean?

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This means that it is performed that your $\phi^T C \phi$ which is N by N , N by N , N by N is also a diagonal matrix, C has to be such that you get this and then this C is equal to $2 \psi \omega M$. So, C matrix has to be such that these are diagonal, and that C is equal to $2 \psi \omega M$. Now, the funny thing that happens is the following and that is that what happens. We have a peculiar situation that if C is of the form this, if C is equal to this or C is equal to this or C is equal to this plus this, since M and K are, so therefore if I substitute this in, let us see if I substitute this, what happens. Mass proportional.

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Damping matrix Mass proportional

$$\underline{C} = \alpha \underline{M}$$

$$\underline{\Phi}^T \underline{C} \underline{\Phi} = \underline{\Phi}^T \alpha \underline{M} \underline{\Phi}$$

$$= \alpha (\underline{\Phi}^T \underline{M} \underline{\Phi})$$

Proportional Damping.

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$$\underline{C} = \beta \underline{K}$$

$$\underline{C} = \alpha \underline{M} + \beta \underline{K}$$

Proportional Damping.

This stiffness if the damping matrix is mass proportional damping matrix, what happens if C is equal to αM ? Then $\phi^T C \phi$ becomes $\phi^T \alpha M \phi$, α is a scalar. So, this becomes $\alpha \phi^T M \phi$. Since, we know this

is diagonal and alpha into diagonal, this is a diagonal matrix. So, C is diagonal and therefore, we can uncouple it, ok.

So, this is what is known. That is the reason why we call it proportional damping that C matrix. Now, note that suppose the C matrix is stiffness proportional, you can do this since you substitute, you will get phi transpose K phi that is uncoupled, ok. So, therefore, we talked about rally damping, right. Remember last to last time we talked about rally damping. Well, it is obvious I am not going to do this. Phi transpose will just become phi transpose M plus phi beta into phi transpose K phi. All of those are diagonal.

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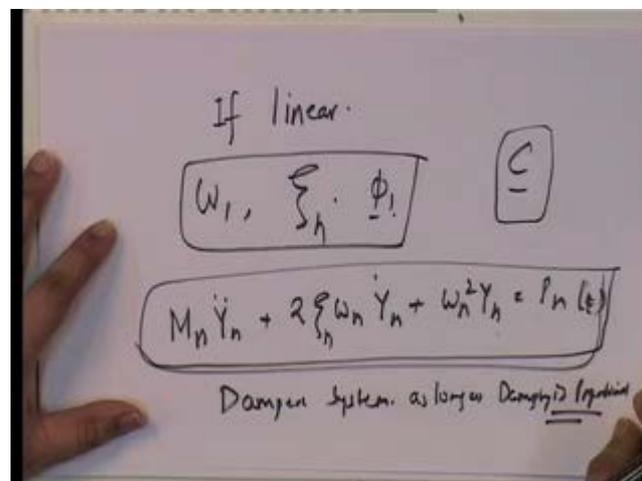
So, therefore, if you have proportional damping, you will be able to use your damp system and now, note that the C matrix is an unknown quantity. C matrix is not known; however, we can say that, look C n is equal to 2 psi omega n M n. So, therefore, extensively understand that as long as we make an assumption on proportional damping, this is fundamental and not derived. What is to be derived is given. This. What is C? Now, you may well ask me that why do I need to find out C. Please understand one thing and that is that we have a situation where if you are using mode superposition for a damped system, you do not need to find out the C matrix. Why? Because you do not extent, you do not require doing this.

The only thing that we say is that look, we do not know what C is, all we say is that the C matrix is a proportional damping such that this is a diagonal matrix, such that we can uncouple the equations. Otherwise, you see if this is not diagonal, what happens even M and K are uncoupled. If Cn are not uncoupled, then through this Cn, you know this C

you get all the other Y_1 Y_2 Y_3 . You know the other Y 's are coming in and then you know then the mode superposition is not really useful because all you are doing is taking it from coupling from one end to another end.

Ok, but remember what I say that look we do not know what damping is, we make an assumption on damping and we also say that look this we do not know. It is sum ψ ; it is not even ψ_n . I do not even know what ψ_n is. How can I find out the damping in every mode? I cannot. I have no clue. All I know is that I define damping in every mode to be given by ψ and note that I do not need to know this C matrix at all.

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So, therefore, if you have a linear system, as long as you are linear, just every mode define this becomes the property of every mode, and if this become the property of every mode, then you see damped system as long as damping is proportional. $M_n \ddot{Y}_n + 2 \psi_n \dot{Y}_n + \omega_n^2 Y_n = P_n(t)$ and note that you know this is again the damped single degree of freedom system. We have already solved as long as the time variation is known, and you find this out by enlarging the time variation of a specific type. So, it is not even as if you have to find out for different ones in different ways.

So, if you have harmonic load, all you need to do is now instead of ω , you have ω_n . So, each mode is $\bar{\omega}_n$ upon ω_n and so that is you know you have to evaluate for each mode if it is periodic well. So, be it has $\bar{\omega}_n$ upon ω_n . So, it has just become much more complicated. That is all, but you know complicated in computation, but not complicated in its basic analytical formulation. It is this level of this

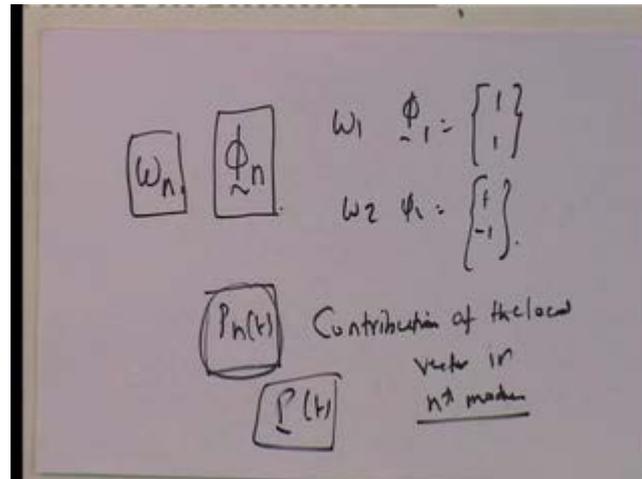
thing is almost as if going from harmonic to periodic. We have to consider ω as n here. You know it is like going from single degree of freedom to multi.

So, it is like going from harmonic to periodic accepting that in periodic, you have infinite here, you have finite. There is nothing else in it and the other point that is there is that I mean if you have pulse load, if you have impulse load, if you have in general arbitrary load, use Duhamel integral, use frequency domain approach for each one. Note that you know there is a complexity. I have not taken an n degree of freedom problem and made it into a single degree of freedom problem. All I have done is, I have taken n degree of freedom problem and broken it in the mode superposition method, broken it up into n single degree of problem.

So, the order of magnitude of the computation goes down one level, but you still have to do n single degree of freedom problems. So, that in a sense is the mode superposition method, and it is equally valid for undamped and damped as long as you have, what. As long as you have your basic concept of proportional damping implies. When proportional damping does implies? Well, typically you know when you have a structure made up of one material; it is by enlarged. We can actually show later on. I am not going to show in this course as it requires an advanced dynamics course where we can actually show that even if it is not made up of the same material, we can innovate. It will not be a ψ here. The ψ becomes just ψ_n big deal. It is still is. I mean I can classically put it in this format. It still does not make the problem more complicated. The problem gets more complicated is when you have what is known is non proportional damping.

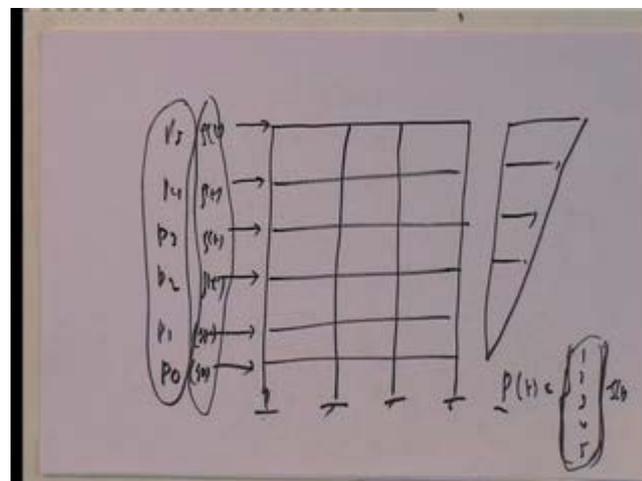
Again in this course, I am not going to tackle. I am just introducing the concept that people have done, have considered the situation where you have non-proportional damping and then you know there are various means of getting it and that is the reason why in a way it is important to get the C matrix. We will see that in the next class, but in essence what we have over here is the following. If you look at this particular problem, the mode superposition technique essentially breaks it down into smaller problems. The other issue that comes in is, can we reduce the problem further and this is something that we see because of the nature of ϕ_n .

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See as you go I have said yesterday, as you go from this phi 1 1 1, the lower mode and phi 2 omega 2, the higher mode. Now, what happens is what P n is? P n is the contribution of the load vector in nth mode. Now, if you look at a load vector P typically, let us look at a kind of a situation. What I mean is let us take wing loading, right.

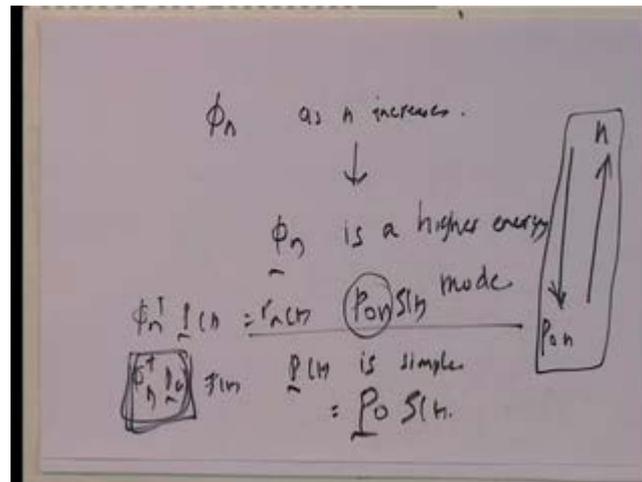
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You have a multi stored building since I am not solving the problem. I can make it as complicated as possible, right and what kind of a load is this being subjected to this, being subjected to wing load. What kind of things that do you have? What is typically done is that you will have a basic. What is this? This is the amplitude and then you have a time variation which is there in very one of them. This is typically how loads are the time variation is independent of this, right.

So, what you have is, how would this be? This would be increased. This would go something like this. This would increase. So, if you look at my P^T vector, it would look like 1, 2, 3, 4, 5 or something other. I am just taking it linearly into ψ of t . So, typical loads are these. If you look at, these are simple kind of loads.

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If you have earth quake, load would be like inertial load, all of those kind that we will complete later, but loads are typically of a form such that if you look at ϕ_n as n increases, what happens is the ϕ_n is higher energy mode. So, if higher energy mode and P^T is typically is simple, if you look at $\phi_n^T P_0$ which is $P_0^T \psi_n$ is given by some vector P into ψ of t . So, this basically becomes $\phi_n^T P_0$ into ψ of t . So, essentially the $\phi_n^T P_0$ which is independent of ψ of t is what. We need to look at. As you get higher energy mode, the mode shape vector also has more and more modes. We will tell you what that means, and this value which essentially is like $P_0^T \psi_n$ is $P_0^T \psi_n$. The amplitude of the load in the n th mode, this actually goes down as you increases n . So, $P_0^T \psi_n$ decreases as you increase n .

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The image shows handwritten notes on a whiteboard. At the top, it says $\vec{v}(0) = \dot{\vec{u}}(0) = \vec{0}$. To the right of this is a box containing $M \ll N$. Below this, a larger box contains $y_n(0) = \dot{y}_n(0) = 0$. At the bottom, it says $\downarrow y_n(t) \text{ as } n \uparrow \downarrow$ and "Neglect the higher modes."

So, what happens is if the higher modes are not being excited because there is no loading and typically, what do you have? Typically you have this kind of a situation. The 0 vector. The 0 vector you can see that Y and 0 is also equal to, this is equal to 0. So, if they are at rest, every mode is at rest because the initial velocity at rest, if it is at rest and it is not being subjected to any load. The higher modes, the Y_n of t as n increases will go down. Y_n will go down. So, therefore, we can neglect the higher modes, and that is actually the beauty of mode superposition because mode superposition, since you can neglect the higher modes, you only consider M modes which are substantially lesser than the n .

So, let us say you have a structure which has thousand degrees of freedom. This is not unusual even in a framed building. You can have thousands of degrees of freedom. So, you take, let us say let us take 1000. So, you take a system which is actually 1000 coupled linear differentiation equations, and you make it into just 5 or 6 single degree of freedom systems because the higher modes are not excited, and this M the number of modes that are excited are typically of the order of 5 or 6 even if it is 1000 degrees of freedom. Now, how you reduced it? You have taken 1000 coupled equations and transformed it into 4, 5, 6 single degree of freedom system problem. That is the huge numerical efficiency that you get from mode superposition method.

So, I shall continue with the dynamic response to single degree of freedom, I mean to multi degree freedom system to various kinds of loads, and you know here I will not look at specific loads like I did dynamic, harmonic, periodic because you see if I bring it to

single degree of freedom, I do not need it to explicitly get it, but what we need to do is, we need to look at some special kinds of loads and see how to get that first. How to get the equations of motion. For example, earth quake. How do I get the equations of motion for an earth quake kind of a situation for a structure?

So, we will do that and once I do that, we will look at you know we still have not talked about, we will only say proportional damping. It works. We have to see how. We can get proportional damping, we will look at damping in a little bit broader sense for multi degree of freedom system problems and finally, we will look at some examples to solve to show you that indeed M is significantly less than n . So, that in essence is what I shall be looking at dynamic response of multi degree of freedom system problems.

Thank you. Bye.