Chemical Principles II
Professor Doctor Arnab Mukherjee
Department of Chemistry
Indian Institute of Science Education and Research Pune
Module 07
Lecture 39
Probability

(Refer Slide Time: 00:16)

So since we said that, that when a system left to itself will go towards the condition of maximum probability we have to talk about what is probability?

(Refer Slide Time: 00:30)

So probability very simply put is number of ways of getting a particular thing divided by the total number of ways.

(Refer Slide Time: 00:38)

So let us say we are talking about probability of x. x is a particular thing, that is the thing we talked about,

(Refer Slide Time: 00:49)

probability of getting a particular thing is number of ways getting that thing. So this is counting number of

(Refer Slide Time: 01:03)

x, or number of getting x divided by total number of ways.

So again this omega denotes number of things, number of ways the

(Refer Slide Time: 01:20)

all possible things can come, all possibilities are sigma.

So these all possibilities are called probability space

(Refer Slide Time: 01:30)

or sample space. So for example,

(Refer Slide Time: 01:36)

so as we mentioned here is the number of ways of getting x and this is total number of ways.

(Refer Slide Time: 01:42)

(Refer Slide Time: 01:43)

So for example if we say that what is the probability of getting a 4 in a dice throw? So here

(Refer Slide Time: 01:52)

we have to see that what are the possibilities that are there in a dice? 1, 2,3,4,5, and 6.

(Refer Slide Time: 02:03)

I have a dice to show you. Here is a dice. As you can see,

and all of you have probably seen the dice but you know it is interesting to see them when

we talk about them.

So this is a dice which has 6 possibilities only.
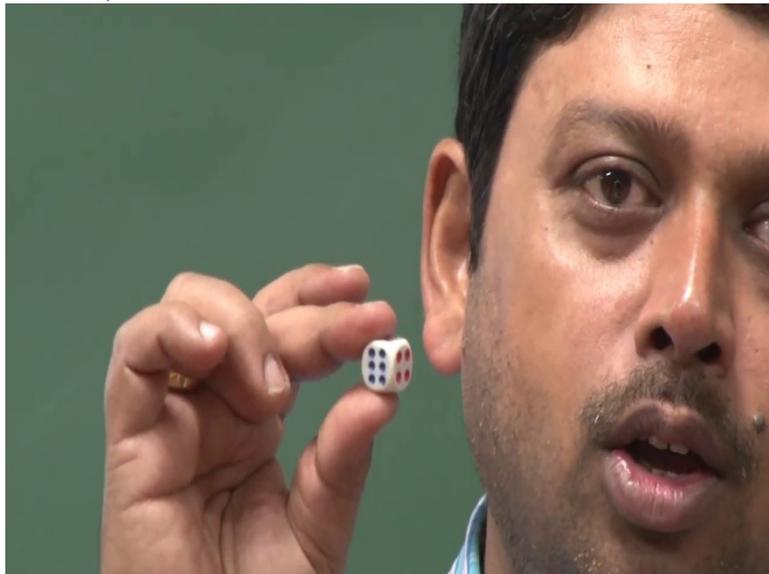
If we throw it you can never get 7 or 8 or any other thing. We can only get the 6. So this is our sample space in which we have to move around. Even if I do a million times I am not going to get a 7 out of this.

I am only going to get either 1 or 2 or 3 or 4 or 5 or 6. That is the constraint that I have, that is all possible outcome in which how many, what is the number of getting a 4?

Only once, one time. I get 4 only one. So x is 4,

(Refer Slide Time: 03:09)

sigma is I have mentioned here, number of ways getting 4 is only one.

(Refer Slide Time: 03:15)

Number of ways getting all of them, let us say, I said you get me either 1 or 2 or 3 or 4 or 5 or 6, there are 6 possibilities, right.

(Refer Slide Time: 03:25)

That is 6 and therefore probability, probability is, as we said that it is the sample size number of getting 6. And that is why probability is 1 by 6.

(Refer Slide Time: 03:39)

So the probability of getting a 4 is 1 by 6. And notice this limit thing, that this is saying that this will not happen if you do just once. If I just throw one or six times I may not get a 4. It will only happen when my, you know sampling is much, much, much more.

So we need this limit, you know omega, this thing, sigma tends to infinity because for the outcome to follow the above probability calculation, we need to try many times.

(Refer Slide Time: 04:19)

So now we are going to do that with a demo coin.

A simpler than dice is a coin, right. The reason is that coin has only 2 possibilities,

(Refer Slide Time: 04:34)

head and tail. So however number of times we do, we can either get a head or get a tail.

And hopefully they are equal. For example I am saying that this is only true when they are of, you know equal probability. That means

they are, both the sides are same, same weighted. If one side is heavier then it will fall on that side and then other side will be seen. So we are not considering that aspect.

So let us say we do this coin toss a few times and see what happens. So again we should get half head and half tail.

So we are going to do two times. We should get one time head and one time tail, right.

So I got a

(Refer Slide Time: 05:25)

tail.

(Refer Slide Time: 05:28)

Ok

second time I got a head.

And I got

a tail again.

And

I got a tail again. So I got 3 tails and 1 head.

If I do a small number of times then I may not get the half, half thing. I will only get half when the number of trials becomes infinite. So we are going to show you that how increasing the number of coin tosses will lead us to half probability.

Because I could do only 3 or 4 times and the statistics is only true when it is done multiple number of times, not 1 or 2 times, right. We are going to just exactly see that. So what we do in computer is that we draw random numbers.

Just like, so we are considering these events as random. Let us say, imagine if push with the same

(Refer Slide Time: 06:32)

force, exactly the same force every time your outcome would be deterministic.

So when you throw that
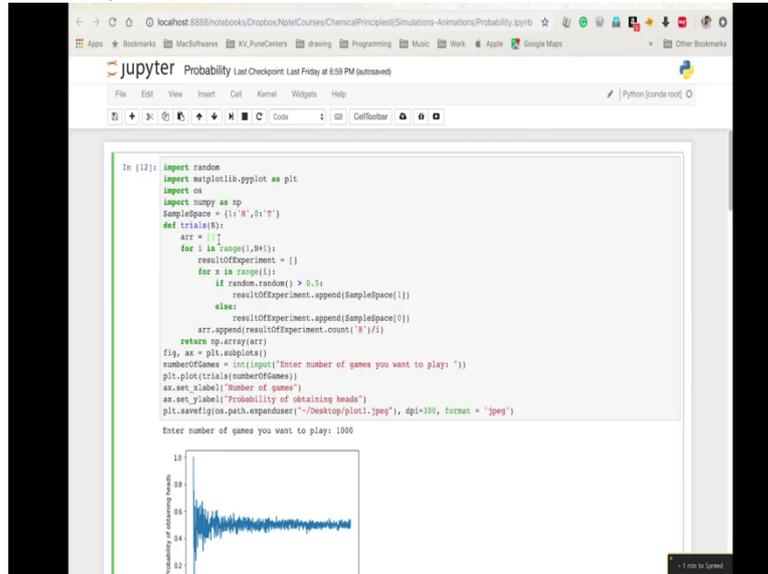
(Refer Slide Time: 06:38)

we assume that every time we are randomly pushing it. Like our force is like kind of random, right.

But in computer we exactly draw random number, almost random number like, it is called pseudorandom number to be more precise but they are random for our own purposes, you can say that. So we can define something is, let us say we are drawing here between 0 and 1.

And something that is below point 5 we say it is head, above point 5 it is tail. So we are totally defining the random number and drawing head and tail. And we can do as many number of times as we want and we will show that probability of getting head is what, as a function of number of trials.

(Refer Slide Time: 07:20)

So you see this is a program that is written in Python. We are going to run the program. And it is asking that enter the number

(Refer Slide Time: 07:32)
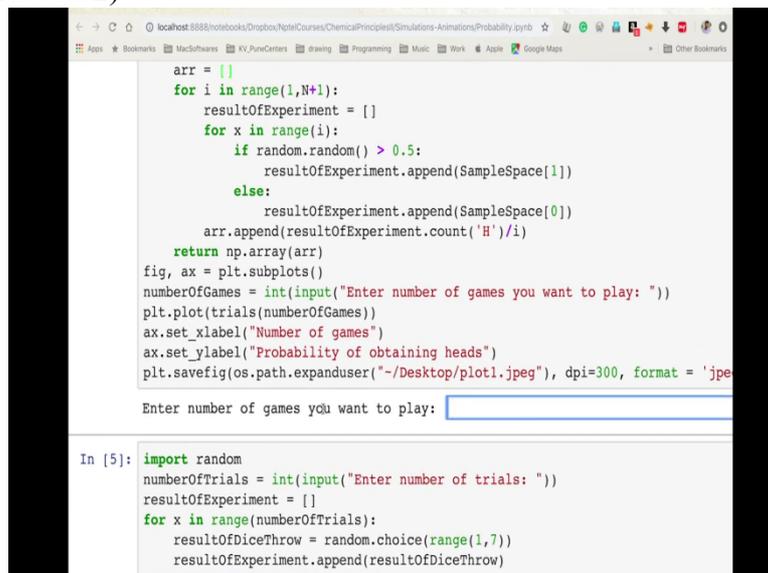
of games you want to play, Ok. We say we want to play only 10.

(Refer Slide Time: 07:39)

(Refer Slide Time: 07:51)

And what you see here, that the probability of obtaining head is increasing. First it was 0, so there was no head. And then increasing to 1 and then slowing coming down, again it is increasing and then coming down, so it is really fluctuating.

So our probabilities are not really half or point 5 with these few trials. Let us some more time. Let us do up to 100.

(Refer Slide Time: 08:22)
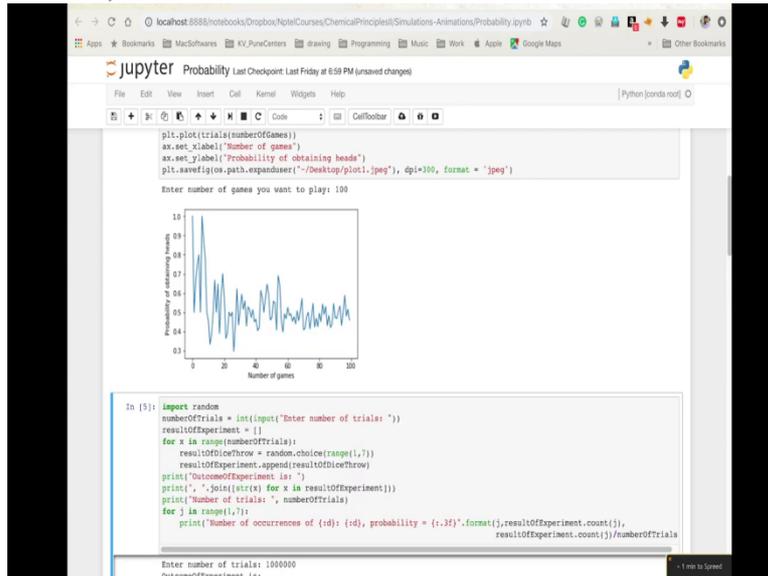
And you see it is still, now point 5 is somewhere here. It is still not there yet, still fluctuating

So let us do now 10000 times

(Refer Slide Time: 08:36)

and you see that the probabilities has converged to the value of point 5. You can see that, right. So this is the point 5 value

(Refer Slide Time: 08:49)

and it as converged to point 5. After oscillating above and below point 5.

So you see this is called law of large number. We have done only 10000 times. We have not done 10 to power 23 times, 10000 you know then you know 10 to the power 23 is almost like million billion billon. Billion is 9, 9 9 18, 18 plus 6 is 10 to power 24.
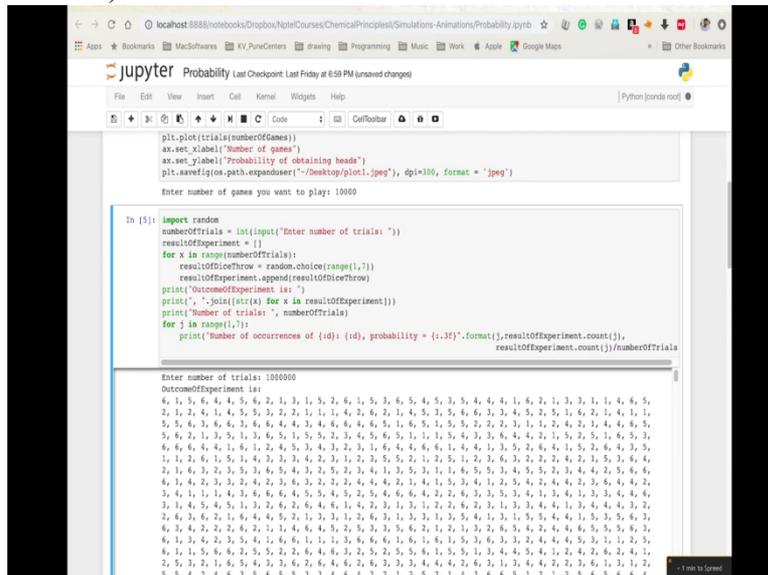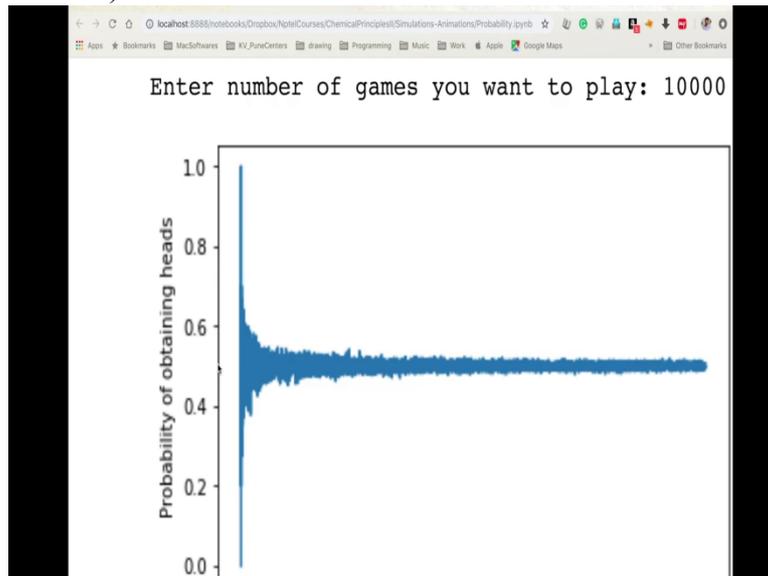
So it will be almost like 10 to power 24. So million billion billion times if we do, you can imagine that how almost accurate that probability will come to.

And that is the number that we deal with when we talk about glass of water or a cylinder of gas. We do not talk about like 10000 particles. Of course we do simulations with 1000 to 10000 particles and still it comes out to be Ok.

So similarly let us do die throw. So now you know that there are 6 possibilities and each of them has 1 by 6 probability, like 1, 2,3,4,5, 6.

If we throw the dice,

let us say for 10 times or 50 times do you think we will get the probabilities of each of them as exactly 1 by 6? We are going to do that the same way as we have done the coin toss using our computer program.

However in this case we have all the 6 possibilities. You can again draw a random number between 1 and 6, an integer random number and we can define each of them as 1, 2,3,4,5, 6 and you can calculate the statistics of it. And that is what we are going to show you in our Python program.

Because only, because we are not really, want to throw a die a million times, it will take time. And secondly we have powerful computer programming tools available in order to achieve the same task in very short time.

So we will do that. See this is again a very small, you know simple Python program and I am running that program. It is asking me number of trials. So I give 6 trials because, you know 6 trials are only required to get all of them as 1 by 6, right. So I give 6 trials and what do I get?

I get 2 times 1 as you can see here, I get 2 times 1,

(Refer Slide Time: 11:11)

I get 0 times 2, so in my 6 trials there was no 2, I get only 1 times 3, and I get only 1 time 4, 1 time 5, 1 time 6. So although these probabilities came fine,

(Refer Slide Time: 11:24)

which is supposed to be point 1 6 7, this and this did not come correctly.

Let us then increase it and make it 100 times.

(Refer Slide Time: 11:38)

And what do I get? Again as you can see, none of the probabilities came out to be right. Because in 100 times I should have around 16, 16 point some times, 16-17 times right

And you can see, number of occurrence of 1 only 8, much, much less than expected; 2, 17, very close to what we expect, and then 3,4, 5 and 6 all came a little

(Refer Slide Time: 12:10)

bit large, little bit more than the number, right. Because we did only 100 times.

Now let us do 10,000 times. 10,000 times is Ballpark number by the way. You can do more than that.

(Refer Slide Time: 12:24)


```
resultOfDiceThrow = random.choice(range(1,7))
    resultOfExperiment.append(resultOfDiceThrow)
print("OutcomeOfExperiment is: ")
print(", ".join([str(x) for x in resultOfExperiment]))
print("Number of trials: ", numberOfTrials)
for j in range(1,7):
    print("Number of occurrences of {:d}: {:d}, probability = {:.3
```

```
Enter number of trials: 10000
OutcomeOfExperiment is:
1, 5, 2, 5, 6, 4, 5, 6, 4, 2, 1, 2, 4, 5, 3, 5, 6, 5, 6, 4, 4, 4,
4, 2, 1, 1, 3, 5, 2, 4, 5, 5, 6, 5, 6, 2, 6, 2, 1, 6, 3, 5, 4, 2,
2, 4, 6, 3, 5, 1, 5, 4, 1, 5, 1, 4, 5, 5, 2, 2, 4, 5, 1, 2, 3, 2,
1, 6, 3, 4, 3, 3, 6, 3, 6, 2, 1, 6, 6, 4, 2, 4, 1, 2, 5, 6, 4, 3,
1, 6, 2, 5, 2, 1, 3, 3, 3, 4, 6, 2, 1, 1, 2, 5, 4, 3, 5, 1, 1, 5,
2, 3, 1, 3, 3, 2, 3, 3, 6, 4, 6, 2, 3, 3, 5, 2, 5, 6, 5, 1, 6, 4,
3, 5, 5, 1, 2, 5, 2, 4, 1, 1, 1, 1, 4, 4, 1, 2, 2, 2, 1, 6, 6, 5,
1, 1, 5, 5, 4, 4, 1, 4, 5, 4, 5, 1, 2, 5, 5, 4, 3, 4, 1, 5, 3, 6,
1, 5, 6, 1, 1, 5, 6, 5, 2, 3, 5, 3, 1, 1, 6, 6, 3, 5, 1, 2, 1, 5,
```

And these are the outcomes that you got. As you can see 1, 5, 2 these are the outcomes that you got. And at the end of our outcome we will have the probabilities which we mention here.

Now you see most of the

(Refer Slide Time: 12:37)


```
        1, 1, 1, 1, 3, 3, 1, 2, 2, 1, 5, 1, 1, 5, 2, 1, 4, 2, 1, 3, 3, 2, 6, 1, 5, 4,
        3, 3, 4, 3, 2, 1, 4, 6, 2, 4, 1, 3, 1, 3, 5, 1, 4, 2, 2, 4, 2, 6, 4, 3, 1, 5,
        2, 1, 2, 4, 4, 6, 5, 4, 4, 5, 1, 6, 5, 6, 6, 3, 4, 1, 3, 5, 4, 4, 6, 4, 1,
        4, 6, 5, 1, 2, 2, 3, 6, 6, 2, 4, 5, 4, 5, 6, 1, 6, 2, 6, 2, 2, 4, 4, 1, 4, 3,
        2, 1, 4, 4, 1, 4, 1, 5, 5, 1, 3, 3, 5, 5, 1, 3, 6, 6, 5, 5, 2, 6, 1, 5, 3, 6,
        4, 4, 6, 4, 5, 5, 3, 2, 3, 2, 3, 2, 2, 2, 4, 1, 6, 1, 2, 5, 5, 5, 6, 3, 5, 3,
        6, 3, 6, 6, 2, 5
Number of trials:  10000
Number of occurrences of 1: 1691, probability = 0.169
Number of occurrences of 2: 1701, probability = 0.170
Number of occurrences of 3: 1634, probability = 0.163
Number of occurrences of 4: 1616, probability = 0.162
Number of occurrences of 5: 1644, probability = 0.164
Number of occurrences of 6: 1714, probability = 0.171
```

```
In [14]: import numpy as np
import os
import matplotlib.pyplot as plt
import random
def trials(N):
    arr=[]
    for i in range(N):
        seq=[1,2,3,4,5,6]
        a=random.choice(seq)
        b=random.choice(seq)
        Y=a+b
```

probabilities are very close to point 1 6 7; point 1 6 9, point 1 7 0 and things like that. Now if you want to be more accurate, so this is nothing but 10000 times right, if I want to be more accurate let us do 100,000, 100,000 times.
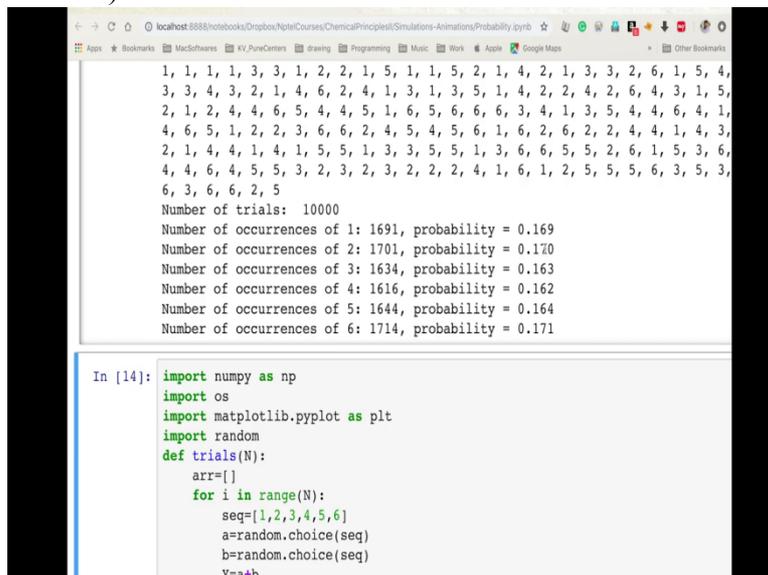
(Refer Slide Time: 12:56)
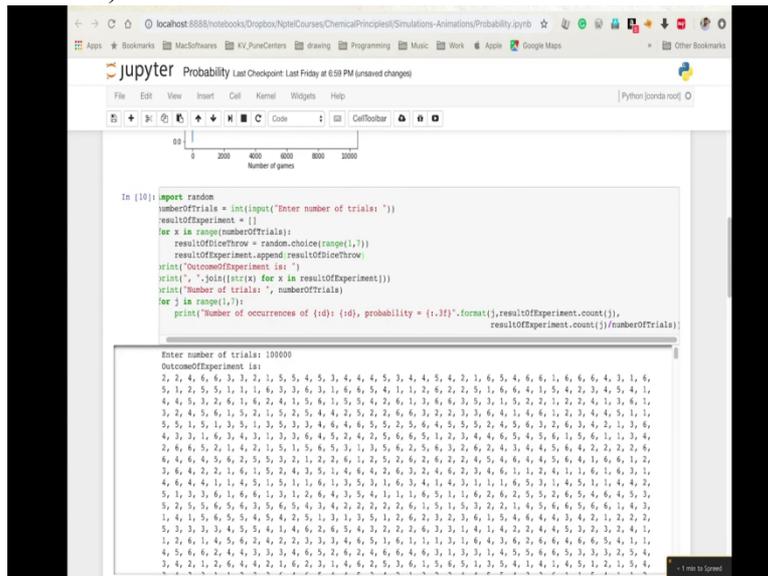
This was superfast and let us see what we got at the end of the program.

You can pull it down. Now you see, with 100,000 we got up to second
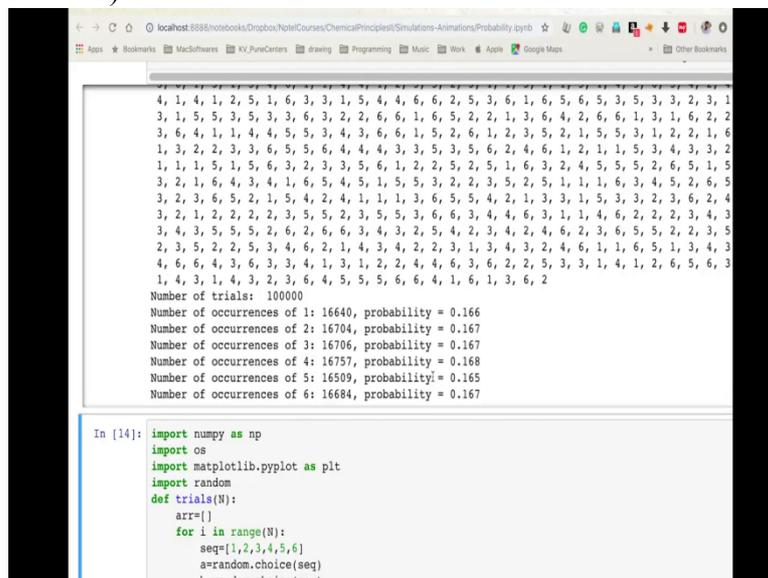
(Refer Slide Time: 13:16)

decimal place correctly; point 1 6 6, 1 6 7, 1 6 7, 1 6 8, 1 6 5, and point 1 6 7. So now we see that only by 100, 000; 100,00 is nothing but 10 to the power 5 only.

We have not reached even a billion thing, we have to go to million billion billion. And you can imagine how accurate those probabilities will be when we talk about molecules, molecular systems, Ok.

So this is the, this is to show you that the statistics will work right when we deal with atoms and molecules for representing the bulk system. It will work out right because it is even working in just 100, 000 trial runs. But it did not when we had very small number of molecules.

So it will be very helpful later on to think that whenever we have doubt on the statistics you imagine that law of large number will play a big role in determining that what will be the outcome of the event.

(Refer Slide Time: 14:18)

Now we will switch back to

(Refer Slide Time: 14:22)

little bit more probability calculations, so we have just saw simple events; simple events meaning either getting 1, 2 and things like that out of a die, head and tail out of the coin throw. Now you can combine and show that how AND probability multiplies.

Remember I told you about multiplication of the microstate just now. We said that if you have W microstates for one system and W for the other system then the combined system will have W into W which is W square.

Where does it come from?

(Refer Slide Time: 14:50)

Combining Probabilities: "AND" probability multiplies

1. What is the probability of getting a "4" in first throw and a "6" in the second throw?

We will be clear here. So what is the probability of getting a 4 in the first throw and, look at this and, I am talking about and probability is not it? Here also there is

(Refer Slide Time: 15:01)

Combining Probabilities: "AND" probability multiplies

1. What is the probability of getting a "4" in first throw and a "6" in the second throw?

an and here.

So what is the probability of getting 4 in the first throw and a 6 in the second throw?

(Refer Slide Time: 15:08)

Combining Probabilities: "AND" probability multiplies

1. What is the probability of getting a "4" in first throw and a "6" in the second throw?

$$P(X \& Y) = P(X)P(Y)$$

It is probability of getting X and Y, is P X

Combining Probabilities: "AND" probability multiplies

1. What is the probability of getting a "4" in first throw and a "6" in the second throw?

$$P(X \& Y) = P(X)P(Y)$$

and we multiply it by P Y.

However again we are assuming that these are independent events. That means getting a 4 in the first throw does not affect anyway the next outcome of the die throw. They are all independent just like coin throw.

So we know the probability of X, right, that is 1 by 6 and the probability of Y is also 1 by 6. Therefore probability is 1 by 36.

Combining Probabilities: "AND" probability multiplies

1. What is the probability of getting a "4" in first throw and a "6" in the second throw?

$$P(X \& Y) = P(X)P(Y)$$

The above is true for independent events where the outcome of the first experiment does not influence the second one. So, the probability P for the above is = 1/6 * 1/6 = 1/36

So similarly what is the probability of getting 2 heads into a row, half into half 1 by 4, 3 heads in a row, half into half into half.

So what is the probability that I will get 10 heads in a row? So that means first toss 10 and second toss 10 and, sorry first toss head and second toss head and third toss head and fourth toss head and fifth toss head, like that up to tenth toss head to be 1 by 2 to the power 10.

So and probabilities multiple. So that is why. What is the probability of getting W microstate for a system and W microstate for the second system is W into W. That is why we had the W square.

(Refer Slide Time: 16:23)

Now what is the probability that we will get a 6 three times in a row? As I said it will be 1 by 6 to the power 3.

(Refer Slide Time: 16:29)

What if the probability is not independent?

So if the probabilities are not independent then the first outcome will affect the second one. However and will still multiply; and will multiply however it will not be the same value. It will not be 1 by 6 like when you had for the dice throw.

We will give an example.

(Refer Slide Time: 16:49)

What is the probability that drawing from a deck of cards we get 2 hearts in a row? Now we know that if you know about the cards there are total 52 cards. Out of which there are 13 hearts, Ok.

(Refer Slide Time: 17:04)

So what is the probability that you will get 2 hearts in a row?

(Refer Slide Time: 17:09)

So you have to get the first heart. What is the probability? It will be 13 by 52.

(Refer Slide Time: 17:18)

For the second heart, I can only select out of 51 cards because there is only 51 cards left, and there is only 12 hearts left

(Refer Slide Time: 17:31)

with that.

(Refer Slide Time: 17:35)

So as I said 13 by 52 multiplied by 12 by 51 giving

(Refer Slide Time: 17:40)

Combining Probabilities: "AND" probability multiplies

1. What is the probability of getting a "4" in first throw and a "6" in the second throw?

$$P(X \& Y) = P(X)P(Y)$$

The above is true for independent events where the outcome of the first experiment does not influence the second one. So, the probability P for the above is = 1/6 * 1/6 = 1/36

2. What is the probability that you will get "6" three times in a row? $P = (1/6)^3 = 1/216$

3. What if the probability is not independent?

What is the probability that drawing from a deck of cards, we get two hearts in a row?
Here, $P = \frac{13}{52} \times \frac{12}{51} = \frac{156}{2652}$

52 cards
13 hearts

13

us 156 by 2652. So here the probabilities are not independent however the multiplication came because we wanted an and condition. So as I said here, probabilities multiply but both probabilities were not

(Refer Slide Time: 17:54)

Combining Probabilities: "OR" probability adds

equal.

Now let us talk about or probability; or probability adds.

(Refer Slide Time: 17:59)

Combining Probabilities: "OR" probability adds

1. What is the probability of getting a "4" or "6" in the throwing of dice?

What is the probability of getting a 4 or 6? See we want 4 or 6. Either of them will be Ok for us. 4, or 6 in the following, in the throwing of a dice.

(Refer Slide Time: 18:11)

Combining Probabilities: "OR" probability adds

1. What is the probability of getting a "4" or "6" in the throwing of dice?

$$P(X \mid Y) = P(X) + P(Y) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

So we put that this sign as an or sign. So x is getting a 4 and y is getting a 6. So it is a x or y, will be added up; P X plus P Y. 1 by 6 plus 1 by 6 is 2 by 6.

And you can understand why. Because let us say in your sample space you have 1,2,3,4, 5 and 6. You want either 4 or 6

whichever is fine. So if you throw you get a 4, or if you get a 6, it is Ok for you.

So it means that we have to add up those probabilities. Because now this, this is 2 out of 3, 2 out of 6 that we are talking about. And that came because each of them has 1 by 6, 1 by 6 probability.

So what is the probability that we get 1 or 2 or 3 or 4 or 5 or 6? 1, because we will get either of one of them. So what is

the probability of tossing 2 coins, that in the tossing of 2 coins either the first one or the second one will be head?

See here also there is or here. Either the first one, so we know tossing the two coin, what will happen? What are the possibiliities? H H, H T, T H and TT, these are all 4 possibilities, total.

So I am saying what is the probability of tossing 2 coins, in tossing 2 coins the first one or the second one, so first one comes head 2 times. Second one comes head 1 time. So there are 3 out of 4 possibilities. How do I get that?

We can get that exactly

(Refer Slide Time: 19:49)

here. So this 2 and this 1 all belong to the condition above. So we say

(Refer Slide Time: 19:54)

that P X or Y, so P X has the possibility 2 by 4, P Y which means head comes later has only possibility so it is 1 by 4 giving us 3 by 4.

(Refer Slide Time: 20:06)

So I can do both the ways. You see,

(Refer Slide Time: 20:08)

so I calculated the individual possibilities, or I count all the possibilities like I did in this case. I counted that, in order to satisfy this above condition how many such events are there. So I am giving a different name for that.

Let us say, I am saying that it is P X prime. P X prime means either

(Refer Slide Time: 20:25)

first one or second one. So there I do not have to add. I straightaway calculate that it is 3 and out of 4 possibilities.

Or I break that into 2 individual events. In one event I say that first one is head. Another one I say that second one is head. And I add those two events probability, events probability and I get this probability. Is this part clear? Right.

(Refer Slide Time: 20:51)

So now combining and and or, that is slightly more complicated.

Combining "AND" and "OR" Probabilities

1. Throw two normal six-sided dice. What is the probability of rolling a "8" (as the sum of the outcome from each dice)?

Throw 2 normal 6-sided dice, what is the probability of rolling a 8 as the outcome of each dice?

Now you see, as I said that with one dice you cannot get 8. However if we throw 2 dice and calculate the sum of the outcome then we can get anywhere from 2 to 12.

Why 2? Because the smallest number is 1, so 2 dice, 1 plus 1 will be 2. So minimum it has to be 2. What is the maximum? Say both of them come 6. 6 by 6 12. So it can be anywhere between 2 and 12.

Now in that I am asking, so what are the outcomes that is possible for 2 dice? So 2,3,4,5,6,7,8,9,10, 11, 12; these, any of these possibilities are there. And I am asking what is the probability of getting

Combining "AND" and "OR" Probabilities

1. Throw two normal six-sided dice. What is the probability of rolling a "8" (as the sum of the outcome from each dice)?

$$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

8.

Now here you see you cannot simply do 8 by 1, 2, 3,4, 5, 6, 7, 8 ,9, 10, 11. 8 by 11 you cannot do. Why? Because there are more number of ways of getting 8. This is only possible if 2 dice are indistinguishable. You do not care about throwing 2 dice and all. You just happen to place those dice.

But here we are talking about first die versus second die; first dice versus second dice. So therefore they are distinguishable dice. And we have to see that how many possible ways we can get the 8. So we can do this 2 ways.

One way is by calculating all possibilities, which means that

Combining "AND" and "OR" Probabilities

1. Throw two normal six-sided dice. What is the probability of rolling a "8" (as the sum of the outcome from each dice)?

$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Two ways:
a. By calculating all the possibilities

(number of ways to roll "8" = 5 { (2,6), (3,5), (4,4), (5,3), (6,2)} [As you can see, we have taken the dice to be distinguishable]

number of ways to roll an 8 is either we roll 2 6, that is first dice is 2 and second dice is 6. Or you roll 3 5 or you roll 4 4 or you roll 5 3 or you roll 6 2. We roll any of these 5, we will get an 8.

So we know all, you know 5 ways of getting a 8

Combining "AND" and "OR" Probabilities

1. Throw two normal six-sided dice. What is the probability of rolling a "8" (as the sum of the outcome from each dice)?

$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Two ways:
a. By calculating all the possibilities

(number of ways to roll "8" = 5 { (2,6), (3,5), (4,4), (5,3), (6,2)} [As you can see, we have taken the dice to be distinguishable]

Total number of outcomes = 6×6 = 36

and what is the total number of outcomes? Total number of outcomes is that my first dice can be any of these 6

(Refer Slide Time: 23:05)

Combining "AND" and "OR" Probabilities

1. Throw two normal six-sided dice. What is the probability of rolling a "8" (as the sum of the outcome from each dice)?

$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Two ways:
a. By calculating all the possibilities

$\{1, 2, 3, 4, 5, 6\}$

(number of ways to roll "8" = 5 { (2,6), (3,5), (4,4), (5,3), (6,2)} [As you can see, we have taken the dice to be distinguishable]

Total number of outcomes = 6×6 = 36

and my second die can be any

(Refer Slide Time: 23:10)

Combining "AND" and "OR" Probabilities

1. Throw two normal six-sided dice. What is the probability of rolling a "8" (as the sum of the outcome from each dice)?

$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Two ways:
a. By calculating all the possibilities

$\{1, 2, 3, 4, 5, 6\}$ $\{1, 2, 3, 4, 5, 6\}$

(number of ways to roll "8" = 5 { (2,6), (3,5), (4,4), (5,3), (6,2)} [As you can see, we have taken the dice to be distinguishable]

Total number of outcomes = 6×6 = 36

of these 6; so which means I can take 1 from here and 1 from there.

I can take 1 from here, 2 from there; 1 from here, 3 from there; 1 from here, 4 from there; 1 from here, 5 from there and 1 from here, 6 from there.

So for 1 I have 6 possibilities. For 2 also I have 6 possibilities. For 3 also I have 6 possibilities. So for 6 of them I have 6 possibilities. Each of them I have 6 possibilities. So 6 into 6, total 36 possibilities are there.

You imagine that you throw a dice and you get any of these possibilities. You throw the second die and you get any of these possibilities. So total possibilities of outcome will be 36. Out of which only 5 will give you 8 as you mentioned here.

So we get 5 by 36.

(Refer Slide Time: 23:59)

But we now will use and and or ways of doing it. Thus the first way is done.

(Refer Slide Time: 24:05)

Now second way.

(Refer Slide Time: 24:06)

By combining the individual probabilities, or probabilities,

(Refer Slide Time: 24:09)

so what you say? That 8 can be obtained from either 2 or 6, or 3 or 5, see I am saying or; or 4 or 4, or 5 or 3 or 6 and 2. I am saying or.

So I am adding up. But when I talk about 2 and 6,

(Refer Slide Time: 24:25)

then I am saying that this is P 2 multiplied by P 6.

(Refer Slide Time: 24:32)

Because I need 2 and 6 in order to get 8. So that is an add probability for each of them whereas between of them there is or probability.

Now we know 2 6, how much, how many ways you can get. 1 by 36 ways, right?

(Refer Slide Time: 24:46)

Because 1 by 6 into 1 by 6.

(Refer Slide Time: 24:50)

So each of them in turn are and probabilities.

(Refer Slide Time: 24:54)

And how much, so I am given the example for that.

(Refer Slide Time: 24:56)

So therefore I can write P 8 as 1 by 6 into 1 by 6 giving us 2 6, this is P 2 6. This is P 3 5, this is P 4 4. This is P 5 3, this is P 6 2.

(Refer Slide Time: 25:12)

Is it clear? So now combining all that we are also getting the same answer, 5 by 36.

This is just two way to show that how to calculate the and and or probabilities, how to calculate in general the probabilities of multiple events which will be definitely useful in later on when we talk about microstates and their distributions and arrangements

And calculations of the two probabilities as we have seen very, very, you know, just now itself that we talked about multiplication. We talked about microstates of two different systems.

So when you combine those microstates of two different systems it is like what is the number of microstates for a dice?

6. What is the microstate for another dice? 6.

Now what is the number of microstate for 2 dice? 36 because 6 possible ways of getting this, 6 possible ways of getting this and when combine that it will become 6 into 6 is 36.

So microstates are multiplicative. Entropies are additive. And that is why the l n function came. So I am just reiterating whatever I said before, after giving this example. So now again we will coming back to the same question that will this 5 by 36 you will observe in 1 or 2 dice? Or not.

So the thing is that, if we, if we really calculate the probabilities of getting a 2, how many possible ways you can get it? How many possible ways you can get the 2? Only one possible ways,
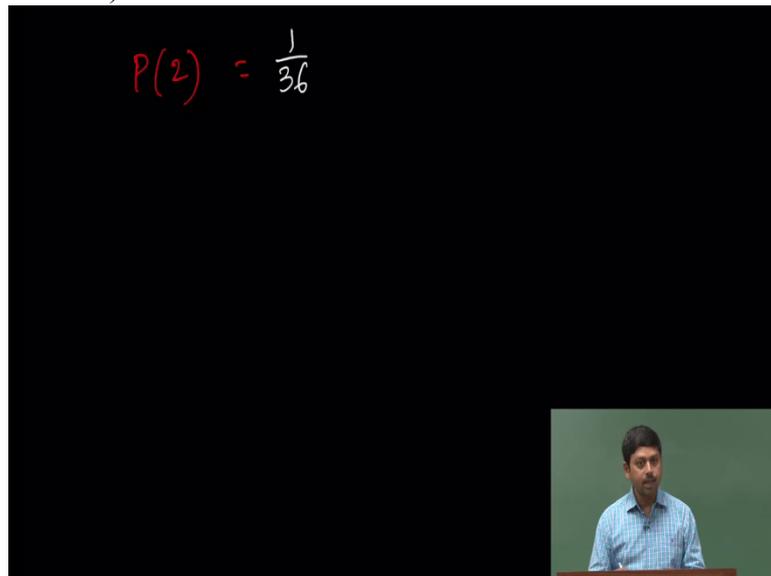
that is correct. How many possible ways you can get 3?

So I will write the possibilities here. 1 1 that gives us
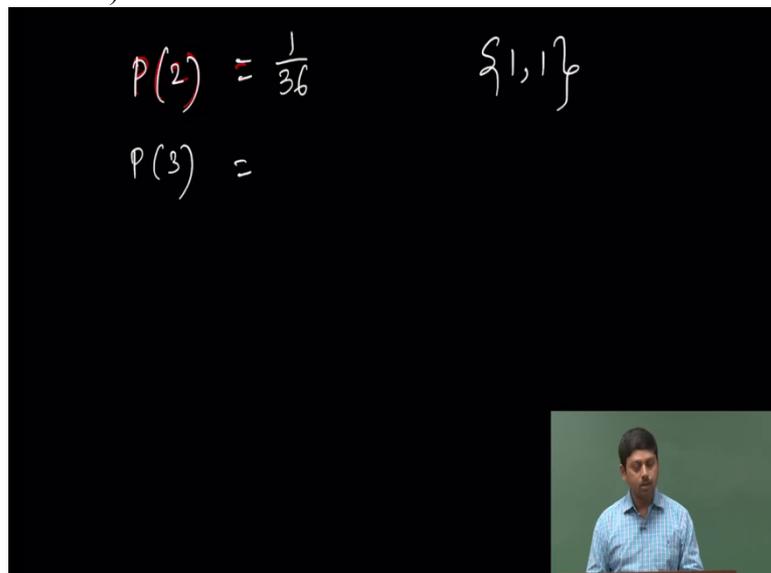
2. How many possible ways we can get 3? 1 2 can give me 3 or 2 1 can give me 3. So 2 possible ways.

(Refer Slide Time: 27:12)

$$P(2) = \frac{1}{36} \qquad \{1,1\}$$

$$P(3) = \frac{2}{36} \qquad \{(1,2), (2,1)\}$$

So how many possible ways we can get 4? 1 3 will give me 4; 2 2; 3 1. Three possible ways.

(Refer Slide Time: 27:28)

$$P(2) = \frac{1}{36} \qquad \{1,1\}$$

$$P(3) = \frac{2}{36} \qquad \{(1,2), (2,1)\}$$

$$P(4) = \frac{3}{36} \qquad \{(1,3), (2,2), (3,1)\}$$

How many possible ways we will get 5? 1 4, 2 3, 3 2; 1 4, 2 3, 3 2 and anything else? 4 1. That is 4 by 36.

(Refer Slide Time: 27:48)

So we see any more possibilities? How many possibilities can we get 6? 1 5, 2 4, 3 3, 4 2, 5 1, 5 possibilities?

(Refer Slide Time: 28:10)

How many ways you can get 6? 7? We will see these 6 possibilities.

(Refer Slide Time: 28:17)

$$P(2) = \frac{1}{36} \qquad \{1,1\}$$

$$P(3) = \frac{2}{36} \qquad \{(1,2),(2,1)\}$$

$$P(4) = \frac{3}{36} \qquad \{(1,3),(2,2),(3,1)\}$$

$$P(5) = \frac{4}{36} \qquad \{(1,4),(2,3),(3,2),(4,1)\}$$

$$P(6) = \frac{5}{36} \qquad \{(1,5),(2,4),(3,3),(4,2),$$
$$\qquad\qquad\qquad\qquad (5,1)\}$$

$$P(7) = \frac{6}{36}$$

I will not do that, and you can do that again. This is 5 possibilities.

(Refer Slide Time: 28:21)

$$P(2) = \frac{1}{36} \qquad \{1,1\}$$

$$P(3) = \frac{2}{36} \qquad \{(1,2),(2,1)\}$$

$$P(4) = \frac{3}{36} \qquad \{(1,3),(2,2),(3,1)\}$$

$$P(5) = \frac{4}{36} \qquad \{(1,4),(2,3),(3,2),(4,1)\}$$

$$P(6) = \frac{5}{36} \qquad \{(1,5),(2,4),(3,3),(4,2),$$
$$\qquad\qquad\qquad\qquad (5,1)\}$$

$$P(7) = \frac{6}{36}$$

$$P(8) = \frac{5}{36}$$

I will just remove this side. And 9 will be 4,

(Refer Slide Time: 28:33)

10 will be 3,

(Refer Slide Time: 28:37)

11 will be 2

(Refer Slide Time: 28:41)

and 12 will be again only 1 way.

So now if I

(Refer Slide Time: 28:45)

plot it, that probability, I will plot it, I will get, this is 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Approximately I am plotting it.

(Refer Slide Time: 29:16)
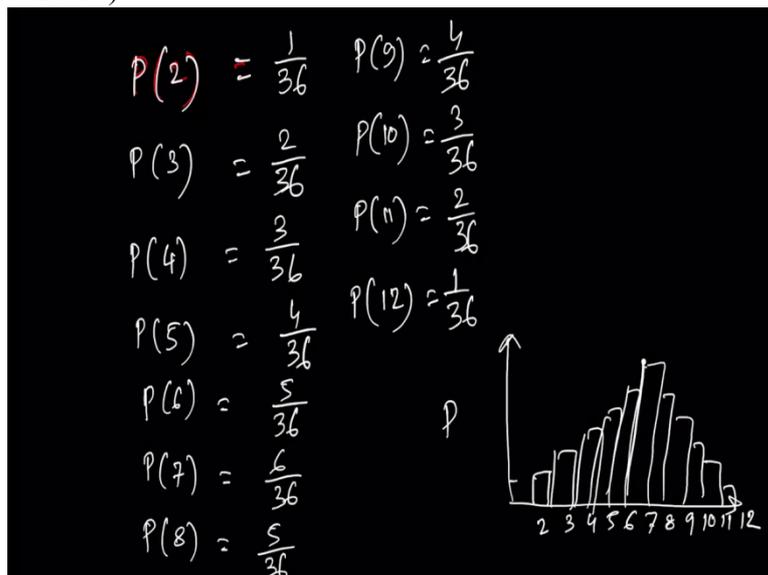
So the peak will be maximum for 7, I will just do this once more.

It will be 2, 3,4,5,6,7,8,9, 10, 11, and 12. So this is for 6 having

(Refer Slide Time: 29:38)
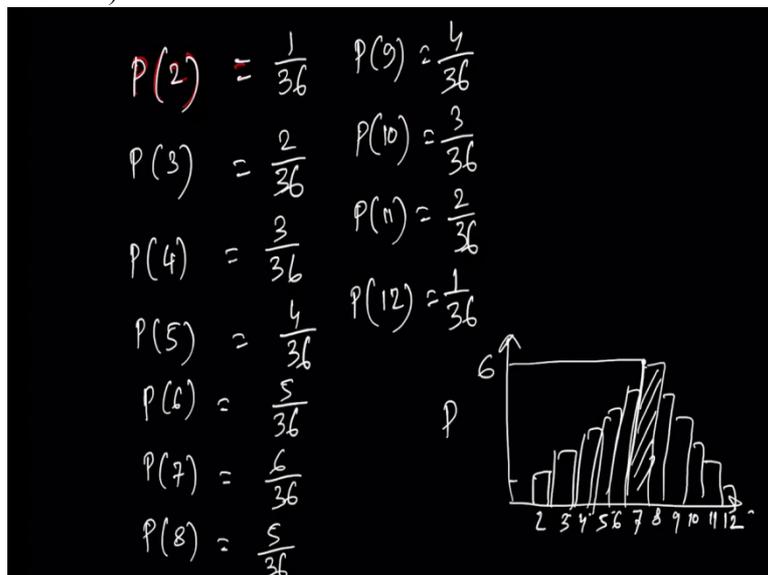
maximum probability of, like 6 times it will occur.

So 6 by 36 is 1 by 6, point 1 6 7 will be the maximum probability, it is symmetric along that. So this is typically called probability mass function, probability mass function in which we count the probability

(Refer Slide Time: 29:57)
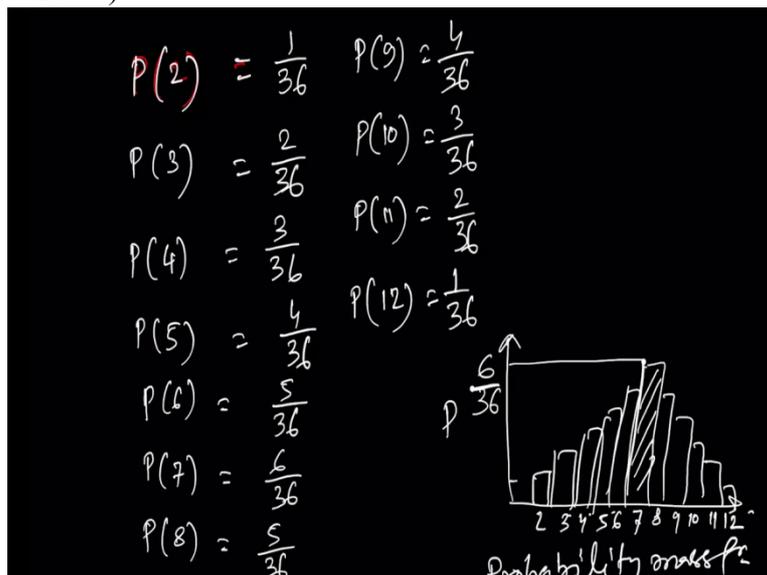
of all possible events.

So what is the most probable situation? 7. That means if roll it many, many, many, many, many times you are going to see 7 more, 7 as outcome which is more in number than 2, Ok and we are going to do the experiment here to show you exactly that.

(Refer Slide Time: 30:22)

So this is again a program that is there. And let us run the program. And put, how many trials we need? At least 10-12 trials we need, right. So let us do it for 50. Just 50.

So it gave us

(Refer Slide Time: 30:38)
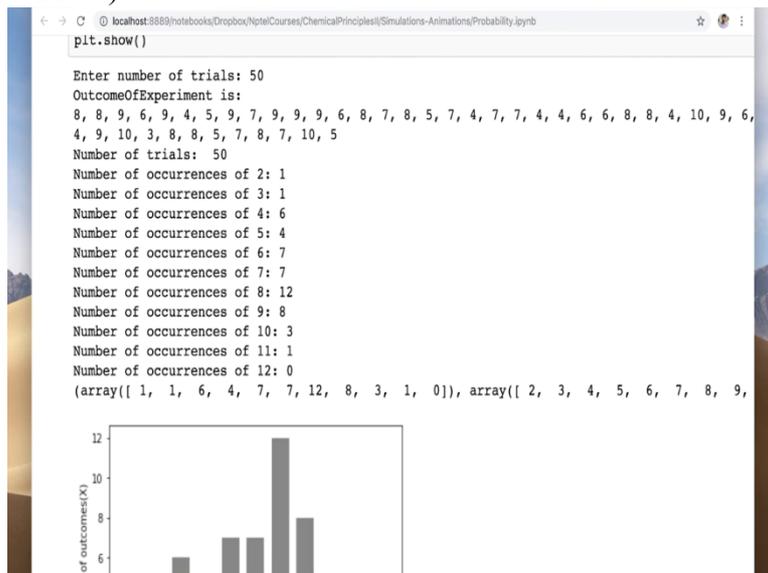
the probability of 2, 3, 4, 5, 6, the occurrences it gave. So probability will be just these occurrences divided by 50, Ok and these probabilities are plotted here.

(Refer Slide Time: 30:50)
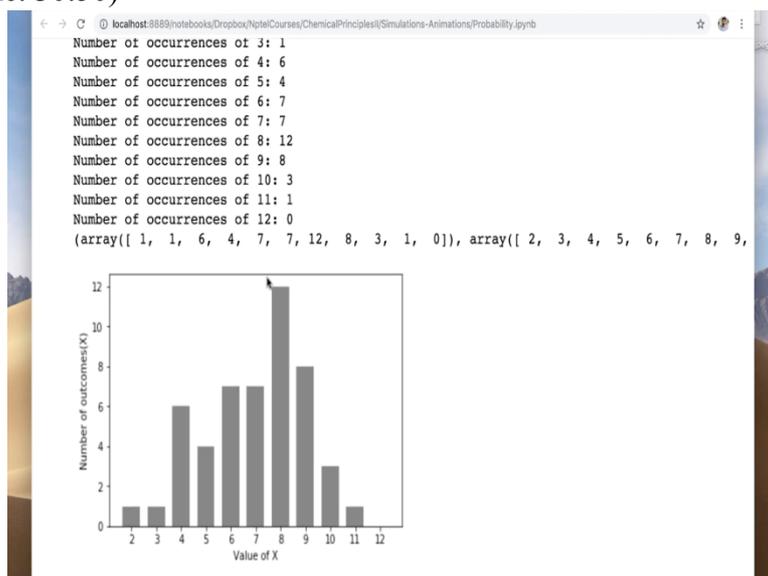
Does it look like the one that I have drawn just now? Does it look like the one

(Refer Slide Time: 30:57)
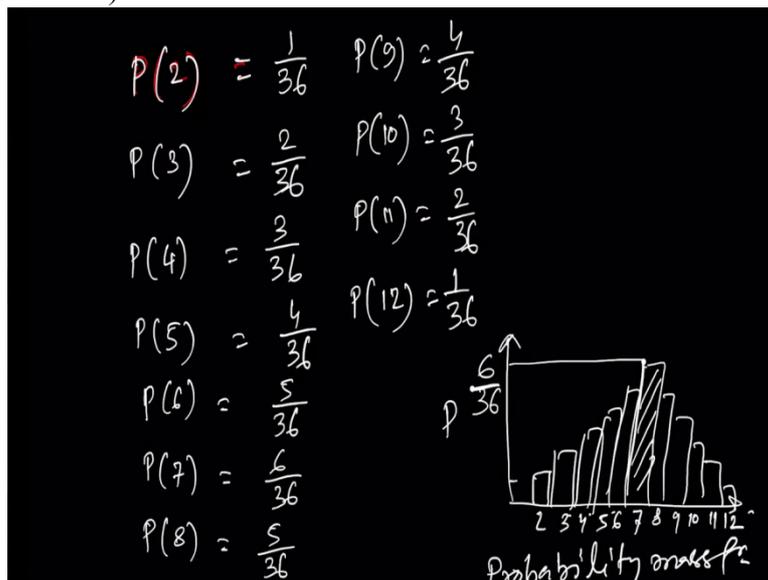
that I show here? Does it look like the one that I am showing here? No, right. Because we have done only 50

(Refer Slide Time: 31:06)
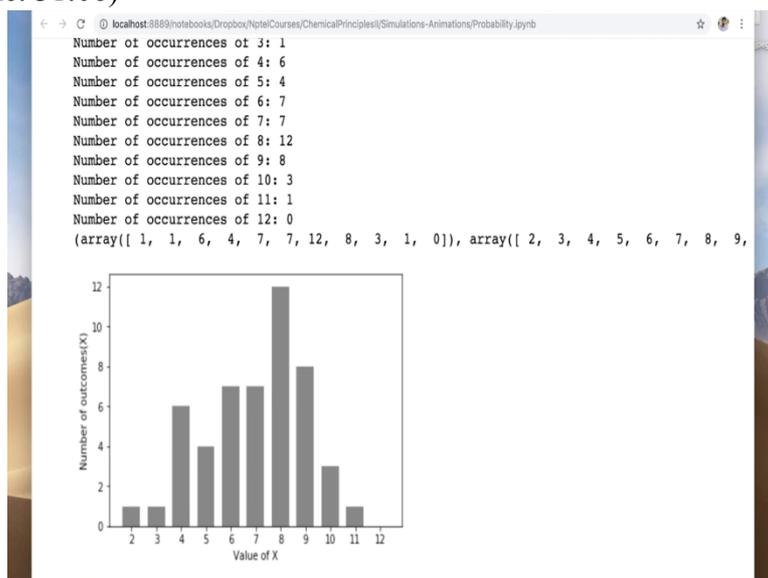
times. Let us now do much more than that.

Shall I do 1000 times? Ok,

(Refer Slide Time: 31:14)

1000 times,

(Refer Slide Time: 31:16)

It is beginning to look like that, beginning; 1000 times. So let us do, there are how many, 11 outcomes, right, this is 2,3, 4, 5,6, 7, 8, 9, 10, 11, 12 no this is 2,3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Yeah there are 11 outcomes, right. So let us do 11000. So that we expect somewhat (()) (31:53)

(Refer Slide Time: 31:45)

(Refer Slide Time: 31:48)

I ran the program now

(Refer Slide Time: 31:59)

and let us calculate the probability. I will reduce a little bit and then I will increase it,

(Refer Slide Time: 32:11)
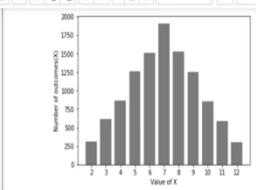
and you see the number of occurrences has been given

(Refer Slide Time: 32:18)
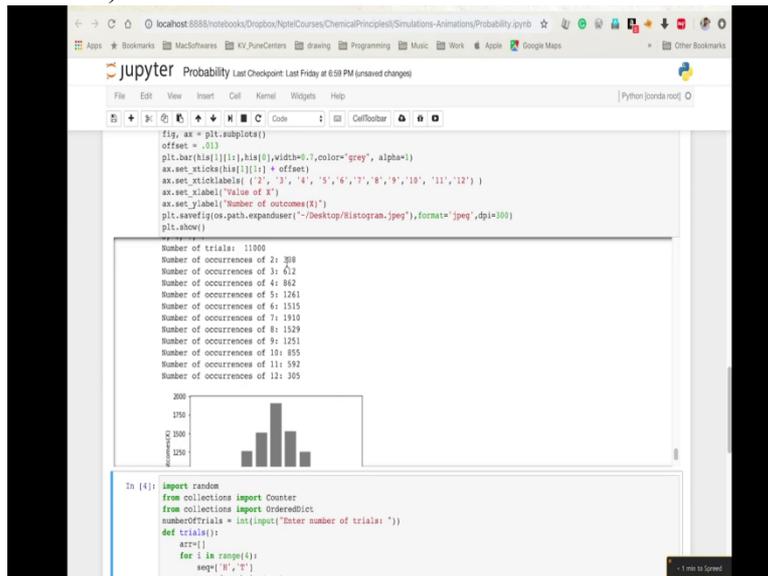
here and as you can see

(Refer Slide Time: 32:20)
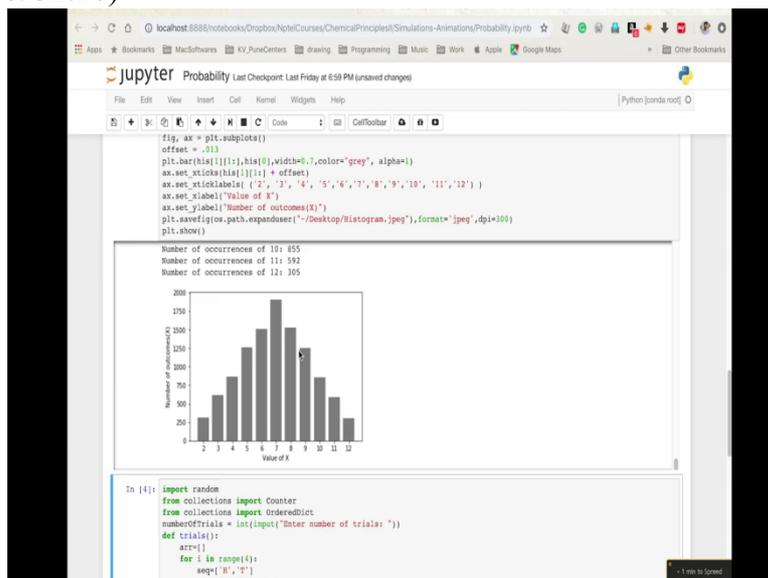
the 7 now has become maximum.

And if you want to calculate the probability of that you can calculate that. So 7 has come 1 9 1 0 times, 1 9 1 0 divided by 11000, is point 1 7 3.

(Refer Slide Time: 32:39)
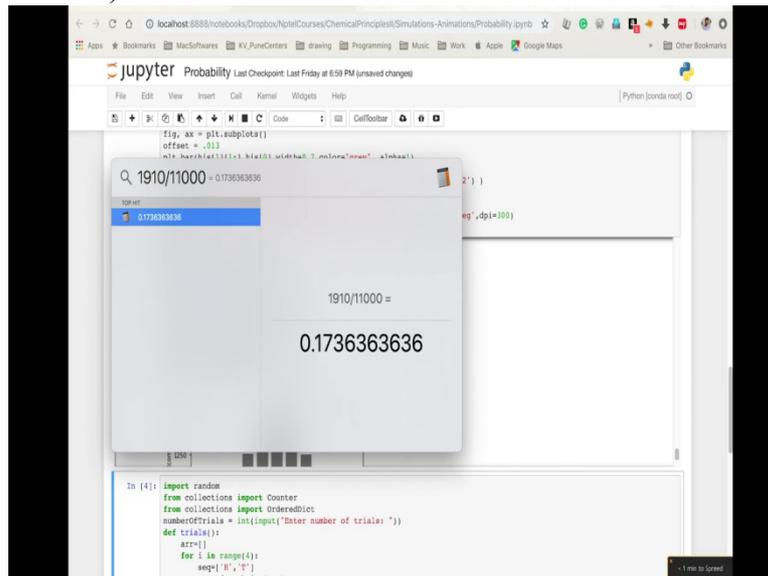
We are expecting point 1 6 7. So although it has become symmetric, it is still not perfect yet. Ok.

So let us, and what is the probability of 2, is 1 by 36. So let us see. 308 divided by 11000 is

(Refer Slide Time: 33:00)
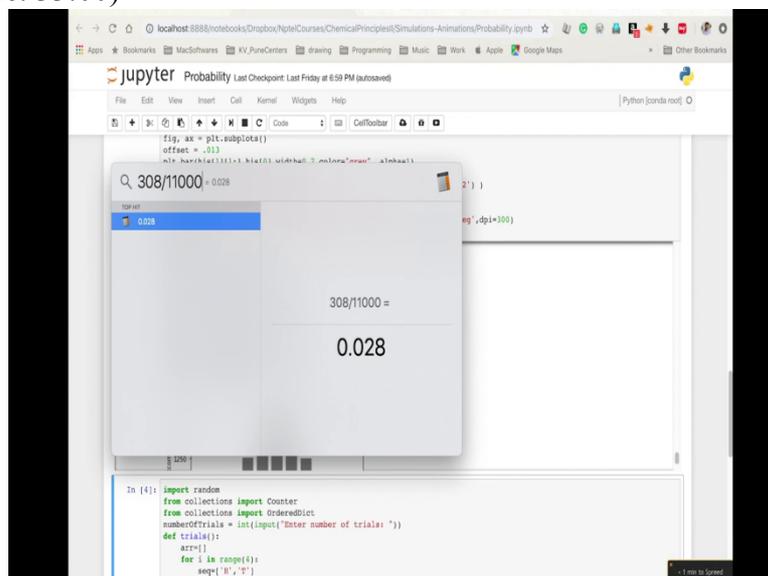
point 0 2 8 and 1 by 36 means

(Refer Slide Time: 33:05)
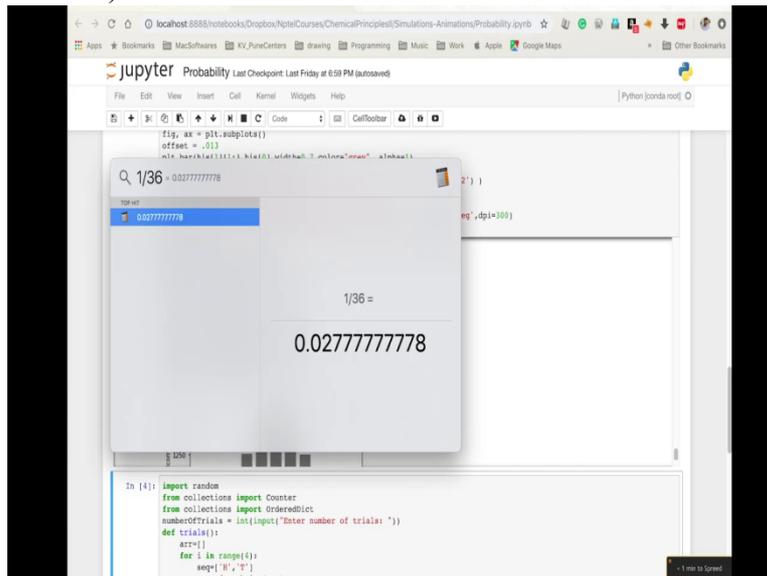
point 0 2 7 7 7 means point 0 2 8. So 2 has come correctly but not the 7.

So in order to get that we have to run some more times, so instead of 11000 I will put 100, 000.

(Refer Slide Time: 33:19)
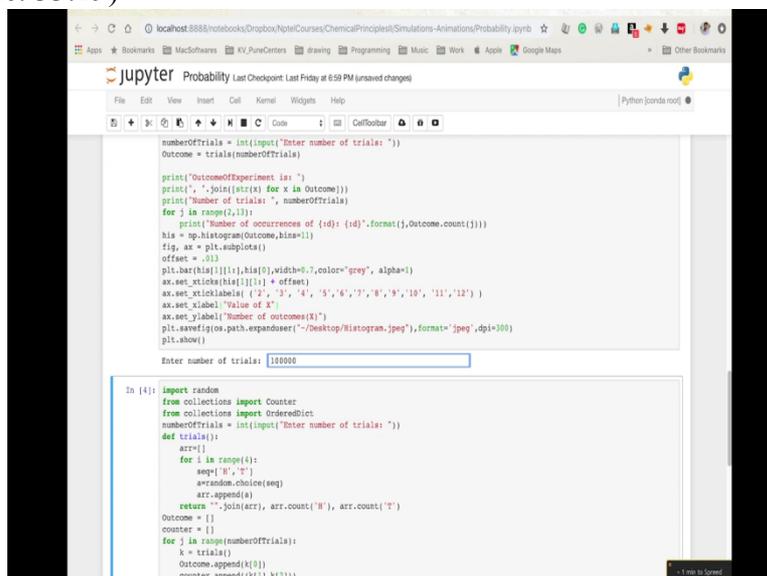
Just like we did before. And I got that, 100,000 result and

(Refer Slide Time: 33:24)
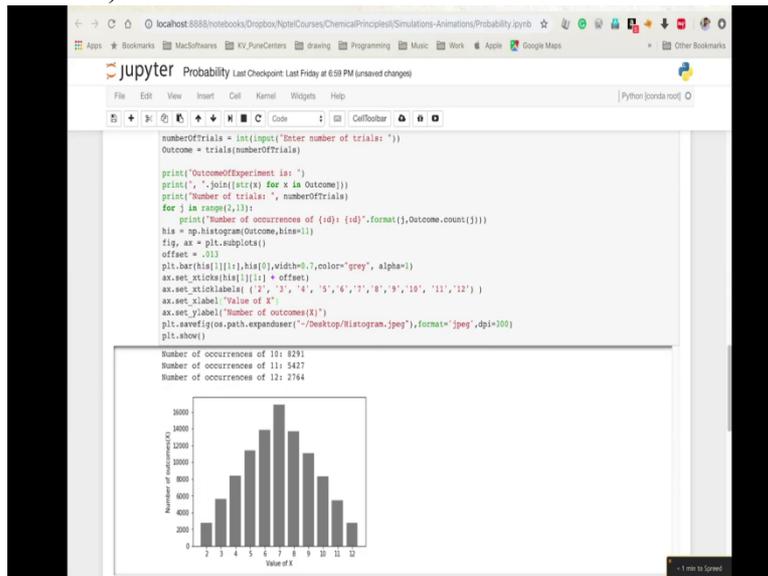
if I go down I can show you the values.

(Refer Slide Time: 33:27)
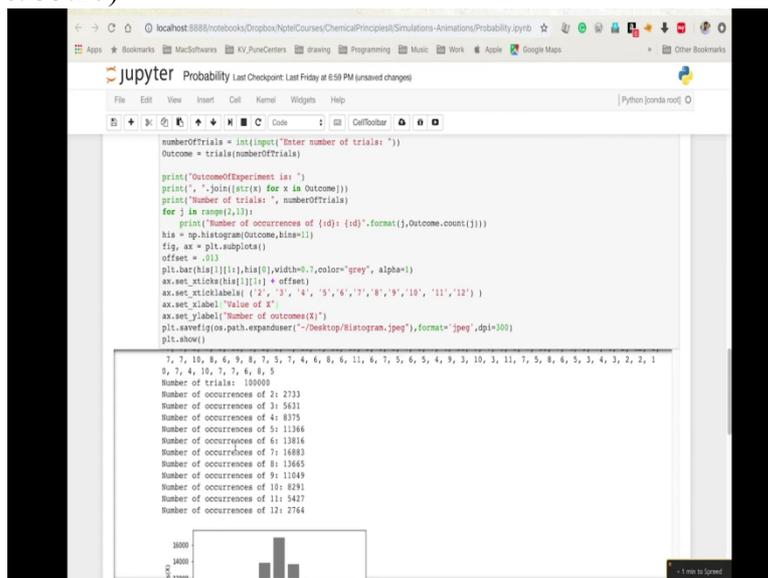
So 7 came 16883

(Refer Slide Time: 33:31)
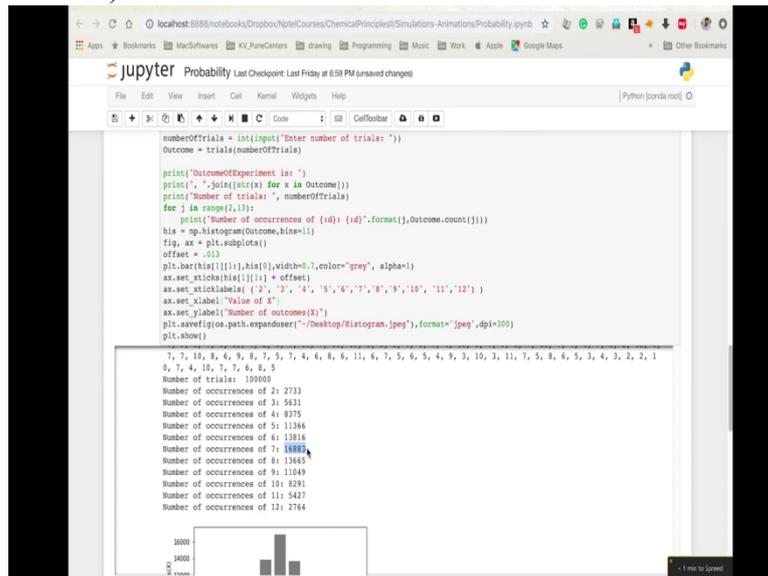
out of 100,000.

So what is the probability? Point 1 6 8m we were expecting point 1 6 7, accurate to almost you know third decimal place. So if I do instead of 100,000 1 million we are going to get more accurate result.

And as you can see it is very, very

(Refer Slide Time: 33:54)
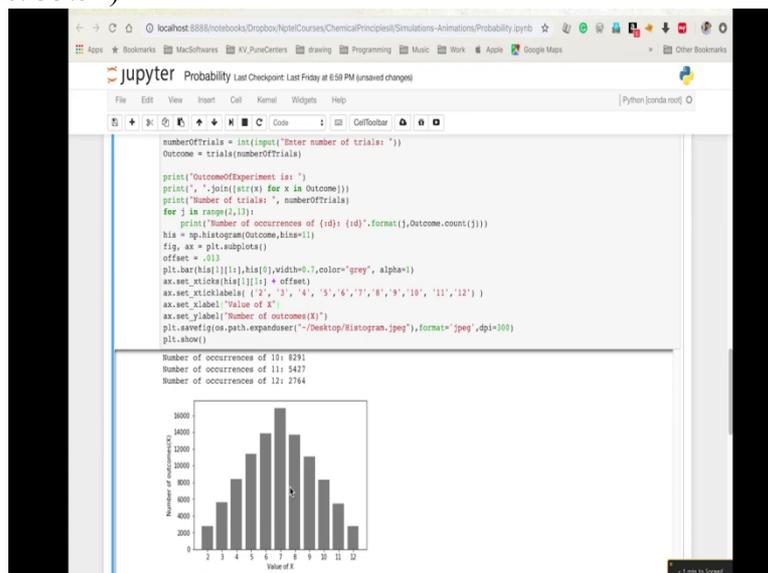
symmetric looking, you know distributions. So again law of large numbers took hold and we are getting the desired,

(Refer Slide Time: 34:08)
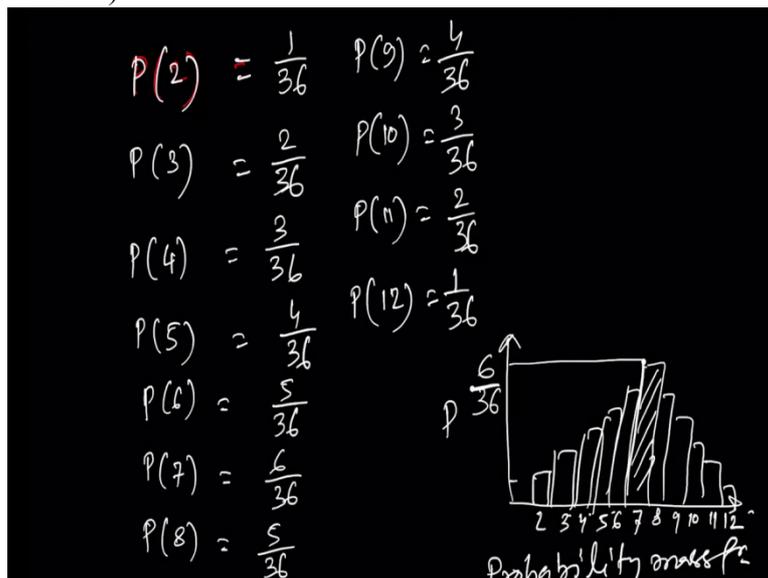
and we are getting the desired outcome from the throw of 2 die, as the way expected. But only thing you remember that this one, is, we can call as most probable distribution.

(Refer Slide Time: 34:32)
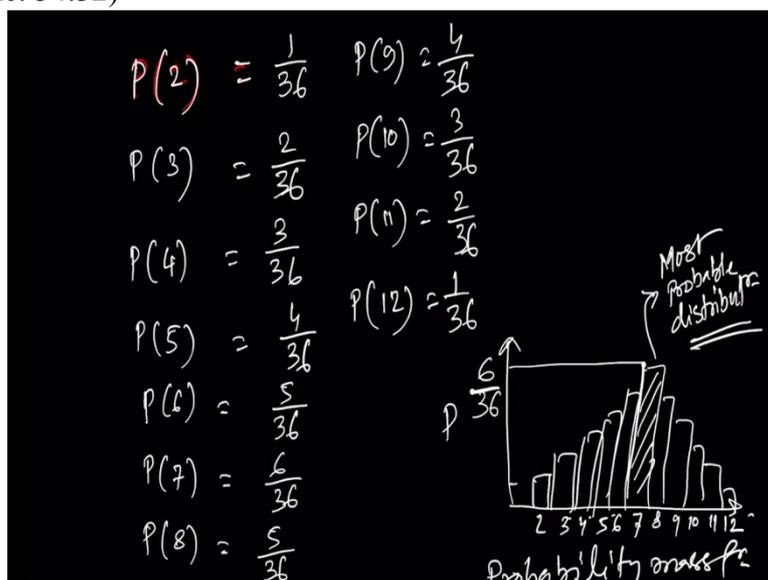
So all those distributions, that means getting 2, all that has some probability, right. But getting a 7 is most probable, right. So that is what we got from this particular

(Refer Slide Time: 34:46)

exercise. Now

(Refer Slide Time: 34:48)

let us say I do one more and and or probabilities combined.

So what is the probability that exactly one of the two top cards of a standard well-shuffled deck of cards is a heart? So now we are not saying both of them have to be heart. We say one of that. We will say one of the two. I can do how many ways?

So if my first card is a heart, the second should not be a heart. My first card can be heart by how many ways, 13 by 52 ways and

(Refer Slide Time: 35:20)

Combining "AND" and "OR" Probabilities $\frac{13}{52} \times$

1. What is the probability that exactly one of the top two cards of a standard well-shuffled deck of cards is a heart?

second card should not be a heart, how many cards are not heart? 39. So 39 by 51

(Refer Slide Time: 35:29)

Combining "AND" and "OR" Probabilities $\frac{13}{52} \times \frac{39}{51}$

is one way of getting. I have to use or probability.

So my first card should be a heart, second card should not be a heart. Or I can say my first card should not be heart, and second card should be a heart. So first card should not be a heart is 39 by 52. Second card should be heart is 13 by 51.

(Refer Slide Time: 35:52)

Combining "AND" and "OR" Probabilities $\quad \frac{13}{52} \times \frac{39}{51} + \frac{39}{52} \times \frac{13}{51}$

Now addition of that will give us the desired number that exactly one heart of the two. So I have done that. The first is a heart. Second is not a heart.

(Refer Slide Time: 36:02)

Combining "AND" and "OR" Probabilities $\quad \frac{13}{52} \times \frac{39}{51} + \frac{39}{52} \times \frac{13}{51}$

1. What is the probability that exactly one of the top two cards of a standard well-shuffled deck of cards is a heart?

→ X= The first is a heart and second is not a heart → $P(X) = \frac{13}{52} \times \frac{39}{51} = \frac{507}{2652}$

While first is not a heart,

(Refer Slide Time: 36:05)

second is a heart.

Now I have X and Y and I can

(Refer Slide Time: 36:09)

combine that any of these, either of these two events will give me the, the desired result, is addition of the two. And this is giving us this.

We can also say that using a not probability. What is a not probability? That I want either of the cards to be heart, right. So I want exactly one out of the two cards should be heart.

So I can say that what is the probability that I get either both of them as hearts or none of them as hearts. If I subtract these two probabilities, then I will get either of them as a heart, you understand?

So to calculate the probability of both of them having a heart, so what is the probability that neither, OK and that we can also get that? Ok so, you understand, so.

Exactly one of them should be heart. So if I get two of them heart, it is not helping. If I get none of them are heart it is not helping. So if I subtract their probability that is only one possibility remains. That is either one of them or exactly one of them is a heart.

(Refer Slide Time: 37:09)

Combining "AND" and "OR" Probabilities $\quad \frac{13}{52} \times \frac{39}{51} + \frac{39}{52} \times \frac{13}{51}$

1. What is the probability that exactly one of the top two cards of a standard well-shuffled deck of cards is a heart?

→ X= The first is a heart and second is not a heart → $P(X) = \frac{13}{52} \times \frac{39}{51} = \frac{507}{2652}$

→ Y= The first is not a heart and second is a heart → $P(Y) = \frac{39}{52} \times \frac{13}{51} = \frac{507}{2652}$

→ Therefore, the actual probability is $P(X) + P(Y) = \frac{1014}{2652}$

2. What is the probability that neither of the two cards is a heart? ("NOT X" is 1 – P(X))

16

Now second one if I say what is the probability that neither of the two cards are heart, neither of the two cards. Which means, again I can do it in two different ways.

I can say that first card is not a heart, is 39 by 51 ways,

second card sorry 52 card is not a is not a heart, ways. Second heart by 38

by 51 ways, because there are 39 not hearts. I am picking any of them. Then I am left with 38 not hearts out of 51 cards, I am taking 13

So this is the probability that neither of the two cards is a heart, Ok.

(Refer Slide Time: 37:46)

But I can do that again with a not probability. Neither of the two cards I can say that 1 minus either one card is a heart or both card is a heart.

(Refer Slide Time: 37:58)

So this is the first one that I did. Second one,

(Refer Slide Time: 38:02)

Combining "AND" and "OR" Probabilities $\frac{13}{52} \times \frac{39}{51} + \frac{39}{52} \times \frac{13}{51}$

1. What is the probability that exactly one of the top two cards of a standard well-shuffled deck of cards is a heart?

→ X= The first is a heart and second is not a heart → $P(X) = \frac{13}{52} \times \frac{39}{51} = \frac{507}{2652}$

→ Y= The first is not a heart and second is a heart → $P(Y) = \frac{39}{52} \times \frac{13}{51} = \frac{507}{2652}$

→ Therefore, the actual probability is $P(X) + P(Y) = \frac{1014}{2652}$

2. What is the probability that neither of the two cards is a heart? ("NOT X" is 1 – P(X)) $\frac{39}{52} \times \frac{38}{51}$

→ X= Neither card is a heart → $P(X) = \frac{39}{52} \times \frac{38}{51} = \frac{1482}{2652}$

→ P(X)=1–(both cards is heart + any one card is a heart)=$1 - \frac{1014}{2652} - \frac{156}{2652} = \frac{1482}{2652}$

16

both card is a heart or any one card is a heart. So any one card is a heart we have already got on the top,

(Refer Slide Time: 38:07)

Combining "AND" and "OR" Probabilities $\frac{13}{52} \times \frac{39}{51} + \frac{39}{52} \times \frac{13}{51}$

1. What is the probability that exactly one of the top two cards of a standard well-shuffled deck of cards is a heart?

→ X= The first is a heart and second is not a heart → $P(X) = \frac{13}{52} \times \frac{39}{51} = \frac{507}{2652}$

→ Y= The first is not a heart and second is a heart → $P(Y) = \frac{39}{52} \times \frac{13}{51} = \frac{507}{2652}$

→ Therefore, the actual probability is $P(X) + P(Y) = \frac{1014}{2652}$

2. What is the probability that neither of the two cards is a heart? ("NOT X" is 1 – P(X)) $\frac{39}{52} \times \frac{38}{51}$

→ X= Neither card is a heart → $P(X) = \frac{39}{52} \times \frac{38}{51} = \frac{1482}{2652}$

→ P(X)=1–(both cards is heart + any one card is a heart)=$1 - \frac{1014}{2652} - \frac{156}{2652} = \frac{1482}{2652}$

16

right. And both card is a heart also we have calculated before. So when we subtract that

Combining "AND" and "OR" Probabilities $\frac{13}{52} \times \frac{39}{51} + \frac{39}{52} \times \frac{13}{51}$

1. What is the probability that exactly one of the top two cards of a standard well-shuffled deck of cards is a heart?

→ X= The first is a heart and second is not a heart → $P(X) = \frac{13}{52} \times \frac{39}{51} = \frac{507}{2652}$

→ Y= The first is not a heart and second is a heart ? $P(Y) = \frac{39}{52} \times \frac{13}{51} = \frac{507}{2652}$

→ Therefore, the actual probability is $P(X) + P(Y) = \frac{1014}{2652}$

2. What is the probability that neither of the two cards is a heart? ("NOT X" is 1 – P(X)) $\frac{39}{52} \times \frac{38}{51}$

→ X= Neither card is a heart → $P(X) = \frac{39}{52} \times \frac{38}{51} = \frac{1482}{2652}$

→ $P(X)=1-(\text{both cards is heart} + \text{any one card is a heart})=1- \frac{1014}{2652} - \frac{156}{2652} = \frac{1482}{2652}$

16

we get the same number as before.

So what I am saying is that there are different approaches to get the probabilities. Once you think about the event in mind, then you will be able to get to that and now that you know the probabilities we are going to see that what will be the probabilities in certain distributions, meaning when we distribute.

Let us say, distribute some chocolates to people, or we distribute some balls to some holes. Or we distribute some energy to particles, what kind of possibilities will arise and what will be the probability of each of those possibilities?

Again our aim is to find out which possibilities are more probable. Just like you saw the two dice throw the probability of, or the possibilities of getting 2 and, possibilities of getting 7 is more probable. Similarly when we distribute the thing, we will see which is more probable and which is not.