Electrochemical Impedance Spectroscopy
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Lecture – 09
Multi-sine, Odd Harmonic, Non-Harmonics, Crest Factor, Spectral Leakage

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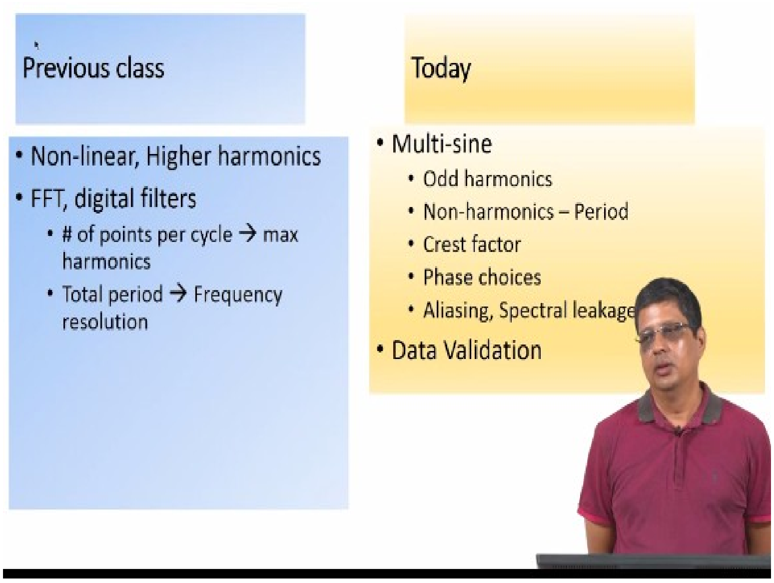

In the previous section, we saw that if you have a nonlinear relationship between current and potential. Also we saw that, when you apply a sinusoidal potential, you would get higher harmonics also. You will get the response at fundamental and you will get response at higher harmonics. Then we moved on to FFT and digital filter; a particular filter called 'freqz' in Matlab and with few examples we saw that if we have more number of points per cycle, you can get higher harmonics and if you have longer duration, you will get higher resolution. Therefore, frequency resolution will be better if you have a longer duration of data acquisition.

In this section, we will look at multi-sine i.e. when you add multiple sine waves together and apply it, you can get the response, and you can do FFT and get the impedance at multiple frequency simultaneously. However, we also need to the pitfalls in doing this or the aspects that one needs to know before applying.

There are different choices; one is to apply sine waves with harmonics i.e. I can apply sine wave at 1 hertz, 2 hertz, 5 hertz, 7 hertz and combine them together and apply as a one wave.

I can say 1, 3, 5, 7, these are odd harmonics and I can mix in any way I want. Another choice is to mix waves with different frequencies where the frequencies are not harmonics. So I can say 1, 1.5, 4.75, whatever number I want and I can choose those frequencies, create waves and apply. In addition, I can give different amplitudes for each wave. I can also give the phase that starting phase to be 0 or something. And each one of this has an implication. When you apply a multi-sine using a commercial software, by and large you will not know what it does. At the best, you would know, it is applying at these frequencies. Beyond this, you would not know. However, you should know at least what the implications are. Therefore, I want to show you what happens when you use harmonics; odd harmonics is one choice that is proposed by few and it is used at least in one commercial machine.

Non-harmonics are also used. Non harmonic means, they are not integer multiples of the base frequency and crest factor is a parameter that one should know. It decides whether nonlinear effect may manifest or may show up in the result or may not show up in the result and when you use non-harmonics, there is a problem. You will have some problem in the measured values; you will have some problem in analyzing the measured values and extracting the impedance correctly at those frequencies. So I want to discuss those also here and if we have time in this section, we will start with data validation. Once you get impedance data you have frequency, magnitude and phase. This is one way. You may have frequency, real and imaginary part. You will have 3 columns in say in Microsoft Excel. If you have a data set it is possible to do some consistency check, to know whether this data has come from a clean system. What do we mean by a clean system? We will come to that a little later. We will also see what the methods of doing this data validation are.

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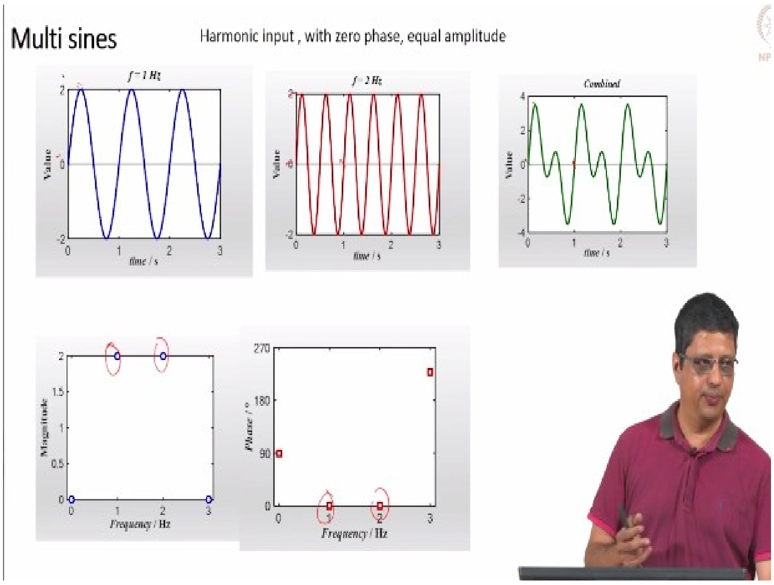

So first, we will take a simple case where you have 2 sine waves; one is at 1 hertz, another is at 2 hertz; 2 hertz means you have 2 sine waves in 1 second, in 3 seconds you have got 6 sine waves, in 3 seconds you have got 3 sine waves in the other case. Starting phase is same, it is 0 for both. They have equal amplitude goes from 0 to 2, 0 to 2, if you add them together it does not go from 0 to 4 because the maximum occurs at one time in one case and the maximum occurs at a different time in the other case .

But you can say, it is a combined wave and this has a period of 1 second which means after 1 second, it ripples. If you give the data from 0 to 1 second and then say copy this, you would be able to generate the full wave. If we take this combined wave and subject this to Fourier transform, assuming that, I have enough number of points given to this data. I have just given you a continuous line here. However, I would actually give data at equal intervals and if I give enough number of data, I can get much number of harmonics. I definitely give at least 1 cycle, I will probably give few cycles, I can get data at 0, 1, 2, 3 and more, but I am just showing you 0 to 3 and you see that magnitude is 2 at 1 and 2 hertz, everywhere else it is 0. Phase is 0 here and these values you can ignore because the magnitude is 0 anyway (refer video).

So this is just to show that you would get the correct values by subjecting it to actually digital filter in this example. I have used freqz here. If you use FFT, also you will get the same result. So if you combine 1 hertz and 2 hertz and sample it up to 1, 2 or 3 seconds and give this, you will not have any problem in retrieving the magnitude and phase of the input wave.

If you do the same thing with output wave, which means the current wave, you would get the correct result. This assumes that this cycle stabilization time is zero. If it stabilizes after some time, we will just assume that we are taking the data after stabilization and then subjecting that current to Fourier transform and you will get these results. When we apply potential, we are the ones who are controlling it, so we can make sure that these potential values are correct and they are steady periodic.

When the response comes, it may take some time to give a steady periodic value. We have seen before that it will oscillate, destabilize after some time. So if you take the data in the beginning and do Fourier transform whether it is a single sine or multiple sine, you will have problem in getting the correct impedance values. If you take the response after it stabilizes, then if the Fourier transform and samplings are done correctly, you will get the correct value.

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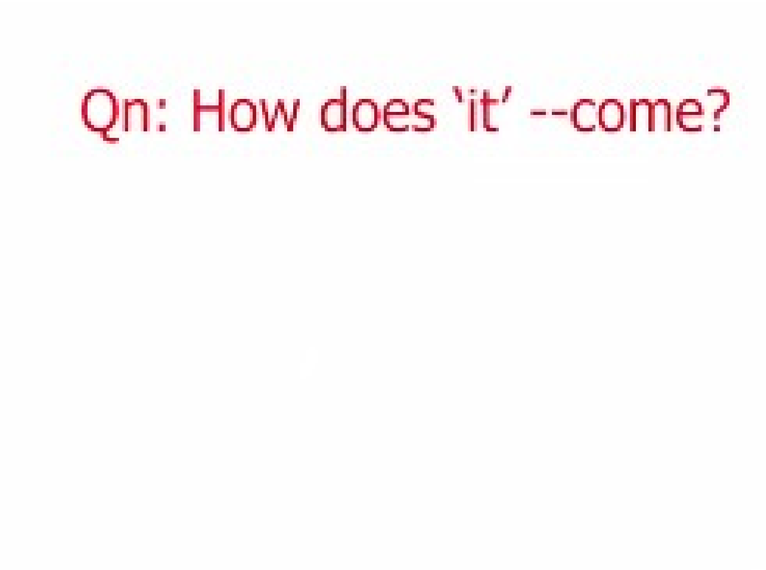

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How does it come? You are asking how does it come here or how does it come this way here top right? Add these points together that is all you need to do. It means, here I have generated this wave by taking time intervals of t. I will just make up this number and say 0.1 second, 0.2 second, 0.3 seconds and so on. Now the first wave is generated using 2sine $(2\pi f t + 0)$, f is 1 hertz. At various time intervals I know the wave 1; y1 will have these values and y2 is going to be 2sine 2π (2t + 0) (f here is 2) and the y here it is going to be y1 + y2. Here I have shown it as a continuous line that means I have given data points at fine grid and asked Matlab to plot it as a continuous line, but actually this is made up of discrete points. If you want to know how the addition of these 2 comes to this, you can try it in any software and see it. So you can do this with the different phases. I have given 0 phase in this example and you can give another phase value as well. One of the typical values is 0. Usually the first wave will be chosen as 0. You can choose any value for these.

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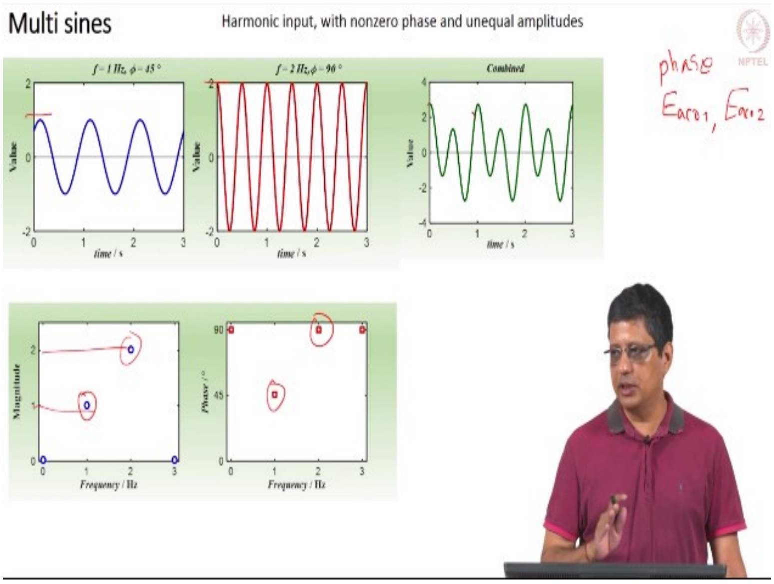

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Now this is wave 1 with 1 hertz and 45-degree phase offset. Second wave is with 90-degree phase offset. When you can generate these, you would get a combined wave, which looks like this. Here, also, after 1 second the cycle repeats. It means the period of the combined wave does not change with the phase and amplitude of each wave. Here the example is with 2 waves (E_{ac01}, E_{ac02}), but you can combine many waves.

The total period of the combined wave will not depend on the phase or the amplitude. It will depend on the frequency. When you have harmonics; whether it is odd harmonic or in general any harmonic, we call it as base frequency and the subsequent frequencies are generated based on this number. Therefore, I have 1 hertz, 2, 3, 4, etc and I can choose any of those. The combined wave will have the frequency of the base frequency. I can choose this as 0.1 then this is going to be 0.2 (refer video). I can choose this as 1 milli hertz, this is going to be 2 milli hertz. As long as the second, third and remaining waves are integer multiples (the frequencies of the remaining waves are integer multiples of the base frequency), the combined wave will have the same period as the first wave.

This is just to show that if you do the Fourier transform or digital filtration, you will get the values as magnitude of 1 here and magnitude of 2 here; the phase value is 45 degree here, 90 degree here, you can try varying it. It will work as long as the input to this filter is correct (refer video for better understanding).

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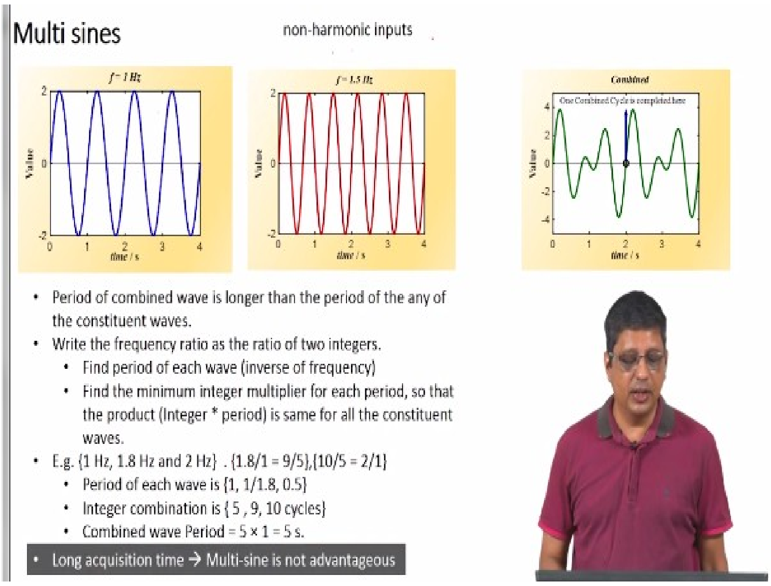

Now I want to look at non-harmonic inputs. Fourier transform tells you the amplitudes and phases at specified frequencies. You can call it as deconvolution. See the wave here is a mix up of these 2 waves (refer video). It looks relatively simple. Assume somebody just gives you this wave and say it seems to be comprised of multiple waves. We cannot tell upfront by looking at it that it comprises of 1 hertz and 2 hertz sine waves with 45 degree and 90 degree. I can call it as a sine wave with 90 degree; I can call this as a cosine wave, with magnitude of 2, for the second wave and magnitude of 1 for the first wave. If we look at the first graph alone on the top left, you can see it looks like a sine or cosine whichever way you want. But it has a period of 1 and it has a magnitude or amplitude of 1. I can think it is a sine wave with 45-degree offset also or I can think of it as a cosine wave with - 45 degree offset. So looking at the individual components, one can say what this is. Looking at the combined wave it is not that easy and this is a clean example with 2 waves. If you have multiple waves, it just not possible, the best thing you can say is, it is a periodic wave with a particular period. Fourier transform helps us decouple this and get it into sine waves of various frequencies with the corresponding magnitude and phase. It comes out neatly because the input waves are sinusoidal. If the input waves are not sinusoidal, they are periodic with a different waveform such as triangular wave.

Fourier transform will give you many frequencies with varying amplitude phases. If this comprises of rectangular waves like what we have seen before; say pulse wave; this is also periodic. I can say starts at a particular point, goes up to this and then it repeats. Likewise, I will get many frequencies. In theory, I will get infinite number of frequencies whereas practically we will get few frequencies. So you will get the pulse wave when u add multiple sine waves with different amplitudes and phases.

Here, it is clean in the Fourier space because we are applying multiple sine waves. If we look at 1 hertz or 1.5 hertz; 1-hertz means in 1 second I get one full wave, 1.5 hertz means in 1 second, I will get 1 and 1/2 wave and in 2 seconds, I have 3 waves. So the time period for the second wave is 1/1.5 seconds that is 2/3, which is 0.67 as a truncation. (Refer video) if I combine them, the combined wave has a period of 2 seconds. So I can apply this wave, I can get the result in 1 second assuming that I can supply 1 cycle and take the result. Here I can supply 1 cycle, it is going to take me 0.67 second, and I will wait for 0.68 seconds. So if I apply this wave and get the impedance; apply the second wave and get the impedance, I will still take only 1.68 seconds. It means that when I apply the combined wave, I need to wait for 2 seconds for the combined wave to come like this. It means the period of the combined wave is 2 seconds here, and is longer than the period of the individual compounds. Earlier case, I can take first individual data in 1 second, I can take second data in half second. I can still take the combined data in 1 second. So this is all that limit the time-period. None of the remaining will add more time to the data acquisition time. When you have harmonic waves, you can look at the base frequency and decide the time-period and that is the time-period necessary for the entire set measurement. When you look at non-harmonics, you will need longer time than just the base. So how do we calculate the total period of the combined wave when this is not harmonic?

So you have to write the frequency ratio, right now I can write it as 1.5/1 or I can write it as 1/1.5 and make it as the ratio of 2 integers. So frequency ratio $f2/f1$ can be 3/2. Same procedure in a slightly different way is:

Find the period of each wave, we have 1 second for one wave and 0.667 seconds for another. Find the minimum integer multiple. It means I need to multiply 1st one by 2, the other one by 3, then the value 1 * 2 gives me 2, and 3/1.5 gives me 2. So it means, in 2 seconds, the first one will complete 2 cycles and second wave will complete 3 cycles and then you can replicate them. If I truncate it at any intermediate value, I will not have completed a full cycle for at least one of these waves. So find the minimum integer multiplier for each period so that the product and the period is the same for all of this. So instead of 1 and 1.5 let us take another example where we add 3 waves with 1 hertz, 1.8, and 2 hertz. You can see the period of 1 hertz is 1 second; period of 2 hertz is half second. This is going to be less than 1 second anyway. So if I do this measurement, I can do the first measurement in 1 second and second measurement in less than 1 second, third measurement in half second. So in 2 and 1/2 second or 3 seconds I am done. Apply the first sine wave, second sine wave, third sine wave and get the data at each time. Here we are ignoring the part where we have to wait. It is to say that wait time is pretty much 0. If I combine the wave, I will need to have 5 seconds to get that wave to repeat itself (the period of the combined wave). So I would write 1, 1.8 and 2 as 9/5, 2/1 and I have to multiply 5 * 1 9/1.8. But I would get the total period as 5 seconds. If I have to combine 1 hertz and 1.01 hertz, I will take 1 second for 1 hertz, for 1.01, I will take close to 1 second. If I combine these 2, I will have to wait for really long time for this wave to repeat. That means after 1 cycle, it will be slightly off, after 2 cycles it is going to be little bit more off, after 3 cycles, 100 cycles, 101 cycles or after many cycles, you would find that they are repeating. So after many cycles, 1 hertz would have taken some number of cycles, 1.01 would have taken 1 less number of cycle and then you can say from here to here you take this and then copy this you will get the same wave. That means when you combine waves which have non integer frequencies, combined wave will take much more longer period compared to the original wave. So when you have longer acquisition time, multi-sine is not advantageous. The reason we use multi-sine is it will take shorter time compared to doing the individual sine waves, but if the combination looks like it will take longer time, there is no point in doing the multi-sine.

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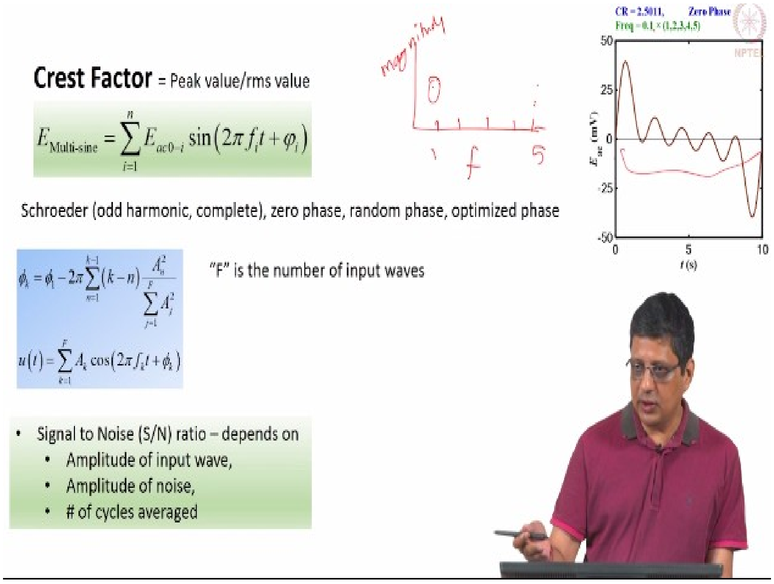

That means if you have harmonics, multi-sine will not take longer time maybe that is a good choice. If you have non-harmonics, it is not a good choice. We should consider another factor or parameter, which is called crest factor. So this is just a mathematical representation of adding different sign waves. Let us take sine wave 1 to n, 1 to 5 or 1 to 10. This is the amplitude of the first sine wave, second sine wave and so on with the corresponding frequency and phase. Crest factor is the maximum of combined wave divide by the RMS value or root mean square value. If you take a pure sine wave, maximum divided by the RMS value is going to be square root of 2. If you combine the sine waves, the maximum divided by RMS will not be square root of 2, it will be more than that. You can optimize the crest factor by adjusting the phase. RMS value will not change, but the peak value can change. There are different choices for the phase values. One choice is to use 0 phase, which means we should not give phase for any of the input values. It is mathematically or implementation wise simple. Another is to give random phase. So some of the publications suggest using random phase for this. Now that you have good optimization software, it is possible to give optimized phase for a given set of sine waves with certain amplitude and frequency. If you tell the software, the sine waves, the amplitudes and/or the frequencies, which you want to give, then you can do the optimization for the phase values. When you change the frequency of the amplitude, the phase value, which gives you the lowest crest factor, will change.

We want a low crest factor because when the crest factor is high, it is effectively going into the nonlinear regime. Now if you use odd harmonic waves say 1, 3, 5, 7 etc for the frequencies, there is a formula called Schroeder phase which gives the crest factor, if you generate the phases like this (first phase choose it arbitrarily). Second phase onwards, you can estimate it using this formula (refer slide or video). Where A here is the amplitude and this is meant for set of cosine waves, but you can choose the first wave to be with the phase of 90 degree and it becomes the sine wave or - 90 degree, it can become sine wave. You can generate this and this is not going to be optimal. But this is going to be better than 0 phase. If you want optimal phases, there is no analytical formula to calculate it as of now. It has to be done using numerical method. If you have analytical formula it, it is lot faster to generate these phases. I think one company uses the Schroeder phase with odd harmonic in the commercial one. If you have n number of waves here or F-number of waves, you can use this formula to calculate k = 1 to F and you can generate the waves from this. This works reasonably well when you combine wave 1, 3, 5, 7, 9. If I want to go from 1 to 13, I should take the full set. I cannot just say I want only few waves. If I take this set, use this formula, and get the phase, I may not get good crest factor. So be careful when you read literature and use that information, they may not tell you that this will not work if you do not have a complete set. They will say if you use odd harmonic, this seems to be a good suggestion although it is not the best case. You should evaluate it yourself and make sure that it works well for your case. So in this example (refer video), if I use 0 phase, I put 1, 2, 3, 4, 5 and base frequency is 0.1. I have combined waves which are 0.1, 0.2, 0.3, 0.4, 0.5 each one with 10 millivolts, I get a nice wave which looks symmetric with period of 10 second. Base frequency is 0.1. So period is 10 seconds and the remaining are all harmonics. Therefore, the combined wave also has a period of 10 second and the crest factor is 2.5. So if you look at the peak versus RMS, it is going to be 2.5. You can calculate it and find it. Now I have 5 wave in this. Each one of them is 10 millivolt as amplitude. The signal-to-noise ratio at each frequency depends on the amplitude of that input wave at that frequency. It also depends on the amplitude of the noise. If noise level is high, you will get poor signal everywhere. It also depends on number of cycles averaged. For example, consider the wave which contains 5 waves with 5 hertz frequency (refer video). It means, in 1 second 1, 2, 3, 4, 5 cycles of 5 hertz with 10 millivolt amplitude is used here. In another wave, I use only 1 cycle of 1 hertz and same 10-millivolt amplitude is given here. If I use many cycles at a given frequency and take the Fourier transform, the signal-to-noise ratio will be better. Although this is a combined wave with 5 frequencies, if I assume, that the noise level is same at all frequencies, then I will get a better quality data at 5 hertz and I will get relatively poorer quality data at 1 hertz because the combined wave essentially uses 1 cycle of 1 hertz and 5 cycle of 5 hertz.

If I take data for 1 cycle, in ideal case you have 0 noise and if you do Fourier transform, you will get what I showed you in the earlier graphs with magnitude and phase without much noise. In real life, you are going to have little bit of noise. Magnitude and phase will not be good precise number. If I take multiple cycles with the same noise level, the Fourier transform will reject the noise better when you have much more number of cycles. The signal strength at this frequency will be good compared to the noise. Noise will have amplitude at all frequencies. When we average over many cycles, you will get better quality data, which means it will come closer to the actual value. Now you can say for any wave, if I take 1 cycle versus n number of cycles, the data quality is better with n number of cycles when you average over this n number of cycles. Even without the sine waves, just imagine that you have a constant, you are expecting a constant signal and the system actually gives you a constant signal. This is 0 whatever this is the signal, it is supposed to be a straight line (refer video), but in measurement there is some noise. Therefore, it looks like this. The noise is going to have certain amplitude and it is going to be +/- some number on the average. If we take data over long period and then average the values, you will definitely get a better average compared to taking data over a small period (refer video for example).

If I take more number of samples and average them, I will get a better average. Likewise, if you do Fourier transform of a data, which contains multiple cycles at a particular frequency, you are likely to get better quality data. Since the combined wave here contains 5 frequencies in this example, the 5 hertz has 5 cycles here, 1 hertz has only 1 cycle here. So in theory when there is 0 noise, I will get clean output from this. In practice, when there is a noise, I will get a response saying this is the magnitude versus function of frequency in the Fourier space. I will get some value 1, 2, 3, 4, 5 hertz here and the quality of data here versus the quality of data here in general this is going to be better. So I would not get the same quality data.

One way to adjust for this is to give larger amplitude here compared to the amplitude here (refer video). So you do not want to necessarily give equal amplitude to all this. For us to understand, I have given equal amplitude and added them here, but that may not be the best way. You may want to give larger amplitude at the lower frequency so that the signal to noise ratio is better in this case. Number of cycle averaged is more for the higher frequency. So in general, you want equally good quality data everywhere. So the point here being when you do single sine, usually you will apply same amplitude everywhere. You will get more cycles averaged at very high frequency. At the medium and low frequency, you will probably have few cycles. But the data you get out of that versus the data you get out of multi-sine may not always match because there are lot of things that are different in the multi-sine versus single sine.

If you take an electrical dummy circuit board, you will probably get similar data whether it is a single sine or multi sine and you will get it faster in the multi sine. It does not mean, in actual electrochemical system, you will get equally good quality data or same data at a shorter time.

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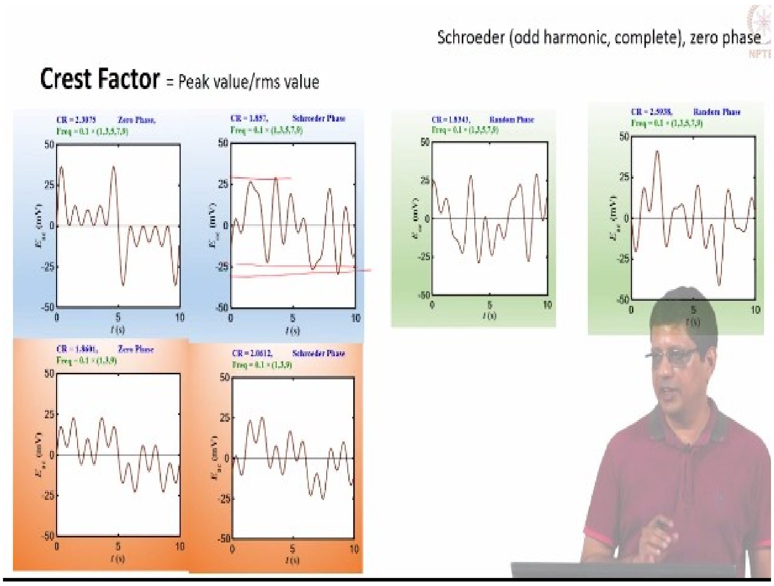

Consider an example where I have used odd harmonics 1, 3, 5, 7, 9 (refer slide or video); at 0 phase, you get a wave periodic at 10. I had probably just truncated 1 point before that, so that I could give it to Fourier transform and 0 phase has crest factor of 2.3. The wave does not look good with Schroeder phase. Looking at this, you can say the first one is nice periodic wave, the other one does not appear so to us. It has a different phase and the crest factor is lower which means it goes up to whatever 35 millivolts in the first graph (refer video). It goes only up to 30 millivolts (second wave). It means from the steady state, we move it up and down when you apply sinusoidal wave and on the left side that graph goes more compared to the right side. So that is effectively saying E_{ac0} on the left side is higher compared to the value at the right side.

Now I also want to show what happens when you use random phase. Each time it will give you a different set of numbers in random number generation. In one case, I got 1.83, which is actually better than Schroder. In another case, I got 2.59, which is worse than 0 phase. So it may give poor or large value of the crest factor in random phase generation depending on your luck. When the crest factor is large, it means you are going into the nonlinear regime, which means you will get higher harmonics. System is nonlinear anyway, but if you apply large amplitude, you will get higher harmonics. If you apply small amplitude, we will get higher harmonics and we can neglect them, as they are small.

Now I want to show you another example where it is odd harmonic, but it is not a complete set (refer slide or video). I want to go from 1 to 9, but I am not taking all the odd frequencies in between, I am taking 1 3 and 9 and I am skipping 5 and 7. 0 phase is actually better than Schroeder here. Crest factor is 2 in the 2^{nd} case and 1.86 in the 1^{st} case. So we should not always assume that if you have odd harmonic, Schroeder phase will give you better crest factor compared to 0 phase or random phase. So if you use partial set, it is better to use an optimization program to get the phase values and calculate the multiple sine wave.

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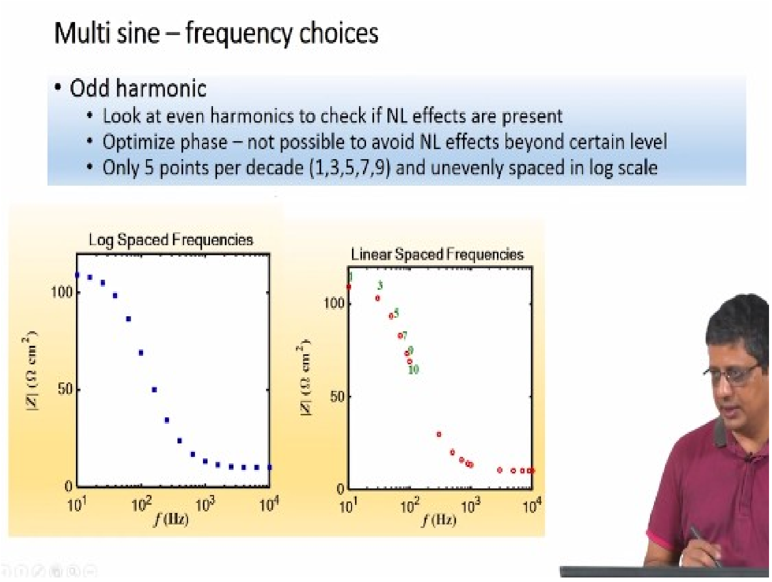

One advantage in using the odd harmonic is, if I supply the wave at 1 hertz, 3 hertz, 5 hertz, 7 hertz, I can also look at the values and when they do the Fourier transform, I will have enough number of points and enough duration to get all the even harmonics also. If the non-linearity shows up here, 1 hertz will give me a data at 2 hertz also and 3 will give me a data at 6 hertz also.

Then it will seem to be in the nonlinear regime and we should redo this experiment. I can change or optimize the phase. However, in terms of optimization, there is only so much you can do. You want a good signal to noise ratio at each 1, 3, 5, 7 frequencies, which means you need to give a minimum amplitude. You cannot give very low amplitude. If you give certain amplitude, no matter how much you optimize, the crest factor is going to add up. It is not going to be 1.4; it is going to be higher than that. So you cannot completely avoid nonlinear effect. All that you can get from this is to know whether it is present or not. If it is present, you can try optimizing the phase, but beyond a limit, you cannot optimize and then you will have to reduce and accept a poor signal-to-noise ratio.

There is another problem. Each decade, which means 1 to 10, 10 to 100, I have only 5 choices; 1, 3, 5, 7, 9. I can say I can take frequency of 0.1 hertz, 0.3, 0.5, 0.7, 0.9, I will start the next again at 1, although it is close to 0.9 we can take 1, 3, 5, 7, 9, 10, 30, 50, 70, 90 and so on. As I choose only odd harmonics, I have only 5 points per decade. If I apply even harmonic also, I cannot tell whether nonlinear effect is present or not. Because I apply input at 1 hertz, 2 hertz and I will get signal at 1, 2, 3 and so on. So I cannot say whether nonlinear effect is present or not. So I cannot use that. We need to see what happens when you use these frequencies. The left side is log space frequency (refer video) with same number of points per decade. I can divide the space of 1 to 10 in a geometric series and get these frequencies. If I do this with the odd harmonic, this is what I get. This is 10 power 1 (refer video), the fundamental is 1. 3, 5, 7, 9, 10 comes close to this. The gap between 1 and 3 is large, 10 and 30s or 100 and 300, 300 to 500, 500 to 700, 700 to 900, and 900 to 1000 is very close in the log space. So if I look at this, we can see this is as a neat graph, I have used this to simulate the impedance for the randel circuit that we have seen before. If you have noise, it is not easy to figure out how this shape is going to be.

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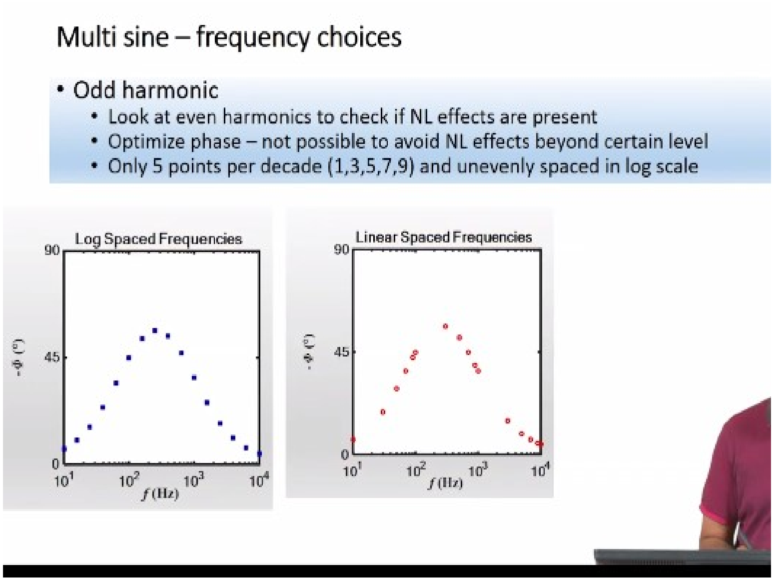

In the same way, if you can look at the phase values, you see large gaps in the data, in the abscissa (refer video/slide); here I have taken 3 points or 5 points per decade. I can take 7 or 10 points per decade. It means, I will get data in a close-packed curve and 1 or 2 data points will go off the chart occasionally. I can delete them stating that there is some noise related problem during data acquisition and I can take the remaining data. You are already in a sparsely spaced region here and when you throw away 2 data points, you will have to really guess what is happening in between. So from that perspective, odd harmonic choice is not really a good choice and if you skip some of the frequencies, it is going to be worse.

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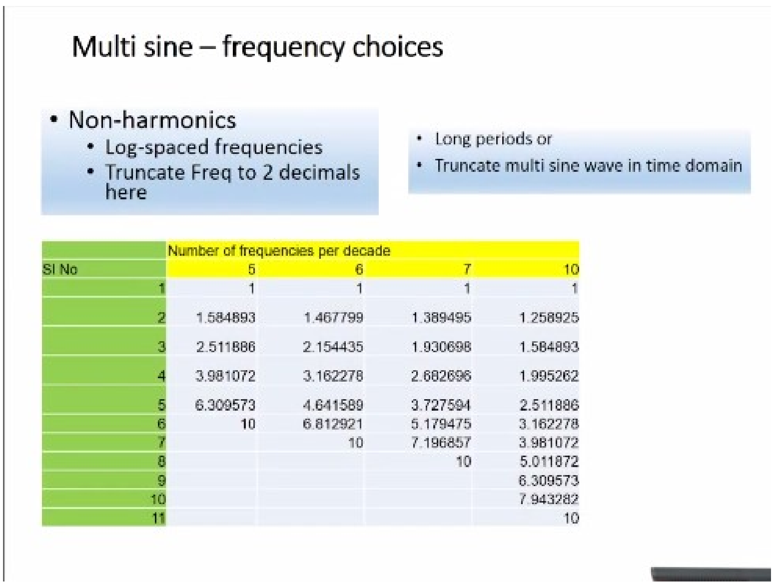

	Number of frequencies per decade			
Sl No	5	6	7	10
1	1	1	1	1
2	1.584893	1.467799	1.389495	1.258925
3	2.511886	2.154435	1.930698	1.584893
4	3.981072	3.162278	2.682696	1.995262
5	6.309573	4.641589	3.727594	2.511886
6	10	6.812921	5.179475	3.162278
7		10	7.196857	3.981072
8			10	5.011872
9				6.309573
10				7.943282
11				10

Here is an example. If I want in real log space, 5 frequencies per decade, I have 1, this is the geometric series, it is 1.584893 (refer video/slide). You do not have to use it this way; you can use to truncate it at 1.58, second can be 2.51, 3.98, and 6.31 and next is going to be 10. You can do seven, 6 frequencies or 10 frequencies per decade and you have to truncate it at few decimal places. When you combine these waves, the period of the combined wave is going to be very long, as these are not integer multiple of the base frequency. If we take more decimals, it is going to be extremely long. Here I will have to take at least 2 decimals so that it looks little different. I cannot take 1 decimal and do this, it would not be that good. So either I have to wait for a very long period or I will combine the waves. But I would not wait that long. I will chop it off in between and replicate. In the time domain, I know it becomes a periodic wave after a long time, but I do not want that. I will have 1 hertz,10 hertz, 1.58, 2.5 etc. So 1 hertz gives 1 cycle in 1 second, maybe if wait for 5 seconds, I will have 5 cycles here; I will have a non-integer value here, but I will have enough number of cycle, so I can average out (refer video).

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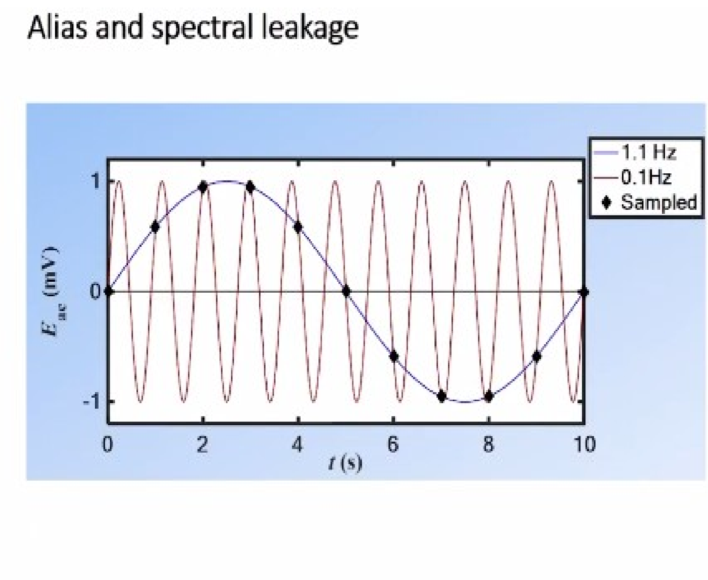

But it leads to certain problems. There are different problems. One is called aliasing; another is called spectral leakage. I will show one example based on aliasing here. Let us say you have a signal, which is given by the brown colour wave, which is a high frequency wave (Note that the legend used in the picture is wrong). This is 1.1 hertz, which means I will have 11 waves in 10 seconds.

In this impedance case, we know what wave we are applying so we will sample it nicely. But let us say, you are taking data from some random source. We sample it every second. If I fit this to a sine wave, it will say this fits nicely to a sine wave of 0.1-hertz frequency. So the blue colour is actually 0.1 and brown colour is 1.1. So I will get a clean fit, it will give me a phase and amplitude and it will tell at this frequency, you have the results. However, it is not the correct result. We are not sampling it fast enough and because of that, what we should get as 1.1 hertz has come to 0.1 hertz and we will think it is clean. If I actually fit a sine wave, it will fit nicely. So this is one type of problem. So different signals are not distinguishable and that is due to poor sampling. You can increase a sampling rate and you will find that this is not at point 1 hertz.

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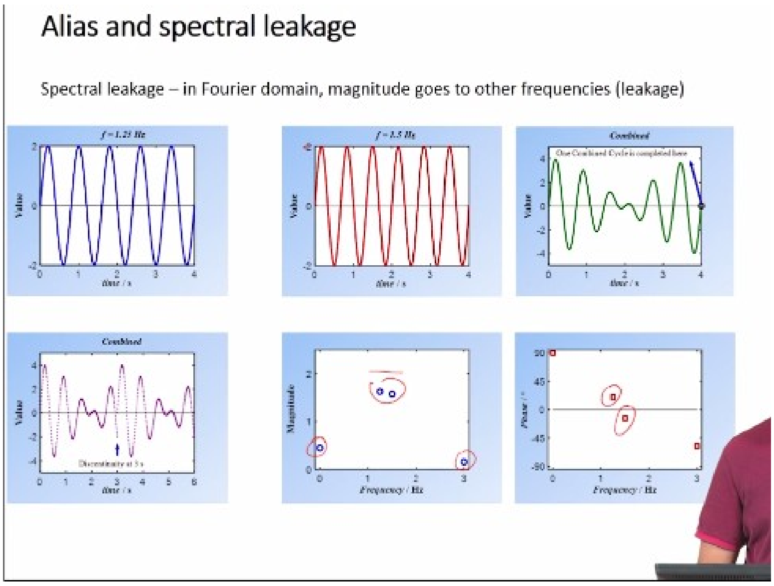

In spectral leakage, the power leaks to other frequencies. For example, we expect that it has to go to a particular frequency (1 hertz, 2 hertz etc), it goes to some other frequency, and this situation is called spectral leakage. When we write the spectrum (magnitude and phase as a function of frequency), it goes to other frequency. So in this example, if we take 2 samples-1.25 and 1.5 hertz, you have to wait for 4 seconds for combined wave. You have to wait only for less than 1 second for the individual waves. If I do not want to wait for 4 seconds, I will take it up to 2 seconds (anyway I have more than 1 cycle here, chop it off at this level and then replicate it). So I will take this graph and chop it off 3 seconds or 2 seconds (In this example I have shown at 3 seconds). I would definitely have 2 cycles of this or more than 2 cycles of this. I am expecting magnitude of 2 as result. However, some of it has gone to DC, some of it has gone to 3 hertz. So power has leaked from 1 hertz, 1.25 and 1.5 and moved away to 0 and 3.

These are 0 phases and phase has gone all over the map. So I cannot chop it off. I have to give full wave. I can chop it off with a trick called windowing. We will continue with that in the next section.