

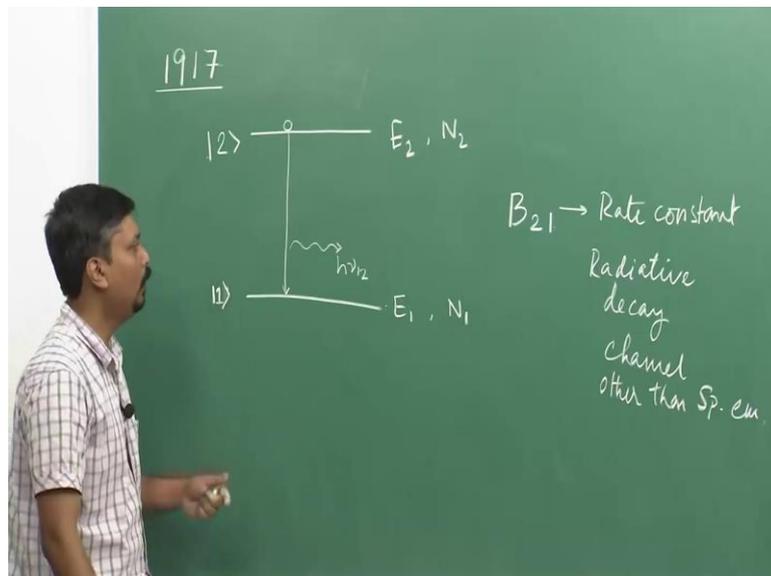
Laser: Fundamentals and Applications
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Lecture – 04
Einstein's Concept of stimulated emission

Hello and welcome back. In the last class we started looking at the laser action. In order to do that what we did we took 2 level systems based on atom and we looked at different optical transitions that are possible. We looked at absorption we looked at spontaneous emission. And then we were discussing about the Planck's law related to blackbody radiation. We also stated that Einstein was trying to use this 2 level oscillator system you know to explain the blackbody radiation. So, he took n number of such 2 level oscillators formulating the overall blackbody and was trying to you know go to the same expression from the same expression as Planck derive.

And we said that at that point of time considering absorption and spontaneous emission he was not being able to do that.

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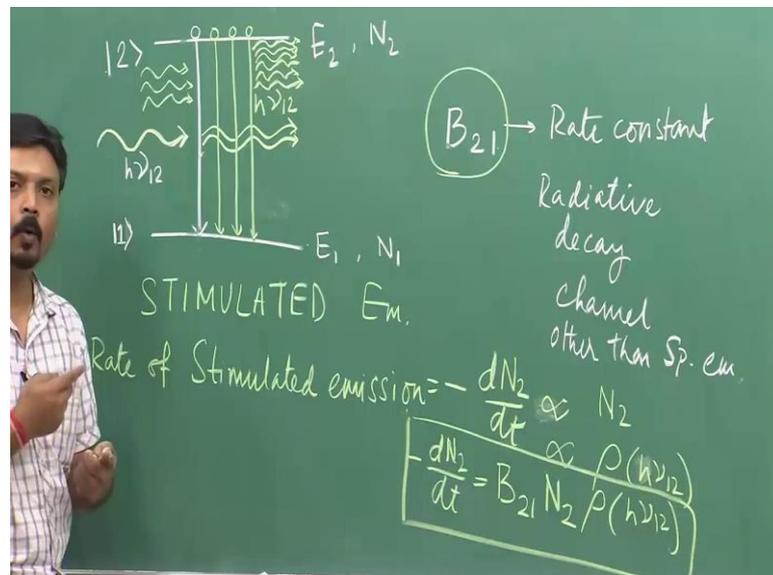


At last in 1917 you have seen this in the second lecture when you are talking about the development of history of laser development. In 1917 Albert Einstein he ultimately proposed something, what did he propose? He said that there must be another you know radiative channel of decay of the excited atom. So, what he meant.

He said that if you take a 2 level system having same energy and let us take same population then Einstein state in 1917 that there must be. We had the process of spontaneous emission without any field and you had the photon emission. And what Einstein said well having this spontaneous emission as the only channel for the atom to decay from excited to ground state is not enough there must be another decay channel such that its rate constant is B_{21} . This is the Rate constant for this separate Radiative decay channel other than spontaneous emission.

So, how do we depict that process let us do that.

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Earlier we did not consider any radiation field for emission processes now if I consider a presence of a radiation field. If this radiation has photon energy equal to the gap of this energy levels then this will trigger the decay of this atom which is at excited energy level to the ground state. In this process what will happen while coming down this will release a photon. So, let me use different color. This will release a photon which again has the energy of $h\nu_{12}$.

This photon coming out is due to the jump that we are seeing due to the optical transition of this atom from 2 to 1. Now what will happen to this photon that is not going anywhere in case of absorption this atom was taking that amount of energy and making this transition, but here this is going and knocking this on off and this knocking of process is actually giving us energy in form of this photon.

This exciting photon is coming out undisturbed. So, this will come out as such and this process of creating a decay of an excited atom to the ground state by using another photon is known as Stimulated Emission. It is quite easy to understand here earlier we did not have any radiation field now I have a radiation field which is stimulating this decay process and which is you know giving rise to a release of a photon and the initial photon that is crossing this stimulated emission is also coming out.

By giving one photon I am getting 2 photon out correct. So, by stimulated emission I can always increase the number of photon right. If I suppose have several molecules here and I give photon for each one and they make a downward transition. I will need total 1 2 3 photon and this 3 photon each one will come out accompanied by another photon coming due to the stimulated emission. So, altogether I will get 3 plus 3, 6 photons. I will see 1 2 3 4 5 6 photons.

Altogether total 4 atoms making a downward transition my input are input is total 4 photons and the output is total 6 plus 2 8 photons. So, you can see stimulated photon will increase the number of photons that will come out come as output very good. Now what Einstein say that there must exist a process like stimulated emission and that has to be associated with a rate constant. The rate of stimulated radiation if we look at, Rate of Stimulated emission how can we write that is just like the other emission process. So, the job of an you know emission is to bring the more atom from excited to ground state, by decreasing the number of population at the excited state.

I can write it is as $\frac{dN_2}{dt}$ and you can separate it out from the other $\frac{dN_2}{dt}$ that we dealt with by writing a super you know subscript as stimulated and spontaneous fine. Here we are specifically writing that this is the rate of the simulated emission minus $\frac{dN_2}{dt}$. So, what are the things that it will depend on it will definitely depend on the number of atoms at state 2 and in this case we are using a photon to stimulate the process. So, definitely it will depend just like absorption process on the density of photon having the frequency ν_{12} all right.

And we have already mentioned here this is the rate constant. So, if we put that, minus $\frac{dN_2}{dt}$ for stimulated emission is equal to the rate constant B_{21} again you notice it is 2 to 1 followed by N_2 multiplied by $\rho_{\nu} h \nu_{12}$, this is by Rate of a Stimulated

emission . Now, I have got 3 processes Absorption, Spontaneous emission, Stimulated emission all 3 processes have their own rates.

Absorption process is upward movement and spontaneous and stimulated emissions are downward movement in this picture fine now if we apply the principle of detailed balance.

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Principle of detailed balance $\rho(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{(e^{h\nu/kT} - 1)}$

Total Rate of absorption = Total Rate of Emission.

$$B_{12} N_1 \rho(h\nu_{12}) = A_{21} N_2 + B_{21} N_2 \rho(h\nu_{12})$$

$$\rho(h\nu_{12}) = \frac{A_{21} N_2}{B_{12} N_1} + \frac{B_{21} N_2}{B_{12} N_1} \rho(h\nu_{12})$$

$$\rho(h\nu_{12}) \left(1 - \frac{B_{21} N_2}{B_{12} N_1}\right) = \frac{A_{21} N_2}{B_{12} N_1}$$

$$\rho(h\nu_{12}) = \frac{A_{21} N_2}{B_{12} N_1} \frac{1}{\left(1 - \frac{B_{21} N_2}{B_{12} N_1}\right)} = \frac{A_{21}}{B_{12}} \frac{e^{-h\nu_{12}/kT}}{\left(1 - \frac{B_{21}}{B_{12}} e^{-h\nu_{12}/kT}\right)}$$

From Principle of detailed balance I can say that the rate of upward transition is equal to the rate of total downward transition. So, the total rates of both the directional movements should be same should be the same as per the principle of detailed balance. If I write explicitly rate of absorption or to be specific if I write Total Rate of absorption must be equal to Total Rate of Emission.

We can use the rate expression that we have already got. So, Rate of absorption is given by B_{12} if you recall in the last class then we have N_1 multiplied by $\rho(h\nu_{12})$ is equal to the rate of spontaneous emission that is A_{21} multiplied by N_2 plus the rate of stimulated emission that we just now we got that is $B_{21} N_2$ multiplied by $\rho(h\nu_{12})$. Einstein was responsible for this you know this whole expression because he brought out the fact that there must be a stimulated emission also along with a spontaneous emission.

So, if that is true and if we remember what he said just few minutes back that in 1917 Einstein could you know use this 2 level oscillators to mimic the blackbody radiation

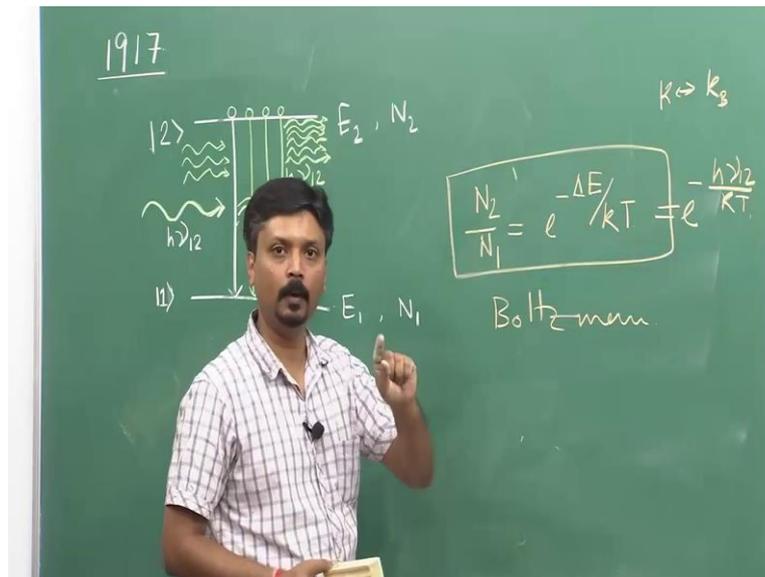
spectrum and he was successful to come to the expression same expression that Planck has got. We should be use; we should be able to use these to do the same thing and you will see that that ultimately will help us in understanding the laser process. So, what we need to do if we remember in Planck's expression. What was the Planck expression, Planck expression was this Planck's law was for a given frequency at a you know a particular temperature T the spectral density was equals to $2 h \nu^3 c^2$ multiplied by $e^{-h\nu/kT}$ this was Planck's for right.

Now if I try to rearrange some of these terms here to get an expression for ρ_{12} ρ_{21} then how should it be. Let us transfer this term on this side first. Let us do that little explicitly, that it is easy for you to understand, this is equal to I am transferring this one on this side. So, A_{21} by B_{12} and then again dividing by N_2 by N_1 plus B_{21} by B_{12} N_2 by N_1 into ρ_{12} .

Now, this term has one ρ and here also we have ρ . If I take this term on this side, what will I get I will get ρ_{12} . If I just rearrange and simply jump a step then I can write it as $1 - \frac{B_{21}}{B_{12}}$ into this and on the right side what we have I have A_{21} by B_{12} multiplied by N_2 by N_1 fine. Now I can take this on the other side and I can get an expression for ρ_{12} finally. So, what I have is A_{21} by B_{12} into N_2 by N_1 and then $1 - \frac{B_{21}}{B_{12}}$ divided by B_{12} multiplied by N_2 by N_1 all right.

I have got some expression for ρ_{12} sorry ρ_{12} , now we have a term called N_2 by N_1 , what is N_2 by N_1 , in other population of atoms at a particular energy state either 1 or 2.

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If you recall what Boltzmann said that the ratio N_2 by N_1 is equal to e to the power minus ΔE by kT where k is a Boltzmann constant you know many occasions you will see it is written as k_B , B stands for Boltzmann.

Here we are not writing that B specifically, we are writing in kT . This we know under equilibrium right, when the equilibrium is established then always I have this relation coming from Boltzmann. If we use this instead of this N_2 by N_1 in this expression I will get a you know the expression in terms of this exponential and here this ΔE . I can write further like e to the power minus $h\nu_{12}$ right because the ΔE here in our case actually refers to E_2 minus E_1 which is equals to h into ν_{12} .

So, use this term here and let us see how we can solve this one. Let me write down a A_{21} by B_{12} into e to the power minus $h\nu_{12}$ by kT divided by 1 minus B_{21} by B_{12} into e to the power minus $h\nu_{12}$ by kT all right.

We have come to some expression of this sort and we will start from here in the next class to see how we can go to the same type of expression as Planck has formulated earlier. And that will help us in understanding how one can get the amplification of stimulated emission or in other words how to get a laser. So, see you tomorrow.

Thank you very much.