

Nuclear stability and nuclear force

B.S.Tomar

Homi Bhabha National Institute

Lecture-3, Module-2

Hello everyone. So in the previous lecture, we discussed about the discovery of nucleus, proton, neutron, and also how the idea about the nuclear radius was obtained from experiments on scattering of high energy electrons and other heavy charged particles by the nuclei. And we also discussed about the nuclear mass and binding energy. Now we'll discuss what are the factors that govern stability of nuclei.



Nuclear stability

Frequency distribution of stable isotopes

A	Z	N	STABLE ISOTOPES
EVEN	EVEN	EVEN	165 ${}^4\text{He}, {}^{12}\text{C}, {}^{24}\text{Mg}, {}^{40}\text{Ca}, {}^{208}\text{Pb}, \dots$
EVEN	ODD	ODD	4 ${}^2\text{H}, {}^6\text{Li}, {}^{10}\text{B}, {}^{14}\text{N}$
ODD	EVEN	ODD	55 ${}^{17}\text{O}, {}^{25}\text{Mg}, {}^{57}\text{Fe}, \dots$
ODD	ODD	EVEN	50 ${}^7\text{Li}, {}^{19}\text{F}, {}^{63}\text{Cu}, \dots$

1. Majority of the stable isotopes: even-Z, even-N. Only 4 o-o nuclei \rightarrow Nucleons like to remain paired.
2. Elements with $Z=20, 50, 82$ have more number of stable isotopes \rightarrow extra stability with magic numbers, 2, 8, 20, 50, 82, 126

So a very simple way of explaining is to see the frequency distribution of stable isotopes, that gives an idea. So what I have made here is a table containing the different configurations of protons and neutrons constituting nuclei.

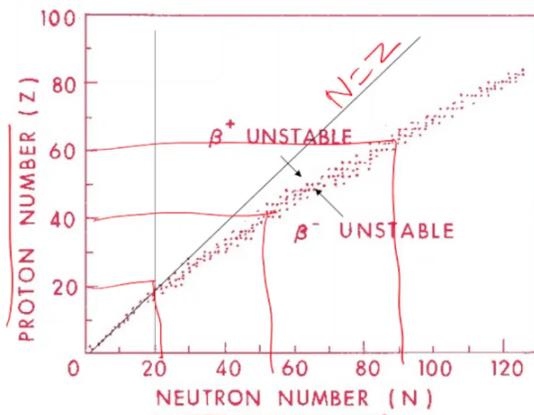
So even mass number can come from even z and even n. And there are 165 nuclei which are stable having even-even configuration. For example, helium-4, carbon-12, magnesium-24, and so on. These nuclei have even protons, even neutrons. We'll call them even-even nuclei. We can also have even mass number from odd-odd configuration. That means odd proton, odd neutron. And you see, there are only four nuclei which are stable having odd-odd configuration like deuterium, lithium-6, boron-10, and nitrogen-14. This gives an idea that the nucleons tend to be paired up. They like to remain paired up. So the majority of the stable isotopes are of even-even type. And there are only four nuclei which are of odd-odd type. Therefore, pairing energy plays a very important role in the stability of nuclei.

Second point is, if you see the table, nuclear table, nuclear chart, how many isotopes, stable isotopes a particular element have, you will find the certain elements having atomic number 20, 50 and 82, corresponding to calcium, tin, and lead respectively. They have a very large number of stable isotopes. So it essentially tells us that there are certain magic numbers which have extra stability associated with these numbers. And later on, we will discuss that the magic numbers are 2, 8, 20, 50, 82, 126. So if a nucleus has got two protons, eight protons, 20 protons, or it has got two neutrons, eight neutrons, 20 neutrons, and so on, such nuclei have extra stability.

So one is that certain magic numbers are more stable, and also the nucleons like to remain paired. So that gives stability to the nucleus.



N/Z Ratio and Nuclear stability



1. For stable nuclei with $Z \leq 20$, $N/Z = 1$
2. For stable nuclei with $Z > 20$, $(N/Z) > 1 \rightarrow$ excess neutrons are needed to stabilize the nucleus.
3. ^{209}Bi ($N=126$) is the heaviest stable isotope.



Another important property governing the stability of nucleus is the neutron to proton ratio, N/Z ratio. So what I have shown here in this graph is the number of neutrons and number of protons for stable nuclei. This distribution gives you how these stable nuclei are distributed with regard to proton number and neutron number. And I have drawn a line where N equal to Z . Now you see here up to proton number 20, N equal to Z . That means up to calcium 40, 20 proton 20 neutron, calcium 40, up to this mass number 40, equal number of neutron and proton can make the nucleus stable. The moment we go more than calcium, more than proton number 20, you require more neutrons to stabilize the nucleus. You can see here for 40 protons you require close to 50 neutrons. For 60 protons you require close to 90 neutrons. You can see here as you increase the number of protons, the number of neutrons to stabilize the nucleus increases further. So for stable nuclei with the Z less than or equal to 20, neutron to proton ratio is equal to 1. But for those nuclei which are stable, having Z more than 20, then it has got certain excess neutrons. So as we increase the proton number in the nucleus, you require more neutron than proton to stabilize the nucleus. And the heaviest nucleus that is stable is 209 Bismuth that is N equal to 126. Beyond this 209 Bismuth, all nuclei are unstable. So both

the certain magic numbers help in stabilizing nucleus. The pairing energy helps if the nucleons are paired up, that nucleus is more stable. And the neutron to proton ratio has to be in certain range to make the stable nucleus.



Nuclear force

What holds the protons and neutrons tightly inside a nucleus ?

Nuclear force

Short range and attractive

Neutrons and protons behave similarly with regard to Nuclear force →

Nucleons

Neutrons and protons are spin half particles → Fermions

1929 P.A.M. Dirac predicts existence of anti-electron

1932 C.D. Anderson discovers positron, the anti-particle of e^- .

Electron, proton and neutron the elementary particles of matter.



Now, so what holds this neutron to proton tightly inside a nucleus? So what is the force that is operating between the nucleons.

So we will use the term nucleon to represent both protons and neutrons in the discussion. Because in fact, we will see later on that nuclear force is independent of whether it is proton or neutron. It is charge independent. So we will see this very soon that the nuclear force in short range, it operates at a very small distances of the order of Fermi's. It is attractive for nucleons. That is how the nucleus remains spherical. The nucleons try to attract each other over a very short range. And the neutron proton behaves similarly with regard to nuclear force. That is why we call them as nucleons.

Neutrons and protons are spin half particles. So they are called as fermions. Spin half particles are called fermions and spin one particles are called bosons. Integral spins particles are called bosons. So the nuclei having half integral spins will follow Fermi Dirac statistics.

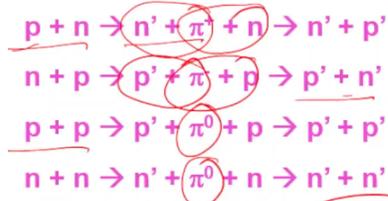
We will also discuss later on the implications of these discoveries in 1929, Paul Dirac in UK predicted that electron has an antiparticle, which we call as the positron. In fact, the thesis by Paul Dirac is a very small thesis wherein he solved the equation of state of electron and predicted that this has got two solutions. The positive solution is electron and so negative solution is antiparticle of electron, which is called as positron. And subsequently in 1932, C.D. Anderson discovered the positron. So gradually we will see that every nuclear particle has an antiparticle and which becomes important in explaining many observations.

So till now what we have understood that any matter is composed of electron, proton, neutron. Therefore, we say that these three particles are the elementary particles of matter. Any matter is made of these fundamental particles, the building blocks of all matter. Later on at the end of this lecture if time permits, I will also show that there is much more than this.



Nuclear force

Exchange of π mesons Yukawa 1935



Meson	Mass (MeV)	Charge	Half life
π^+	140	+e	2.5×10^{-8} s
π^-	140	-e	2.5×10^{-8} s
π^0	264	0	2.6×10^{-16} s

π meson survives for $\sim 10^{-24}$ s, Range of nuclear force $\sim 10^{-15}$ m.

Mass of π meson ~ 140 MeV

Does it not violate law of mass energy conservation?

$\Delta E \cdot \Delta t \geq \hbar \rightarrow$ Heisenberg's uncertainty principle



Okay, so let us go into further. So what holds the nucleons together in the nucleus? In 1935, Yukawa gave the exchange theory of π mesons. So he proposed that inside the nucleus, the protons and proton or proton, proton, neutron, neutron are exchanging a particle, which he called as the meson. And so it is like two players playing volleyball. So if the volleyball is exchanged very fast, you will see like a banana between the two players that binds the two. Similarly, in this, the proton gets converted into a neutron and π^+ positive π meson. And then this positive π meson, so it is actually this, this is from the proton. Later on, this neutron captures this pi plus meson to get neutron and proton pair. So this is how a neutron and proton exchange. A proton becomes π^+ + neutron and so.

Similarly, a neutron can convert to proton and π^- . And this π^- is captured by proton will give you neutron. So neutron proton exchanging π^- meson again become proton neutron. The proton and proton will exchange neutral π meson and the neutron and neutron will exchange neutral π meson. So they are constantly exchanging these π mesons between them.

A pair of nucleons will exchange a π meson depending upon whether it's proton neutron or neutron proton or proton, the charge of the π meson will be different. The masses of these π mesons are given here, π^+ meson, π^- and π neutral. Their charges and their half-life. So they are very short half-life particles. They don't survive for a long time.

Now inside the nucleus, when this π meson is released from one proton, it survives for a very short time. And based on the time of survival, you can calculate how much distance it will travel within this much time. And that gives you the range of nuclear force. So the range of nuclear force happens to be of the order of 1 Fermi. And you will see the corollary of this.

The important point is that how we are able to produce a particle of 140 MeV between a proton and neutron when they are remaining intact. A pair of proton neutron remain proton neutron pair, but in the process generate a mass of 140 MeV. Does it not violate the law of mass energy conservation? Apparently you feel that it is not possible to generate mass from a pair of proton neutron when they are remaining intact. But then that is where the Heisenberg's uncertainty principle comes to our rescue. The Heisenberg principle of energy and time, this product of $\Delta E \times \Delta t$ has to be more than equal to the Planck's constant $\hbar/2\pi$.

And from this you can predict that if you produce a particle of mass, equivalent to 140 MeV energy, then you can see it will survive for a time which is possible by complying with the Heisenberg's uncertainty principle. So that explains that, that is how we could explain that the π -meson exchange theory is valid because this π -meson does not survive, it is captured by another nucleon inside the nucleus and so this is how what forms the bond between the nucleons inside that nucleus. So to list the different properties of this nuclear force and later on we will see how we can, when we make a model to explain the different properties of the nucleus, this nuclear force properties will become useful. So what we found out from the previous discussion about the frequency of stable nuclei, we found that the nuclear force is charged independent, how do we say that? We found that the even proton, odd neutron and odd proton, even neutron, the number of stable nuclei are nearly same, 55 and 50. If you recall the table of table isotopes that even proton, odd neutron, 55, even neutron, odd proton, 50.



Nuclear force

1. It is charge independent: $[e-o]_{\text{stable}} = [o-e]_{\text{stable}}$

$n-p \sim p-p \sim n-p$

2. It is spin dependent

Ground state of ${}^2\text{D}$ is a triplet state

3. It has short range

$m_{\pi} = 140 \text{ MeV}$, ΔE . $\Delta t \sim \hbar \rightarrow \Delta t \sim 10^{-24} \text{ s}$

$\Delta x = v \Delta t = 3 \times 10^8 \times 10^{-24} \sim 10^{-15} \text{ m}$

4. It is saturated

$B/A \sim \text{constant}$, If each nucleon interacted with all other nucleons, $B \propto A(A-1) \rightarrow B/A \propto A$



So the number of stable nuclei having even odd or odd even configuration is nearly same. That means the nuclear force is charge independent, that means NP or PP or NN exchange is of the same order. The second property of this nuclear force is that it is spin dependent and the evidence for this came from the ground state of deuterium. The ground state of deuterium is a triplet state, deuterium has one proton and one neutron. Now proton and neutron can be paired up or it can be unpaired.

What is found that in the ground state they are unpaired and the paired state is unstable. So that means depending upon the state, the spin state of the two nucleons, nucleus can be stable or unstable. That gives an idea that the nuclear force is spin dependent. Third property is nuclear force is short range. we discussed this just now that the mass of the pi meson which is 140 MeV which is surviving for 10^{-24} seconds.

So what is the range of nuclear force? The distance travelled by π meson within this time, assuming that it is moving velocity of light, so velocity into time, so we have 10^{-15} meter. So it is a short range, it operates at the Fermi's and that is why when you say the nucleus dimension 10 Fermi, then it is not operating at 10 Fermi, it is operating at 1 Fermi. And that is why each nucleon is interacting only with those nucleons which are in the immediate vicinity of it. There comes the property of saturation. So the binding energy per nucleon or the average binding energy is constant.

That means every nucleon is interacting with only those nucleons which are in its immediate vicinity. So that cause the saturation of the nuclear force. The saturation of nuclear force also it comes out from the constancy of the average binding energy. There are other properties of the nuclei other than the nuclear force is which we will discuss later on is nuclear spin.



Nuclear properties

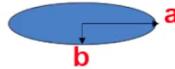
1. **Nuclear spin:** odd – A → 1/2 integral, Even – A → I=zero (e-e) or integral.

2. **Magnetic Moment:** Spin and orbital motion of nucleon → $\mu = J \mu_N$, $\mu_N = \frac{eh}{4\pi m_N}$

$$\mu_e = 1\mu_B, \mu_p = 2.79\mu_N, \mu_n = -1.91\mu_N$$

3. **Quadrupole Moment:** Asymmetric distribution of charge

$$Q = (2/5) Z (a^2 - b^2)$$



4. **Statistics:** Fermi Dirac (s=1/2), Bose Einstein (s=1)

5. **Parity:** Reflection symmetry

So every nucleus, because neutrons have spin half, protons have spin half and nuclei have spin, it can be half integral or it can be integral.

So if it is an odd mass number, nucleus is odd mass, that means either protons are odd or neutrons are odd, then this nucleus will have half integral spin. If it is an even mass nucleus, it could be even proton even neutron or odd proton odd neutron, then this nucleus will have integral spin. And in that also if it is even even nucleus, then I equal to 0. If it is odd-odd nucleus, then I is equal to some integral.

This is for the ground state of the nucleus. Ground state spin of nuclei follows this. There are other properties, nuclei have their magnetic dipole moment. So the magnetic dipole moment comes from one is the spinning of the proton and neutron along its own axis. So if a proton spins, like electron has a Bohr magneton, proton has got a nuclear magneton, $\frac{eh}{4\pi m_n}$

So not only the spin, spinning of the proton and neutron gives you the magnetic moment of proton and neutron, but the orbital motion of proton will also give you its magnetic dipole, but not that of the neutron.

So important point is that the neutron has got a magnetic dipole moment, but it is spinning around its own axis. And that tells lot about whether the neutron, proton are fundamental particles or they have certain structure. So proton has got a magnetic dipole moment of 2.79 nuclear magneton. Neutron has got -1.91 nuclear magneton. So nuclei are magnetic particles, you must have read about nuclear magnetic resonance where the magnetic dipole moment interacts with the applied magnetic field, gives you the splitting of the levels. Similarly nuclei have what you call as the electric quadrupole moment. The electric quadrupole moment comes from the asymmetric distribution of charge in the nucleus. So if the nucleus is having a distribution like if it can be a prolate or oblate

having semi-measure and semi-minor axis a and b , then this is a classical formula for quadrupole moment, but from the nuclear charge distribution inside nucleus, you can calculate the quadrupole moment of the nucleus.

And there are techniques, the hyperfine interaction based techniques whereby one can experimentally find out the magnetic dipole moment, electric quadrupole moment. So these are the properties nuclei possess. Nuclei also follow certain statistics for half integral nuclei you will have Fermi-Dirac, for integral spin you will have the Bose-Einstein statistics. And they also have parity.

So parity is nothing but the reflection symmetry. If you change the wave function from x to minus x , if the wave function changes sign, you say it is odd parity. If it does not change the sign, we say it is even parity. So nuclei also have parity. We will discuss this parity when we discuss the spin states of nuclei. Now I just told about the proton and neutron and electron are elementary particles.



Discovery of strange particles

Cosmic rays or in accelerators

Different particles can be grouped into two classes

Hadrons → strongly interacting particles.

Baryons

n, p

Λ, Σ, Ξ

Δ

Half-integral spin

Mesons

π ✓

K ✓

ρ ✓

Integral spin

Leptons → weakly interacting particles.

e^-, ν_e ✓

μ, ν_μ ✓

τ, ν_τ ✓

Half-integral spin

p, n are not the elementary particles?



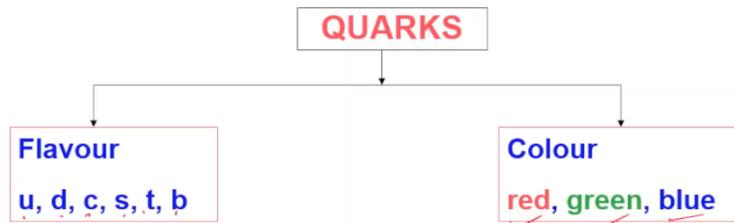
Till early 1930s, it was all very well accepted. But then what happened? So we can say that the type of particles of which any matter is made of is proton, neutron and electron. But in the 1930s and 40s, in accelerators, as we had high energy accelerators, and even in the cosmic rays, different types of particles are formed and they were called as the strange particles. These strange particles could not be explained in terms of the neutron, proton and electron. And very detailed experiments have been done for the last so many years.

I will not go into details of these different discoveries. But what happens here, I give you a list of particles. Apart from neutron, proton, there are other particles lambda, sigma, delta and so on. And these particles are also like proton, neutron, but their masses are

higher. And there are also particles apart from pi meson, there were k mesons, there are rho mesons and they were discovered experimentally in the cosmic rays or even in the accelerators. Similarly, apart from electron, there are neutrinos, there are muon and there are tau lepton.

So there are different types of particles which were discovered subsequently. And so it was, it was no, there is a doubt whether proton, neutron are the fundamental elementary particles. And so there was a classification of these different particles into neutrons. Neutron means which are interacting by strong interaction. And the pions are this spin half particles and half particles spin particles, neutron, proton, lambda, delta particles.

Then the mesons are actually integral spin particles, but they are made of certain things which we will discuss very soon. And these are the leptons, light particles, leptons means light particles which are interacting by weak interaction. So we have particles interacting by strong interaction, particles interacting by weak interaction. And there, that is how the scientists started feeling that proton, neutron may not be the fundamental particle, building blocks of matter. And therefore, in 1964, when a good number of discoveries of strange particles have happened, 1964 Murray Gell-Mann and Zweig predicted, this is one of the, you know, where the theory precedes the experimental evidence.



M. Gell-Mann and G. Zweig (1964)

There are six different kinds or flavours of quarks.

$$Q/e = +2/3 \quad \begin{matrix} u & c & t \\ -1/3 & d & s & b \end{matrix}$$

u = up; d = down; c = charm; s = strange; t = top; b = bottom.

Proton $\underline{u} \underline{u} \underline{d}$, $Q = +2/3 + 2/3 - 1/3 = +1$
Neutron $\underline{u} \underline{d} \underline{d}$, $Q = +2/3 - 1/3 - 1/3 = 0$

So these two scientists predicted independently that the protons and neutrons and different baryons. So when we say baryons, baryons are made of strongly interacting particles and they are made of quarks. So the quark concept, quark structure of nucleons was proposed by Gell-Mann and Zweig. And these quarks are of different type, they are called different flavors and different colors. So we have up quark, down quark, charm quark, strange quark, top quark and bottom quark.

And each quark, when it is inside a nucleon or a particle, it can have different colors, red, green. So that is only for the understanding. They don't really have that color, but to explain certain properties, you give them some nomenclature. So these quarks are having different colors, different flavors and colors and they have different charges also.

So the first time there were particles having fractional charges. We have never heard about fractional charge. So up quark, charm quark and top quark were proposed to have $+2/3$ charge and the down quark, strange and bottom were proposed to have $-1/3$ charge. And it was experimentally later on proved that a proton is made of uud, 2 up quark and 1 down quark. So you can see up quark is $+2/3$, $+2/3$, $-1/3$ for d quarks, the charge of proton is $+1$. Neutron is udd, 1 up quark and 2 d quark, $+2/3$, $-1/3$, $-1/3$, 0.

So it could explain the charge, not only the charge, the spins and many other properties of the nucleons. So not nucleon, they are called baryons. Apart from proton, neutron, there are other particles, lambda, delta, sigma type particles. All of them could be explained and they have been all discovered by this time.



Leptons

Leptons or light particles are not made of quarks.

Participate in electromagnetic, weak interactions but not in strong interactions.

$$\begin{array}{ccc} Q/e = -1 & \frac{e}{\nu_e} & \frac{\mu}{\nu_\mu} & \frac{\tau}{\nu_\tau} \\ Q/e = 0 & & & \end{array}$$

Masses of leptons are much less than those of quarks with $m_e c^2 = 0.511$ MeV, $m_\mu c^2 = 106$ MeV and $m_\tau c^2 = 1784$ MeV.

Neutrinos are very much lighter and their rest mass may even be zero.

$m_{\nu_e} \leq 30$ eV, $m_{\nu_\mu} < 0.25$ meV and $m_{\nu_\tau} < 70$ meV.

Leptons are involved in β decay and other weak transitions.



Another types of particles called leptons means light particles and they are not made of quarks and they participate in the weak interactions and not in the strong interactions.

So these 6 leptons are having electron, muon, tau having minus 1 charge and their corresponding neutrinos, electronic neutrino, muonic neutrino, tau neutrino, 0 charge. And these are very small particles that have very less mass, 0.511 MeV for electron, similarly for muon and tau leptons. And then their corresponding neutrinos are still much lower masses. In fact, still there are experiments going on to find out the mass of neutrino.

So it is predicted that the electronic neutrino will have about less than 30 electron volt mass, muonic neutrino less than 0.25 milli electron volt and tau neutrino less than 70

milli electron volts. So there are experiments going on to find out the masses of the neutrino, these leptons. So leptons, that means these light particles like electron and neutrino, they are involved in beta decay and other weak interactions. We will see later on how the beta decay of a neutron gives you electron and anti-neutrino and the beta decay of a bound proton gives you positron and a neutrino.

So these are the kind of observations which have been made, which is a domain of high energy physics. So I will not go into the details of this particular area, but just to give you a feel that apart from neutron proton and electron, there are now many, many other particles which have been discovered and the scientists are trying to in fact explain all the particles that are discovered in high energy physics, heavy ion reactions, high energy reactions or even in the cosmic rays. So there are many, many particles which are being observed and they have been now explained in terms of six quarks and six leptons. So apart from the nuclear physics which deals with proton neutron electron, there is another domain of nuclear science that is high energy physics where one deals with the other particles which are baryons, leptons and undergoing strong and weak interactions. That is all I have to say. Thank you very much. Thank you.