Fundamentals of Transport Processes II
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Lecture - 7
Kinematics

So, this is lecture number 7 of the course on fundamentals of transport processes. In the past four lectures, we had looked at some mathematical preliminaries. As I said, our final objective was to derive conservation equations for fluid flow, momentum conservation equations. And since we wanted to treat velocity as a vector rather than looking at the individual components, we were doing some fundamentals of vectors and vector calculus, tensors and tensor calculus. In this lecture, we will try to apply this to fluid flows, in order to get some physical understanding of what these things mean, so we will do this without reference to the forces acting on the fluids or the moment of transport.

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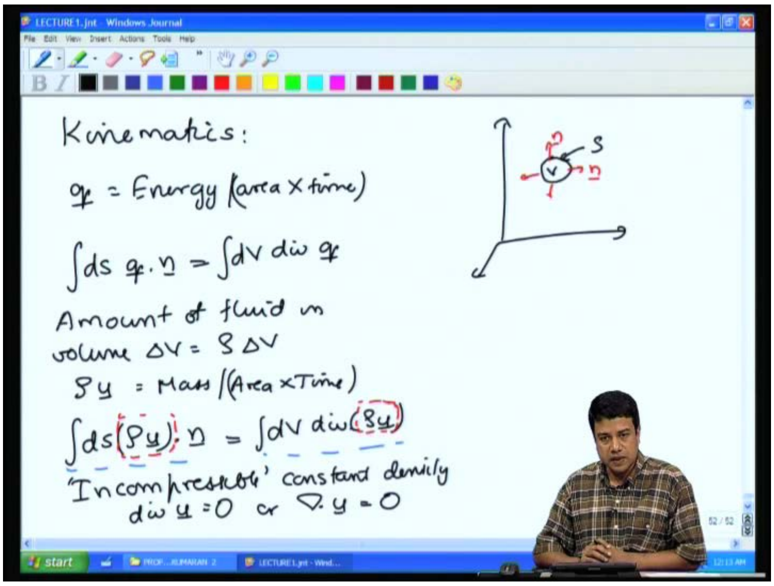

But rather the description of the fluid flow itself, so what is called in fluid mechanics as a kinematics, the description of fluid flow without reference to the forces that are acting on the fluids. So the simplest operator that you can look at is the divergence operator acting on the velocity field, divergence of, if you know that when we deal with, when we deal with heat and mass transfer. The flux q is the amount of energy transported per unit area per unit time, per area into time, so this flux is a local quantity, it tells you how much energy is transported per unit area per unit time across some surface, it gives you the direction of the flow of energy.

So, I take this and integrated over the surface of some volume, I have a volume v and I integrate over the surface, you know that the total energy coming out of this surface energy perpendicular to the surface is q dot n; where n is the unit normal to the surface, n is the unit normal to the surface. So therefore, the total energy that comes out of the surface per unit time is integral over the surface of q dot n, integral of the surface of q dot n, and you know by the divergence theorem, that this can be written as integral of the volume of the divergence of q. Note that, this is an integral tells you, how much net energy is coming out from inside the surface to outside? if there is no net energy source within the surface, there is no energy that is coming out from inside to outside.

Therefore, the integral over the volume of divergence of q has to be equal to 0, so divergence of q basically tells you, if you integrate with this divergence of q over the volume, it basically tells you whether there is a net source of energy within this volume or not. If there is no net source of energy within this volume, integral of d s q dot n has to be equal to 0 therefore, the integral of the volume divergence of q has to be equal to 0. This is true for any volume however, if I take the volume going to 0, then I have the divergence of q at that particular point is 0, so there is no source at that point ok.

Similarly, if the divergence of q is 0 everywhere, there is no source at any rotation basically, it tells you for a given volume whether, there is an energy source within that volume or not. Same thing can be done for fluid flow, the amount of fluid coming out, the amount of fluid at any location is equal to of fluid in volume, delta v is equal to rho times delta v. So, rho is the fluid density, the mass per unit volume of fluid, for this same surface rather than looking at the heat that is coming out, I wanted to know, what is the mass of fluid that was coming out of this volume? The mass flux that is coming out of this volume is rho times u, which is mass per unit area into time.

There is a flux of mass that is coming out of this volume, it is the amount of mass coming out per unit area per unit time, so if I want to find out the total mass, that is coming out of this volume from inside to outside. If the total mass that is coming out of the volume from inside to outside, that has to be equal to integral d s of rho u dotted with the unit normal n. And from the divergence theorem, I know that this is equal to integral d v of divergence of rho u, note that the divergence theorem. If I have some quantity dotted with n, I have integral of the surface of something dotted with n is equal to the volume integral of the divergence of that whole thing, you cannot just take divergence of part of that. So, it has be the divergence of the whole thing, so this gives me the the total mass of fluid that is coming out.

Obviously, if there is fluid mass coming out from inside to outside, there has to be some mass source inside this volume. If there is no mass source within this volume, then this thing has to be equal to 0 which means that this also has to be equal to 0, so if this known mass source within the volume, then this has to be equal to 0. Further, if the fluid is also what is called? Incompressible, if there is no mass source anywhere, and if the fluid is also incompressible that is constant density, if the fluid is also incompressible which means that the density is a constant. Then, I require that divergence of u is equal to 0 or del dot u is equal to 0, for an incompressible fluid, if there are no sources of mass anywhere within the volume, that means that the divergence of u has to be equal to 0.

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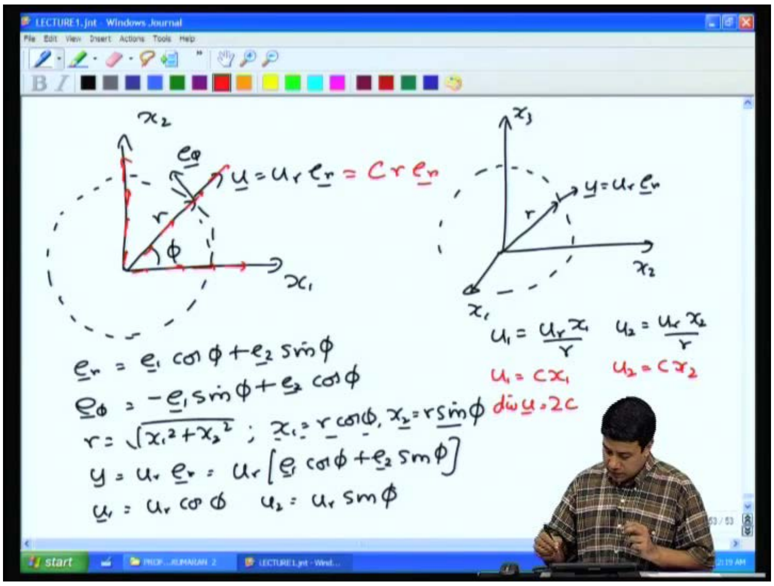

So, let us look at what the divergence means, if I have some spherical surface, if I have some spherical surface around the origin at some location, I have some this is distance r, and I have a velocity field u vector. If this velocity field is along the direction of the radius vector, that means that u vector is equal to u r e r, it says a for us to do it in just two dimensions to make the point, so rather than doing at three dimensional system. I will just restrict attention to a two dimensional system which has two coordinates, because easier to make the point in two dimensions. This is x 1 and x 2 and I have some surface, some circular surface of constant distance from the origin, and the velocity u vector is equal to u r times e r.

Now, this e r vector can be resolved in terms of the two components, e r can be resolved in terms of its two components, so if this is the angle made with respect to the x 1 axis of this radius vector. Then you know that e r is equal to e 1 cos phi plus e 2 sin phi, e phi is perpendicular to this; in this polar coordinate system e phi is perpendicular to this, which means that e phi is equal to minus e 1 sin phi plus e 2 cos phi; e r and e phi are mutually perpendicular vectors.

Now, I can write u in terms of e r and e phi, also note that this radius vector is r, this radius vector is r, that means that r is equal to square root of x 1 square plus x 2 square; and x 1 is equal to r cos phi, x 2 is equal to r sin phi. So, these are the components of the x 1 and x 2 in terms of r and phi. So, u vector is equal to u r times e r, so u vector is equal to u r times e r is equal to u r into e r is e 1 cos phi plus e 2 sin phi, which means that u 1 is equal to u r cos phi and u 2 is equal to u r sin phi ok.

So, writing cos phi and sin phi in terms of x 1 and x 2, if you are writing cos phi and sin phi in terms of x 1 and x 2; you get that u 1 is equal to u r times x 1 by r plus and u 2 is equal to u r times x 2 by r. That is the velocities along the outward radius vector, if the velocity increases proportional to r itself, if the velocity increases proportional to r itself, that is if this is equal to some constant times r times e r. This is equal to some constant times r times e r, that is as we go outward the velocity is increasing with distance outward, it is increasing proportional to r itself. So, then for this case, you will get u 1 is equal to c x 1; u 2 is equal to c x 2, and what is the divergence of u? it is partial u 1 by partial x 1 plus partial u 2 plus partial x 2.

So, that means the divergence of u is equal to partial u 1 by partial x 1 plus u 2 by partial x 2 is equal to 2 times c. So, the divergence is non zero only, if the velocity is increasing as you go outwards ok. As you go outwards the velocity keeps increases, that means the divergence of u is non zero, that means the velocity field is diverging from this point as you go outwards. If divergence of u 0 there is no net flow out of this surface, so that is the physical meaning of divergence, there can be a net flow out of a surface only, if there is some source within that surface, which is generating that net flow. So, the divergence is non zero only when you have a net source of fluid somewhere within the surface.

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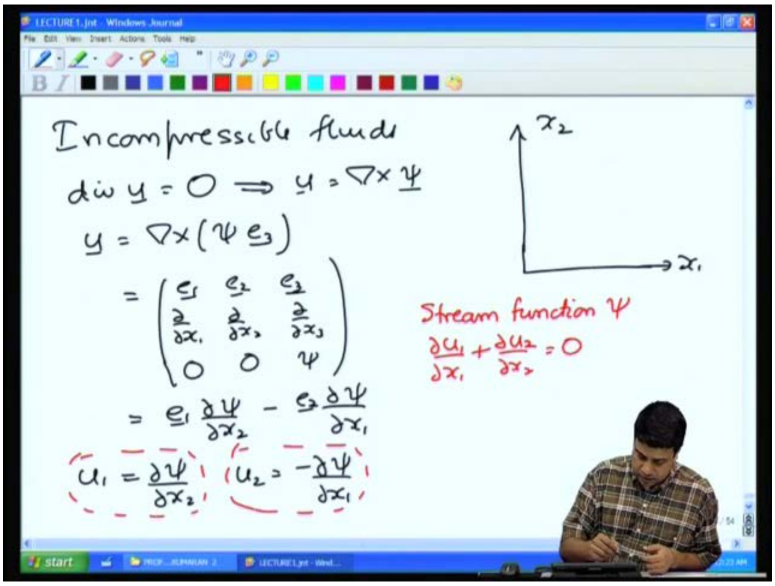

For incompressible fluids, divergence of u is equal to 0, if the divergence of u is equal to 0 that is the fluid is incompressible, if you recall what I said in the last lecture, this means that u can always be written as the curl of some vector. Let me say that u, can always be written as the curl of some vector, so this does not have a simplification in general. Because, all that means is that you are writing one vector in terms of another, one vector is being expressed in terms of some other vector, so you are not reducing the complexity of the problem. However, it does result in a significant simplification, if the flow is two dimensional ok.

If the flow is two dimensional, so that the velocity is only in one of two directions, so velocity is only in one of two directions, we can have a velocity in the x 1 x 2 plane only, if I take the curl of a vector that is perpendicular to that. Because, the curl of a vector is perpendicular to that vector itself, so if I have to take the curl of a vector and get a velocity in the x 1 x 2 plane, that means this vector itself has to be perpendicular to that. So, that means that in the x 1 x 2 plane, this velocity can be written as del cross of a vector, that is in the third direction, so what this will mean is that u 1 is equal to… Let us, let us expand this out in order to make the point clear, so this will be equal to e 1 e 2 e

3 d by d x 1 d by d x 2 d by d x 3. And this vector has a component only in the x 3 direction, this component has this vector has a component only in the x 3 direction ok.

So, this gives me e 1 partial psi by partial x 2 minus e 2 partial psi by partial x 1, e 1 partial psi by partial x 2 minus e 2 partial psi by partial x 1. This is in a cartesian coordinate system, in other coordinate systems you just use the equivalent of the curl that, I derived for you in the last lecture in terms of the scale factors. So, this means that the velocity in the one direction is equal to partial psi by partial x 2 and u 2 is equal to minus partial psi by partial x 1.

This psi for these two-dimensional flows, this is what is called the stream function, this psi for these two dimensional flows is, what is called the stream function, you can easily see that with these substitutions. The velocity field identically satisfies partial u 1 by partial x 1 plus partial u 2 by partial x 2 is equal to 0, it identically satisfies that. Because when I take partial by partial x 1 of partial psi by partial x 2 plus partial by partial x 2 of minus partial psi by partial x 1, which identically satisfies that equal to 0 good.

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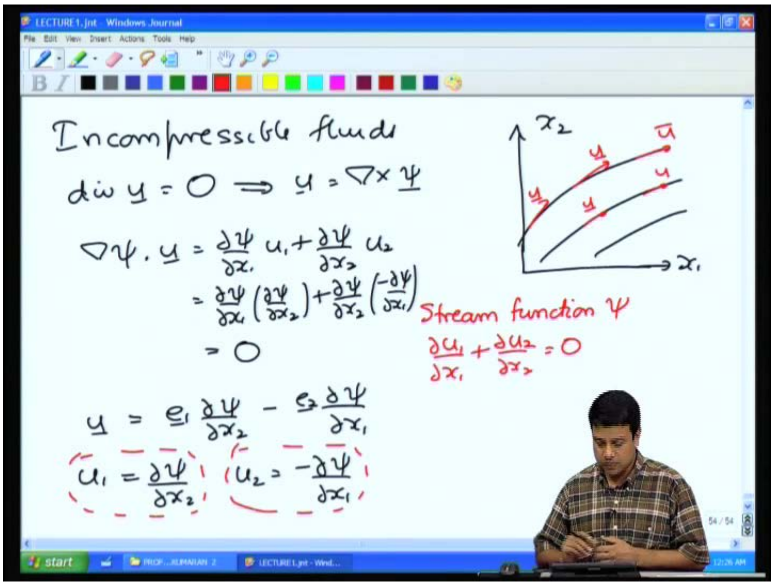

So, what does the stream function physically mean? what does the stream function physically mean? let us look at if I take the gradient of psi and dotted with the u vector, if I take the gradient of psi and dotted with the u vector. This becomes partial psi by partial x 1 u 1 plus partial psi by partial x 2 u 2; u 1 is partial psi by partial x 2, and u 2 is minus partial psi by partial x 1, this is equal to 0. So, that means that grad psi dotted with u is equal to 0, grad psi note e is the direction of maximum variation of psi, it is perpendicular to surfaces of constant psi.

In this case, since we are dealing with a two-dimensional coordinate system, it is perpendicular to lines of constant psi, because the surface in two dimensions is just a line. That, if grad psi dot u is equal to 0, grad psi dot vector is 0 only, if that vector is along surface of constant psi that means that u is a surface of constant psi, u is parallel to a surface of constant psi ok. Because psi if you take grad psi dotted with delta x, that will be 0, only if delta x is along the direction in which psi is a constant grad psi dot u is equal to 0, that means that u is along the direction; where psi is a constant.

That means, if the velocity vector is tangent to lines of constant psi, the velocity vector is tangent to lines of constant psi. So, these things are called stream lines, lines of constant stream function or called the stream lines, the velocity is tangent to these lines of constant psi, which are the stream lines. So, if the velocity vector is parallel to the stream lines, that means that there is no flow perpendicular to the stream lines, so along the stream line there is no flow perpendicular to the stream lines.

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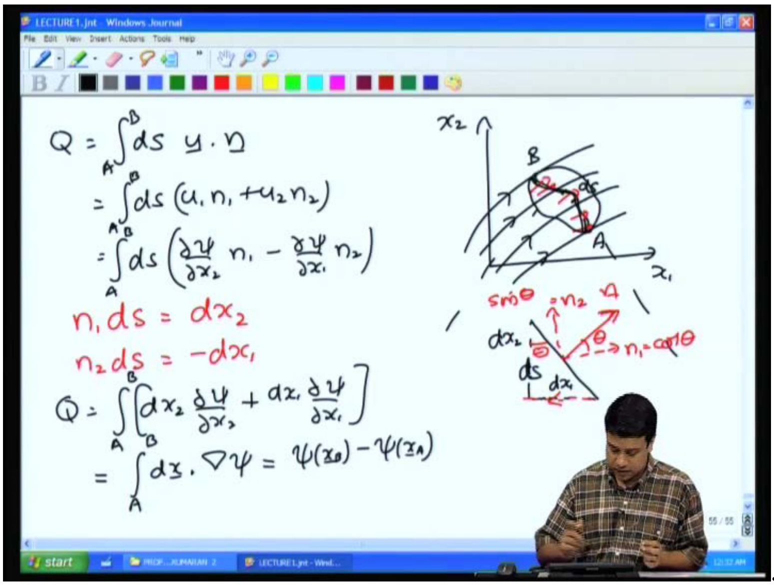

The stream function also has an another important physical implication for two dimensional flows, if I take a bunch of stream lines, and I take two points and I take some surface, some surface between these two locations. Note that, the velocity is along these stream lines, so what is the integral between A to B, integral between A to B of, so if I want to find out, what is the net fluid flow? That is coming out of this, as you do not find out, what is the net fluid flow? That is coming out of this surface.

The net fluid flow were in two dimensions, so this is per unit length in the perpendicular direction, so if I had the three dimensional system, this may be an integral over the surface, it is in this two-dimensional, it is only over a line. Q is equal to integral d x where x is let me write it as s to avoid confusion, integral d s that is along the path some differential element along this path is d s. So, integral d s of u dot m, integral d s of u dot m, so this I can write in the two directions as d s u 1 and 1 plus u 2 into surface of u 1 is partial psi by partial x 2 and 1 plus minus. Now, if I take a small differential element along this path, and I expand it out, so this is my element d s, let me write it little higher.

So, this is my element d s with unit normal n with unit normal n with components n 1 and n 2 now, this d s represents an increase or decrease in x 1 and increase in x 2. So, there is a displacement. There is a displacement along the x 1 direction of minus d x 1, because going on negative direction, so d x 1 is negative, and so displacement d x 1 is going in the negative direction. And then, there is a displacement in the x 2 direction d x 2. So, this if this is the angle theta, so this is the angle of theta between n 1 and n, s is perpendicular to n 1, and x 2 is perpendicular to n 1, and s is perpendicular to n. So, this is also equal to theta ok, so clearly for this particular configuration since the angle between n 1 and n is the same as the angle between x 2 and s; you require that n 1, n 1, n 1 d s is equal to d x 2 ok.

That is n 1 is equal to cos theta, cos theta and n two is sin theta ok, n 2 is sin theta, n 1 is cos theta. So, n 1 d s is equal to d x 2 and n 2 d s, n 2 d s will be minus d x 1, because when n 2 is positive, the displacement x 1 is in the negative direction. So, n 2 d s is equal to minus d x 1, substituting these into the expression for n 1 and n 2, you get Q equal to integral n 1 d s is d x 2 partial psi by partial x 2, and n 2 d s is minus d x 1; you will get d x 1 partial psi by partial x 1 between these two locations A and B. This you can easily identify, this is integral between A and B of d x dotted with grad psi, d x vector dotted with grad psi in two dimensions, so this is equal to integral between A and B of d x vector dotted with the gradient of the stream function. So, this we know from the integral theorem for gradients is just equal to psi at x B minus psi at x A.

So, the difference in stream function between two positions tells you, how much net fluid flow? There is for any surface that connects those two positions from the, from the integral theorem for the gradient, that flow along any surface has to be exactly the same; you take u dot n across along any surface, so it joints these two positions A and B to be this surface, it will be that surface to be any other surface. I take the net flow across that surface u dot n d s, that will be the same, and it will be equal to the difference in the stream functions between the two end points.

Of course, along a stream line itself, the stream function is the same, that means that there is no net flow that is going across that stream line. Because, the difference in stream function on two points on the stream line is 0, so there is no net flow going across the stream line ok. So, that is stream function and stream line, so they those emerged from the definitions of the divergence of the velocity filed.

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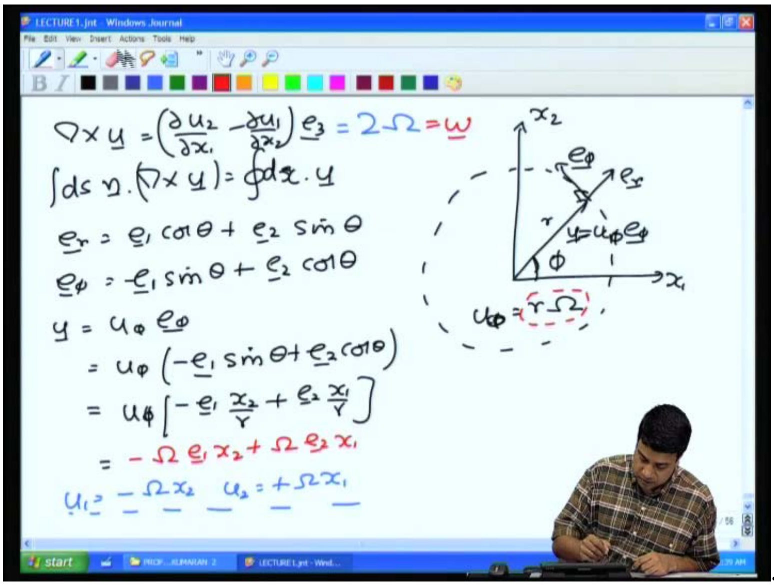

The other thing, that emerges is from that curl of the velocity field, so if we take del cross u, once again we will do it in a two dimensional coordinate system. Let us say that a along some surface, I have a velocity that is now tangential to the surface, so that u is equal to u phi times e phi. So, the velocity is in the phi direction, this is r, and this is phi now once again, the integral theorem for this is that, integral d s n dot del cross u equal to integral over surface d x dot u along the contour. That is probably in that surface, so you take any surface in the three-dimensional plane whose contour is a circle along the x y direction. So, let us do a little calculation once again for this particular value for this particular velocity field, what will be the curl?

So, we know that in this coordinate system, I have my coordinates and my coordinates e r e phi, and I have relations between e 1 and e 2 and e r and e phi, e r is equal to e 1 cos theta plus e 2 sin theta; e phi is equal to minus e 1 sin theta plus e 2 cos theta. Now, if I take a body that is in solid body rotation, that means if the linear velocity at any distance r, the linear velocity u phi at any distance, r is equal to r times the angular velocity. The angular velocity omega times r is the linear velocity at any position, distance from the center. So, this is a solid body that is rotating about this central axis, so velocity field is given by u is equal to u phi e phi; which is equal to u phi into minus e r sin theta plus e theta cos theta. Oh sorry, I should write e 1 sin theta and this should be 1 minus e 1 sin theta plus e 2 cos theta.

Since, sin theta is x 2 by r u phi u 2 minus e 1 x 2 by r plus e 2 x 1 by r, because x 1 is equal to r cos theta, and x 2 is equal to r sin theta. Now, I should write these as some scalars, and if I use this form for solid body rotation form for phi, I will get this is equal to minus omega e 1 x 2 plus omega e 2 x 1, because I used u phi is equal to r times omega; which means that the velocity field is u 1 is equal to minus omega x 2, u 2 is equal to plus omega x 1. So that is the velocity field in the Cartesian coordinate system, one thing you can immediately see for this velocity field the divergence of the velocity is equal to 0; partial u 1 by partial x 1 plus partial u 2 by partial x 2 is identically equal to 0.

Now, next if I take the curl of the velocity field, del cross u is equal to if I can expand it out, for because I do not have velocity in the x 3 direction. So, I just have velocity in the x 1 and x 2 direction, this will turn out to be partial u 2 by partial x 1 minus partial u 1 by partial x 2 in the e 3 direction. I take once again e 1, e 2, e 3, d by d x 1, d by d x 2, d by d x 3, and u 1, u 2, u 3; u 3 is of course, equal to 0 and there is no derivative with respect to x 3 direction. So, I will get only the partial u 1 by partial x 2 minus partial u 2 by partial x 1 times e 3. As expected, this curl is perpendicular to the direction of the plane of flow, it is perpendicular it is in the x 3 direction perpendicular to the velocity direction, because the curl of a vector is always perpendicular to that vector itself.

And if I substitute these values, you can very easily see that, this becomes just equal to u 2 is proportional to x 1, so partial u 2 by partial x 1 will be omega, and partial u 1 by partial x 2 gives you minus omega. so this will add up to give you 2 omega, so the curl of the velocity field at a point at for a solid body rotation on any contour; note that this curl that I got for this solid body rotation is independent of r now, its independent of distance from the origin.

It is perpendicular to the direction of the velocity variation, and it is equal to 2 times the angular velocity, this is also called omega the verticity, the local verticity at the flow. So, the solid body rotation is equal to half the local verticity, if I take the curl of the velocity locally calculate that, the angular rotation the solid body rotation around a point is equal to 1/2 the verticity. And of course, for this particular case the stokes theorem, the theorem for curl should apply and you can easily show the vectors.

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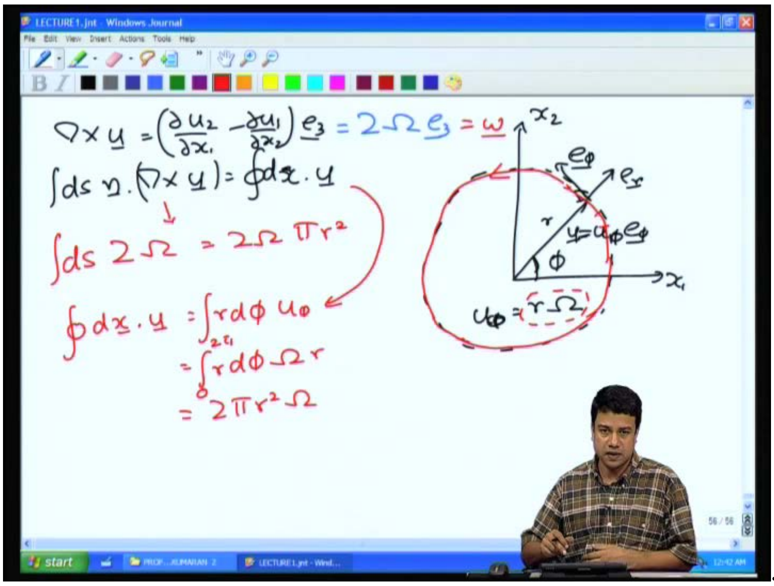

Integral of n dot del cross omega will be equal to integral of the surface of n is the unit vector perpendicular to the surface, in this particular case the unit vector perpendicular to the surface must give this, the unit vector. The unit vector perpendicular to the surface in the e 3 direction, so I just get 2 omega e 3 dotted with e 3 itself, this just becomes 2 times omega. Integral of the surface and for this circular surface, this just becomes equal to 2 omega times pi r square that is plus 1.

If I calculate the circulation, so this thing is what is called the circulation? Integral of d x dotted with u, d x now is along the perimeter of the surface area, d x now is along the perimeter of the surface. Then, the incremental distance along this perimeter is integral r d phi of u phi, which was equal to omega times r, and you can easily see this phi goes from 0 to 2 pi as goes on a circle. So, phi goes from 0 to 2 pi and you will get 2 pi r square omega, which is exactly what I had, when I get that integral of the curve over the surface. Because this is basically stokes theorem for this particular case, so there is a net verticity only, if there is circulation around the surface.

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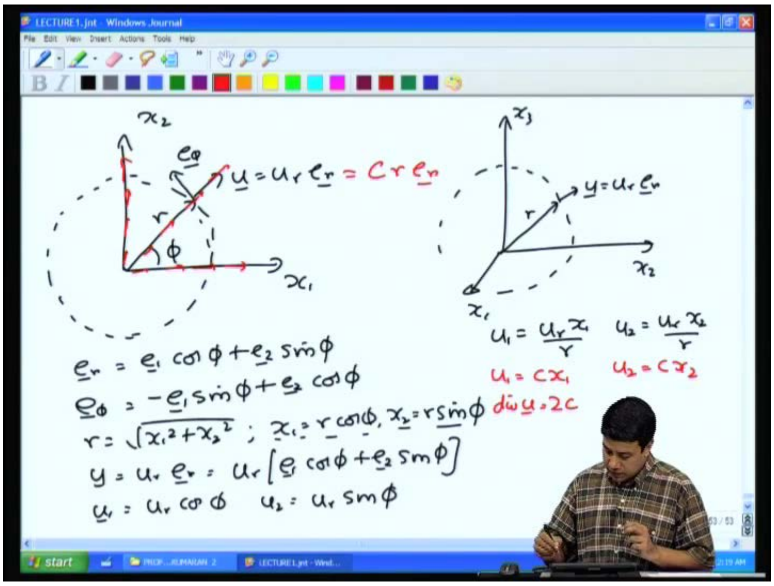

And I taken my previous velocity field, the previous velocity field which had a divergence, if I had this outward velocity field, and if I integrate that over the entire surface. And if the curl of that, I have integrated with I have to yet got identically equal to 0. So, divergence tells you whether there is fluid coming in or out from the surface, curl tells you, if there is fluid that is circulating around that surface, so those are the physical interpretations of divergence and curl.

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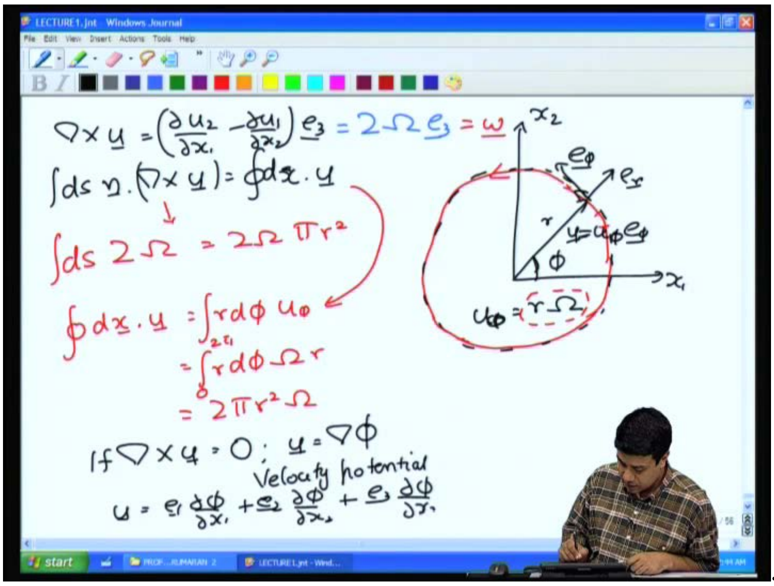

If the curl of the velocity field is equal to 0, I told you in the previous lecture, that if the curl of any vector is equal to 0, that vector can always be represented as the gradient of a scalar function. So, if the flow is irrotational, so that the curl of the velocity field is equal to 0, if the flow is irrotational, so that the curl of the velocity field is equal to 0. That means that if del cross u is equal to 0, then u can always be written as the gradient of some scalar function. So, for irrotational flows for which the curl of the velocity field is equal to 0, that velocity field can always be written as the gradient of some function, this function is called the velocity potential.

It is called the velocity potential, the gradient of this potential gives you the velocity field at any point, for irrotational flows the velocity potential is well defined in three dimensions. So, in three dimensions we get the same result for the velocity potential, as in two dimensions, so this for irrotational flows alone we can write a velocity potential. And you can show that the potential lines is the lines along which the potential varies, and the lines for the stream function or both are troubling to each other. So, if I expand out velocity potential, I will get u vector is equal to e 1 partial phi by partial x 1 plus e 2 by partial x 2 plus e 3 partial phi by partial x 3.

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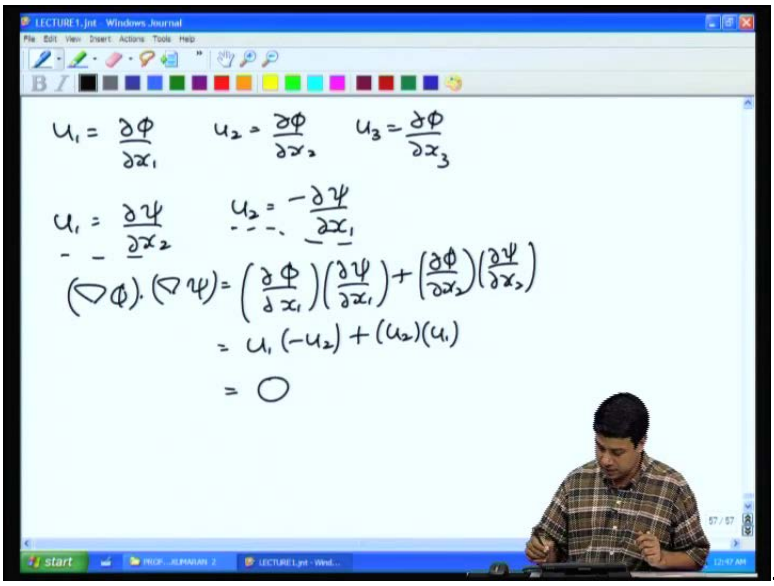

Therefore, I have the velocity components u 1 is equal to partial phi by partial x 1, u 2 is equal to partial phi by partial x 2, u 3 is equal to partial phi by… Now, when I had expressed my velocity in terms of stream function, when I had expressed my velocity in terms of stream function, I said that u 1 is equal to partial psi by partial x 2, u 2 is equal to minus partial psi by partial x 1. So, if I take the dot product of the velocity gradient and the gradient of the stream function, if I take the dot product of the velocity gradient and the gradient of the stream function.

You find that grad phi dotted with grad psi is equal to partial psi partial, partial phi by partial x 1 is u 1, partial psi by partial x 1 here is minus u 2, partial psi by partial x 1 is minus u 2 plus partial phi by partial x 2 is u 2, partial psi by partial x 2 is equal to plus u 1. So, this is identically equal to 0 therefore, the gradients of the stream function or the gradients of the potential are perpendicular to each other, or since the lines of constant stream function are perpendicular to the gradients. Lines of constant potential are perpendicular to the gradient of the potential, the lines of constant potential, and the lines of constant stream function always intersect perpendicular to each other in the fluid flow ok.

This is not surprising, because I showed you earlier, that the velocity was equal to the gradient of the potential, it was equal to the curl of the stream function for two dimensional flows. So, it is, it is not surprising these two are perpendicular to each other ok. So, this gives you some ways of understanding, what are what are the the the divergence and the curl of the velocity? What they physically represent? The next step is to look at the gradient of the velocity. As I said you can always take the gradient divergence curl of any vector tensor, divergence is restricted to vectors and tensors, gradients you can take of scalars as well but, you cannot take that the gradient of a vector as well.

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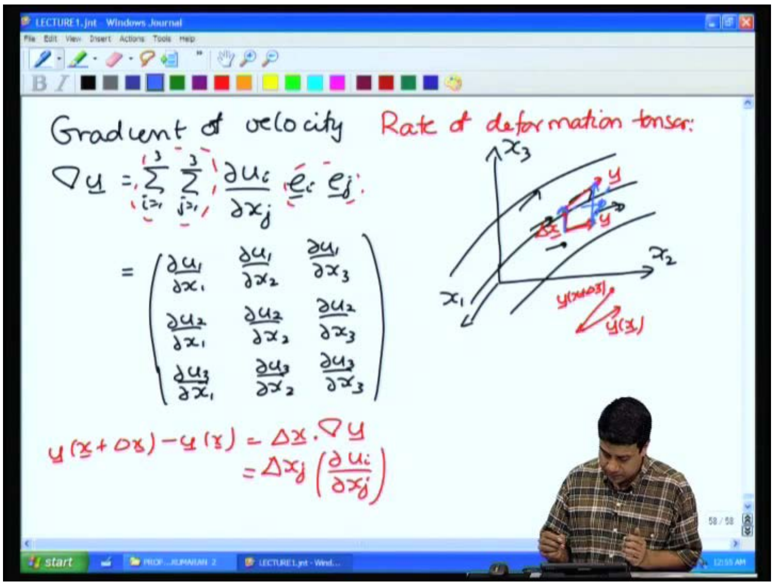

So, gradient of velocity grad u, it has now 9 components; so if I, if I write it out indicial notation is equal to partial u i by partial x j, there is no repeated index. That means that this equal to summation i is equal to 1 to 3, summation j is equal to 1 to 3 of partial u i partial x j, e i e j the gradient of the velocity field. We do not usually write these two out in long form, where we just do indicial notation, because there are two unrepeated indices, which understood that there are two unit vectors and two summations. I can write it in matrix form partial u 1 by partial x 1, you can write out in matrix form as well, this is also what is called the rate of deformation tensor, is called the rate of deformation tensor; what is the physical implication of this tensor? What is the implication of this tensor?

If I have a coordinate system, there is some velocity field with a velocity vector, different at each point in the flow; this is some fluid flow that is taking place, if I look at one particular location. If I look at one particular location has some velocity u, I have velocity u, I go small distance delta x in some direction, I go small distance delta x in some direction, there is a small change in the velocity u. So if I take those velocity vectors, there is a velocity u here at x, and I go small distance away, this velocity has changed a little bit. So this is become u at x plus delta x, and you know from the definition of the gradient, that u at x plus delta x minus u at x is equal to delta x dotted with grad u.

A small change in velocity, when I travel a small distance velocity itself is a vector but, still I can define the change in velocity by subtracting out two vectors; where my initial velocity vector, the final velocity vector I subtract of the 2 and I get the difference. That is equal to delta x dotted with grad u, I have to be careful here, because we are dotting a vector with a tensor, so I have to write this in indicial notation, as delta x j times partial u i by partial x j. That is the dot product is with the displacement vector, and the gradient vector not the displacement vector, and the velocity vector.

So, this gives me the difference in velocity for two points that are nearby, what is the implication of this? if this grad u, u are equal to 0; there is no change in velocity as you move. So, we have one point which is going with some velocity, and adjacent point is moving with exactly the same velocity. So, as time progresses the difference between those two points will remain exactly the same; that means that the material line that connects these two points does not change it, does not stretch or rotate. On the other hand, if there is a difference in velocities between those two points; that means that the material line element; that is joining those two points is either expanding contracting or rotating.

Now, there is a principle of Galilean invariance which states that, if you have reference frame that are moving with respect to constant velocities with respect to each other, you cannot generate internal stresses within, within a system. If you just move the whole system at a constant velocity, in other words if I was having some fluid flow experiment that was going on in some container. If I moved that container with constant velocity, every point within the flow rate moves with that exact same velocity. So, there is no deformation taking place, because the fluid moves with a constant velocity. And you require deformation of material elements in order to generate stresses within the flow rate. That means the stresses cannot be generated, if the entire equipment is moving with a constant velocity.

If I have stresses in a certain fixed reference frame, I take that same experiment, and I move it, move the reference stream with a constant velocity, I cannot generate stresses within the system. If the gradient of the velocity is equal to 0, that means that the velocity does not vary, as I go from point to point within the system, velocity is a constant, their cannot be stresses. There can be stresses only, if there are gradients, so when there are gradients; that means that nearby points are not moving with the same velocity. They are moving with different velocities relative to each other, and when nearby moved with different velocities relative to relative to each other. Then there are stresses that are generated due to the stressing of material elements, due to the deformation of material elements.

So, this tensor which basically gives us the rate, at which nearby points are moving with respect to each other, gives you the rate at which deformation is taking place, and that is why it is called the rate of deformation tensor. And obviously, any stresses that are generated within the flow, cannot be related to the velocity itself, because the velocity if you move reference stream with a constant velocity, there should be no internal stresses, that are generated during the flow.

So, any stresses that are generated within the flow have to be related to the rate of deformation tensor, probably so I have this rate of deformation tensor, and I have expressed in terms of its components. It is a little bit central in our discussion of fluid mechanics, because all the stresses ultimately will be related to this form. So, if the difference in velocities has a component along the line joining the centers, along the distance between the two elements. If there is a component along that, that means that this material element is prettily being stretched, if the difference in velocities has a component along the direction, this material element is being stretched.

On the other hand, if the difference in velocities has a component, that is perpendicular to this direction, perpendicular to the direction, that means that this element is trying to get rotated, it trying to get rotated. So, as different kinds of deformation, one is stressing the other is rotation. Now, how do you describe these kinds of deformations? without reference to a fundamental underline coordinate system, so that will be next topic of our discussion.

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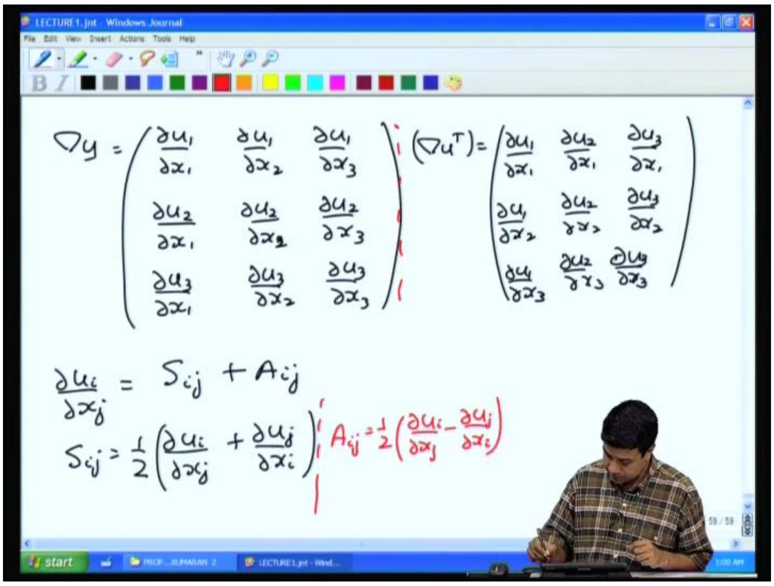

So, this rate of deformation tensor, this is second order tensor, it has a total of 9 elements, it is a second order tensor; which has a total of 9 elements. This matrix any matrix can be written as, a sum of a symmetric and an anti-symmetric product, any matrix can be written as, the some of the symmetric and an anti-symmetric product. So, if I write this in indicial notation, this becomes partial u i by partial x j, can always be written as the sum of a symmetric and then anti symmetric products; S i j plus A i j, where S is a symmetric part, and A is the anti-symmetric part.

So, we write for example, if I write this matrix, as grad u this can be written as S matrix plus A matrix, how do you get the symmetric matrix? We take the matrix grad u add the transpose of this matrix and divide by 2. So, if I want the symmetric matrix, what I do is S i j, I take this matrix, and I add the transpose, what does transpose mean? That means that I change the rows and columns. The row becomes the column and the column becomes row, so what would be the transpose of this matrix? So, let me write this as is the transpose, grad u transpose is equal to partial u 1 by partial x 1 that is the transpose. In indicial notation, I am changing the rows and columns, in this particular matrix the row index was i, because u i by partial x j, and the column index was j, so I just interchange the two.

And so, the transpose will simply becomes partial u j by partial x i, and take half of this, so this is the symmetric part. Similarly, you can get the anti-symmetric part by just taking the matrix taking its transpose, subtracting its transpose, and dividing by 2; A i j is equal to half partial u i by partial x j minus partial u j by partial x j. For a symmetric matrix there are 9 elements of course, but since the half diagonal elements are equal to each other; you get total of 6 independent elements. That is the 3 diagonals plus 3 half diagonal elements, the other 3 are equal to the first 3, because the transpose of the matrix equal to x i.

For the anti-symmetric matrix, the transpose is equal to negative of itself therefore, the diagonal elements are 0, and the 3 half diagonal elements are equal to the negative of the other 3 half diagonal elements. So, that means there are only 3 independent elements in an anti-symmetric matrix, 6 in a symmetric matrix, sum of these two gives me total 9 elements in the original matrix. And so, this rate of deformation tensor decompose in the symmetric and anti-symmetric matrices, and I can further decompose the symmetric matrix into two parts; one is what is called a trace an isotropic matrix, and the other is what is called a symmetric trace less matrix.

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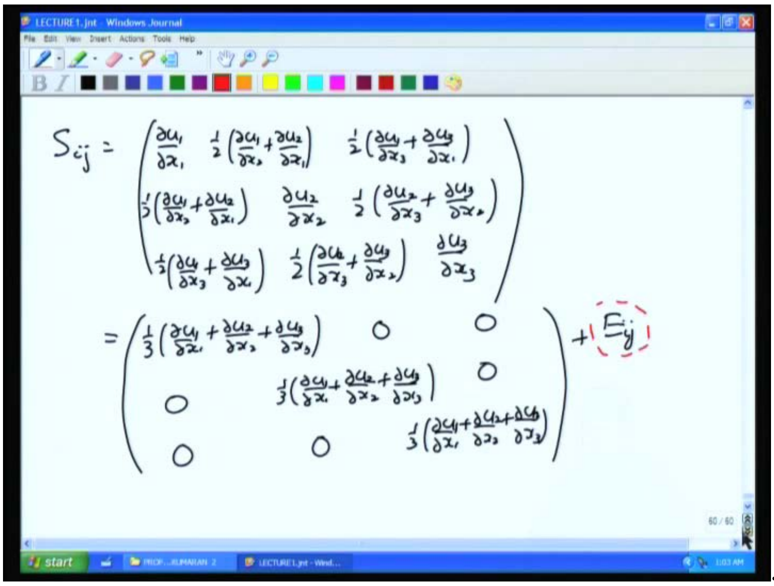

So, S i j if I write it out on long hand, this is partial u 1 by partial x 1, 1/2, it is the symmetric matrix, and I can write it as the sum of two parts, one is, one matrix which is just an identity matrix, just one-third. This is what is called an isotropic matrix; it is proportional to the identity matrix, the diagonal terms are all equal off diagonal terms are all 0 plus the second matrix, which I will call E i j. Note that, the sum of the diagonal elements of this matrix, this isotropic matrix is identical to the sum of the diagonal elements of the symmetric matrix. So, we subtract out this from this symmetric matrix, the sum of the diagonal elements of that matrix will be identically equal to 0.

Therefore, if the sum of the diagonal elements of a matrix is called the trace of that matrix therefore, the trace of this matrix which is basically the symmetric matrix minus this isotropic matrix is identically equal to 0. So, it is called as symmetric trace less matrix, so I have decompose my rate of deformation tensor into a symmetric trace less, an anti-symmetric, and an isotropic matrix. If you recall originally, I decomposed into a symmetric an anti-symmetric, that symmetric further I decomposed into an isotropic and a symmetric trace less matrix. Why did I do this? Because each of these matrices represents a different type of deformation; we will discuss it in the next lecture, what types of deformations that these represent? These types of deformations are independent of the underline coordinate system, that you using to analyze the problem.

So, if I have this rate of deformation tensor, I find out separation to these three different matrices, these types of deformation will be independent of the coordinate system used. Even though, in individual components of this rate of deformation tensor, may depend upon the types of components used, so the next lecture we will continue this.

First we look at a two dimensional system, and see what kinds of deformation each of these matrices represent, and then we will progress to a three dimensional system. And then look at what are the dimensions? What are the types of deformations in three dimensions? This is central, because alternately the stresses will all be related to this rate of deformation tensor later on in the flow. So, kindly go through this, and brush up on your knowledge of matrices, so that I can proceed with this, relate this to the rate of deformation tensor in the next lecture, we will see with that.