Fundamentals of Transport Processes

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Lecture No. # 11

Unidirectional Transport Cartesian Co-ordinates- IV (Separation of Variables)

Welcome to this, the eleventh lecture in the series on fundamentals of transport processes, where we were trying to formulate a framework for analyzing transport processes of heat, mass, and momentum transfer. We were looking at unidirectional transport. That is transport takes place only in one particular direction. As I said in general, in a general flow field, in a reactor or in a heat exchanger, the velocity is a function of all three coordinates, as well as the function of time. And that is complicated, because it results in partial differential equations for the governing equations for the concentration, momentum or temperature fields.

So, we are trying to solve first the simpler cases - the simpler cases of transport only in one direction. And we were trying to formulate a framework for how to analyze in a common framework, mass, momentum and energy transfer.

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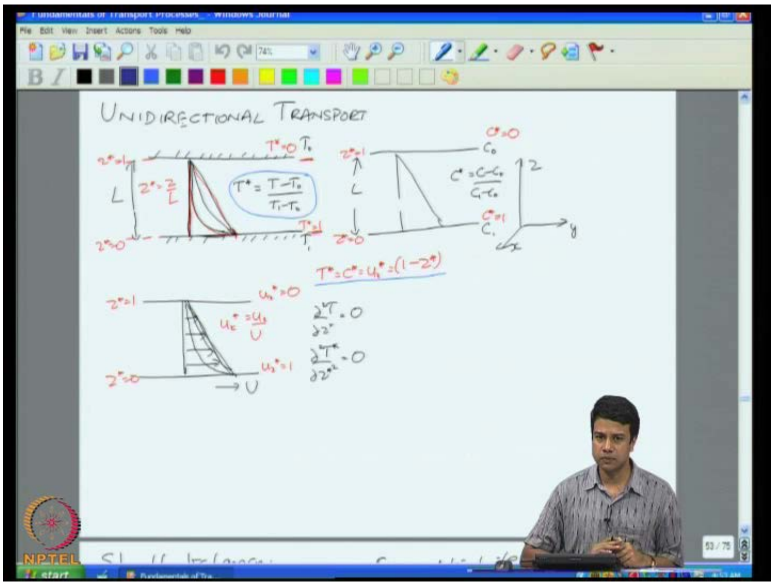

So, the basic configuration that we are looking at, look something like this. We are looking at transport in what is called Cartesian coordinates. The x and y coordinates are along the plane. The z coordinate is perpendicular to the plane. And we considered flat surfaces, the system to be bounded by flat surfaces, at in the z direction. So that, everything there is nothing that is changing in the x, and y directions, everything is the constant in those two directions, there is transport taking place in the z direction. I will go through the derivation of transport equations once again, because there are some additional things that we will need as we go along in this lecture, which were not covered previously for simplicity.

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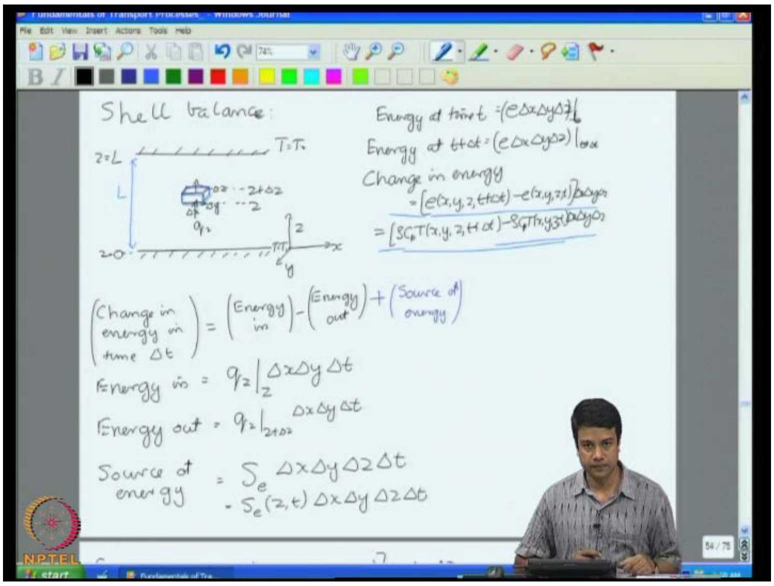

So, the basic shell balance equation was change in energy in the time delta t is equal to energy in, within the time minus energy out, with in the (( )) same time delta t. This equation is valid, so long as there is no protection or dissipation of energy within this differential volume. If there is a protection of consumption of energy, it is necessary to add on additional term in this energy balance equation. One could envisage, for example, reaction taking place within this differential volume. And if it is exothermic, it will generate heat. If it is endothermic, it will consume heat, thus, going to be either a source of energy or sink of energy during this differential volume.

There could be dissolution processes. Dissolution processes can also be a exothermic or endothermic. One could have phase change, in which case, there is a latent heat of phase transformation which has to be included. And there could even be mechanical sources of energy.

For example, in a pipe if you have a viscous fluid flowing, then there is fluid friction, shear stresses. And that itself generates heat. And all the energy that goes into pumping that flow gets converted into heat energy, and thereby increases the temperature of the system. So, in general, if one more to include these energy production or consumption rates, one would have to modify this equation to include the additional term of the form, source of energy within this differential volume.

So, this additional source of energy, the energy there is produced within the differential volume, within the time delta t. And now the specific form of this term, will general depend upon the nature of the production of energy in this case. So, if it is viscous heating, it will have one form, if it is due to a reaction, it will have another form. But in all cases, this rate of production of energy within this differential volume is going to first of all is going to depend upon the volume itself. Because if you have energy produced within volume delta x, delta y, delta z, as I shrink this volume the amount of conversion going on due to reaction decreases. It is going to decrease in proportion to the volume, if that volume small compared to macroscopic skill. So, this source of energy is going to be proportional to volume itself, and the amount of energy generated is going to be proportional to delta t.

So, this additional source of energy, I will write it as source. It is going to be some function, times delta x, delta y, delta z, in the volume itself, times delta t, the time interval. Now, this S e could be in general a function of position as well as time. For example, if energy is produced due to a chemical reaction, the rate at which energy is produced is going to depend upon the local reaction rate. The local reaction rate may change, because the concentration changes, it may change because the temperature changes. So, therefore this rate of production of energy is going to depend upon the local value of the reaction rate. And therefore, it is in general, it is function of position as well as time.

However, in this series of lectures, we are considering a situation whether is variation only in one particular direction. And therefore, the rate of production of energy can be written as S e of z and t. So, it is going to be a function of both z coordinate in general as well as time.

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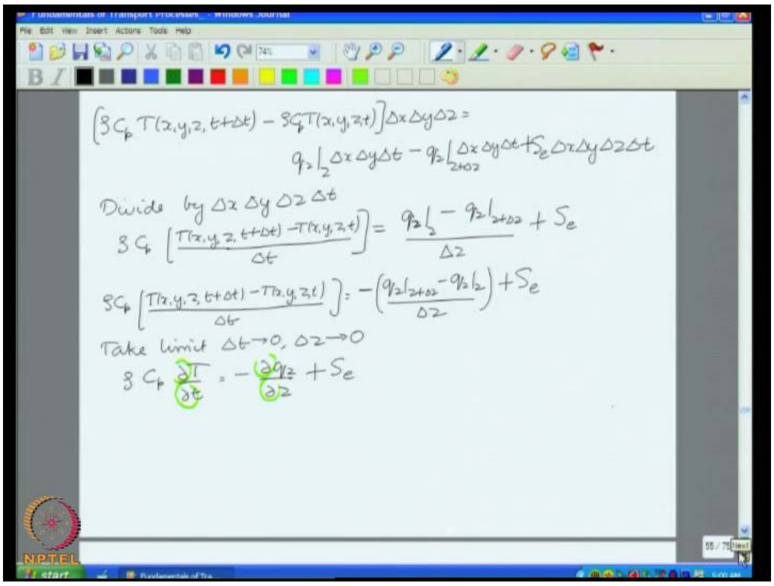

How does that change the balance equation that I had earlier? I have to add another term plus S e delta x delta y delta z delta t. This additional term due to the source, in this energy balance equation. And when I divide by delta x delta y delta z delta t, I just end up with this energy swifts. So, this is the additional source of energy that appears in the differential equation. And if I take the limit as the delta t goes to 0, and delta z goes to 0, this remains. This is this is the source of energy within the differential volume. Note that the amount of energy produced is equal to S times volume times time. Therefore, S is the amount of energy produced per unit volume per unit time, this amount of energy produced per unit volume per unit time.

If the energy will produced due to reaction, then we know what is the rate of reaction? The reaction rate per unit volume per unit time, times the the delta h of the reaction will give the source of energy. In similar manner, one can get the source for other kinds of (( )) heating or cooling due to absorption, due to dissolution, due to phase change, due to viscous heating. And all of these will have this particular form, S is the rate of production of energy per unit volume per unit time, locally within the flow.

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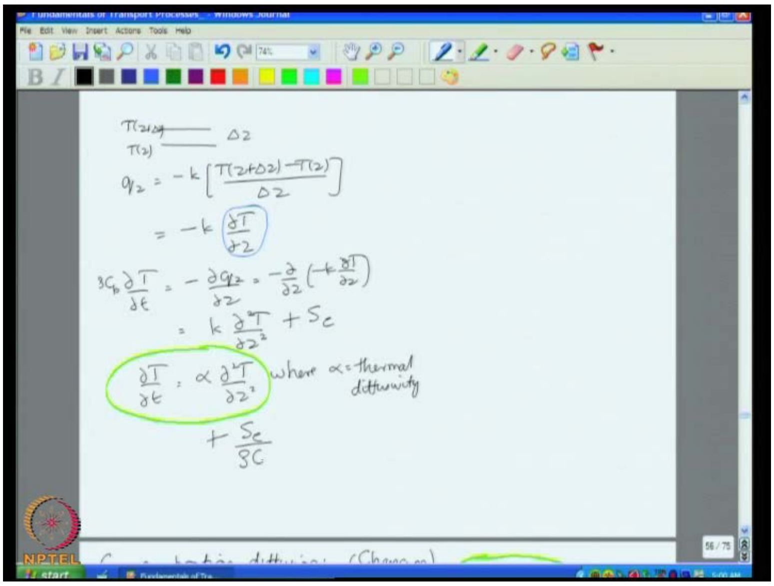

So, this is the modification that we have to make, whenever there is a source power dissipation of energy. And that results in modification of the conservation equation, and plus S e divided by rho C p. So, there is this additional source term in the energy balance equation. Now, similar source terms will appear in the mass balance equation as well.

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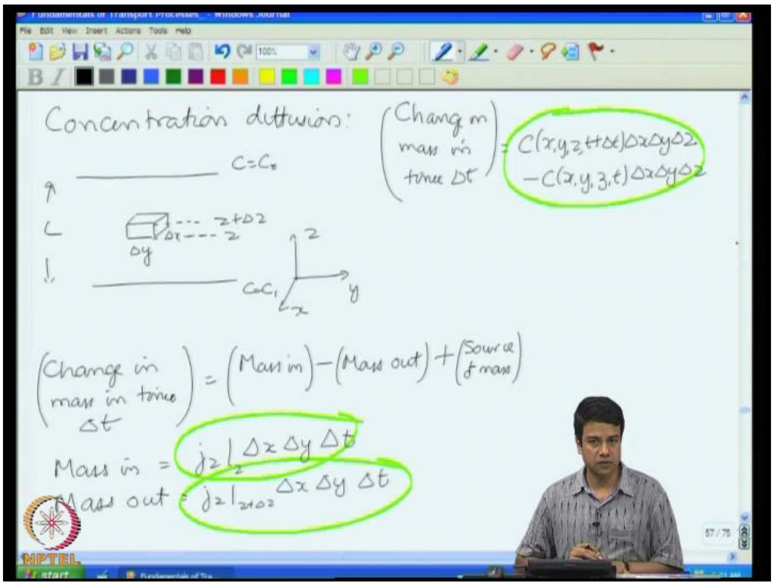

So, for example in this particular case, change in mass, in time delta t equal to mass in minus mass out plus the source. So, if the these species whose concentration is given by c, if that species is produced in a reaction, the source will be positive. If it is consumed in a reaction, the source will be negative. So, this is additional source of mass that also comes in to the mass conservation equation. That source of mass has the form, it is equal to some function S, which is in general function of position, times the volume, times delta t.

So, if the rate of reaction is given by r. So, that gives rate at which mass is produced per unit volume, per unit time within that differential volume. That is the local rate of production. So, that additional source of mass will be proportional to the volume itself, in the limit as the volume goes to 0. It is proportional to the time that I wait, because as the reaction proceeds, the amount produced is going to be proportional to delta t, the limit as the delta t is equal to 0. So, this is the additional source of mass that comes in due to reaction.

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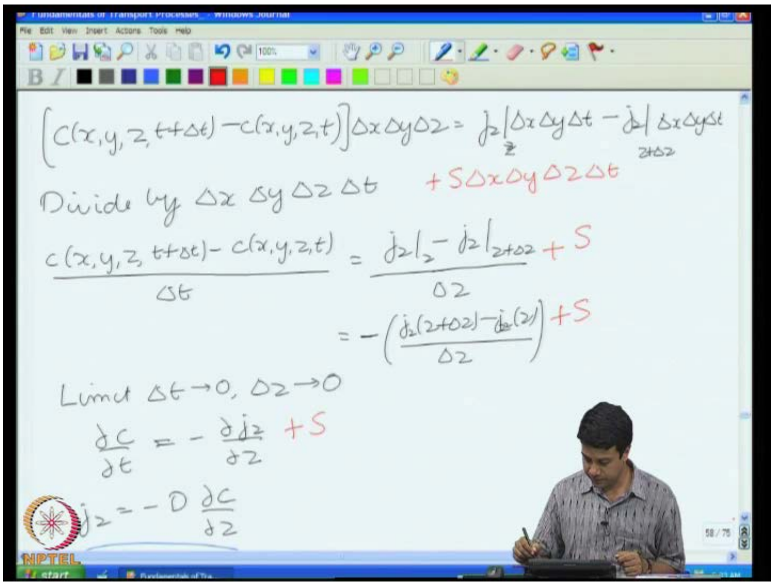

And in this mass conservation equation, this source comes in as plus S times delta x delta y delta z delta t. And if I divide throughout by delta x delta y delta z delta t, I get something that is equal to plus S, the rate of production of mass. And this; therefore this means the concentration field, an additional production term. And this source of mass is the rate of production of mass per unit volume, per unit time once again. So, this is the rate of production per unit volume per unit time. And you can see the dimensionally the center equation (( )) dc by dt, c is the concentration mass per unit volume, dc by dt is mass per unit volume per unit time. Therefore, S also has to have dimensions of mass per unit volume per unit time.

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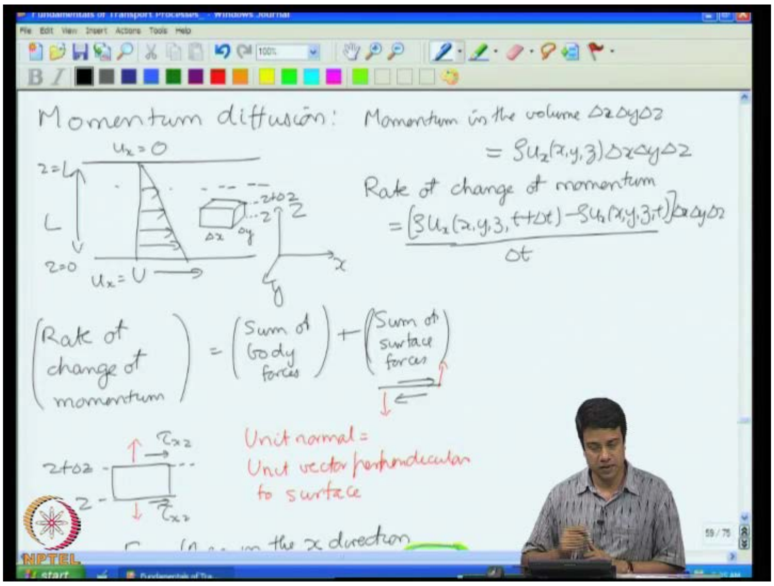

What is the equivalent source in the momentum conservation equation? That, the source the momentum conservation equation comes in the form of body forces. I wrote in the last two lectures go the some of the applied forces, forces are of two types: One is what are called surface forces, at any differential volume, they act at the surfaces at that volume. And the force applied is equal to the stress times to surface area or the pressure times to surface area. So, the some of the applied forces, I can divide it into two parts. One is sum of applied, sum of body forces plus sum of surface. The body forces act on the entire volume itself. An example is the gravitational force, the centrifugal force, various other kinds of forces, such as electrical or magnetic forces. They act on the entire volume.

So, if I had a body of mass m, the gravitational forces is the total mass time acceleration due to gravity. If I have a differential volume of fluid of volume delta v, then the gravitational force is equal to mass time acceleration due to gravity, can also be written as the density times the volume, times the acceleration due to gravity. So therefore, these body forces are proportional to the volume itself. The surface forces are proportional to the surface area. So, in that lecture we looked at surface forces, the shear stress acting on the top and bottom surfaces. I defined tau x z as a force by area in the x direction, acting it a surface with (( )) normal in the z direction, and from that I got for you, the forces on the top surface, and force in the bottom surface.

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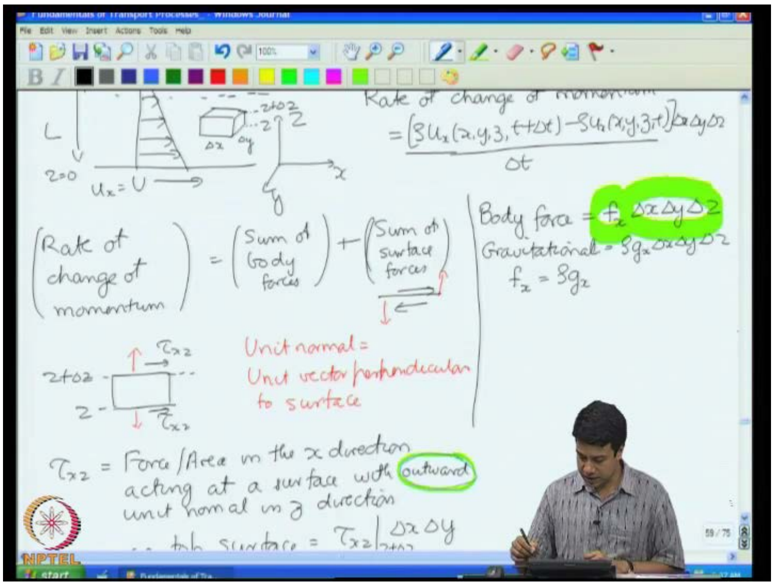

Now, what about the body forces? The body force will in general of the form, some force in this particular case, we looking at momentum balance in the x direction, for the x component of the velocity. Therefore, the body force will be of the form x, f x times delta x delta y delta z. The force the x component of body force, times delta x delta y delta z; so, it has the form of force density - force by unit volume times the volume itself. If this was the gravitational force, gravitational force will have the form rho times g x times delta x delta y delta z. g x is the component of the acceleration due to gravity in the x direction or in the flow direction. Recall, we were writing the momentum balance equation for the x direction or the flow direction. So, therefore the gravitational force will be rho g x which means that for the gravitational force, f x is equal to rho times g x. So, this has the form of force, per unit volume, the force density - the force density times that volume itself. So, this is the form of force density times the volume itself.

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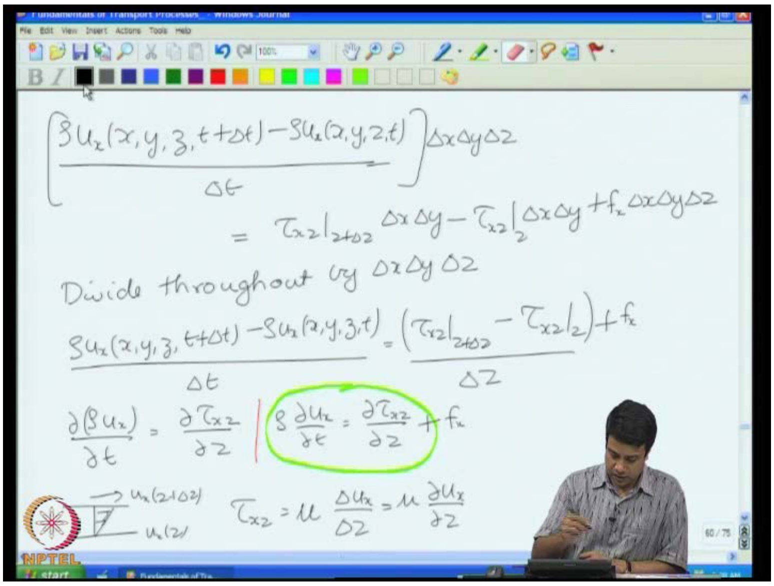

So, this can be put into the differential equation which I had earlier, plus f x delta x delta y delta z. I have a similar force form here, plus f x delta x delta y delta z, and this carries through to this equation plus f x, I am sorry, I am write this should be no delta x. When I divide throughout delta x delta y delta z, this delta x delta y delta z cancels, and I will just get f x.

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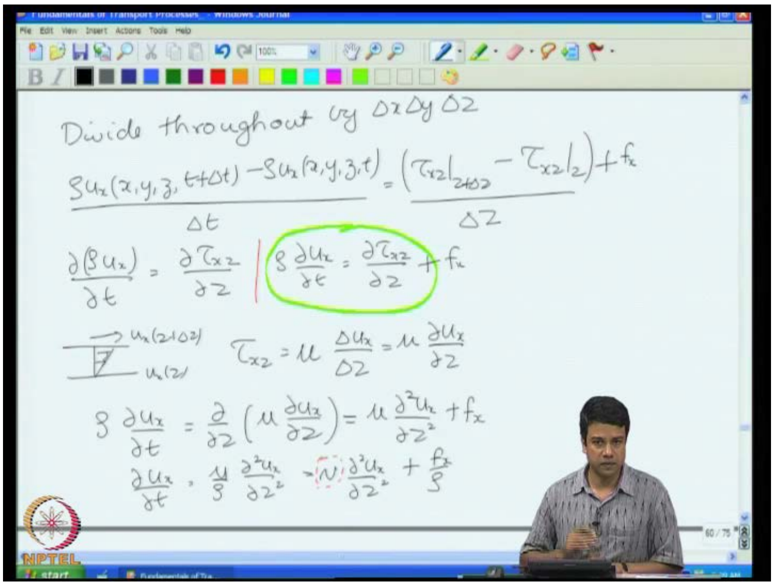

And therefore, by momentum conservation equation finally, is of the form, this plus f x or if we (( )) throughout by density f x by rho. Note once second, f x is the force acting per unit volume per unit time. I am sorry, the force acting per unit volume. In the previous cases, so force acting per unit volume is equivalent to momentum change per unit volume per unit time.

In the case of mass transfer, there was a source term, mass increase per unit volume per unit time. In the heat transfer problem has the source term, heat transfer per unit volume per unit time. In this case, it is a force per unit volume, which is momentum per unit volume per unit time. So, this is the effect of additional sources or sinks of body forces in the mass momentum energy conservation. In all three cases their exactly the same form. Even though, we calculated the momentum conservation equation, there is slightly different framework from the mass energy conservation equations.

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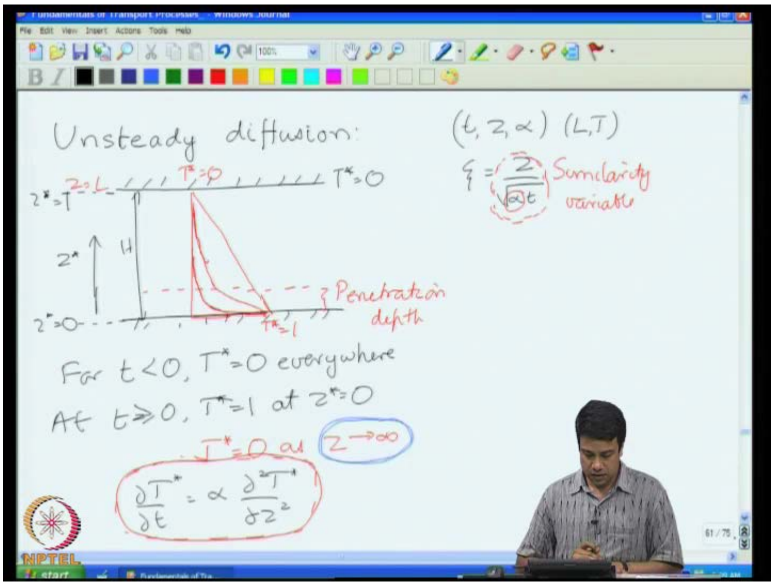

So, and then we looked at the unsteady diffusion. Of course, at steady state keep keep the temperature, temperature difference between two surfaces. In the final steady state, the solution is just the linear profile. We got that in terms of the scaled variables T star equal to 1 at z equal to 0, and T star equal to 0 at z equal to 1.

And the steady profile for exactly the scheme same, when expressed in terms of scaled temperature, concentration and velocity fields, T1 minus z. So, having got the steady state solutions, you keep two surfaces at two different temperatures, what is the temperature variations in between. Two surfaces at two different concentrations, what is the concentration variations in between? Two surfaces moving with different velocities, what is the velocity variations in between? At steady state, they have exactly the same form, that is because the equations of motion that we got in all three cases are exactly at the same form. And then we went on to look at the unsteady problem.

So, this was the problem, where I have two surfaces at equal temperature, the scaled temperature T star was equal to 0. The temperature in between the two surfaces was also equal to 0. So, everything was exactly the same temperature, the two surfaces as well as the medium in between. At time t is equal to 0, I increase the temperature of the bottom surface to a higher temperature T 1. So, instantaneously at T is equal to 0, the entire fluid is still at 0, the medium as well as the top plane is still a 0 temperature. However, instantaneously I am increase the bottom surface temperature to 1, and I want to know, how the temperature in the gap between the two surfaces evolves with time.

At very long time, I would expect that temperature should reach a linear profile, but at very short time, initially you will have a temperature that looks like, some kind of step profile. And that will slowly evolve over time, until finally in the long time limit you get the linear temperature profile. In this case, there are no sources or sinks within the volume. And therefore, the temperature equation is just dt by dt is equal to alpha d square T by d z square, without the source or sink time.

And we are trying to get a solution for this, in the limit where the penetration depth, the depth to which the temperature fields penetrate is small, compared to the thickness between the plates. So, if this distance H is large compared to the region over, which you have a disturbance due to the presence of the bottom depth at higher temperature. Then, you can effectively consider the boundary condition to be T star equal to zero as z goes to infinity. Because, the temperature field here locally is not, the the top plate is too far away to influence the temperature field here locally. So, the early requirement the temperature field has goes to zero, as z goes to infinity rather than at a fixed value of z. So, that was the requirement for temperature field.

So, to solve this problem the first thing, we try to do as look for some way to scale the variables. You have a dimensional length z, dimensional time T. Now, if the presence of the top plate for important, then I would simply have scaled z by the total height H. But, I consider the case for the penetration depth is small compared to H. So that, the length scaled in this case should not depend upon it. So, that was the basic idea. So, then I have z and t. I also have one other dimensional parameter, which is the diffusability alpha. And from that I can make only one-dimensional less variable, z by square root of alpha t. Because, z, alpha and t, three-dimensional parameters, they contain two dimensions. Two (( )) dimensions per length t. And from that I can get one-dimensional less variable, that was the basic idea. And if this assumption is correct, then from this dimension less variable, I should be able to rewrite, the conservation equation, in terms of this variable is alone. Because, once I have non-dimensionalized everything, the final equation that I get should not depend individually on z, t, and alpha. It should depend only upon the parameter chi. So, that was the basic idea.

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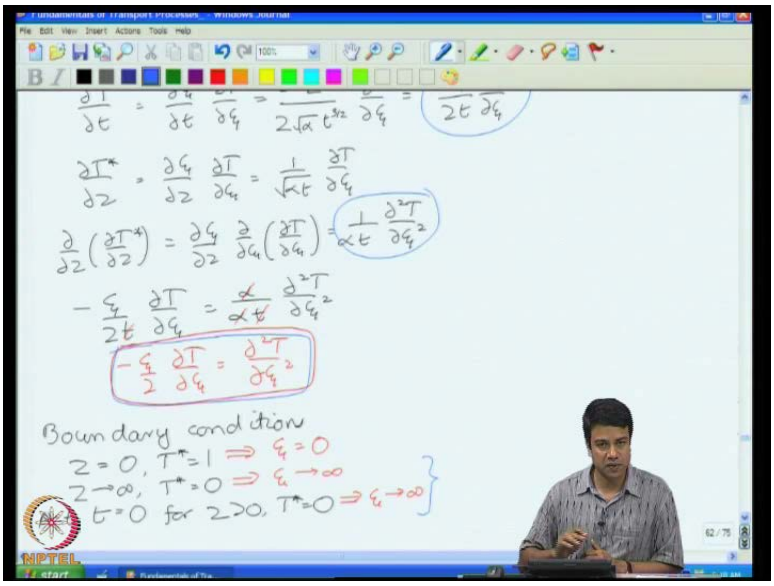

And so we got, we did expressed T star in terms of chi by differentiation by chain rule. And after we implemented this, by use chain differentiation to express dt by dt in terms of dt by d chi, as well as d square by dz square in terms of d square z by d chi square. So, after you implemented these two, the final equation ended up, as we expected we can equation only in terms of chi, not individually in terms of z and t. And the boundary conditions, at z is equal to temperature the T star is equal to 1, z is equal to 0 is equivalent to chi equal to 0, because chi equal to z by square root of alpha t. As z goes to infinity, T star equal to 0; that is chi going to infinity. The limit of chi going to infinity, which is very far away from the bottom plate, the temperature should go to temperature that was the initial temperature in the entire fluid. The third condition was an initial condition. Initially, for all z greater than 0, we have imposed the condition that the temperature T star equal to 1 is switched on at time t is equal to 0.

Since, it switched on time t is equal to 0, this temperature everywhere else in the fluid expect for the bottom plate, it is still T star is equal to 0. So therefore, for z is greater than 0 at time t is equal to 0, T star has to be 0 everywhere for also, except for the bottom plate at T is equal to 0. So, that was the third condition. Now, T is equal to 0 for z greater than 0 is equivalent to chi going to infinity, because chi was equal to z by root alpha t. So, if I take the limit t going 0, chi goes to infinity. So, in the limit of chi goes to infinity, I require the T star is equal to 0.

So, clearly here you can see, I started off with an equation in the second order partial differential equation in z, and the first order in time. I required two boundary conditions in z, one initial condition in time for getting a solution. I expressed in terms of the dimensionless coordinate chi. Once, I have done that, two of the boundary conditions; that is one boundary condition z going to infinity, and initial condition t is equal to 0, both of these end up being identical to each other. In the final equation that I have in terms of chi, I can impose only two boundary conditions, because it is only the second order equation chi. So, two of the… So, one boundary condition, and one initial condition turnout to be same thing, when expressed in terms of chi.

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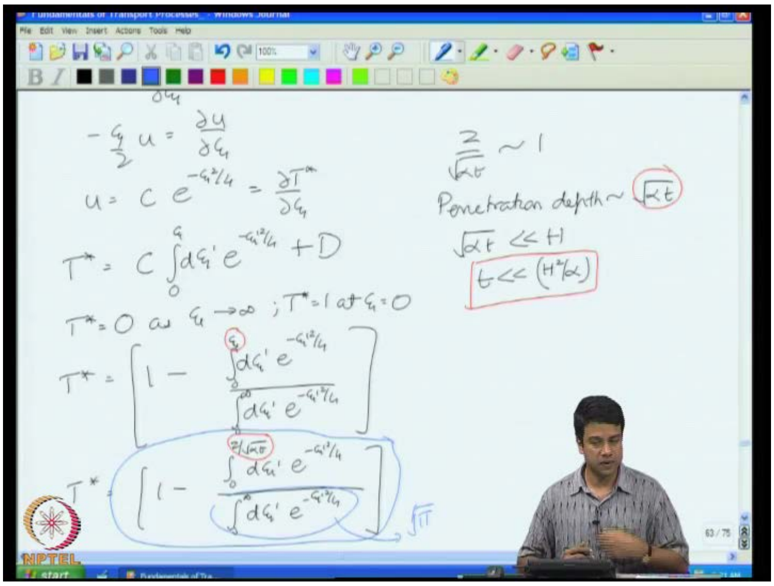

So, now I have only two boundary conditions to solve for chi. And we looked at the solution for that, the solution is of the form integral zero to chi of e power minus chi prime square by 4, divide by integral 0 to infinity. This integral 0 to infinity is actually just given by, it is just a constant. It is actually equal to 2 square root of phi, equal to 2 square root of phi. So, this just a constant, and the variable chi only comes in the upper limit of integration.

Now, in the limit is chi goes to 0, the temperature goes to 0. So, the temperature disturbance is non-zero, only when chi is order one. For example, if you actually calculate this function, So, actually calculate this function for different values of z by square root of alpha t, you will find that z by root of alpha t is equal to 1, is about 0.56 at z by root alpha t is equal to 2, it goes down to 0.08, and 3 it goes down to 0.04. So, decays over distance comparable over over chi, approximately equal to z by square root of alpha t, of order 1; 1, 2, 3, etcetera. That means the thickness to which it penetrates is given by square root of alpha t.

Because, this temperature disturbance is is is significant, only when z by square root of alpha t is 1 or when the thickness is approximately root of alpha t. By the time the thickness the distance becomes three times root alpha t, the temperature disturbance already becomes small. So, therefore this is the penetration depth, square root of alpha t, by the time the the distance goes to 1, one time the penetration depth, the temperature is already decayed to about half the value, at the bottom volume. So, therefore, this is the penetration depth and initially, we had assumed that the penetration depth is small compared to the distance between the plates.

So, therefore the square root of alpha t has to be small compared to H. Therefore, this solution is valid only at the very initial time, when t is small compared to H square by alpha. So, once t becomes of the same magnitude, H square by alpha, the fact is that now the plate there, it is starting to affect the heat conduction in domain. And presence of other plate has to be taken in to account, when we solve the problem.

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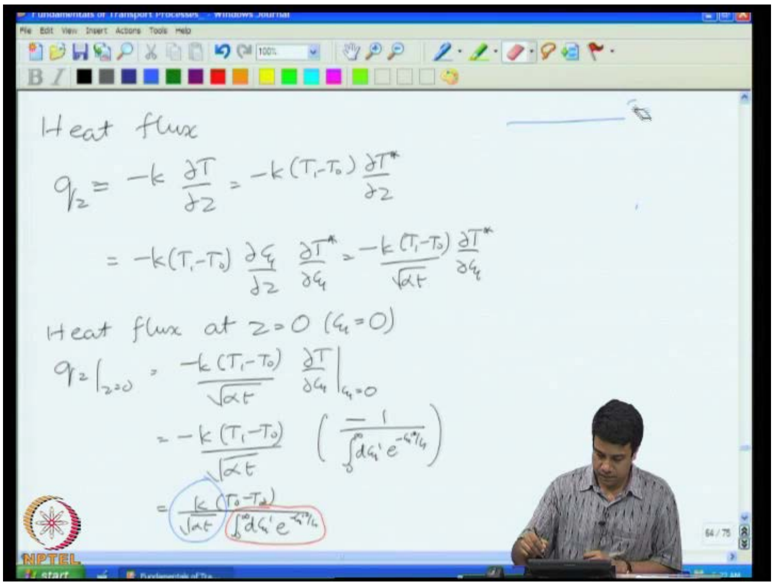

This is easily extended to heat transfer problems. I have T 1 at the bottom. I have I have I have a domain in which t is equal to t naught, t is equal to t 1, and t is equal to t naught everywhere, for time t less than 0. As a exact analog t transfer problem, and we get the exact analogous solution. You can you you can actually get the flux as a function of time. And if you can see the flux actually goes 1 over square root of alpha t.

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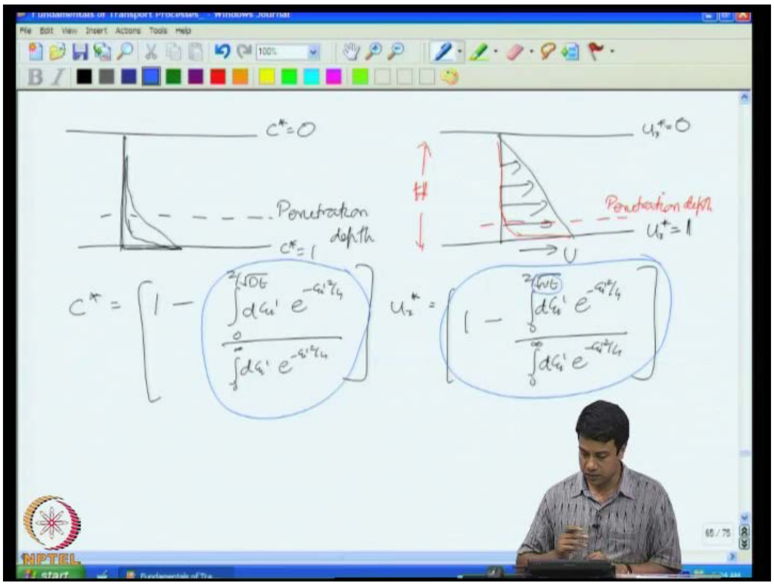

The mass transfer problem, exactly analogous, have two plates with equal concentration of solute, C star is equal to 0 everywhere, in both on both surfaces as well as within the fluid for time T less than the 0. At time T is equal to 0, I instantaneously change C star to finite value 1, and then I want to know, how the concentration field progresses with time. And this case I get exact same solution C star is equal to 1 minus this whole thing. The only difference I do is to replace alpha by d, the thermal diffusion coefficient, the mass diffusion coefficient within this expression, exactly the same for momentum conservation. So, I have fluid which is initially at rest, both surfaces are at rest. At time t is equal to 0, I instantaneously start moving the bottom plate with the velocity u, in terms of the scaled velocities, I get the exact same solution, except that instead of the mass diffusability, I have the momentum diffusability or the kinematic viscosity here, that is the only difference. So, the solution when expressed in terms of dimensionless, temperature concentration velocity are exactly the same, except that you have to use appropriate diffusion coefficient, whether it is mass diffusability, thermal diffusability or kinematic viscosity as the case may be (( )). So, all three cases exactly the same.

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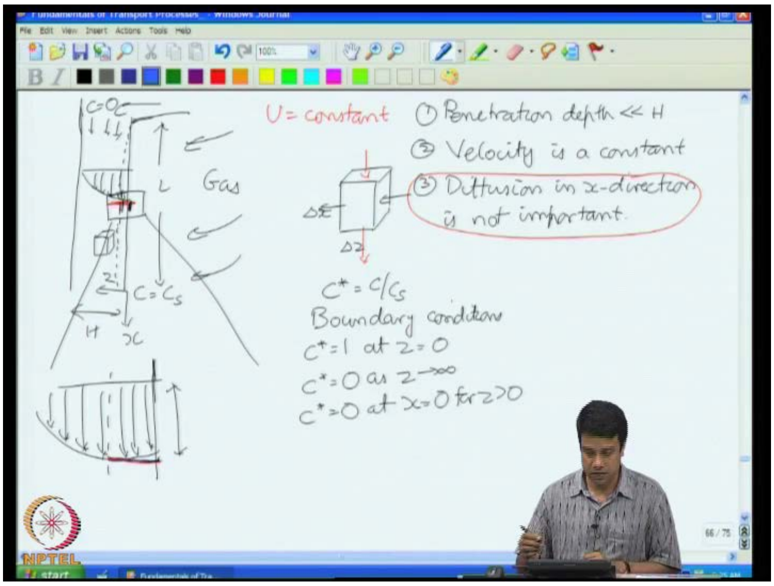

And last class I had also actually solved for you a problem which is not exactly unidirectional flow. I solved for you the problem of the diffusion into a falling flow. And in this case, the flow is, the the the transport is not unidirectional. There is transport both in the z direction, by cross stream direction as well as the convection due the mean flow downwards in the x direction. And, so you have to write two boundary conditions, for z is equal to 0 and z goes to infinity. Once again we are assuming that penetration depth is small compared to the thickness of the fluid. You also have a condition that the concentration goes to 0, at the very entrance of the channel, because the fluid has not yet come in contact with the gas.

But, we wrote a differential equation for this differential volume, at steady state, under conditions there is no variation with time in the concentration field. However, there is variation with respect to the stream vise directions, that is mass coming in and leaving the differential volume in the stream vise direction. There are diffusion fluxes in the cross stream direction. We neglect the diffusion in the stream vise direction in comparison to convection. We came back and saw, and what conditions that is valid.

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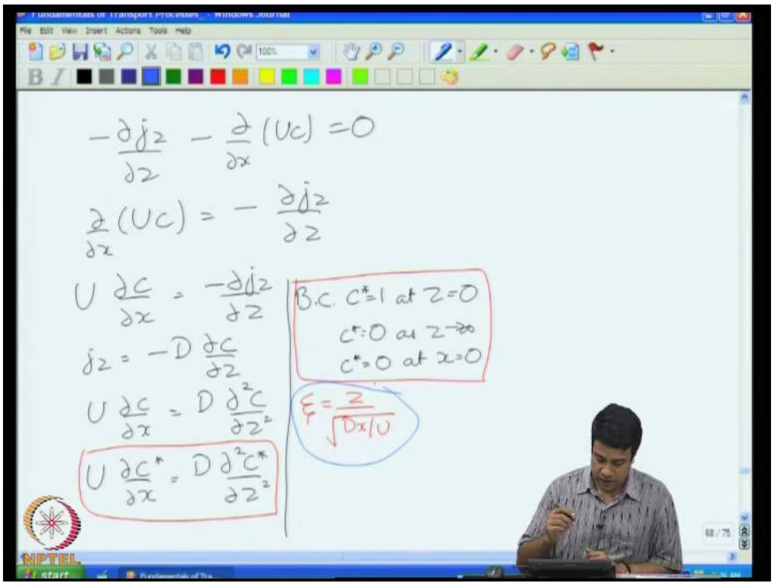

So, this mass conservation equation, gave me a resultant equation, that is exactly the same form as the unsteady equation, except that instead of dc by dt, I have U times dc by dx, note that U in this case was approximated by a constant velocity. U is independent of position. So, I have U times dc by dx instead of dc by dt. So, the solution is exactly the same, except I had to replace t by x by U. Then, I get exact solution. So, if I define a dimensionless variable of this form, where t is replaced x by U, the solution that I got for the unsteady problem can be written down in exactly the same manner for this steady state problem.

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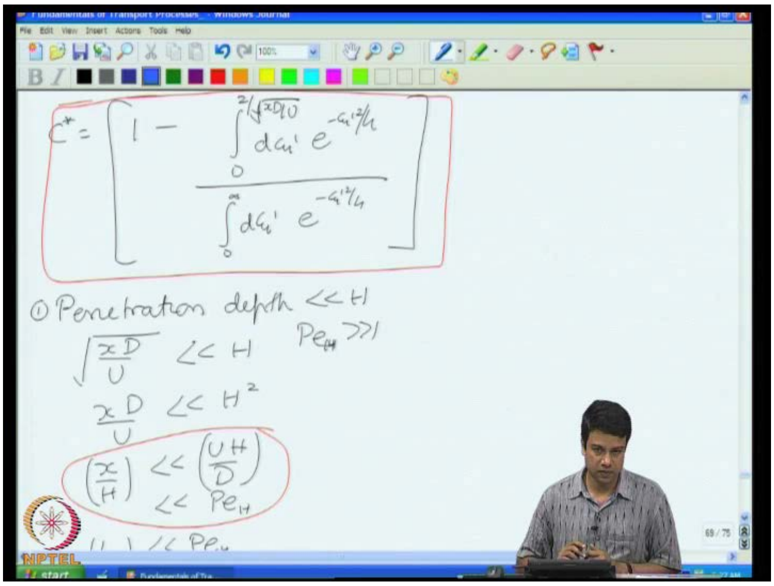

And I have got out that solution for you. In this case, the penetration depth is private of x D by U. Note that in this case, this similarity variable z by square root of x D by U is not a dimensional dimensional necessity. Because, I have now four parameters z, x ,D and U, two dimensions length, and time. So, I can form at least two dimensionless loops, but just using the analogy between the unsteady problem, and the present steady state problem, I was able to write down the solution for this as well. The penetration depth is square root of x D by U, the solution is valid only when this is small compared to the total thickness H. And we got condition that x by H which has to be small compared to a peclet number, based upon H and the mass diffusability.

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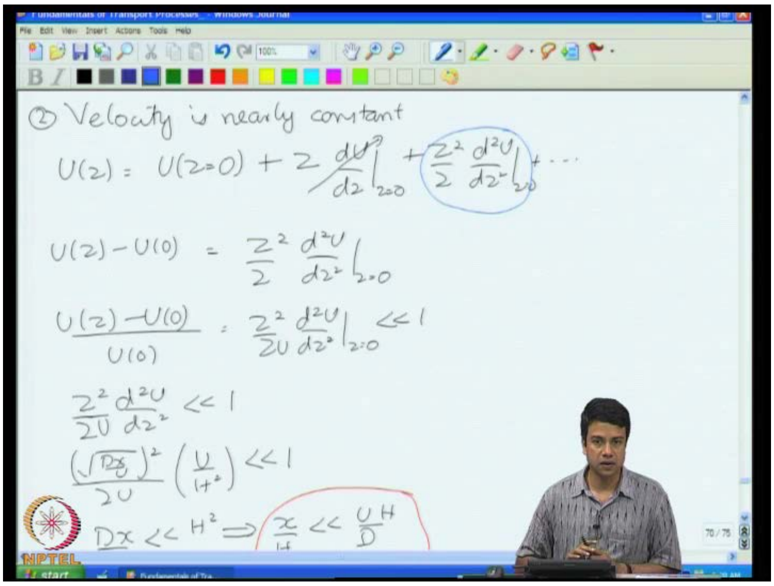

There was a second condition, that the velocity is nearly a constant. The velocity in this case is not actually a constant. We will come back and actually, calculate the velocity field in this problem, may be the next lecture, or the lecture after that. But then, there is velocity, its is the slope of the velocity at the surface is 0, because the shear stress has to be 0 from the stress balance condition. However, there is a correction which causes the second derivative, the curvature of the velocity field; that is non-zero. And therefore, the variation velocity will go as the curvature time z square. And we estimated that variation over lengths comparable to the penetration depth.

And on that basis, we found the condition that d x by U has to be small comparable to h square or x by H small comparable to peclet number. The exact same condition that we had for the penetration depth to be small comparable to thickness H.

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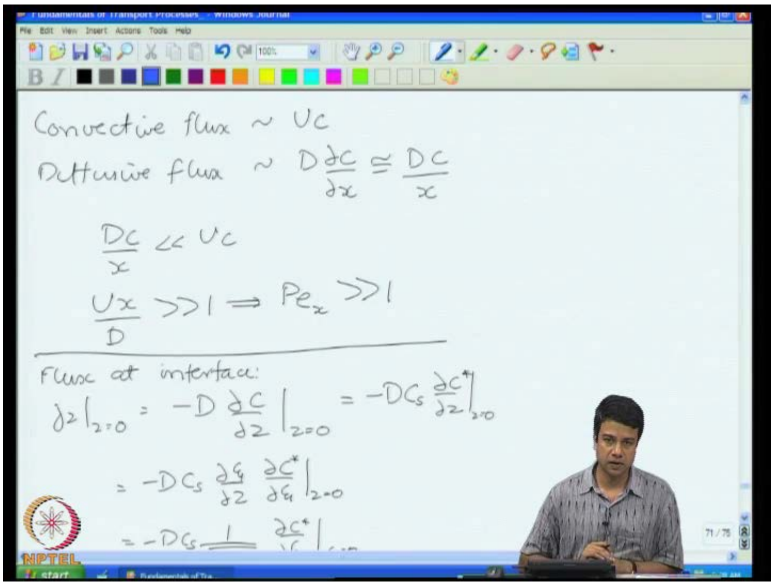

And finally, the convector flux and the diffusible flex, ratio has to be small. That happens, when the peclet number based upon the downstream distance x is large compared to y, so these are the conditions.

And this enabled has to get for the first time the correlation between the nusselt number, and the peclet number for this particular problem. We calculated the average flux as an integral over entire the downstream distance from 0 to l of the local flux. And from that we got a correlation for the peclet number of this form, 2 C s root over L power half, and the nusselt number had this form.

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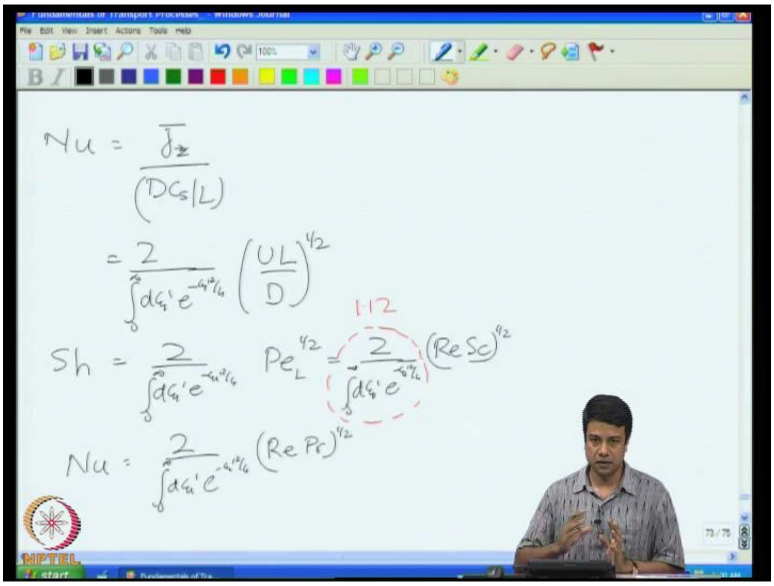

It goes (( )) peclet number per half times some constant. This constant can be evaluated; this constant turns out to be approximately 1.12. So, this times R e power half S e power half is the shower number or the nusselt number in this case. Exact same analogy will work for heat transfer. If I had a heated gas near the fluid which is flowing down, which is a cool fluid, commonly encountered configuration and for example, cooling towers. In that case, I know want know what is the depth to which the temperature profile goes, and what is the flux of heat at that interface. I did exactly the same result. In that case, the nusselt number will be equal to 2 by integral 0 to infinity d chi prime e power minus chi prime square by 4, times the Reynolds number times the prandtl number (( )) It was the exact same analogy for heat transfer.

Not a similar analogy for momentum transfer, because we had body forces active system, we will come back to momentum transfer problem a little later. So, that was a class of solutions called similarity solutions, where effectively I have transport into infinite fluid. And I use that fact to reduce the problem based upon dimensional analysis or analogy to dimensional analysis problem, and then use that to get a solution. How about the channel is of finite width? That was the problem that we started in the previous class.

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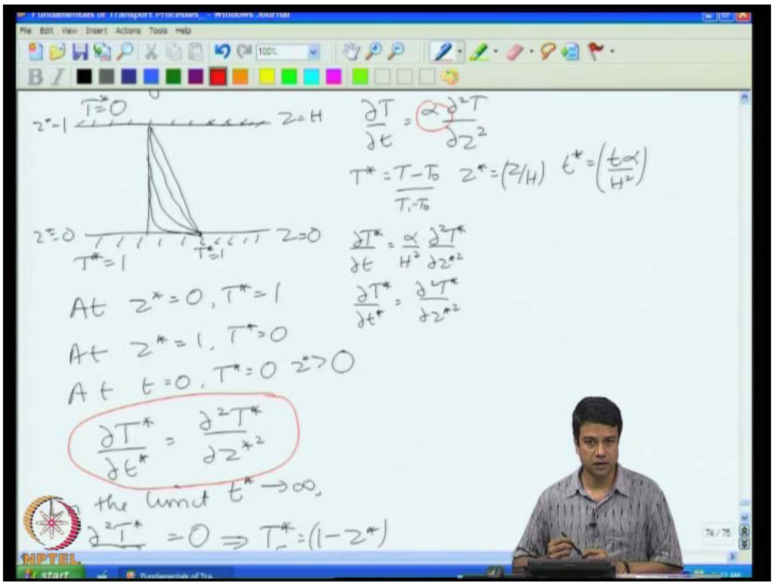

I have T is equal to 1 at bottom, T is equal to 0 at top. The bottom z equal to 0, the top z equal to 1. Now, differential equation in this case is going to be of the form dt by dt is equal to d square t by dz square. I had used scaling in order to reduce the number of dimensional groups, I effectively removed thermal diffusability from the problem by appropriate scaling, so that I can get in just terms of scaled variables. T star was defined as T by minus T naught minus T 1 by T naught. So, T star is 1 at the bottom is equal to 0 on top. z star is z by H, it is 0 at the bottom 1 on top.

We are scaling everything in such a way, that everything varies approximately in the range 0 to 1, throughout the entire domain. So, that we do not have to worry about the actual dimensions in the problem. And if I put this in, I get the scaling for the temperature.

Since, only alpha and H, and only parameters that I have, I can get only one scale temperature T star, which T times alpha by H square. And if I put that in a finally, I get an equation in terms of the scaled variables which contains no dimensional parameters. Now, in the limit as time goes to infinity, I would expect to have linear temperature profile. We have already solved that linear temperature profile about three lectures ago. In this particular case, linear profile that varies between one at the bottom, and 0 on top is 1 minus z star. So, d square T by dz square star is equal to 0, I get the steady temperature profile as 1 minus z star. Now, I separate all the temperature into two parts:

one is the steady part, and other is the transient part. We will see later the reason, why we do this separation, It is important, it is important that we separate it out in to two parts, one is the steady part, and other is the transient part.

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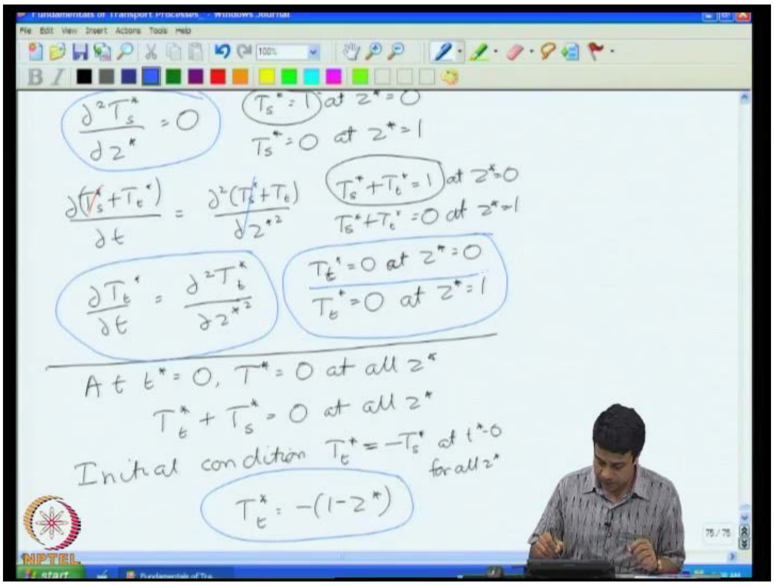

The steady part satisfies the equation d square T steady by dz square is equal to 0 with boundary conditions T steady is equal to 1 at z is equal to 0, and T steady is equal to 0 at z is equal to 1. So, those are the boundary conditions for the steady part. The total temperature field, so T star equal to T steady plus T transient, satisfies the equation the T steady plus T transient by dt is equal to d square T steady plus T transient by dz square. These equations are linear in the temperature. The total temperature field has boundary conditions, T steady plus T transient is equal to 1 at z equal to 0, and T steady plus T transient is equal to 0 at z equal to 1.

Now, this steady temperature is by construction independent of time. So, the time derivative of the steady temperature is identically equal to 0. In addition, I have my equation for the steady temperature as the second derivative. (( )) this d square T steady by dz square is also equal to zero. Therefore, the transient part of the temperature I have dt transient by dt is equal to d square T transient dz square. So, that is the equation for the transient part of the temperature. What about the boundary conditions? At z is equal to 0, T steady is equal to 1, and T steady plus T transient is also equal to 1.

The total temperature is also equal to 1; that means, I have to have T transient is equal to 0 at z equal to 0. So, therefore the transient part of the temperature has to be identically equal to 0, at z is equal to 0. How about z is equal to 1; T steady is 1, the sum of the two, T steady plus T transient is also equal to 1. Therefore, I need to have T transient is equal to 0 at z star equal to 1. Now, for the steady part there was no time derivative. So, it is sufficient to have just two boundary conditions.

For the transient part, there is the time derivative in first order in time. Therefore, you require an initial condition as well. What is that initial condition? At the initial time T star equal to 0, t equal to 0 at all z. So, when I just switched on the temperature of the bottom surface, the temperature everywhere else within the flow is equal to 0. So, therefore the temperature is 0 at T star equal to 0 for all z; however, the steady part, this means that T transient plus T steady is equal to zero at all z. The steady part is the independent of time, the steady part is time independent, it is it is it is only a steady solution. Therefore, the initial condition is that T transient is equal to minus T steady at T star equal to 0 for all z. So, the transient part is equal to minus T steady, T steady was equal to 1 minus z, (( )) for that required T star transient is equal to minus of 1 minus z for all z.

So, this is the differential equation. These are the two boundary conditions. This is the initial condition for this unsteady problem. Note that, because we separated it out the temperature field, it was a steady plus a transient part, in for the transient part the boundary conditions are T star is equal to 0, both at z is equal to 0 as well as z is equal to 1. In other words, we have homogeneous boundary conditions, in the two special coordinates for the unsteady problem.

If I just had T star is equal to zero an all boundaries, then of course, the temperature field would just be T star equal to zero everywhere. So, the temperature field will be non-zero only for forcing it somewhere, and that forcing is coming in at the initial time in this problem. So, we have a problem in which the temperature is zero on both boundaries for the transient part. So, for the transient part alone the temperature on both surfaces is equal to 0. But there is forcing at the initial time. And that is because even though the temperature is 1 at the bottom surface in the original problem, I subtracted out the steady part. The steady part temperature is equal to 1 on the bottom surface.

And therefore for the transient part the temperature is equal to 0, both at the bottom and the top. So, if the transient part is identically equal to 0, in the long time limit, the temperature would just identically equal to 0. However, the transient part is being forced at initial time, because the total temperature is zero. Therefore, T transient has to be minus of T steady. And that is what is forcing the temperature field in this unsteady problem.

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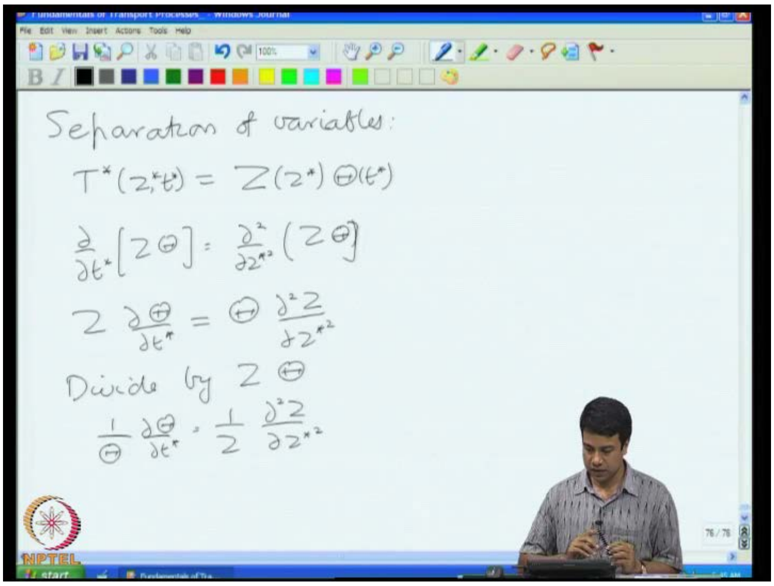

So, how do you solve this equation? We solve it by a methods called separation of variables. I have a temperature field which is the function of z star, and T star. This temperature field, I write it as the product of two functions - z of z star times, I call it as theta of T star. My original equation was a partial differential equation, the original equation that I have here, this a partial differential equation of a function that depends both upon z and t.

Now, I am separating it out into two parts: one which depends only upon z, and other which depends only upon t. So, I can insert this into the differential equation, and what I will get is d by dt star of z times theta equal to d square by dz star square of z times theta. However, z is only a function of z star, and theta is only function of T star. Therefore, I can simplify this to get z times partial theta by partial T plus is equal to theta into d square z by partial z star square. And divide by z times theta to get 1 over theta, partial theta by partial t equal to1 by z, partial square z by partial z star square. Now, so now, I got the equation of this form, and if I look at this equation the left hand side of the equation is only a function of time. The right hand side of the equation is only a function of z star. So, if this equation were true, this has to be true for all values of T star, and z star. What that implies is that if this equation were true, then both the right and left hand sides have both got to be equal to constants.

Let us assume that is true for one particular z star and T star. Then I can change z star and keep T star a constant, and then the equal to will be destroyed, because, the left hand side is only a function of T star, and right hand side is only a function of z star. So, if I change z star and keep T star a constant, only the right side will change, the left side will not change, and the quality is no longer valid. Alternatively, I could do it the other way. I could keep z star a constant, and change T star; this case only the left changes.

So, the only way that this equality will be valid for all values of z star, and T star is if both sides are equal to constants. So, by this separation of variables procedure I reduced this equation which is originally a partial differential equation, I wrote the dependent variable T star as the product of two terms. One depends only upon z star, the other only depends only upon T star. I insert it that into the equation divided by z times theta, and I got an equation which the left hand side depends only upon T star, and the right hand side depends only upon z star. And because of that both of this individually have to be equal to be constants.

So, now we have two differences two different equations to solve, both of them are equal to constants. And we have to solve this, in the domain, in order to find out the solution for temperature field. That solution for the temperature field, we will continue in the next lecture, but before we leave let me just reemphasis, if you important points here.

First thing is, I had an unsteady diffusion problem, and in the limit of long time, you expect the temperature has to go to the final steady solution for the temperature field. And that final steady solution for the temperature field is given by 1 minus z star, so the limit as T goes to infinity. I expect the temperature field to take a steady value. So, I separate it out the transient part from the steady part. The steady part is what is there in the limit as T goes to infinity. The transient part is the correct to steady part as I am approaching infinity, time starts from zero and time progresses, there is steady part which is going to reach in the long time limit.

And there this transient part which is the difference between the actual temperature, and steady part. And the reason I did that correction was, because I want to get homogeneous boundary conditions for T star at both boundaries here. I want to get homogeneous boundary conditions for both and for T star, the transient part at both boundaries. Initially of course, the temperature is zero which means the transient part is actually non-zero, so because of this I am getting the forcing at initial time, and I have homogeneous boundary conditions at both z is equal to 0, and z is equal to H. And then, I briefly told you the procedure for separation of variables. This one we will review once again in the next class, before we go on to the solution for the entire temperature profile.

So, this separation of variables procedure is the second example, how we solve this partial differential equations within the domain. The first example, that I showed you was the the similarity solutions, where we made use of the fact, that there are no dimensions, with a deficits of dimension, in order to convert from partial differential equation to an ordinary differential equation. In this case, we will get two partial differential equations, and we will see how to solve those. So, we will continue this in the next class. Please keep in mind, what has being done so far in the separation of variables in this lecture, and we will see you next time. Thank you.