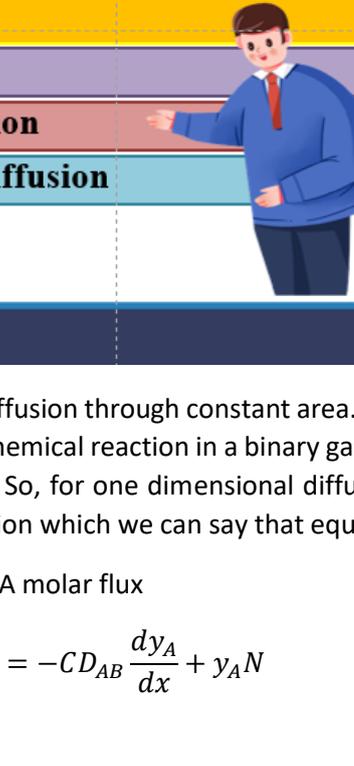


**Polymer Process Engineering**  
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**Department of Chemical Engineering**  
**Indian Institute of Technology-Roorkee**  
**Lecture – 22**  
**STEADY-STATE DIFFUSION IN POLYMERS**

Hello friends, welcome to the study of state diffusion in polymers under the areas of polymer process engineering. Now, in the previous segments, we studied about the mass transfer operation, then discuss about the mechanism of mass transfer, molecular diffusion, then we studied about the Fick's law of molecular diffusion, diffusion velocities, unsteady state diffusion and we ended with the Fick's second law of diffusion. In this particular segment, we are going to discuss about the steady state diffusion through constant area, then we will discuss about the steady state diffusion through non-diffusing components, steady state diffusion through variable area, then we will have some problems to solve and then diffusion from a sphere and we will discuss about the accumulation counter diffusion and non-accumulation counter diffusion.

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Now, let us talk about the steady state diffusion through constant area. Now, assume the steady state diffusion in the x direction without any chemical reaction in a binary gaseous mixture of species A and B for one dimensional diffusion species. So, for one dimensional diffusion of a species A, the molar flux can be given by this particular equation which we can say that equation number 1.

For one dimensional diffusion of species A molar flux

$$N_A = -CD_{AB} \frac{dy_A}{dx} + y_A N$$

Where,  $N = N_A + N_B$

On separating the variables in the above equation, we will get

$$\frac{-dy_A}{N_A - y_A N} = \frac{dx}{CD_{AB}}$$

### Steady state diffusion through constant area

- Assume that **steady state diffusion in the x direction** without any chemical reactions in a binary gaseous mixture of species A and B for one dimensional diffusions of species A.
- For one dimensional diffusion of species A, **molar flux:**

$$N_A = -CD_{AB} \frac{dy_A}{dx} + y_A N \quad \dots (1)$$

Where,  $N = N_A + N_B$

- Separating the variables in eq (1), we get:

$$\frac{-dy_A}{N_A - y_A N} = \frac{dx}{CD_{AB}} \quad \dots (2)$$



Now, where the  $n_A$  is equal to  $n_A + n_B$ , the A and B are the species as we described earlier. Now, if you separate the variables in the equation 1, this particular equation, then we get  $\frac{-dy_A}{n_A - y_A n} = \frac{dx}{cd_{AB}}$ , this we can say the equation number 2. So, for gaseous mixture at constant pressure and temperature, the concentration  $c$  and the diffusion coefficient  $d_{AB}$  they are constant and independent of the position and composition. So, all the molar fluxes are constant in that in this particular equation.

### Steady state diffusion through constant area

- For the gaseous mixture at constant pressure and temperature **the concentration (C) and diffusion coefficient  $D_{AB}$  are constant and independent of position and composition.**
- Also, **all the molar fluxes are constant** in equation 2. Therefore, integrating eq 2 between boundary conditions as follows:

$$\begin{aligned} \text{At, } x=x_1, & \quad y_A=y_{A1} \\ \text{At, } x=x_2, & \quad y_A=y_{A2} \end{aligned}$$

Where,

- 1** indicates **the start of diffusion path**
- 2** indicates **the end of diffusion path**



All the molar fluxes are constant

At,  $x=x_1$ ,  $y_A=y_{A1}$

At,  $x=x_2$ ,  $y_A=y_{A2}$

**Steady state diffusion through constant area**

- If we integrate equation (2) with the boundary conditions we get:

$$\int_{y_{A1}}^{y_{A2}} \frac{dy_A}{N_A - y_A N} = \int_{x_1}^{x_2} \frac{dx}{CD_{AB}} \quad \dots (3)$$

Let,  $N_A - y_A N = z$ ;  $-dy_A N = dz$ ;  $-dy_A = dz/N$

Putting all the values in equation 3, we get:

$$\int_{z_1}^{z_2} \frac{dz}{Nz} = \int_{x_1}^{x_2} \frac{dx}{CD_{AB}} \quad \dots (4)$$

On integrating equation 2 with the boundary conditions we will get

$$\int_{y_{A1}}^{y_{A2}} \frac{dy_A}{N_A - y_A N} = \int_{x_1}^{x_2} \frac{dx}{CD_{AB}}$$

Let

$$N_A - y_A N = z; \quad -dy_A N = dz; \quad -dy_A = dz/N$$

On putting all the values in the above equation 3 we get

$$\int_{z_1}^{z_2} \frac{dz}{Nz} = \int_{x_1}^{x_2} \frac{dx}{CD_{AB}}$$

Therefore, if we integrate this equation number 2 between the boundary conditions, like at  $x$  is equal to  $x_1$ ,  $y_A$  is equal to  $y_{A1}$ , and at  $x$  is equal to  $x_2$ , the  $y_A$  is equal to  $y_{A2}$ . Now, 1 and 2, they indicate the start of diffusion path and 2 indicates the end of diffusion path. So, if we integrate this equation, then we get this particular equation with that particular boundary condition. Now, let us say that  $n_A - y_A n = z$  and  $-dy_A n = dz$  because it is being consumed over the period of time that is equal to  $dz/n$ . So, if you put all the values in this particular equation, we get this equation that is integration from  $z_1$  to  $z_2$   $dz$  over  $nz$  equal to  $y_1$  to  $y_2$ ,  $x_1$  to  $x_2$   $dx$  over  $CD_{AB}$  that is equation number 4.

## Steady state diffusion through constant area

$$\frac{1}{N} (n_2 z_2 - n_1 z_1) = \frac{1}{CD_{AB}} (x_2 - x_1)$$

$$\ln\left(\frac{z_2}{z_1}\right) = \frac{N}{CD_{AB}} (x_2 - x_1)$$

$$\ln\left(\frac{\frac{N_A - Y_{A2}N}{N_A - Y_{A1}N}}{\frac{N_A - Y_{A1}N}{N_A - Y_{A1}N}}\right) = \frac{N}{CD_{AB}} (x_2 - x_1)$$

$$\ln\left(\frac{N_A - Y_{A2}N}{N_A - Y_{A1}N}\right) = \frac{N}{CD_{AB}} (x_2 - x_1) \quad \text{--- (5)}$$

So, if we integrate this particular equation, we get  $\frac{1}{N} \ln z_2 - \ln z_1$  is equal to  $\frac{1}{CD_{AB}} (x_2 - x_1)$  and  $\ln z_2$  over  $z_1$  is equal to  $\frac{N}{CD_{AB}} (x_2 - x_1)$  which comes out to be  $\ln \frac{N_A - Y_{A2}N}{N_A - Y_{A1}N}$  over  $N_A - Y_{A1}N$  which is  $\frac{N}{CD_{AB}} (x_2 - x_1)$ . Now, this is  $\ln \frac{N_A - Y_{A2}N}{N_A - Y_{A1}N}$  over  $N_A - Y_{A1}N$ . This is  $\frac{N}{CD_{AB}} (x_2 - x_1)$ . So, if you multiply both the sides by  $N$  in the equation then it becomes  $N \ln \frac{N_A - Y_{A2}N}{N_A - Y_{A1}N}$  is equal to  $\frac{N^2}{CD_{AB}} (x_2 - x_1)$ . This is equation number 5.

## Steady state diffusion through constant area

$$N_A = \frac{N_A}{N} \frac{CD_{AB}}{x_2 - x_1} \ln \left[ \frac{\frac{N_A}{N} - Y_{A2}}{\frac{N_A}{N} - Y_{A1}} \right] \quad \text{--- (6)}$$

Therefore, after integrating with the boundary condition the equation of for diffusion for the set condition can be expressed as  $N_A = \frac{N_A}{N} \frac{CD_{AB}}{x_2 - x_1} \ln \frac{N_A - Y_{A2}N}{N_A - Y_{A1}N}$ . That is equation number 6. So, for study state one dimensional diffusion of A through non-diffusing B the  $n_B$  equal to 0 and  $n_A$  equal to constant. So, therefore, we can represent  $\frac{n_A}{n}$  is equal to  $\frac{N_A}{N} + \frac{n_B}{N}$  that is equal to 1. Therefore, the equation

number 6 which we describe here this can become like this  $n_A$  over is equal to  $c D_{AB}$  over  $x_2$  minus  $x_1$  into  $1 - y_{A2}$  over  $1 - y_{A1}$  that is equation number 7.

### Steady state diffusion through non-diffusing component

- For steady state one dimensional diffusion of A through non diffusing B,  **$N_B$  equal to 0 and  $N_A$  equal to constant.** Therefore,
 

$$\frac{N_A}{N} = \frac{N_A}{(N_A + N_B)} = 1$$
- Hence, equation (6) becomes:
 

$$N_A = \frac{CD_{AB}}{x_2 - x_1} \ln \left[ \frac{1 - y_{A2}}{1 - y_{A1}} \right]$$

..... (7)

1-D diffusion of A through non-diffusing B

**$N_B$  equal to 0 and  $N_A$  equal to constant**

$$\frac{N_A}{N} = \frac{N_A}{(N_A + N_B)} = 1$$

Hence from equation 6;

$$N_A = \frac{CD_{AB}}{x_2 - x_1} \ln \left[ \frac{1 - y_{A2}}{1 - y_{A1}} \right]$$

And this is the equation 7.

For an ideal gas;  $C = P_t/RT$

For mixture of ideal gases;  $y_A = p_A/P_t$

Hence, equation 7 will become;

$$N_A = \frac{D_{AB}P_t}{(x_2 - x_1)RT} \ln \left[ \frac{P_t - p_{A2}}{P_t - p_{A1}} \right]$$

Where,

**$P_t$  is the total pressure**

**$p_{A1}$  and  $p_{A2}$  is the partial pressures of A at point 1 and 2, respectively.**

## Steady state diffusion through non-diffusing component

- Here, for an ideal gas,  $C = P_t/RT$
- For a mixture of ideal gases,  $y_A = p_A/P_t$
- Hence, equation (7) becomes:

$$N_A = \frac{D_{AB}P_t}{(x_2 - x_1)RT} \ln \left[ \frac{P_t - p_{A2}}{P_t - p_{A1}} \right] \dots (8)$$

Where,

$P_t$  is the total pressure

$p_{A1}$  and  $p_{A2}$  is the partial pressures of A at point 1 and 2, respectively



So, for non-diffusing component for an ideal gas we can put the  $C = P_t/RT$  for a mixture of ideal gas quite obvious that it needs to be addressed with respect to the mole fraction. So,  $y_A = p_A/P_t$ . Therefore, the equation 7 can be represented like  $N_A = \frac{D_{AB}P_t}{(x_2 - x_1)RT} \ln \left[ \frac{P_t - p_{A2}}{P_t - p_{A1}} \right]$ . This is equation number 8 where  $P_t$  is the total pressure and  $p_{A1}$  and  $p_{A2}$  they are the partial pressure of partial pressures of A at point 1 and point 2. Now, for the diffusion under the turbulent condition, the flux is usually calculated based on linear driving force for this purpose the equation this can be become manipulated or rewrite in terms of a linear driving force.

## Steady state diffusion through non-diffusing component

- For diffusion under turbulent conditions** the flux is usually calculated based on **linear driving force** for this purpose the equation e can be manipulated to rewrite in terms of a linear driving force.
- Since for binary gas mixture of total pressure  $P_t = p_A + p_B$

Therefore,  $P_t - p_{A2} = p_{B2}$  and  $P_t - p_{A1} = p_{B1} \implies p_{A1} - p_{A2} = p_{B2} - p_{B1}$

- Hence, equation (8) becomes :

$$N_A = \frac{D_{AB}P_t}{(x_2 - x_1)RT} \left[ \frac{p_{A1} - p_{A2}}{p_{B2} - p_{B1}} \right] \ln \left[ \frac{p_{B2}}{p_{B1}} \right] \dots (9)$$



Total pressure

$$P_t = p_A + p_B$$

Therefore;

$$P_t - p_{A2} = p_{B2} \text{ and } P_t - p_{A1} = p_{B1}$$

$$p_{A1} - p_{A2} = p_{B2} - p_{B1}$$

Hence equation 8 becomes

$$N_A = \frac{D_{AB} P_t}{(x_2 - x_1) RT} \left[ \frac{p_{A1} - p_{A2}}{p_{B2} - p_{B1}} \right] \ln \left[ \frac{p_{B2}}{p_{B1}} \right]$$

Since for binary gas mixture with the total can be given as  $p_t$  is equal to partial pressure of A plus partial pressure of B. Therefore,  $p_t$  minus  $p_{A2}$  is equal to  $p_{B2}$  and  $p_t$  minus  $p_{A1}$  is equal to  $p_{B1}$  and this can be represented as the partial pressure of A at A1 minus partial pressure of A2 is equal to partial pressure of B at point 2 and partial pressure of B at point 1. Therefore, the previous equation these 8<sup>th</sup> equations can become or can be represented which is equation number 9.

## Steady state diffusion through non-diffusing component

- Equation (9) can be re-written as:

$$N_A = \frac{D_{AB} P_t}{(x_2 - x_1) RT p_{BLM}} (p_{A1} - p_{A2}) \dots (10)$$

- $p_{BLM}$  is **the logarithmic mean of partial pressure of species B** which is

$$p_{BLM} = \frac{(p_{B2} - p_{B1})}{\ln \left( \frac{p_{B2}}{p_{B1}} \right)} \dots (11)$$

- The component A diffuses by **concentration gradients** which is  $-dy_A/dx$

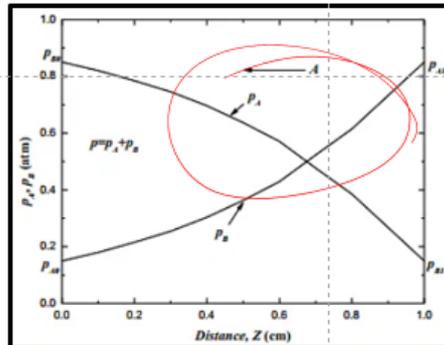


$$N_A = \frac{D_{AB} P_t}{(x_2 - x_1) RT p_{BLM}} (p_{A1} - p_{A2})$$

$$p_{BLM} = \frac{(p_{B2} - p_{B1})}{\ln \left( \frac{p_{B2}}{p_{B1}} \right)}$$

## Steady state diffusion through non-diffusing component

- **Flux is inversely proportional** to **the distance** through which the diffusion occurs and **the concentration of the stagnant gas** that is in terms of the logarithmic mean of partial pressures of species B ( $p_{BLM}$ )
- **As  $x$  and  $p_{BLM}$  resistance increases and flux decreases.**



So, based on the previous aspect the equation 9 can be rewritten as  $n_A = \frac{D_{AB} p_t}{X} \left( \frac{p_{A1} - p_{A2}}{p_{B1} - p_{B2}} \right)$  and this has become the equation number 10. So, here this  $p_{BLM}$  is the logarithmic mean of the partial pressure of a species B which can be represented like this  $p_{BLM} = \frac{p_{B2} - p_{B1}}{\ln \frac{p_{B2}}{p_{B1}}}$ .

This can be given as equation number 11. Now the component A diffuses by the concentration gradient and which is given as  $\frac{dA}{dX}$ . Now flux is inversely proportional to the distance through which the diffusion occurs and the concentration of the stagnant gas that is in terms of the logarithmic mean of the partial pressure of a species B that is  $p_{BLM}$ . So, as  $X$  and  $p_{BLM}$  resistance increases the flux decreases this can be very well understood in this particular figure.

### Problem-1

**Question:** Carbon dioxide ( $\text{CO}_2$ ) is diffusing through non diffusing air under steady state conditions at a total pressure of 1 atm and temperature of 300 K. The partial pressure of carbon dioxide is 20 kPa at one point and 5 kPa at other point the distance between the points is 5 cm. Calculate the flux of carbon dioxide.

Given at 300 K at and at 1 atm the diffusion coefficient  $D_{\text{CO}_2\text{-air}}$  is  $2 \times 10^{-5} \text{ m}^2/\text{s}$ .

Now let us take up another problem that is carbon dioxide is diffusing through non-diffusing air under the steady state condition at a total pressure of 1 atmosphere and a temperature of 300 Kelvin.

$$Q_{CO_2} = D \cdot N_{CO_2} = \frac{D_{CO_2 \text{ air}}}{RT(x_2 - x_1)} \times \frac{P_t}{P_{B,M}} (P_{CO_2,1} - P_{CO_2,2})$$

$$D_{CO_2} = 2 \times 10^{-5} \text{ m}^2/\text{s} \text{ @ } 300 \text{ K } 1 \text{ atm}$$

$$P = 1 \text{ atm} = 101.3 \text{ kPa}$$

$$P_{CO_2,1} = 20 \text{ kPa}$$

$$P_{CO_2,2} = 5 \text{ kPa}$$

$$P_{B,1} = P_t - P_{CO_2,1} = (101.3 - 20) \text{ kPa} = 81.3 \text{ kPa}$$

$$P_{B,2} = P_t - P_{CO_2,2} = (101.3 - 5) \text{ kPa} = 96.3 \text{ kPa}$$

$$P_{B,M} = \frac{P_{B,2} - P_{B,1}}{\ln \frac{P_{B,2}}{P_{B,1}}} = \frac{96.3 - 81.3}{\ln \frac{96.3}{81.3}} = 88590 \text{ Pa}$$

The partial pressure of the carbon dioxide is 20 kilo Pascal at one point and 5 kilo Pascal at other point the distance between these 2 points are 5 centimetres. So, 1 and 2 they are given and you need to calculate the flux of carbon dioxide which is given that at 300 Kelvin and at 1 atmosphere the diffusion coefficient  $D_{CO_2 \text{ air}}$  is  $2 \times 10^{-5}$  metre square per second. Now assume the ideal gas and let air is equal to B. So,  $n_{CO_2}$  is equal to  $D_{CO_2 \text{ air}}$  over  $RT \times (x_2 - x_1)$  into  $p_t$  over  $p$  logarithmic mean  $p_{CO_2,1} - p_{CO_2,2}$ . Now it is given that this  $D_{CO_2}$  is given which is equal to  $2 \times 10^{-5}$  metre square per second at 300 Kelvin and 1 atom atmosphere.

$$N_{CO_2} = \frac{2 \times 10^{-5} \times 101.3 \times 10^5 \times 15000}{8314 \times 300 \times 0.05 \times 8890}$$

$$= 2.75 \times 10^{-6} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

p is equal to 1 atmosphere or this is p is equal to 1 atmosphere which is equal to 101.3 kilo Pascal and t is equal to 300 Kelvin. So, the p CO2 1 is 20 kilo Pascal and p CO2 at the station 2 is 5 kilo Pascal. So, p B 1 this is equal to p t minus p CO2 1 which is equal to 101.3 minus 20 kilo Pascal which is comes out to be 81.3 kilo Pascal and p B 2 this is p t minus p CO2 which is 101.3 minus 5 which is comes out to be 96.3 kilo Pascal. Now if we talk about p B l m if you substitute then it comes out to be p B 2 minus p B 1 over ln p B 2 over p B 1 which is 96.3 minus 81.3 over ln 96.3 over 81.3 which is 88590 Pascal. So, n CO2 if you substitute all the values in this particular equation we get 2 into 10 to the power minus 5 into 1.013 into 10 to the power 5 into 15000 over 8314 into 300 and consistency of the unit must be addressed t 590 and this is 2.75 into 10 to the power minus 6 kilo mole per meter square second and this is our answer.

### Steady state equimolar counter diffusion

- This is the case for **the diffusion of two ideal gases** where an equal number of moles of the gases **diffusing counter currently** to each other.
- In this case,  $N_B = -N_A = \text{constant}$  and  $N_A + N_B = 0$
- the molar flux in equation a at steady state can be written as:

$$N_A = -CD_{AB} \frac{dy_A}{dx} + y_A N$$

Where,  $N = N_A + N_B$

- Hence, the above equation becomes:

$$N_A = -CD_{AB} \frac{dy_A}{dx}$$

..... (12)

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For diffusion of two ideal gases where an equal number of moles of gases diffusing counter currently to each other so,

- $N_B = -N_A = \text{constant}$  and  $N_A + N_B = 0$

The molar flux in equation a at steady state can be written as

$$N_A = -CD_{AB} \frac{dy_A}{dx} + y_A N$$

Where,  $N = N_A + N_B$

Then the above equation becomes

$$N_A = -CD_{AB} \frac{dy_A}{dx}$$

Now, let us talk about the steady state equimolar counter diffusion. This is the case of the diffusion of 2 ideal gases where an equal number of moles of gas diffusing counter currently to each other in this case n B is equal to minus n A and the constant and n A plus n B is equal to 0. So, the molar flux in the equation at steady state can be written as n A is equal to minus C D AB d Y A over d X plus Y A n where n A n A plus n B is equal to n. Therefore, if we substitute then this equation can become the equation number 12 which is n A is equal to minus C D AB over d Y A over d X.

## Steady state equimolar counter diffusion

- For ideal gas,  $C = P_t/RT$

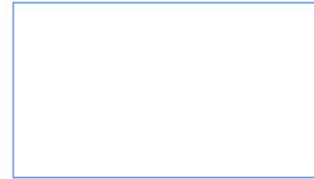
So,

$$N_A = -\frac{D_{AB}P_t}{RT} \frac{dy_A}{dx} \quad \dots (13)$$

- Integrating eq 13 with the boundary conditions at  $x = x_1, y_A = y_{A1}$  and  $x = x_2, y_A = y_{A2}$ . The equation of molar diffusion for steady state equimolar counter diffusion can be:

$$N_A = \frac{D_{AB}P_t}{(x_2 - x_1)RT} (y_{A1} - y_{A2})$$

$$N_A = \frac{D_{AB}}{(x_2 - x_1)RT} (p_{A1} - p_{A2})$$



For ideal gas

$$C = P_t/RT$$

So,

$$N_A = -\frac{D_{AB}P_t}{RT} \frac{dy_A}{dx}$$

$$N_A = \frac{D_{AB}P_t}{(x_2 - x_1)RT} (y_{A1} - y_{A2})$$

$$N_A = \frac{D_{AB}}{(x_2 - x_1)RT} (p_{A1} - p_{A2})$$

So, for ideal gas C is equal to  $P_t/RT$  and if we substitute to this particular equation so, thus  $n_A$  can become equation number 13. So, if you integrate this this particular equation with the boundary condition at  $X$  is equal to  $X_1$  and  $Y_A$  is equal to  $Y_{A1}$  and  $X$  is equal to  $X_2$  and  $Y_A$  is equal to  $Y_{A2}$  the equation of the molar diffusion for a steady state a cumular counter diffusion can be written like this.

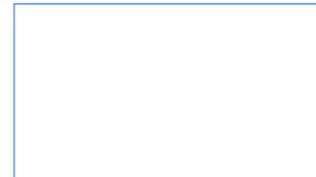
## Problem-2

**Question:** Carbon dioxide (CO<sub>2</sub>) is diffusing at steady state through a straight tube of 0.5 m long with an inside diameter of 0.05 m containing nitrogen (N<sub>2</sub>) at 300 K and 1 atm pressure. The partial pressure of carbon dioxide at one end is 15 kPa and 5 kPa at the other end.

Given that at 300 K and 1 atm pressure,  $D_{\text{CO}_2\text{-N}_2} = 4 \times 10^{-5} \text{ m}^2/\text{s}$ .

Calculate the following for the steady state equimolar counter diffusion

- the molar flow rate of carbon dioxide
- the molar flow rate of nitrogen



Now, let us take up another problem. Now, carbon dioxide is diffusing at a steady state through a straight tube of say 0.5-meter-long with an inside diameter of 0.05 meter containing the nitrogen at 300 Kelvin and 1 atmosphere pressure. The partial pressure of carbon dioxide at one end is 15 kilo Pascal and 5 kilo Pascal at the other end. Now, you are supplied with the diffusion coefficient at 300 Kelvin and 1 atmosphere pressure you need to calculate the following for the steady state a counter diffusion that is a molar flow rate of the carbon dioxide and the molar flow rate of a nitrogen. Now let us assume the ideal gas in a counter diffusion of CO<sub>2</sub> flux. So,  $n_{\text{CO}_2}$  can be written as  $\frac{D_{\text{CO}_2}}{RT(z_2 - z_1)}$  which is into  $p_{\text{CO}_2,1} - p_{\text{CO}_2,2}$  at the station number 2.

$$n_{\text{CO}_2} = \frac{D_{\text{CO}_2} \cdot p_{\text{CO}_2,1} - p_{\text{CO}_2,2}}{RT(z_2 - z_1)} \quad (p_{\text{CO}_2,1} - p_{\text{CO}_2,2})$$

$$p_{\text{CO}_2,1} = 15 \text{ kPa}$$

$$p_{\text{CO}_2,2} = 5 \text{ kPa}$$

$$R = 8.314$$

$$p_{\text{CO}_2} = 1 \text{ atm} = 1013 \text{ kPa}$$

$$T = 300 \text{ K}$$

$$D_{\text{CO}_2} = 4 \times 10^{-5} \text{ m}^2/\text{s}$$

$$z_2 - z_1 = (15000 - 5000)$$

$$n_{\text{CO}_2} = \frac{4 \times 10^{-5}}{8.314 \times 300 \times 0.5} \times (15000 - 5000)$$

$$= 3.21 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

$$A = \frac{\pi \cdot D_i^2}{4} = \frac{\pi}{4} \times (0.05)^2 = 7.96 \times 10^{-3} \text{ m}^2$$

$$\text{Molar flow rate of CO}_2 = 3.21 \times 10^{-3} \times 7.96 \times 10^{-3}$$

$$= 6.29 \times 10^{-6} \frac{\text{kmol}}{\text{s}}$$

$$n_{N_2} = \frac{D_{CO_2 N_2}}{RT(x_2 - x_1)} (p_{N_2 1} - p_{N_2 2}) = 86300 \text{ Pa}$$

$$p_{N_2 1} = p_t - p_{CO_2 1} = (101.3 - 15) \text{ kPa} = 86300 \text{ Pa}$$

$$p_{N_2 2} = p_t - p_{CO_2 2} = (101.3 - 5) \text{ kPa} = 96300 \text{ Pa}$$

$$D_{CO_2 N_2} = \frac{\Delta n_{CO_2}}{\Delta p_{CO_2}} = \frac{0.4}{124700} = -3.21 \times 10^{-7}$$

$$4 \times 10^{-5} \times 8314 \times 300 \times 0.5 = 71808 \text{ mol/m}^2 \cdot \text{s}$$

$$\text{molar flow rate of } N_2 = -3.21 \times 10^{-7} \times 1.96 \times 10^{-3} \text{ m}^2 = -6.29 \times 10^{-10} \frac{\text{kmol}}{\text{s}}$$

Now, it is given this this  $d_{CO_2}$  is given  $p_t$  is equal to 1 atmosphere which is equal to 101.3 kilo Pascal,  $t$  is given as 300 Kelvin,  $p_{CO_2 1}$  is given at 15 kilo Pascal and  $p_{CO_2}$  at station number 2 is given as 5 kilo Pascal and  $R$  is equal to 8.314. So,  $n_{CO_2}$  is equal to again if we substitute all these things to this particular formula, then it becomes  $n_{CO_2} = \frac{D_{CO_2 N_2}}{RT(x_2 - x_1)} (p_{N_2 1} - p_{N_2 2})$  into  $10$  to the power minus  $5$  over  $8314$  into  $300$  into  $0.5$  into  $15,000$  minus  $5000$  if we make the consistency of the unit which comes out to be  $3.21$  into  $10$  to the power minus  $7$  kilo mole meter square per second. Now, molar flow rate of  $CO_2$  is equal to  $CO_2$  into  $A$  that is  $A$  is the cross-sectional area of the tube and given that the internal diameter of the tube is  $0.05$  meter. So, the cross-sectional area of the tube can be given as  $\frac{\pi D_i^2}{4}$  and  $\frac{\pi}{4}$  into  $0.05$  and this is  $1.96$  into  $10$  to the power minus  $3$ -meter square and molar flow rate, rate of  $CO_2$  is equal to  $3.21$  into  $10$  to the power minus  $7$  into  $1.96$  into  $10$  to the power minus  $3$ , which comes out to be  $6.29$  into  $10$  to the power minus  $10$  kilo mole per second and this is the first part. Now, if we talk about the nitrogen which is  $d_{CO_2} n_2 = R T (x_2 - x_1)$  that is  $p_{N_2}$  station 1 to  $p_{N_2}$  station 2.

Now,  $p_{N_2 1}$  is given as  $p_t$  minus  $p_{CO_2 1}$  this is  $101.3$  minus  $15$  kilo Pascal that is comes out to be  $86,300$  Pascal. Similarly,  $p_{N_2 2}$  at station 2 that is  $p_t$  minus  $p_{CO_2}$  at station 2, this is comes out to be  $101.3$  minus  $5$  kilo Pascal which comes out to be  $96,300$  Pascal. Now, we know the equimolar counter difference of ideal gas this  $d_{CO_2} n_2$  is equal to  $d_{N_2} n_2$ .

So, if we substitute that comes out to be  $4$  into  $10$  to the power minus  $5$  over  $8314$  into  $300$  into  $0.5$  and  $86,300$  minus  $96,300$  and this comes out to be  $0.4$  over  $1247$  minus  $3.21$  into  $10$  to the power minus  $7$ . So, the mass flow molar flow rate of  $n_2$  is equal to minus  $3.21$  into  $10$  to the power minus  $7$  kilo mole per meter square into  $1.96$  into  $10$  to the power minus  $3$ -meter square and that comes out to be  $6$  point minus  $6.29$  into  $10$  to the power minus  $10$  kilo mole per second and that is our answer. Now, let us talk about the steady state diffusion through the variable area. Now, consider a component  $A$  which is diffusing at a steady state through a cumular triangle here which is tapered uniformly.

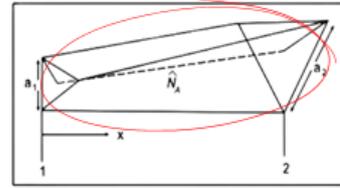
## Steady state diffusion through variable area

- Consider a component A is diffusing at steady state through a equimolar triangle conduit which is tapered uniformly.
- A is diffusing through stagnant non-diffusing B component.
- Equilateral triangle, the formula for area, where a is the length of one side can be written as:

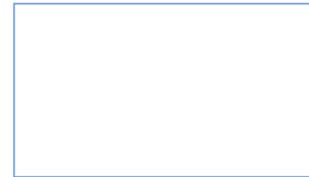
$$A = \frac{1}{2} \times \text{side} \times \text{altitude} = \frac{1}{2} \times a \times (\sqrt{3}/2 a)$$

$$\text{Altitude} = (\sqrt{3}/2 a)$$

$$A = \sqrt{3}/4 a^2$$



Schematic of diffusion of A through a uniformly tapered geometry.



Now, A is diffusing through the stagnant non-diffusing B component and equilateral triangle, the formula of air where area where A is the length of one side can be written as A is equal to half of the side x altitude and this can be represented as like this. Now, if returning to the Fick's law of formula at a position x the flux of A through a triangle of stagnant B can be written as per this equation. Now we are having this  $n_A$  is equal to  $\dot{N}_A$  over A is equal to  $4$  over square root of  $3$   $n_A$  A square where this dot  $n_A$  that is the rate of diffusion of a kilo moles per second and if we modify then it can become the equation number 14. Now, before limits are imposed it must be remembered that A is a function of x as the size of triangle uniformly tapered with the distance along the duct. So, it can be represented like A is equal to  $A_1$  plus  $A_2$  minus  $A_1$  over  $x_2$  minus  $x_1$  x minus  $x_1$  and thereby like this. Now, let us if we substitute the x and x for A and integrating with the limits of partial pressure of component A at 1 and the p A at a triangle of side A and the partial pressure of component A at point 2.

## Steady state diffusion through variable area

- Returning to **the Fick's law formula**, at position x the flux of A through a triangle of stagnant B can be written as:

$$N_A \left(1 - \frac{p_A}{P_t}\right) = \frac{-D_{AB}}{RT} \frac{dp_A}{dx}$$

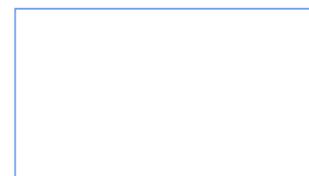
Now,

$$N_A = \frac{\dot{N}_A}{A} = \frac{4}{\sqrt{3}} \frac{\dot{N}_A}{a^2}$$

Where  $\dot{N}_A$  = Rate of diffusion of A kmol/s

$$-\frac{RT}{D_{AB}} \times \frac{4}{\sqrt{3}} \frac{\dot{N}_A}{a^2} dx = P_t \frac{dp_A}{P_t - p_A}$$

.....(14)



Fick's law formula

$$N_A \left(1 - \frac{p_A}{P_t}\right) = \frac{-D_{AB}}{RT} \frac{dp_A}{dx}$$

$$N_A = \frac{\dot{N}_A}{A} = \frac{4}{\sqrt{3}} \frac{\dot{N}_A}{a^2}$$

Where  $N_A$  = Rate of diffusion of A kmol/s

$$-\frac{RT}{D_{AB}} \times \frac{4}{\sqrt{3}} \frac{\dot{N}_A}{a^2} dx = P_t \frac{dp_A}{P_t - p_A}$$

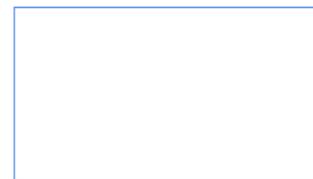
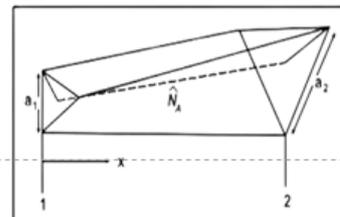
### Steady state diffusion through variable area

- Before limits are imposed it must be remembered that  $a$  is a function of  $x$ , as the size of the triangle uniformly tapered with distance along the duct.

$$a = a_1 + \frac{a_2 - a_1}{x_2 - x_1} (x - x_1) = a_1 + \frac{a_2 - a_1}{x_2 - x_1} x - \frac{a_2 - a_1}{x_2 - x_1} x_1$$

$$\frac{da}{dx} = 0 + \frac{a_2 - a_1}{x_2 - x_1} (1) - \frac{a_2 - a_1}{x_2 - x_1} (0) = \frac{a_2 - a_1}{x_2 - x_1}$$

$$dx = \frac{x_2 - x_1}{a_2 - a_1} da$$



$$a = a_1 + \frac{a_2 - a_1}{x_2 - x_1} (x - x_1)$$

$$= a_1 + \frac{a_2 - a_1}{x_2 - x_1} x - \frac{a_2 - a_1}{x_2 - x_1} x_1$$

$$\frac{da}{dx} = 0 + \frac{a_2 - a_1}{x_2 - x_1} (1) - \frac{a_2 - a_1}{x_2 - x_1} (0) = \frac{a_2 - a_1}{x_2 - x_1}$$

$$dx = \frac{x_2 - x_1}{a_2 - a_1} da$$

## Steady state diffusion through variable area

$$\begin{aligned}
 & \frac{P_{A1}}{P_{A2}} \\
 & - \frac{RT N_A x_2 - x_1}{D_{AB} a_2 - a_1} \int_{a_1}^{a_2} \frac{y}{\sqrt{3} a^2} da = P_t \int_{P_{A1}}^{P_{A2}} \frac{dP_A}{P_t - P_A} \\
 & \frac{4RT N_A}{\sqrt{3} D_{AB}} \frac{x_2 - x_1}{a_2 - a_1} \left[ \frac{-1}{a} \right]_{a_1}^{a_2} = \ln \left( \frac{P_t - P_{A2}}{P_t - P_{A1}} \right) \\
 & \frac{4RT N_A}{\sqrt{3} D_{AB}} \frac{x_2 - x_1}{a_2 - a_1} \left( \frac{1}{a_1} - \frac{1}{a_2} \right) = \ln \left( \frac{P_t - P_{A2}}{P_t - P_{A1}} \right)
 \end{aligned}$$

So, the  $p_{A2}$  and  $p_{A1}$  then the triangle is at is of side  $A_2$  this we can write as  $n_A$  is equal to dot  $n_A$ . Now, if we can write  $Rt n_A x_2 - x_1$  over  $d_{AB} A_2 - A_1$  integration from  $A_1$  to  $A_2$   $4$  square root of  $3 A^2 d_A$  is equal to  $p_t p_{A1}$  to  $p_{A2}$  the integration and  $d p_A$  over  $p_t - p_A$ . So,  $4 R t n$  over  $3 d_{AB}$ . So,  $x_2 - x_1$  over  $A_2 - A_1$  minus  $1$  over  $A$  and integration from  $A_1$  to  $A_2$  that is minus  $\ln p_t - p_A$  and  $p_{A1}$  to  $p_{A2}$ . So, this can become the  $4 R t n_A$  over  $3 d_{AB} x_2 - x_1$  over  $A_2 - A_1$   $1$  over  $A_1$  minus  $1$  over  $A_2$   $n$  which is equal to  $\ln p_t - p_{A2}$  over  $p_t - p_{A1}$ .

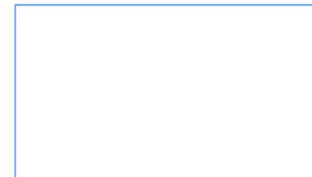
## Steady state diffusion through variable area

$$\begin{aligned}
 N_A \left( \frac{a_1 - a_2}{a_1 a_2} \right) &= \frac{\sqrt{3} D_{AB} P_t}{4RT} \left( \frac{a_2 - a_1}{x_2 - x_1} \right) \ln \left( \frac{P_t - P_{A2}}{P_t - P_{A1}} \right) \\
 N_A &= \frac{\sqrt{3} D_{AB} P_t}{4RT} \frac{a_1 a_2}{x_2 - x_1} \ln \left( \frac{P_t - P_{A2}}{P_t - P_{A1}} \right)
 \end{aligned}$$

So, if we substitute all those things, then it becomes the  $n_A A_1 - A_2$  over  $A_1 A_2$  which is equal to  $3 d_{AB} p_t$  over  $4 R t$  and  $A_2 - A_1$  over  $x_2 - x_1$  and  $\ln p_t - p_{A2}$  over  $p_t - p_{A1}$ . So,  $n_A$  is equal to square root of  $3 d_{AB} p_t$  over  $4 R t A_1 - A_1$  over  $A_2 x_2 - x_1$   $\ln p_t - p_{A2}$  over  $p_t - p_{A1}$ .

### Problem-3

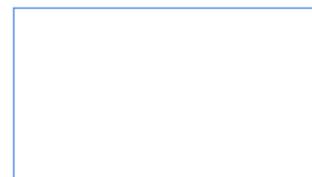
**Question:** The carbon dioxide (CO<sub>2</sub>) is diffusing through non-diffusing nitrogen (N<sub>2</sub>) at steady state in a conduit of 2 m long at 300 K and a total pressure of 1 atm. The partial pressure of carbon dioxide at the left end is 20 kPa and 5 kPa at the other end. The cross section of the conduit is in the shape of an equilateral triangle being 0.025 m at the left and tapering uniformly to 0.05 m at the right end. Calculate the rate of transport of carbon dioxide (CO<sub>2</sub>). The diffusivity is  $D_{AB} = 2 \times 10^{-5} \text{ m}^2/\text{s}$ .



This is the desired formula. Now, let us take up a question and that is the carbon dioxide CO<sub>2</sub> is diffusing through a non-diffusing nitrogen and to a steady state at in a conduit of 2 meters long at 300 Kelvin and a total pressure of 1 atmosphere. The partial pressure of the carbon dioxide at the left end is 20 kilo Pascal and other end 5 kilo Pascal and the cross section of the conduit is in the shape of an equilateral triangle of 0.025 meters at the left end tapering uniformly to 0.05 meter the right end. You need to calculate the transport of the carbon dioxide and diffusivity is given. So, we have given the diffusivity.

$R = 8314$     $T = 300 \text{ K}$     $p_t = 1 \text{ atm} = 101.3 \text{ kPa}$   
 $p_{A1} = 20 \text{ kPa}$     $p_{A2} = 5 \text{ kPa}$     $a_1 = 0.025 \text{ m}$     $a_2 = 0.05 \text{ m}$   
 $x_2 - x_1 = 2 \text{ m}$

$$N_A = \frac{\sqrt{3} D_{AB} Q_{A2}}{4AL} \ln \left( \frac{p_t - p_{A2}}{p_t - p_{A1}} \right)$$

$$N_A = \frac{0.22 \times 10^{-2}}{997680} \ln \left( \frac{96.3 \times 10^3}{81.3 \times 10^3} \right) = 3.74 \times 10^{-11} \text{ kmol/s}$$


So, we are having the value of R 8.3140 is equal to 300 Kelvin and p t is equal to 1 atmosphere which is equal to 101.3 kilo Pascal and p A 1 is equal to 20 kilo Pascal and p A 2 is equal to 5 kilo Pascal. A 1 is equal to 0.025-meter, A 2 is equal to 0.5 meter and x 2 minus x 1 is equal to 2 meters. So, if we substitute, then we get n A is equal to the square of 3 d A B, which is the formula which we previously



integrate with the limits of  $p_{A2}$  at  $r_2$  and  $p_{A1}$  at  $r_1$ , this gives  $\ln \frac{p_{A2}}{p_{A1}}$  and that is the equation number 16. So, as  $r_1$  is less than  $r_2$  and then  $\frac{r_1}{r_2}$  is almost equal to 0. So, if you substitute then it becomes  $\frac{4\pi r_1^2 n_A D_{AB}}{r_1^2}$  is equal to  $n_A$  and that is the flux at the surface.

### Diffusion from the sphere

$$-\frac{RT N_A}{4\pi r^2 D_{AB} p_t} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) = \ln \left( \frac{p_t - p_{A2}}{p_t - p_{A1}} \right) \quad \text{--- 16}$$

$$\frac{N_A}{4\pi r_1^2} = D_{AB} \frac{p_t (p_{A1} - p_{A2})}{RT p_{BLM} r_1} = N_A \quad \text{--- 17}$$

= the flux at the surface





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This is the equation number 17. So, this is equation number 17 can be simplified if  $p_A$  is small  $p_{A1}$  is small compared to  $p_{total}$ . So, then  $p_{BLM}$  is almost equal to  $p_{total}$ . So, we can set  $2r_1$  is equal to  $d_1$  that is diameter and  $c_{A1}$  is equal to  $p_{A1} / RT$  then this equation 17 can become  $N_{A1}$  is equal to  $2 D_{AB} / d_1 (c_{A1} - c_{A2})$ . This is equation number 18.

$$N_{A1} = \frac{2 D_{AB}}{d_1} (c_{A1} - c_{A2}) \quad \text{--- 18}$$

$p_{A1} \ll p_t$       $p_{BLM} \approx p_t$       $c_{A1} = \frac{p_{A1}}{RT}$

$2r_1 = d_1$  (Diameter)

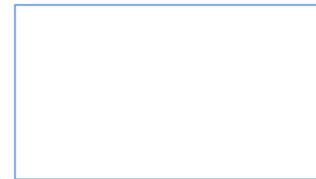




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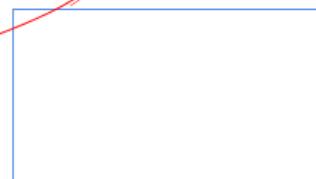
### Problem-3

**Question:** A sphere of naphthalene having a radius of 5 mm is suspended in a large volume of still air at 310 K and 1 atm. The partial pressure at the surface of naphthalene at 310 K is 50 Pa. Assume dilute gas phase. The diffusion coefficient of component that is DAB of naphthalene in air is at 310 K is given as  $6 \times 10^{-6} \text{ m}^2/\text{s}$ . Calculate the rate of evaporation of naphthalene from the surface.



Now, let us take up a problem that is a sphere of naphthalene having radius of 5 mm which is suspended in a large volume of still air at 310 Kelvin and 1 atmosphere. The partial pressure of the surface of naphthalene at 310 Kelvin is 50 Pascal and assuming that dilute gas phase the diffusion coefficient of the component that is  $D_{AB}$  of the naphthalene in air is at 310 Kelvin is given as  $6 \times 10^{-6}$  meter square per second. You need to calculate the evaporation of naphthalene from the surface.

$$\begin{aligned}
 D_{AB} &= 6 \times 10^{-6} \text{ m}^2/\text{s} & p_{A1} &= 50 & p_{A2} &= 0 & r_1 &= \frac{5}{1000} \text{ m} \\
 p_{B1m} &= p_t = 1 \text{ atm} & & & & & R &= 8.314 \\
 &= 101.3 \text{ kPa} & & & & & & \\
 \frac{N_A}{4\pi r_1^2} &= \frac{D_{AB} p_t (p_{A1} - p_{A2})}{R T p_{B1m} r_1} = N_{A1} \\
 \frac{N_A}{4\pi r_1^2} &= \frac{(6 \times 10^{-6}) (1.013 \times 10^5) (50)}{(8.314) (310) (1.013 \times 10^5) (0.005)} \\
 &= 0.23 \times 10^{-6} \text{ kmol/m}^2\text{s}
 \end{aligned}$$



Now, here  $D_{AB}$  is given that is  $6 \times 10^{-6}$  square meter per second  $p_{A1}$  is 50,  $p_{A2}$  is equal to 0 and  $r_1$  is equal to 5 over 1000 meter and  $R$  is as usual 8.314 and  $p_{B1m}$  is equal to  $p_t$  is equal to 1 atmosphere which is equal to 101.3 kilo Pascal. So, if you use the formula then  $N_{A1}$  over  $4\pi r_1^2$  that is  $D_{AB} p_t (p_{A1} - p_{A2})$  over  $R T p_{B1m} r_1$  which is equal to  $N_{A1}$ . So, this can be represented as  $4\pi r_1^2$  this is equal to  $6 \times 10^{-6}$  into 1.013

into 10 to the power 5 into 50 over 8.3, 8314 into 310 into 1.013 into 10 to the power 5 into 0.005 which comes out to be 0.23 into 10 to the power minus 6 kilo mole per meter square and that is our answer.

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So, dear friends in this particular segment we discussed about the mass transfer aspects and diffusion which are very essential in the polymeric systems and for your convenience we have enlisted variety of references and which can be used for the further studies. Thank you very much.