

Polymer Process Engineering
Prof. Shishir Sinha
Department of Chemical Engineering
Indian Institute of Technology-Roorkee
Lecture – 11
Applied polymer rheology: Fluid behaviour

Hello friends, welcome to the applied polymer rheology. Here we are going to discuss about the fluid behaviour. Now, in this particular segment, we are going to discuss about the Newtonian and simple non-Newtonian fluids, complex non-Newtonian fluids, fluids influenced by the external force fields, non-continuous fluids, we are going to discuss about the relativistic fluids. Now, let us talk about the rheology, the basic introduction. The rheology term is used to examine the flow and deformation of all matters, both solid and fluids. The key to understanding polymer processing is provided by the rheology.

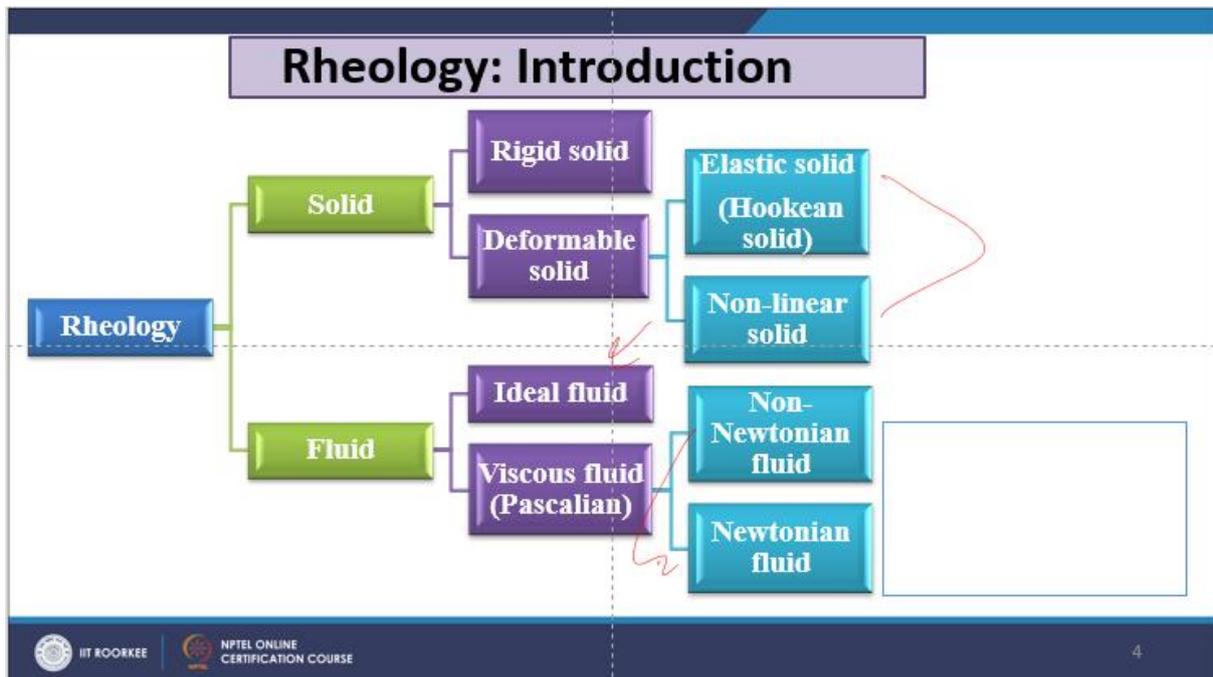
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The structure and characteristics can be related like here you can see this aspect. Additionally, the flow and deformation, they are the part of every polymer processing procedure. Now, rheology can be divided into two aspects, one is the solid and second one is the fluid. Now, when we talk about the solid, there may be two types of solids, one is the rigid solid, where you cannot change the shape and second one is the deformable solids where you can easily change the shapes they may acquire or they do not acquire the original shape.



Now, if we talk about the deformable solids, thereby, these can be further subdivided into two categories, elastic solid, usually attributed to the Hookean solid and nonlinear solids. Now, if we talk about the fluids, there are two types of fluids, one is the ideal fluid and second one is the viscous fluid. So, if you talk about the viscous fluids, again, this can be subdivided in two different aspects, one is the non-Newtonian fluid and second one is the Newtonian fluid. So, let us talk about the Newtonian liquid model or Newtonian fluids. These Newtonian liquids are a type of a fluid that exhibits a linear relationship between the shear stress and shear rate.

Newtonian Liquid Model

- Newtonian liquids are a type of fluid that exhibit a linear relationship between shear stress and shear rate.
- Shear stress (τ) is the force per unit area applied parallel to the surface of the liquid, while shear rate (du/dy) represents the velocity gradient perpendicular to the direction of flow.
- The equation that describes the relationship between shear stress and shear rate is known as Newton's law of viscosity:

$$\tau = \mu * (du/dy)$$
 where τ is the shear stress, μ is the dynamic viscosity, and (du/dy) is the shear rate.

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$$\tau = \mu * (du/dy)$$

Now, shear stress sometimes referred as a tau is the force per unit area applied parallel to the surface of the liquid, where while shear rate sometimes referred as $\frac{du}{dy}$ or $\frac{dv}{dy}$, this represents the velocity gradient perpendicular to the direction of flow. The equation that describes the relationship between the shear stress and shear rate is known as Newton's law of viscosity and mathematically it can be represented as $\tau = \mu \frac{du}{dy}$ or $\tau = \mu \frac{dv}{dy}$, where tau is the shear stress and mu is the dynamic viscosity and $\frac{du}{dy}$ or $\frac{dv}{dy}$ is the shear rate. The dynamic viscosity mu, this represents the resistance of the liquid to flow and it is a measure of the internal friction. So, whenever liquid flows, then there is some resistance attributed within the liquid and within the surface. So, this mu represents the resistance of liquid to flow.

Newtonian Liquid Model

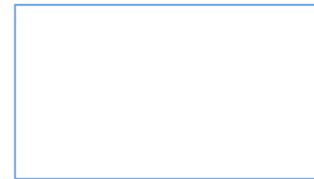
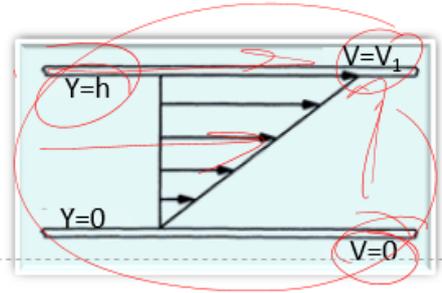
- The dynamic viscosity (μ) represents the resistance of the liquid to flow and is a measure of its internal friction.
- A higher viscosity indicates a thicker or more resistant fluid, while a lower viscosity implies a thinner or less resistant fluid.
- The unit of dynamic viscosity is typically expressed in Pascal-seconds (Pa·s) or poise (P).
- The Newtonian liquid model assumes that the dynamic viscosity remains constant regardless of the shear rate applied.

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A high viscosity, this indicates the thicker or more resistance fluid, while a lower viscosity implies a thinner or less resistant fluid. The unit of dynamic viscosity is typically expressed in Pascal second or PAS or Poise. The Newtonian liquid model, this assumes that the dynamic viscosity remains constant regardless of the shear rate applied. This implies the liquid's flow behaviour is independent of the applied stress and deformation rate. The shear rate $\frac{du}{dy}$ or $\frac{dv}{dy}$ can be calculated by dividing the velocity difference between the two fluid layers, here these are the two fluid layers.

Newtonian Liquid Model

- Consider a fluid layer with height (h) and width (w) between two parallel plates.
- The top plate is moving with velocity (v_1), while the bottom plate is stationary ($v_2 = 0$).
- The shear rate (du/dy) is the velocity gradient between the two plates, which can be approximated as $(v_1 - v_2)/h$.



So, the velocity difference between the two fluid layers by the distance between those layers. So, this is the, this one. Now, let us talk about the Newtonian liquid model in which they consider a fluid layer, this is the fluid is flowing in this particular direction. Now, consider a fluid layer with a height h and width w between the two plates and these two plates are parallel in nature. The top plate is moving with the velocity v_1 while the bottom plate is stationary with the velocity v_0 , v_1 is equal to v_0 , v_2 is equal to 0 .

Newtonian Liquid Model

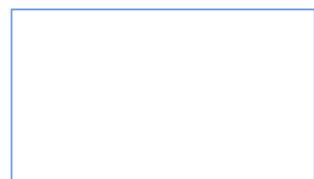
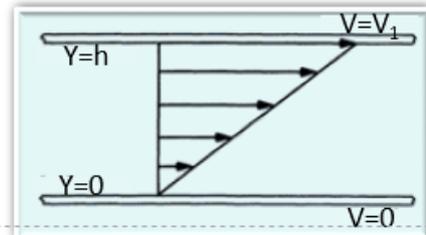
- The shear stress (τ) is the force required to maintain the relative motion of the plates, which is proportional to the applied force (F) and inversely proportional to the area (A) of contact.

Therefore, $\tau \propto F/A$.

- By rearranging the terms and considering the definition of viscosity, we get

$$\tau = \mu * (du/dy),$$

which is Newton's law of viscosity.



$$\tau = \mu * (du/dy),$$

The shear rate $\frac{du}{dy}$ is the velocity gradient between the two plates which can be approximated as $\frac{v_1 - v_2}{h}$. The shear stress τ is the force required to maintain relative motion of the plates which is proportional to the applied force and inversely proportional to the area of the contact. Therefore, τ is proportional to $\frac{f}{a}$. Now, if we rearrange the terms and considering the definition of the viscosity, we get $\tau = \mu \frac{du}{dy}$ which is the Newton's law of viscosity. Let us talk about the simple non-Newtonian things.

Simple non-Newtonian

- The "simple" non-Newtonian fluids are those whose flow behavior is affected by the rate of shear.
- **Shear-thinning:** The fluid becomes less resistant to flow as the applied force or shear rate is increased.
- **Shear-thickening:** fluid becomes more resistant to flow as the applied force or shear rate is increased

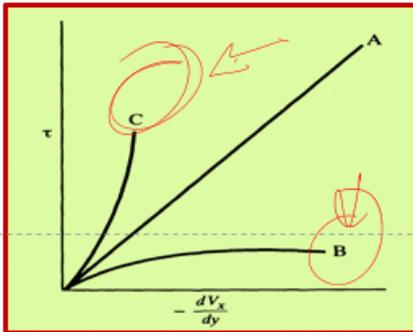
The diagram illustrates the classification of simple non-Newtonian fluids. On the left, two circles represent 'Shear-thinning' (blue) and 'Shear-thickening' (green). A plus sign (+) is placed between them, with an arrow pointing to a larger orange circle on the right labeled 'Simple non-Newtonian'. Red scribbles are drawn around the circles and the plus sign, and a blue box is present below the diagram.

**Turonov, J., Kolesnikov, N., Zhdanov V., et al. Flow structures and heat transfer on dimples in a staggered arrangement. Int. J. Heat Fluid Flow. 2012;35:168-175

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The simple non-Newtonian fluids, they are those fluids whose flow behaviour is affected by the rate of shear. Now, shear thinning, the fluid becomes less resistant to the flow as applied force or shear rate is increased. Now, shear thickening, the fluid becomes more resistant to flow as the applied force or shear rate is increased. So, this is the shear thickening and this becomes the simple non-Newtonian fluid. Now, here if you see that this particular figure, the shear stress versus shear rate comparison for a Newtonian, shear thinning and shear thickening fluid.

Simple non-Newtonian



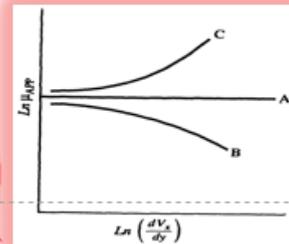
- B and C are simple non-Newtonian.
- **Shear-thinning** fluids (B) are also called pseudoplastic,
- **Shear-thickening** fluids (C) are termed dilatant.

Fig. 2. Shear stress vs. shear rate comparison for (A) Newtonian, (B) shear-thinning, and (C) shear-thickening fluids.

This one is the shear thickening fluid. So, this B and C, they are the simple non-Newtonian and shear thinning fluid B, this one is also called the pseudo plastic and shear thickening fluid, they are termed as dilatant. Now, this particular figure, this is the logarithmic of apparent viscosity versus logarithmic of shear rate of Newtonian, shear thinning and shear thickening fluids. So, if you rewrite the Newton's law of viscosity to get the apparent viscosity, sometimes referred as μ_{app} , this is equal to τ_{yx} is equal to minus μ_{app} $\frac{dv_x}{dy}$ and we get μ_{app} as τ_{yx} over $\frac{dv_x}{dy}$ and μ_{app} for a Newtonian is a constant. Now, shear thinning fluid show a decreasing apparent viscosity with increasing shear rate and shear thickening fluid, this shows an increasing apparent viscosity with an increasing shear rate.

Simple non-Newtonian

Fig. 2. Logarithm of apparent viscosity vs logarithm of shear rate for (A) Newtonian, (B) shear-thinning, and (C) shear-thickening fluids.



- Rewriting the Newton's law of viscosity to get **apparent viscosity μ_{App}**

- $\tau_{yx} = -\mu_{App} \frac{dV_x}{dy}$, and we get,

- $\mu_{App} = \frac{\tau_{yx}}{\left(\frac{dV_x}{dy} \right)}$

- μ_{App} for a Newtonian is a constant.

$$\tau_{yx} = -\mu_{App} \frac{dV_x}{dy}$$

$$\mu_{App} = \frac{\tau_{yx}}{\left(\frac{dv_x}{dy}\right)}$$

Now, simple non-Newtonian, these can be expressed in another form using Oswald-De Waele power law and this is τ_{yx} is equal to minus $k \left(\frac{dv_x}{dy}\right)^{n-1} \frac{dv_x}{dy}$, where $\frac{dv_x}{dy}$ is the absolute value of the rate of shear and k is a consistency index and n is the flow behaviour index, this one. So, if we write the above expression in the form of τ_{yx} is equal to $K \left(\frac{d\gamma}{dt}\right)^n$, where $\frac{d\gamma}{dt}$ is the rate of strain and $\frac{d\gamma}{dt}$ is equal to minus $\frac{dv_x}{dy}$. Now, this law applies only when $\log \tau_{yx}$ versus $\log \frac{d\gamma}{dt}$ plot a straight line. Now, n is the slope of each plot and the existence of shear thinning or shear thickening behaviour in the material depends on the composition and interaction between its constituents. Now, you see that the ketchup is a non-Newtonian fluid, it does not follow the rules.

Simple non-Newtonian

- Simple non-Newtonian can be expressed in another form using the **Oswald-De Waele power law**:

$$\tau_{yx} = -K \left| \frac{dv_x}{dy} \right|^{n-1} \frac{dv_x}{dy},$$
- Where, $\frac{dv_x}{dy}$ is the absolute value of the rate of shear, K is the consistency index, and n is the flow behavior index.
- Writing the above expression in the form

$$\tau_{yx} = K \left(\frac{d\gamma}{dt} \right)^n$$
- Where $\frac{d\gamma}{dt}$ is the rate of strain and $\frac{d\gamma}{dt} = - \frac{dv_x}{dy}$.



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$$\tau_{yx} = -K \left| \frac{dv_x}{dy} \right|^{n-1} \frac{dv_x}{dy}$$

$$\tau_{yx} = K \left(\frac{d\gamma}{dt} \right)^n$$

Now, if we have in this particular table, we have enlisted different type of fluids with the n values with some simple examples, like if we talk about the shear thinning fluids, the n is less than 1 and the examples of polymer solutions, polymer melts, foods, all those things, then Newtonian n is equal to 1, their water and organic fluids, they are the best examples, then shear thickening, where n is greater than 1, they are the continuous and dispersed phases, water, sand, cement, water, corn, starch, etcetera. Let us talk about the complex non-Newtonian fluids. Now, the complex non-Newtonian fluids are subset of non-Newtonian fluids and that exhibits more intricate rheological behaviour beyond simple shear thinning or shear thickening. These fluids often possess a combination of different flow characteristics making their flow behaviour more complex and different or difficult to model compared to the Newtonian fluid. Now, the complex non-Newtonian fluid they find application

in various fields, this includes the material science, chemical engineering, food processing, pharmaceutical and cosmetics.

Simple non-Newtonian		
Fluid type	n	Examples
Shear-thinning	< 1.0	Polymer solutions, polymer melts, foods
Newtonian	1.0	Water and organic fluids
Shear-thickening	> 1.0	Continuous and dispersed phases (water-sand, cement, water corn starch, etc.)

The complex non-Newtonian fluid exhibit time parameter as an influencing factor, it means that they change their properties with respect to the time and this can be expressed as $\tau = \phi \dot{\gamma}$. Now, there are various type of a complex non-Newtonian fluids, each with its own unique characteristics. Some of the common examples of these complex non-Newtonian fluids, the shear thinning fluid, these fluids are also known as the pseudo plastic fluids. They exhibit the decrease in the viscosity with increase in shear rate. As the shear rate increases the fluids resistance to flow decreases and resulting in thinner consistency.

Complex non-Newtonian
<ul style="list-style-type: none"> • Complex non-Newtonian fluids are a subset of non-Newtonian fluids that exhibit more intricate rheological behaviors beyond simple shear-thinning or shear-thickening • These fluids often possess a combination of different flow characteristics, making their flow behavior more complex and difficult to model compared to Newtonian fluids. • Complex non-Newtonian fluids find applications in various fields, including materials science, chemical engineering, food processing, pharmaceuticals, and cosmetics.

Complex non-Newtonian

- Complex non-Newtonian fluids exhibit time parameter as a influencing factor. It means that they change their properties with respect to the time, and can be expressed as;

$$\tau = \phi \left(\frac{dy}{dt}, t \right)$$

$$\tau = \phi \left(\frac{dy}{dt}, t \right)$$

The example of shear thinning fluids is includes the ketchup, toothpaste, certain type of paints. When these fluids are at rest, they have higher viscosities, but they flow more easily when subjected to shear stress. Then shear thickening fluids, in contrast to shear thinning fluids, the shear thickening fluids also known as the dilatant fluids experiences an increase in viscosity with increasing in shear rate. And these fluids become thicker and resist flow when subjected to higher shear rates, like silicote and a mixture of corn starch and water sometimes called as obec. They are the example of shear thickening fluids.

Complex non-Newtonian

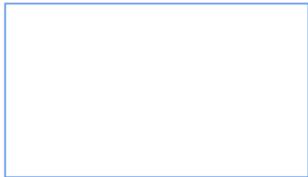
- **Shear-Thickening Fluids:** In contrast to shear-thinning fluids, shear-thickening fluids, also known as dilatant fluids, experience an increase in viscosity with increasing shear rate.
- These fluids become thicker and resist flow when subjected to higher shear rates.
- Silly putty and a mixture of cornstarch and water (sometimes called oobleck) are examples of shear-thickening fluids.

Bingham plastic fluids, Bingham plastic fluids such have a yield stress, meaning that they require a certain amount of applied stress before they start to flow. Once the yield stress is exceeded, these

fluids exhibits linear viscous behaviour similar to the Newtonian fluid. Common example of Bingham plastic fluid includes the certain type of drilling mud, mayonnaise, tomato sauce, etc. Then thixotropic fluids, the thixotropic fluids exhibit a time dependent decrease in viscosity when subjected to continuous shear stress. With prolonged shear stress, the fluid gradually becomes less viscous and flows more easily.

Complex non-Newtonian

- **Bingham Plastic Fluids:** Bingham plastic fluids have a yield stress, meaning that they require a certain amount of applied stress before they start to flow.
- Once the yield stress is exceeded, these fluids exhibit linear viscous behavior similar to Newtonian fluids.
- Common examples of Bingham plastic fluids include certain types of drilling mud, mayonnaise, and tomato sauce.



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However, when the shear stress is removed, the fluid recovers its original viscosity over time. Some paints, gels and certain printing displays thixotropic behaviour. Rheopectic fluids, the Rheopectic fluids behave oppositely to the thixotropic fluids and they exhibit a time-dependent increase in viscosity under continuous shear stress. The longer the shear stress is applied, the more viscous the fluid becomes. However, once the shear stress is removed, the fluid gradually returns to its original viscosity.

Complex non-Newtonian

- **Rheopectic Fluids:** Rheopectic fluids behave opposite to thixotropic fluids. They exhibit a time-dependent increase in viscosity under continuous shear stress.
- The longer the shear stress is applied, the more viscous the fluid becomes. However, once the shear stress is removed, the fluid gradually returns to its original viscosity.
- Some clay suspensions and certain lubricants exhibit rheopectic behavior.



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Some clay suspensions, certain lubricants, they exhibit the reopatic behaviour. Let us, while considering the complex Newtonian fluids, let us discuss about some related models like power law model. The power law model is a commonly used mathematical expression for describing non-Newtonian fluid. It relates the shear stress τ to the shear rate γ through a power law relationship. Now, this is represented in mathematically like this where this k is the consistency index, n is the flow behaviour index and the consistency index represent the fluids resistance to flow and the flow behaviour index indicates the degree of shear thinning and, we usually n is less than 1 or the shear thickening when n is greater than 1.

Complex Non-Newtonian Fluids: Related models

1. Power Law Model:

- The power law model is a commonly used mathematical expression for describing non-Newtonian fluids. It relates the shear stress (τ) to the shear rate ($\dot{\gamma}$) through a power law relationship:

$$\tau = K \dot{\gamma}^n$$

- where K is the consistency index and n is the flow behavior index. The consistency index represents the fluid's resistance to flow, and the flow behavior index indicates the degree of shear-thinning ($n < 1$) or shear-thickening ($n > 1$) behavior.



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$$\tau = K \dot{\gamma}^n$$

The Herschel-Burkley model, the Herschel-Burkley model, this extends the power law model by including a yield stress τ_0 which represents the minimum stress required for the fluid to start flowing. The relationship between the shear stress τ and shear rate γ is given by this particular relationship. Now, this model captures the behaviour of fluids that require a certain amount of force to initiate flow. Bingham plastic model, the Bingham plastic model, this describes the fluids that exhibit a yield stress but have a linear relationship between the shear stress τ and a shear rate γ . Once the yield stress is exceeded, the model can be expressed like this where the τ_0 is the yield stress and μ_0 are the plastic viscosity.

Complex Non-Newtonian Fluids: Related models

2. Herschel-Bulkley Model:

- The Herschel-Bulkley model extends the power law model by including a yield stress (τ_0), which represents the minimum stress required for the fluid to start flowing. The relationship between shear stress (τ) and shear rate ($\dot{\gamma}$) is given by:

$$\tau = \tau_0 + K \dot{\gamma}^n$$

- This model captures the behavior of fluids that require a certain amount of force to initiate flow.

$$\tau = \tau_0 + K \dot{\gamma}^n$$

Complex Non-Newtonian Fluids: Related models

3. Bingham Plastic Model:

- The Bingham plastic model describes fluids that exhibit a yield stress but have a linear relationship between shear stress (τ) and shear rate ($\dot{\gamma}$) once the yield stress is exceeded. The model can be expressed as:

$$\tau = \tau_0 + \mu_0 \dot{\gamma}$$

- where τ_0 is the yield stress and μ_0 is the plastic viscosity. The fluid behaves as a rigid solid below the yield stress and as a Newtonian fluid above the yield stress.

$$\tau = \tau_0 + \mu_0 \dot{\gamma}$$

The fluid behaves as a rigid solid below the yield stress and as a Newtonian fluid above the yield stress. The Casson model, the Casson model is used to describe the behaviour of fluid with a yield stress and a non-zero yield strain. It relates the square root of the shear stress τ to the square root of the shear rate. Now, this can be represented mathematically like this where the τ_0 is the yield stress and μ_0 is the plastic viscosity. This model accounts for the non-linear relationship between the stress and strain in yield stress flow.

Complex Non-Newtonian Fluids: Related models

4. Casson Model:

- The Casson model is used to describe the behavior of fluids with a yield stress and non-zero yield strain. It relates the square root of the shear stress (τ) to the square root of the shear rate ($\dot{\gamma}$) through the equation:

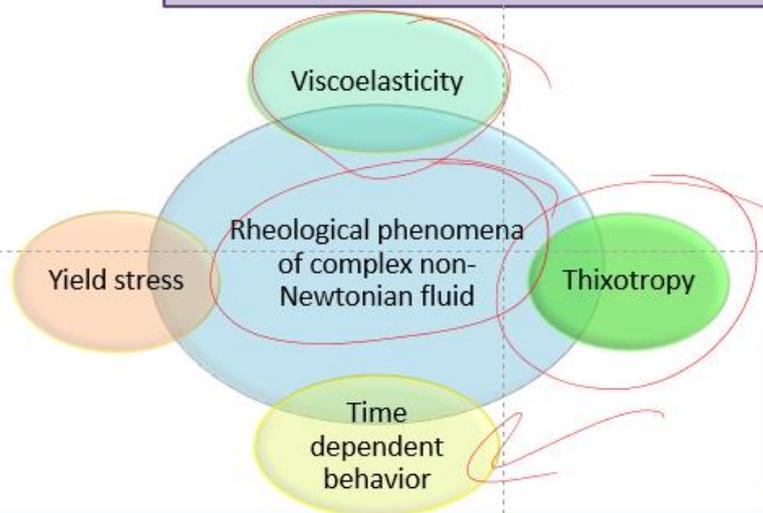
$$\sqrt{\tau} = \sqrt{\tau_0^2 + \mu_0^2 \dot{\gamma}^2}$$

- where τ_0 is the yield stress and μ_0 is the plastic viscosity. This model accounts for the non-linear relationship between stress and strain in yield stress fluids.

$$\sqrt{\tau} = \sqrt{\tau_0^2 + \mu_0^2 \dot{\gamma}^2}$$

Now, here let us talk about the complex non-Newtonian model. This is the rheological phenomena of a complex non-Newtonian fluid where we are going, we can have the thixotropic behaviour, viscoelasticity behaviour and the yield stress and the time dependent behaviour. Now, let us talk about the viscoelasticity. Now, for understanding the viscoelasticity model, we need to understand the non-Newtonian liquid model which is represented over here. Delta X between the two layers and the ideal elastic solid that is the Hookean model which is represented over here.

Complex non-Newtonian



Now, this particular figure shows the Newtonian liquid or a dashpot. This shows the elongational deformation along the X axis, the dashpot. This can also be used as a shear deformation. This can be written as γ_{xy} is equal to ΔX over h . This figure shows the deformation is the time dependent.

In a Newtonian fluid, the relationship between the time and deformation is a linear. So, we can express the shear strain rate as $\dot{\gamma}_{xy}$ is equal to $\dot{\gamma}_{xy} \Delta t$. the linear elasticity or the Hookean spring, the force F is directly proportional to the deflection that is ΔX . So, F is equal to $K \Delta X$. K is the constant of proportionality or spring constant and this is the schematic representation of the Hookean solid.

Viscoelasticity

- Figure shows elongational deformation along the x-axis, the dashpot can also be used for shear deformation, written as

$$\gamma_{xy} = \frac{\Delta X}{h}$$
- Figure shows that deformation is time dependent. In a Newtonian fluid, the relationship between time and deformation is linear. We can express shear strain rate as a

$$\dot{\gamma}_{xy} = \dot{\gamma}_{xy} \Delta t$$

Newtonian liquid (dashpot)

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$$\gamma_{xy} = \frac{\Delta X}{h}, \dot{\gamma}_{xy} = \dot{\gamma}_{xy} \Delta t$$

Viscoelasticity

- Linear Elasticity or the Hookean Spring:**
 - The force, F , is directly proportional to the deflection, Δx .
 - $F = k \Delta x$
 - k is the constant of proportionality or the spring constant.

Fig: A schematic representation of a Hookean solid

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Now, this equation can be expressed as the following in terms of stress and strain τ_{yx} is equal to $\gamma_{xy} G$ and τ_{yx} is the shear stress and γ_{xy} is the corresponding shear strain. Now, the spring depicts the body that is stretching, but it can also be used to depict the shear. As soon as the load is applied, the spring deforms and this deformation lasts as long as the load does. At the time Δt when the load is relieved, the material element will almost immediately take on its original shape. James Clark Maxwell, he developed the viscoelastic model initially and he presented a system model that incorporates the elastic and viscous effects.

Viscoelasticity

- **Linear Elasticity or the Hookean Spring:**
 - The spring depicts a body that is stretching, but it can also be used to depict shear.
 - As soon as the load is applied, the spring deforms, and this deformation lasts as long as the load does.
 - At time Δt , when the load is relieved, the material element will almost immediately take on its original shape.

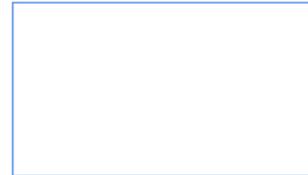
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The Maxwellian model today, which is graphically represented by instantaneous change in the spring in a time dependent reaction. This is based on this model and the linear differential equation that result that or relates the stress and strain. So, the Maxwell model and White model, they are the viscoelastic model. Now, this particular figure, this shows the characteristics of both the Newtonian fluid and elastic fluid. Now, here you see the difference between the Maxwellian model and White model.

Viscoelasticity

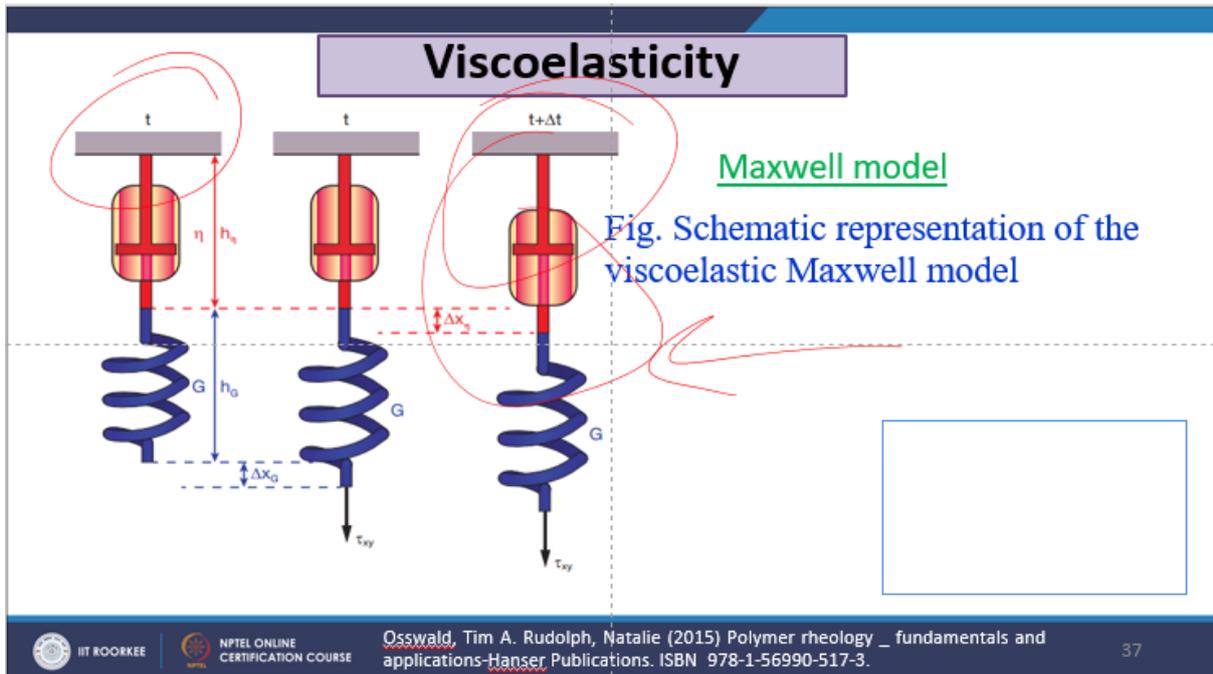
Viscoelastic model:

- James Clerk Maxwell presented a system model that incorporates both elastic and viscous effects.
- The Maxwell model of today, which is graphically represented by an instantaneous change in the spring and a time-dependent reaction, is based on his model and the linear differential equations that result that relate stress and strain.
- Both Maxwell model and Voigt model are viscoelastic models.

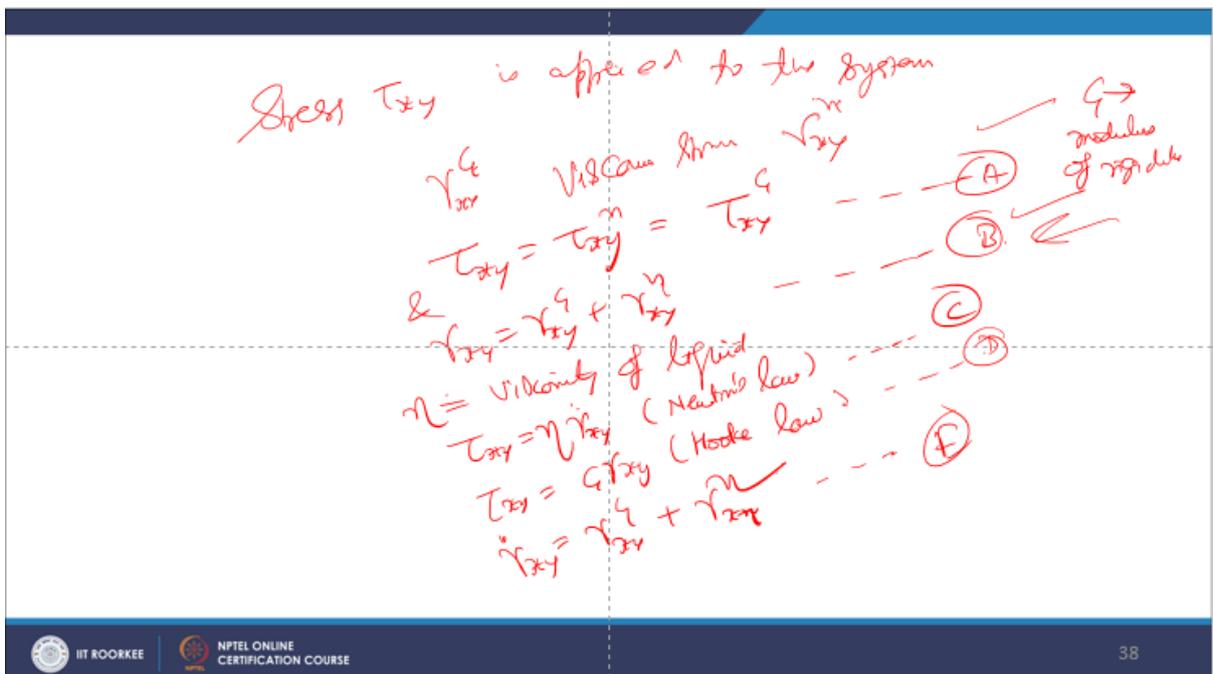


This is the viscoelastic model of elastic and Newtonian and this one is the ideal elastic solid spring and Newtonian that Newtonian liquid that is a dashpot. So, here you see the difference. So, both viscous that is a resistance to flow and elastic that is ability to recover from the any kind of deformation. These qualities are present in these fluids. So, when subjected to an applied stress or deformation, they exhibit both instantaneous elastic responses and time dependent viscous responses, gels, polymer solutions, various biological fluids, they are the best example of viscoelastic fluids.

Now, here we are going to discuss about the Maxwell model. So, this is the schematic representation of the viscoelastic Maxwell model. Here this is the t and then if we stretch this thing, then can be represented $t + \Delta t$. Now, stress τ_{xy} is applied to the system. Stress is same in both the fluids, stress usually is same in both the fluids and a solid element.



Total strain is the sum of elastic strain that is γ_{xy} and the viscous strain γ_{xy}^v . So, τ_{xy} is equal to τ_{xy}^v is equal to τ_{xy}^e . This is let us say equation A and γ_{xy} is equal to γ_{xy}^e plus γ_{xy}^v . This is let us say equation number B. Now, if we take this, this is the viscosity of liquid.



So, $\tau_{xy} = G \gamma_{xy}$ that is Hooke's law and $\tau_{xy} = \eta \dot{\gamma}_{xy}$ that is Newton's law. And G is the modulus of rigidity. So, if we differentiate this equation that is equation number B, it becomes $\dot{\tau}_{xy} = G \dot{\gamma}_{xy} + \eta \ddot{\gamma}_{xy}$. So, if we combine equation A, B and E with equations C and D, so it becomes $\dot{\tau}_{xy} = G \dot{\gamma}_{xy} + \eta \ddot{\gamma}_{xy}$ is equal to $G \dot{\gamma}_{xy} + \eta \ddot{\gamma}_{xy}$ minus $\tau_{xy} = G \gamma_{xy}$, where λ is equal to η / G .

$\frac{d\tau_{xy}}{dt} = G \frac{d\gamma_{xy}}{dt} - \frac{\tau_{xy}}{\lambda} \quad \lambda = \frac{G}{\eta}$

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Let us talk about the thixotropy. Now, this thixotropic fluid, they demonstrate a time dependent decrease in viscosity when subjected to continuous shear stress. Now, this implies that the fluid viscosity reduces when it is repeatedly stirred out shear. Now, this represents the thixotropic fluid behaviour. This is plotted with the shear stress versus rate of shear and this is a time dependent model. However, when fluid eventually regains its initial viscosity after the shear stress is eliminated, so the various paints, gels, adhesive, they frequently contain the thixotropic fluids.

Thixotropy

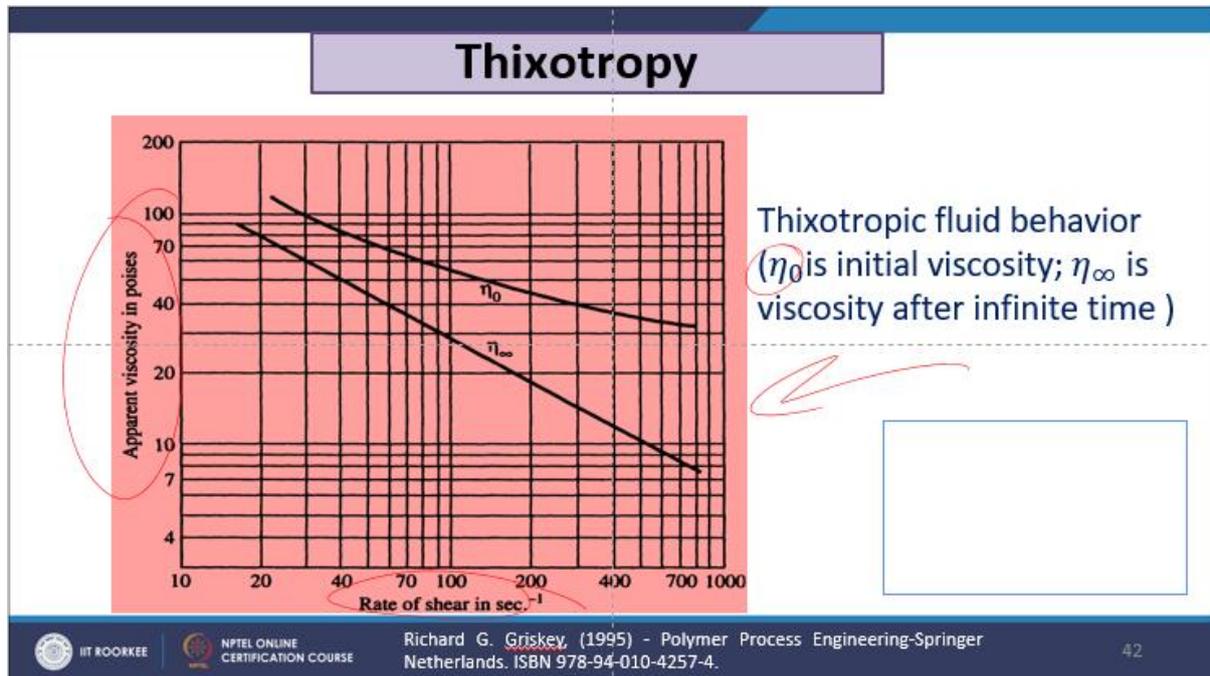
- **Thixotropic fluids** demonstrate a **time-dependent** decrease in viscosity when subjected to continuous shear stress.
- This implies that the fluid's viscosity reduces when it is repeatedly stirred or sheared.

Fig: Thixotropic fluid behavior

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These fluids can be thought of as a fluid in which τ is equal to $\phi \frac{d\gamma}{dt}$. This is the thixotropic fluid behaviour where η_0 is the initial viscosity and η_∞ is the viscosity after infinite time. So, this is the generalized rate of shear versus apparent viscosity. Now, let us talk about the yield stress and time dependent behaviour. The yield stress, these fluids have a threshold stress known as

yield stress and that must be exceeded before they start to flow and these fluids behaves as a solid or remain static below the yield stress.



There are some of the examples which we have enlisted like blood, toothpaste, crude oil, preemie fluids, gels, foams, all these things are the example of yield stress fluids. They decrease the viscosity and start the flow once the yield stress is exceeded. So, toothpaste, mayonnaise, certain colloidal suspensions, they are again the best example of yield stress fluids. Now, fluids influenced by external force field. Electrically conducting field play a very important role in rheology.

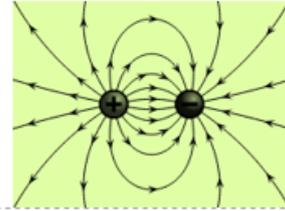
$$\tau = \phi \left(\frac{d\gamma}{dt}, t \right)$$

$$\tau = \phi \left(\frac{d\gamma}{dt}, field \right)$$

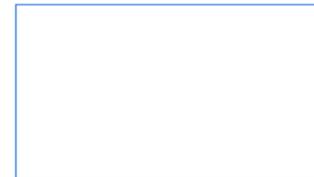
Fluids influenced by external force fields

- Electrically conducting fields play an important role in rheology. Stress is a function of strain and applied electrical field, and can be expressed as;

$$\tau = \phi \left(\frac{d\gamma}{dt}, field \right)$$



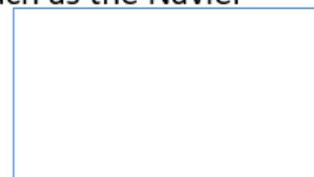
- Example include magnetohydrodynamics.
- Originally applied to situations involving plasmas and ionised gases.
- Their velocity profile behaviour is comparable to that of "simple" non-Newtonian.



Stress is a function of strain and applied electrical field and can be expressed as $\tau = \phi \left(\frac{d\gamma}{dt}, field \right)$. The example includes the magnetohydrodynamics originally applied to a situation involving the plasmas and ionized gas. Their viscosity profile behaviour is comparable to that of the simple and non-Newtonian. Let us talk about the non-continuum fluid. Non-continuum fluids also known as the rarefied, rarefied gases, rarefied fluids or concerned gas, these refer to which, in which the mean free path of the molecule is comparable to or a larger than the characteristic length of the fluid.

Non-Continuum fluid

- Non-continuum fluids, also known as rarefied gases, rarefied fluids, or Knudsen gas refer to fluids in which the mean free path of the molecules is comparable to or larger than the characteristic length of the flow.
- Unlike continuum fluids, such as liquids and gases at standard conditions, non-continuum fluids exhibit behavior that cannot be described by classical fluid dynamics equations, such as the Navier-Stokes equations.
- Instead, their behavior is governed by statistical mechanics and kinetic theory.



Unlike continuum fluids such as liquids and gases at standard conditions, non-continuum fluids exhibit behaviour that cannot be described by the classical fluid dynamics equation, such as Navier-Stokes equation. Instead, their behaviour is governed by the statistical mechanics and kinetic theory. Fogs, aerosols, spray, smoke, rarefied gases, they are the example of non-continuum fluids. In this regime,

the Navier-Stokes equation which are the corner stone of classified fluid dynamics are no longer valid. There are some key points about the non-continuum fluids like mean free path.

In non-continuum fluids, the average distance a molecule can travel without colliding with another molecule, this is called as a mean free path, which is significant compared to the size of the system. Now, this implies that the molecular collision between becomes less frequent and individual molecular behaviour becomes important. Knudsen number: this number defined as the ratio of the mean free path to the characteristic length scale of the system and is used to characterise the non-continuum behaviour. When the Knudsen number is large and Kn is greater than 0.

Non-Continuum fluid

- **Knudsen Number:** The Knudsen number, defined as the ratio of the mean free path to the characteristic length scale of the system, is used to characterize the non-continuum behavior. When the Knudsen number is large ($Kn > 0.01$), the fluid is considered non-continuum.
- **Molecular Interactions:** In non-continuum fluids, molecular interactions become more significant. The behavior of individual molecules, such as their collisions, diffusion, and thermal effects, cannot be ignored.

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01, the fluid is considered as a non-continuum. The molecular interaction, in non-continuum fluids, molecular interaction becomes more and more significant and the behaviour of individual molecules such as their collision, diffusion and thermal effects cannot be ignored. Departure from continuum assumptions, the continuum assumptions such as no slip boundaries or no slip boundary condition and the assumption of local equilibrium breakdown in non-continuum fluids. The fluid may exhibit slip flow where the velocity at the boundary differs from that of the predicted by the no slip conditions. The rarefaction effects, the rarefaction effects become prominent in non-continuum fluid and these effects arise due to the low density of molecule and can include thermal creep, pressure driven flow and velocity slips. Knudsen paradox, the non-continuum fluid behaviour can often contradict the predictions of classical fluid mechanics.

Non-Continuum fluid

- **Departure from Continuum Assumptions:** The continuum assumptions, such as the no-slip boundary condition and the assumption of local equilibrium, break down in non-continuum fluids. The fluid may exhibit slip flow, where the velocity at the boundary differs from that predicted by the no-slip condition.
- **Rarefaction Effects:** Rarefaction effects become prominent in non-continuum fluids. These effects arise due to the low density of molecules and can include thermal creep, pressure-driven flow, and velocity slip.

For example, in micro scale or a nano scale channels, the flow rate may increase with a decreasing pressure gradient and contrary to the behaviour observed in the continuum fluid. The modelling approaches to describe the non-continuum fluid behaviour, various modelling approaches are used like Boltzmann equation, direct simulation Monte Carlo methods, molecular dynamic simulations and these methods account for individual molecular interaction and provide insights into the non-continuum phenomena. Now various applications are attributed. The non-continuum fluid dynamics is relevant in many engineering and scientific applications. This includes the micro fluidic vacuum system, gas flow in the non-porous material, spacecraft reentry and rarefied gas dynamics.

Non-Continuum fluid

- **Modeling Approaches:** To describe non-continuum fluid behavior, various modeling approaches are used, such as the Boltzmann equation, direct simulation Monte Carlo (DSMC) methods, and molecular dynamics simulations. These methods account for the individual molecular interactions and provide insights into non-continuum phenomena.

Understanding the flow behaviour of non-continuum fluid is essential for designing efficient processing techniques such as extrusion, injection moulding and a film casting to achieve the desired polymeric product properties. The non-continuum fluids in polymer rheology, they are also used in

study phenomena such as polymer melt spinning, fibre formation and the behaviour of polymer blends and composites. The characterisation of these non-continuum fluids involves measuring properties like viscosity, shear stress, shear rate, viscoelastic properties using techniques such as rotational rheometry, capillary rheometry and dynamic mechanical analysis are sometimes referred as DMA. Let us talk about the relativistic fluids. The branch of fluid dynamics that deals with the behaviour of the fluids at relativistic speed, where the velocity approaches to the speed of light.

Non-Continuum fluid

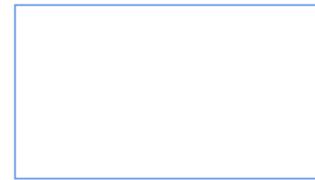
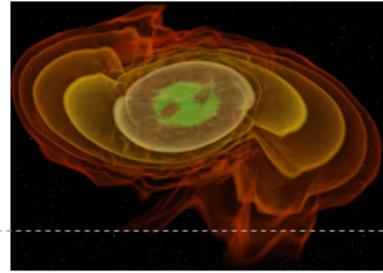
- Non-continuum fluids in polymer rheology are also used to study phenomena such as polymer melt spinning, fiber formation, and the behavior of polymer blends and composites.
- The characterization of non-continuum fluids involves measuring properties like viscosity, shear stress, shear rate, and viscoelastic properties using techniques such as rotational rheometry, capillary rheometry, and dynamic mechanical analysis (DMA).

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The relativistic dynamics modifies the equation of classical fluid dynamics such as conservation, equation of mass, momentum and energy by incorporating the Lorentz transformation and relativistic energy momentum relations. The relativistic fluids, this exhibits the unique properties and phenomena compared to their non-relativistic counterparts. For example, energy and mass are interconvertible. The relativistic energy momentum relation $E = mc^2$ allows the conversion of mass into energy and vice versa and the enhanced particle velocities. The enhanced particle velocities, the relativistic effects allow the particles within the fluid to achieve the velocities approaching to the speed of light resulting then the relativistic shocks and generation of particle jets.

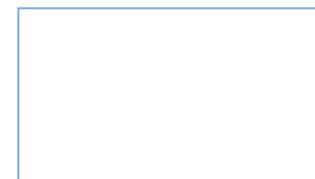
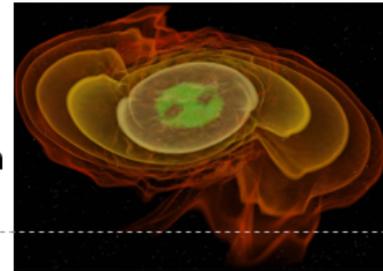
Relativistic fluids

- Branch of fluid dynamics that deals with the behavior of fluids at **relativistic speeds**, where velocities approach the **speed of light (c)**.
- **Relativistic hydrodynamics** modifies the equations of classical fluid dynamics, such as the **conservation equations of mass, momentum, and energy**, by incorporating the **Lorentz transformations and relativistic energy-momentum relations**.



Relativistic fluids

- **Enhanced particle velocities:** Relativistic effects allow particles within the fluid to achieve **velocities approaching the speed of light**, resulting in relativistic shocks and the generation of particle jets.
- **Relativistic temperature:** The concept of temperature in relativistic hydrodynamics is modified to account for **relativistic energy and momentum**, leading to different thermodynamic properties and equations of state.



The relativistic temperature, the concept of the temperature in the relativistic hydrodynamics is modified to account the relativistic energy and momentum. This led to the different thermodynamic properties and equation of state. So dear friends, in this particular segment we discussed several rheological models, rheological approaches which are very much useful in the polymeric system especially when we go for any kind of moulding operation, injection whether injection moulding or a compression moulding operation.

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And for your convenience we have enlisted several references which you can utilize as per your requirement. Thank you very much.