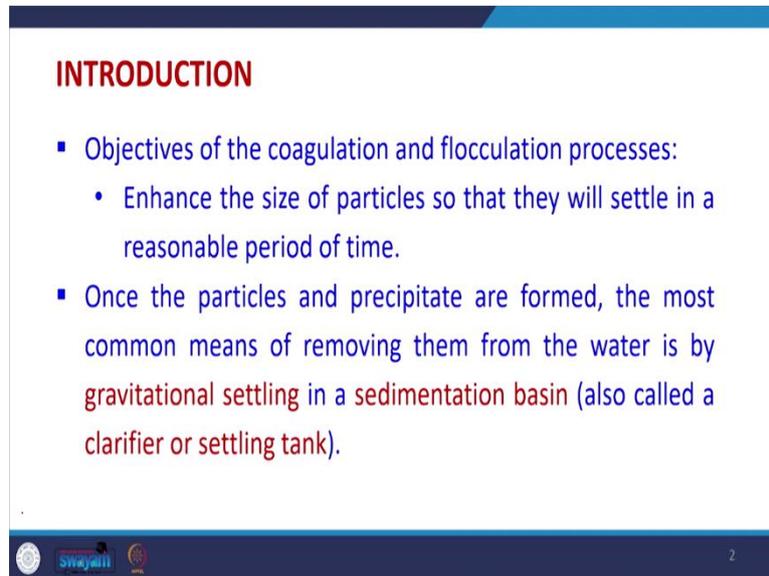


Biological Process Design for Wastewater Treatment
Professor Vimal Chandra Srivastava
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Lecture 19
Coagulation, Flocculation, and Sedimentation - I

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INTRODUCTION

- Objectives of the coagulation and flocculation processes:
 - Enhance the size of particles so that they will settle in a reasonable period of time.
- Once the particles and precipitate are formed, the most common means of removing them from the water is by **gravitational settling in a sedimentation basin** (also called a **clarifier or settling tank**).

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Welcome everyone in this NPTEL online certification course on Biological Process Design for Wastewater Treatment. So, in the previous lectures we studied regarding the wastewater treatment and some of the unit operations that are carried out for treatment of such wastewaters. Now, we started learning about the first step that is the flow equalization and thereafter, the aeration in the previous lecture.

Now, we will be continuing further and we will be studying regarding coagulation, flocculation and sedimentation. So, we will start with the introduction section of this. So, we generally carry out coagulation and flocculation for two purposes, one is to enhance the size of particles, so that they will settle down in a reasonable period of time the objective is to settle the particles which by themselves do not settle down and they should also settle in a very reasonable period of time.

So that the design of the coagulation, flocculation system requires minimum volume possible. So, for doing this, we use coagulation, flocculation with effective dosage of coagulant and flocculent if required, and also we take care that the what types of particles are there, what is the size or what is the zeta potential on the particles. So, all these things are very important when we are studying coagulation and flocculation. Once the particles are settled and

precipitate are formed, they settled down and most common methods of removing them from water is by gravitation or settling in the sedimentation basin. So, that is why coagulation, flocculation sedimentation, they are generally performed in sequence and the unit operations are in sequence.

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Coagulation and flocculation Density
Size

Coagulation and flocculation units are used for

- Removing non-settleable (suspended) solids/particles (within some time domain) present in water
- Remove infectious agents
- Remove toxic compounds that have adsorbed to the surface of particles
- Remove precursors to the formation of disinfection by-products

[Eckenfelder, 2000] 3

Now, studying further coagulation, flocculation units are used for removing non-settleable suspended solids or particles within some time domain present in the water. So, we want to remove the non-settleable suspended solids or particles, which by themselves do not settle down and the reason may be that their density is smaller or their size is very smaller. So, if any of these two factors are present, that density of the solids is small and or their size is small, they will not settle down. So, they are within some limit they are their size is within some limits. So they it cannot sit her down by itself. And similarly, density is also a smaller if possible.

So then also it will not settle down. Now, to remove these solid particles may be infectious. So, we can remove these infectious agents, they may be toxic compounds also and they can be removed via adsorption to the surface of the particles which are formed via coagulation coagulants. So, we can remove toxic compounds, we can remove infectious agents, and we can remove precursors also which lead to the formation of disinfectant by-products. So, through this we can remove all these materials via coagulation and flocculation.

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Coagulation

- During coagulation, chemicals/coagulants with charges opposite those of the suspended solids are added to the water to neutralize the negative charges on non-settleable solids (such as clay and color-producing organic substances).

[Eckenfelder, 2000] 4

So, during coagulation chemicals or coagulants with charges opposite to those of suspended solids are added to the water to neutralize the negative charge on the non-settleable solids, such as clay colour producing organic substances. So, what happens is that in the collides which are present in the natural water suppose this is one of the collide.

So, generally at its natural pH in the water and wastewater it will be negatively charged. So, this will be negatively charged. Now, there is another collide which is again in the same water and which is of size similar to that or maybe a little bit different, it is also negatively charged. So, both are negatively charged. So, there is always a force of repulsion between them. And since their sizes are also small, they cannot settle down.

Now, what is done is that we will add some coagulants which have charges opposed to those are suspended solids. So, that means we will be adding some coagulants which will impart positive charge like suppose Alum sulphate is there. So, from Alum sulphate Al_3 plus will come out. Now, this Al_3 plus that means it is positively charged overall. So, this positive charge will be attracted and it will go to the surface of the these collides and thus the charge of the collide is neutralized. So, here we neutralize the negative charge and non-settleable solids.

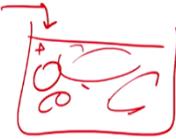
So, thus since both are now neutral and if we can somehow force them to come together, then vander wall force of attraction will come into picture and they will form a bigger flock which will be combined together and now, since both are virtually neutral and their size has become a greater, so, they have a lot of probability of settling down. So, this is what is done in their

coagulation we always add coagulants with charges opposite to those of suspended solids present in the water. So, this is the idea.

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Coagulation

- Once the charge is neutralized, the small suspended particles are capable of sticking together.
- Rapid-mixing is done to properly disperse coagulant, and promote collisions among coagulants and suspended particles to achieve good coagulation.
- Over-mixing does not affect coagulation, but insufficient mixing will leave this step incomplete.



[Eckenfelder, 2000] 5

Now, once the charge is neutralized, the small suspended particles are capable of sticking together as shown in the previous slide. Now, rapid-mixing is done to properly disperse coagulants. So, as soon as suppose this is a water tank, and as soon as the coagulants are added, so, they will break into respective positive and negative side.

So, what is done is that, we do rapid-mixing, so, that the coagulants disperse all throughout the solution. Now, and also through rapid-mixing, we try to force the coagulant to reach out to the negatively charged particles so, that it may be neutralized. So, this rapid-mixing is done to properly disperse coagulants and promote collisions among the coagulants and suspended particles to achieve good coagulation. Over mixing does not affect coagulation, but insufficient mixing will leave this step incomplete.

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Flocculation

- A gentle mixing stage, increases the particle size from submicroscopic **microfloc** to visible suspended particles.
- Microfloc particles collide causing them to bond to produce larger, visible flocs.
- Floc size continues to build with additional collisions and interaction with added coagulant.
- Once floc has reached its optimum size and strength, water is ready for sedimentation.

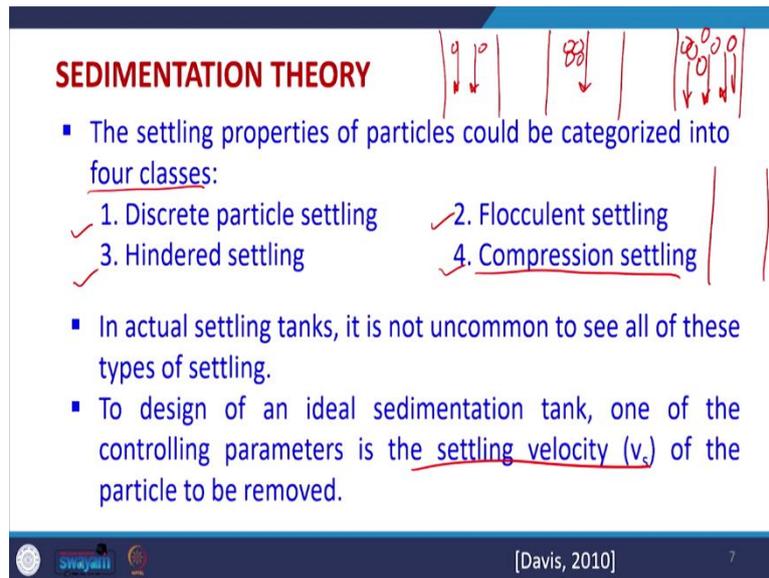


[Eckenfelder, 2000] 6

So, up to charge neutralization, we tried to do using the coagulation step after that we have Flocculation step. So, a gentle mixing state which increases the particle size from sub microscopic microfloc to visible suspended particles. So, this is done. So, in this case, what we do is that we try to do the gentle mixing. Now, since, two collides have come together, we want the third collide to also come and form a bigger floc but we do not want very rapid-mixing, otherwise these flocs may separate out.

So, that is why only gentle mixing is done gentle mixing increases the particle size from sub microscopic microfloc to visible suspended particles, these microfloc particles collide causing them to bond to produce larger visible floc, these floc size continues to build up with additional colleges and interaction with added coagulant once the flocs have reached the optimum size and strength the water is ready for sedimentation and then they will settle down in the sedimentation stage. Some other flocculants are added also which help in increasing the size this will depend you can refer to the another of my lecture on physico chemical methods for wastewater treatment, where I have discussed the coagulation and flocculation in detailed so that you can further study these topics for this biological process design, I think this is enough.

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SEDIMENTATION THEORY

- The settling properties of particles could be categorized into four classes:
 - ✓ 1. Discrete particle settling
 - ✓ 2. Flocculent settling
 - ✓ 3. Hindered settling
 - ✓ 4. Compression settling
- In actual settling tanks, it is not uncommon to see all of these types of settling.
- To design of an ideal sedimentation tank, one of the controlling parameters is the settling velocity (v_s) of the particle to be removed.

[Davis, 2010] 7

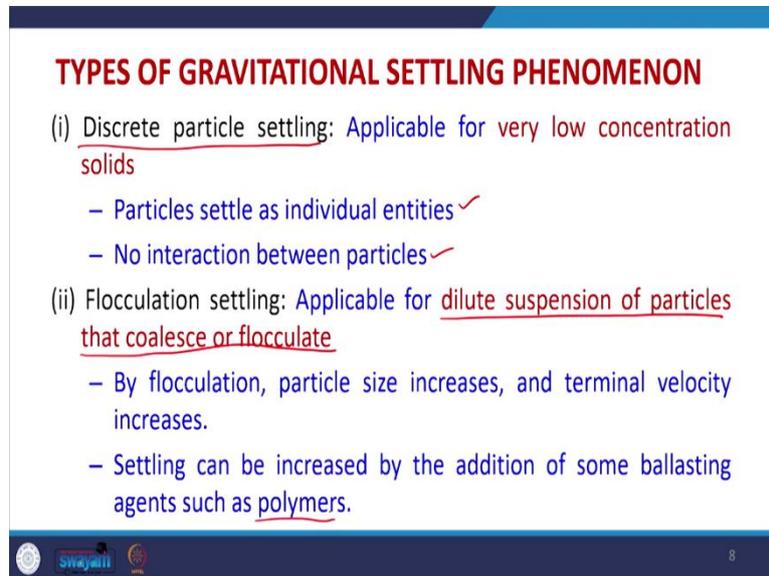
Now, further going on sedimentation theory, the settling properties of the particle could be categorized properties can be categorized into four classes. So settling once it is done settling in the is then after coagulation flocculation after biological treatment also that is called secondary settling and then some other places also settling is possible during treatment itself. So, sedimentation or settling is very important not only for physico chemical treatment, but for biological treatment also, now settling properties of particles can be categorized into four category one is called discrete particle settling, then Flocculent settling, Hindered settling and then the compression settling and what is the difference between these four?

So discrete particles settling as the name suggests here only suppose this is a beaker is there so, we have only few particles which are settling down and they are not affecting each other. So, we have discrete particle settling. So, generally we start the theory using the discrete particle settling, then we have flocculent settling where the particles are coming together and forming bigger flocs and then they are settling down. So, this is flocculent settling in the Hindered settling the amount of particles are such large that they actually affect the settling of each other.

So, this is hindered settling and in the fourth estate, compression settling the amount of particles is much larger the particles themselves compress the particles which are at the bottom and does the settling occur. So, which we have compression settling in actual settling tank It is not uncommon to see all of these type of settling may happen the design of ideal sedimentation tank one of the controlling parameter is the settling velocity of the particle to

be removed and this theory can be obtained from the discrete particle settling itself. So, we will try to study sedimentation or settling in detail because it is very important for our course.

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TYPES OF GRAVITATIONAL SETTLING PHENOMENON

- (i) Discrete particle settling: Applicable for very low concentration solids
 - Particles settle as individual entities ✓
 - No interaction between particles ✓
- (ii) Flocculation settling: Applicable for dilute suspension of particles that coalesce or flocculate
 - By flocculation, particle size increases, and terminal velocity increases.
 - Settling can be increased by the addition of some ballasting agents such as polymers.

Now, types of Gravitational Settling Phenomenon So, we can have discrete particle settling I suggest is applicable for very low concentration solids, particles settle as individual entities and no interaction occurs between the particles. So, this is discrete particle settling then we have flocculation settling applicable for dilute suspension of particles that coalesce our flocculate, by flocculation the particle size increases and the terminal velocity also increases. So, settling can be increased by addition of some ballistic agents such as polymers or some other flocculants also.

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(iii) Hindered settling:

- For suspension of intermediate settling.
- In this case, particles are such close together that the inter-particle force due to one hinders the settling of another particle.
- The particles remain in a fixed position with respect to each other, and particles settle as a whole.

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So, this is possible then hindered settling for suspensions of intermediate settling. So, this is hindered settling where for suspension it is used in this case the particles are such close together that the inter particle force due to one hinders the settling of the another particle. So, that means we have lots of particles and the settling of one is actually affecting the settling of another. The particles remain in the fixed position with respect to each other and particles settle as a whole this is the hindered settling.

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(iv) Compression settling:

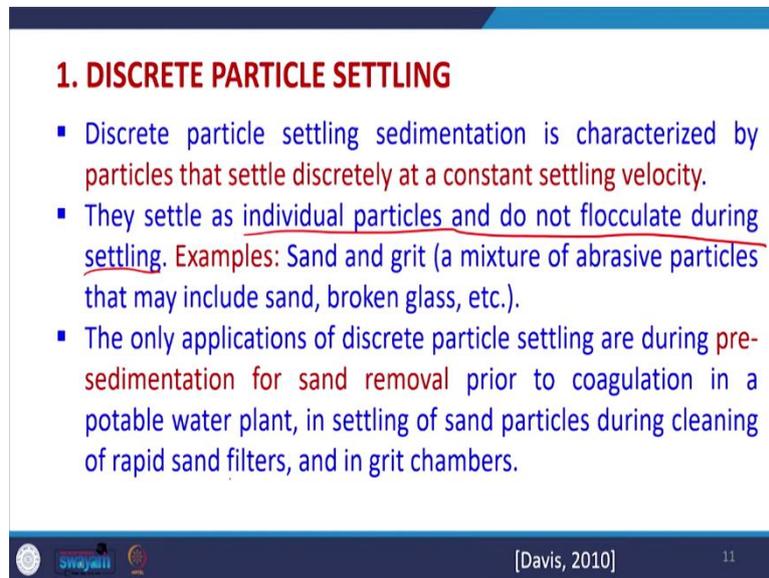
- Case in which particles are in such high concentration that a whole structure is formed.
- Compression takes place due to the weight of the whole mass, which continuously increases.
- A clear water is formed above the compression zone

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Then, we have compression settling in this case the particles are set in such high concentration that the whole structure is formed and compression takes place due to the

weight of the whole mass which continuously increases, the clear water is formed above the compression zone that means that clear water is above and all the settle particles are at the bottom.

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1. DISCRETE PARTICLE SETTLING

- Discrete particle settling sedimentation is characterized by particles that settle discretely at a constant settling velocity.
- They settle as individual particles and do not flocculate during settling. Examples: Sand and grit (a mixture of abrasive particles that may include sand, broken glass, etc.).
- The only applications of discrete particle settling are during pre-sedimentation for sand removal prior to coagulation in a potable water plant, in settling of sand particles during cleaning of rapid sand filters, and in grit chambers.

[Davis, 2010] 11

Now we will start with the first in greater detail. The discrete particle settling that discrete particles settling sedimentation is characterized by particles that settle discretely at a constant settling velocity. They settle as individual particles and do not flocculate during the settling examples like sand, grit, etcetera are a mixture of abrasive particles that may include sand, broken glass etcetera. The only application of discrete settling are due to pre sedimentation for sand removal prior to coagulation in a portable water plant or in settling of sand particles during cleaning of rapid sand filters or in grit chambers etcetera.

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PARTICLE SETTLING THEORY

- When particles settle discretely, the particle settling velocity can be calculated, and the basin can be designed to remove a specific size particle.
- A particle settling in a fluid experiences following force balance:

$$m \frac{du}{dt} = F_e - F_D - F_b$$

where

- m is the mass of the particle
- u is the settling velocity of the particle in the fluid
 - a_e is the acceleration force,
 - $a_e = g$ for Gravitational settling; $a_e = rw^2$ for settling under centrifugal action
 - F_D is the drag force; F_b is the buoyancy force

A particle settling in a fluid experiences following force balance:

$$m \frac{du}{dt} = F_e - F_D - F_b$$

Now, but with the discrete settling we have the particle settling theory and this is very important because it helps us in designing the settling or sedimentation basins later on. So, we have a particle settling theory which is based upon the will discrete particle settling. So, when particles settle discretely the particle settling velocity can be calculated and the basin can be designed to remove the specific size particle, suppose that particle is settling in a fluid then under that condition experiences following forces.

So, this is the particle which is settling down now suppose it is settling down at the bottom so it will experience a force in the same direction which is labelled as F_e but this is opposed this a_e is equal to gravitational force, force of gravity. So this is actually F_g the force of gravity. Now, since the particle is settling down, there will be a buoyancy force which is opposite to the settling.

So, this buoyancy force will be going up towards upward direction and this F_b is called buoyancy force, then there will be a drag force also which will be dependent upon the velocity with which it is settling down and this is a drag force which will be acting opposite

to the settling direction Now, overall the force balance can be written like this $m \frac{du}{dt}$ is equal to F_e which is downwards minus F_d minus F_b .

So, F_d is the drag force and F_b is the buoyancy force. So, an m is the mass of the particle u is the settling velocity of the particle in the fluid which is here given an a is the acceleration force this a_e can become equal to g for gravitational settling equal to $r \omega^2$ for settling under centrifugal action, but we will be considering mostly the gravitational settling. So, a_e equal to g that will be there.

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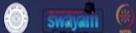
- F_D is the drag force, and F_b is the buoyancy force are given as:

$$F_D = C_D \frac{\rho_f u^2}{2} A_p$$

$$F_b = m \left(\frac{\rho_f}{\rho_p} \right) a_e$$

where

- C_D is the drag coefficient
- ρ_f and ρ_p are the density of fluid and particle, respectively
- A_p is the projected area of the particle
- m is the mass of the particle


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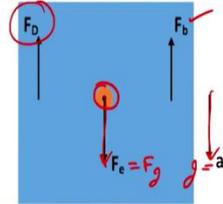
PARTICLE SETTLING THEORY

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A particle settling in a fluid experiences following force balance:

$$m \frac{du}{dt} = F_e - F_D - F_b$$

Now, going further the drag force and the buoyancy force can be given as follows. So, buoyancy force easily can be written as m into ρ_f by ρ_p into a_e . So, ρ_f is the density of the fluid and ρ_p is the density of the particle respectively. Now, a_e already we have defined that drag force can also be defined like this, drag force is equal to C_D into $\rho_f u^2$ by 2 into A_p . So, C_D is the drag coefficient, we will be learning regarding this this is very important, then, A_p is the projected area of the particle. So, for spherical particle we can easily calculate otherwise, we have to find out the sphericity and then we can determine m is the mass of the particle and ρ_f is already defined here ρ_f is the density of the fluid, u is the settling velocity. Now, what we do is that we put the value of F_d and F_b in the previous equation. So, the question becomes like this.

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Cont....

- For spherical particles having diameter (D_p), the value of A_p and m is given as:

$$A_p = \frac{\pi D_p^2}{4}, m = \left(\frac{\pi D_p^3}{6} \right) \rho_p$$

$$F_e = F_g + F_b$$

- For particles settling with terminal velocity (u_t) under the force of gravitational force ($a_e = g$), $(du/dt) = 0$.

For spherical particles having diameter (D_p), the value of A_p and m is given as:

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Now, for spherical particles having diameter D_p , the value of A_p and m can be given as follows A_p you got two pi D_p square by 4, this is projected area and mass can be calculated by using the volume into density. So, that volume is pi D_p cube by 6 into ρ_p so, we can substitute A_p and m using this now, for particles settling with terminal velocity suppose the velocity terminal velocity has been reached. So, under that condition du/dt will be equal to 0 and the gravitational force we are keeping a is equal to g . So, under this condition, what we have is F_e is equal to F_d plus F_b and we will put all other values in this equation.

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Cont....

- Putting the values of different forces, the terminal velocity (u_t) by Newton's method is given as:

$$u_t = \sqrt{\frac{2mg(\rho_p - \rho_f)}{A_p C_D \rho_p \rho_f}}$$

$$u_t = \sqrt{\frac{4(\rho_p - \rho_f)gD_p}{3\rho_f C_D}} \quad (\text{for Spherical particle})$$

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Cont....

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$F_e = F_g + F_b$

- For particles settling with terminal velocity (u_t) under the force of gravitational force ($a_e = g$), $(du/dt) = 0$.

$U = u_t$

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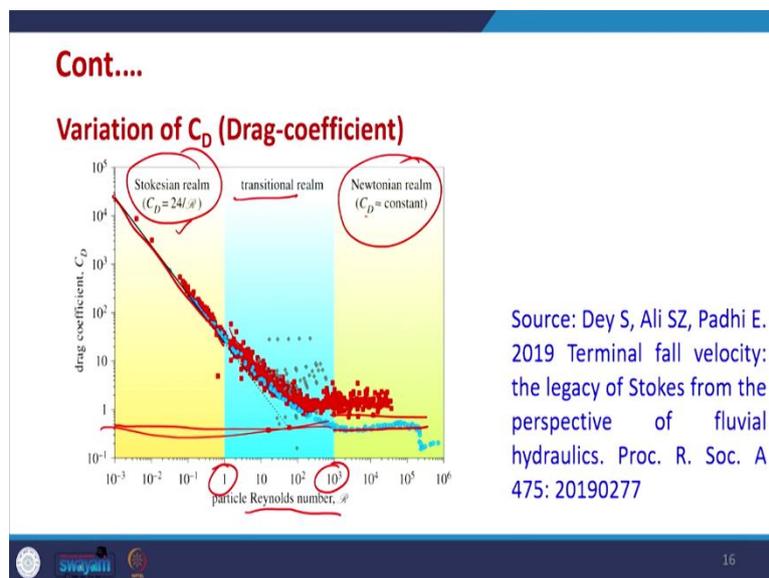
For spherical particles having diameter (DP), the value of AP and m is given as:

$$A_p = \frac{\pi D_p^2}{4}, \quad m = \left(\frac{\pi D_p^3}{6} \right) \rho_p$$

So, if we put on the values and try to find out the we keep also u is equal to u_t under terminal velocity condition and if you try to solve the equation we will be getting these two equations in one of the equations we have kept as such there is A_p term there is m term. And there is C_D term in the second equation both are essentially same, this is for the spherical particles that means A_p and m have been replaced for spherical parties.

So, we have our two equations, this is more common equation which is used because generally what we consider is that the particles which are settling down are spherical in nature. So, this is a spherical particles. So, this is the equation which is most commonly used. Now, in this equation, there is a term for C_D and this is very important that what is the drag coefficient and because drag coefficient is dependent upon the velocity also.

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So, this is variation of drag coefficient with respect to particle Reynolds number. So, particle Reynolds number we can calculate using the velocity and other parameters viscosity or diameter and the fluid parameters also. So, drag coefficient C_D varies like this So, C_D is like it decreases linearly up to a certain range, which is around the particle Reynolds number of 1. So, up to this range or 1 or 2, so, this this is linear after that in the later stages when the

Reynolds number particle Reynolds number is greater than 10 Raise to 3 this C_D is virtually constant.

So, it is fixed it is a certain value which is fixed in between this. So, this first region where there is linear decrease it is called Stoke's law region and in the case where the C_D is constant it is called Newtonian reason. And in between we have transitional region transition where different equations have been given to model the variation of C_D with particles Reynolds number.

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Variation of C_D (Drag-coefficient)

- In the laminar zone, Stoke's law is applicable

$$C_D = \frac{24}{Re}; \quad 0.01 \leq Re \left(= \frac{\rho_f u_t D_p}{\mu_f} \right) \leq 0.1 \approx 1$$

$$u_t = \frac{g(\rho_p - \rho_f)D_p^2}{18\mu_f}$$

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Now, how these can be modelled in the laminar region where the Stoke's law is applicable and where the C_D decreases linearly with the Reynolds number the C_D is given a C_D is equal to 24 by Re and this is applicable for Reynolds number which is given particles Reynolds number which is given ρ_f into u_t into D_p by μ_f , which is the fluid viscosity. Now, if the Reynolds number is between 0.01 to 0.1 Sometimes it is taken as 1 also. So, under up to this condition, a C_D is taken as 24 by Re and this Re can be replaced by this term. So, overall the

equation will become u_t is equal to $g \rho_p$ minus $\rho_f D_p$ square into $18 \nu_f$. So, this is the equation which is used in the laminar zone.

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Cont....

For transition zone: $0.1 \leq Re \leq 1000$

$$C_D = \frac{a}{Re^n} = \frac{18.5}{Re^{0.6}}$$

$$C_D = \frac{24}{Re} + \frac{3}{\sqrt{Re}} + 0.34$$

For turbulent zone $500 \leq Re \leq 200,000$

- C_D is independent of Re and $C_D = 0.44$

Re Range	α	β
Stokes range ✓	24.0 ✓	1.0
Intermediate range	18.5 ✓	0.6 ✓
✓ Newton range $C_D = 0.44$	0.44	0.0

$C_D = \alpha Re^{-\beta}$

For transition zone: $0.1 \leq Re \leq 1000$

$$C_D = \frac{a}{Re^n} = \frac{18.5}{Re^{0.6}}$$

$$C_D = \frac{24}{Re} + \frac{3}{\sqrt{Re}} + 0.34$$

For turbulent zone $500 \leq Re \leq 200,000$

C_D is independent of Re and $C_D = 0.44$

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Cont....

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$$u_t = \frac{g(\rho_p - \rho_f)D_p^2}{18\mu_f}$$

Now, we have a transition zone also and we have a turbulent zone also in the transition zone various equations have been given and this transition zone can be taken from 0.1 Reynold number to up to 1000. Now, there are different types of equations like one of the equation is C_D is equal to a by Re raise to n and generally a is taken as 18.5 and this n values taken as 0.6.

So, we can use this equation. Similarly, C_D is also given the drag coefficient is also given by this type of equation which is reported in the literature. So, we can use any of the equation depending upon the type of water wastewater etcetera. Then for turbulent zone there is another equation that Reynolds number around 500 are from 1000 to 2,00,000, we can take C_D is equal to 0.44 That means C_D is equal to 0.44.

Now, if you take this equation, this equation and previous C_D is equal to 24 by Re , we can generalize the value of C_D this equation can be considered as common for all three now, C_D is equal to αRe raised to minus β . So, this is the equation now, in this case for a stokes law range we have α is equal to 24 and β is equal to 1. So, that means, it becomes 24 by Re , so, this equation is correct.

Now, for Newton's law range, we have beta is equal to 0 and alpha is equal to 0 point 44. That was C_D is equal to 0.44 So, this equation is correct. Similarly, for intermediate range the alpha is taken as 18.5 and beta is equal to 0.6 So, this equation is generic equation representing the C_D variation with particles Reynolds number. So, this is this equation can be used.

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For non-spherical particles

- The formula for Reynold number and settling velocity calculation are modified using the shape factor (ϕ):

$$Re = \phi \frac{\rho_f u_t D_p}{\mu_f}$$

$$u_t = \sqrt{\frac{4(\rho_p - \rho_f) g D_p}{3 \rho_f \phi C_D}}$$

$\rho_p = \rho_f = \rho$ $C_D = 0.44$ $\mu_f = \mu$ $\beta = 1$ $C_D = ?$
 Steady calculation, Laminar u_t
 1 u_t $Re_p \rightarrow C_D =$
 2 u_t Re_p $C_D =$
 3
 4 u_t

Source: Metcalf & Eddy [2003] 19

For non-spherical particles:

$$Re = \phi \frac{\rho_f u_t D_p}{\mu_f}$$

$$u_t = \sqrt{\frac{4(\rho_p - \rho_f) g D_p}{3 \rho_f \phi C_D}}$$

Cont....

For transition zone:

$$0.1 \leq Re \leq 1000$$

$$C_D = \frac{a}{Re^n} = \frac{18.5}{Re^{0.6}}$$

$$C_D = \frac{24}{Re} + \frac{3}{\sqrt{Re}} + 0.34$$

For turbulent zone

$$500 \leq Re \leq 200,000$$

C_D is independent of Re and $C_D=0.44$

$$C_D = \alpha Re^{-\beta}$$

Re Range	α	β
Stokes range ✓	24.0 ✓	1.0
Intermediate range	18.5 ✓	0.6 ✓
✓ Newton range $C_D = 0.44$	0.44	0.0

For transition zone: $0.1 \leq Re \leq 1000$

$$C_D = \frac{a}{Re^n} = \frac{18.5}{Re^{0.6}}$$

$$C_D = \frac{24}{Re} + \frac{3}{\sqrt{Re}} + 0.34$$

For turbulent zone $500 \leq Re \leq 200,000$

C_D is independent of Re and $C_D=0.44$

$$C_D = \alpha Re^{-\beta}$$

Cont....

Variation of C_D (Drag-coefficient)

In the laminar zone, Stoke's law is applicable

$$C_D = \frac{24}{Re} \quad 0.01 \leq Re \left(= \frac{\rho_f u_t D_p}{\mu_f} \right) \leq 0.1 \approx 1$$

$$u_t = \frac{g(\rho_p - \rho_f)D_p^2}{18\mu_f}$$

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Now, for non-spherical particles, the formula for Reynolds number and settling velocity calculations are modified using the shape factor ψ and which is for the Reynolds number it is multiplied by ψ and here we add a term ψ . So, this equation can now be used in place of other equation, if we know the shape factor value the shape factor may be 0.9 1.1 1.2.

So, depending upon that we can find out the value of settling velocity. Now, in general suppose we have to solve the equation for finding out the Reynolds number and we for a discrete particle where everything is given. So, what we have to do the general steps is that if go further we should know that different values so, like First thing is that we should know the diameter of the particle.

So, it should be known to us in micrometre or something. So, this should be known to us, then we should know the diameter the density of the fluid or we can take this to be density of water, because for wastewater generally it may be taken as our water. So, this is density of water, then we should know the density of particle also.

So, they should be known, similarly, we should know the viscosity of the fluid, our viscosity of the water. So, this should also be known to us. So, suppose everything is known to us, all these four parameters are known to us. So, under this condition what we can do is that we have to find out the settling velocity. Now, since all these known are knows, but we do not know the value of C_D , because we do not have the value of suppose shape factor is also known for purely spherical particle it will become 1.

So, now, what is the value of C_D we do not know until unless we do not know the C_D value, we cannot find out the terminal velocity. So, for doing this what is done is that we try to find out the value of we considered that the settling is actually happening in that laminar region. So, for starting the calculation starting calculation we assume that there is a laminar zone settling which is happening. So, that means, we have the C_D value is equal to we are

assuming that it is in the range of Reynolds number 1 or we can assume this to be in the range of Re is equal to 1 and we directly apply this particular formula for calculating the u_t value.

So, here as soon as we assume that laminar zone is there, we can directly use this particular formula. And once we are using this particular formula, we do not require the C_D value because it is fixed. So, if we assume laminar zone, we can find out the u_t using the equation which is given here, and then we can find out the u_t once u_t value has been found out, we can find out the value of Reynolds number. So, once u_t is known, we can that means u_t should be known, then we can find out the value of particle Reynolds number and once particle Reynolds number is known, we can find out the value of C_D and since C_D is known as suppose C_D value comes out to be this Reynolds number comes out to be in the range of any of these.

So, if it comes out to be less than 1 or less than 0.1 that means, our initial assumption that it is laminar range is correct, and we can find out we can be sure that u_t is correct. Now, suppose it is possible that this Reynolds number is not in the laminar range. So, it is in the transition range or it is in the turbulent range, then we have to find out the value of C_D and from C_D we calculate back u_t . So, the first calculation is until unless this u_t and this Reynolds number they should fall in the same transition zone that means a C_D calculated assume and the Reynolds number should be in the same transition zone and we keep on calculating these because the Reynolds number will go on changing.

So, this is the as we write like this, this is the 1 iteration then 2, then 3, 4, because the Reynolds number will keep changing to C_D value will also be keep changing, though they may be in the same zone also. So, this is done until that two consecutive u_t values are same. So, this is a iteration procedure which is adopted for finding out the terminal velocity using the terminal velocity theory which has been given here. Now, the problem here is that, we have to do lot of iteration and we have to continuously modify the value of Reynolds number and does the value of C_D until unless two consecutive u_t 's are very similar and that then that u_t can be assumed.

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Non-iterative method of determining terminal velocity

- Multiplying $v = fd_p^2 \rho_p / 18\mu$ by $(d_p \rho / \mu)$, we get $\rho_p > \rho_f$

$$f = C_D = 0.44$$

$$K = d_p \left(f \rho_p \rho / \mu^2 \right)^{1/3}$$

$$Re = K^3 / 18; \quad Re < 2.0$$

$$Re = 1.74 K^{1.5}; \quad Re \geq 500$$

$K < 3.3$ Stokes' law range

$43.6 > K > 3.3$ Intermediate range

$2360 > K > 43.6$ Newton's law range



$$K = d_p \left(f \rho_p \rho / \mu^2 \right)^{1/3}$$

$$Re = K^3 / 18; \quad Re \leq 2.0$$

$$Re = 1.74 K^{1.5}; \quad Re \geq 500$$

Cont....

Variation of C_D (Drag-coefficient)

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$$u_t = \frac{g(\rho_p - \rho_f) D_p^2}{18\mu_f}$$

Cont....

For non-spherical particles

- The formula for Reynold number and settling velocity calculation are modified using the shape factor (ϕ):

$$Re = \phi \frac{\rho_f u_t D_p}{\mu_f}$$

$$u_t = \sqrt{\frac{4(\rho_p - \rho_f)gD_p}{3\rho_f \phi C_D}}$$

$D_p = C_g = \rho_w C_p \frac{\mu_f = \mu_w}{\rho = 1} \mid C_g = ?$
 Stokes calculation, Laminar u_t
 1 u_t $Re_p \rightarrow C_D =$
 2 u_t Re_f $C_D =$
 3
 4 u_t



Source: Metcalf & Eddy [2003]

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For non-spherical particles:

$$Re = \phi \frac{\rho_f u_t D_p}{\mu_f}$$

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Now, this problem can be solved using the non-iterative method for determining terminal velocity. This is done initially; this was the formula if you remember this is the formula which was there for u_t here only differences that we are using. So, we have the terminal velocity which is given and the difference here is that we are assuming here it is being assumed that ρ_p is much greater than ρ_f , which is actually correct.

So, if we multiply this equation by this and then actually we get a term where the K value will be defined the K value will be this. Now, suppose the C_D is equal to 0.44 and we use this particular K equation. So, what we do is that, this all these equations for different ranges will be modified the in case of C_D we use the key term and for Stokes law region the K value which will be less than point 3 point 3 for intermediate range.

The K value is greater than 3 point 3 but less than 43 point 6 and for Newton's law in the K value is greater than 43 point 6 up to 2360. So, this K value is used. Now, the using this K factor also that Reynolds number the equations for different ranges are also modified. So, this equation can be changed as follows for less than 2 in the Stokes law region it is the Reynolds number will be given by K cube by 18 and similarly, for greater than 500 in the transition

zone Reynolds number can be given by this equation. Now, the beauty of this equation is that, we do not need to perform the iteration once the values are known like here DP , ρ_f , ρ_p , ν_f etcetera are known, we can directly find out the K value and once K value is known, we know that K value is in the stock law region or in the newton's law region or intermediate region, because from K value we can directly find out the region and when this is used, we can directly go for finding out the respective equation and from there we can solve to find out the value of that terminal velocity. So, this is non iterative method of finding out that terminal velocity of as compared to previous equation. So, with this will end today's lecture will continue further. Thank you.