

Biological Process Design for Wastewater Treatment
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Lecture no. 14
Bacterial Growth Kinetics - II

Welcome everyone in this NPTEL course on Biological Process Design for Wastewater Treatment. So, in the previous lecture we studied regarding the bacterial growth and it is a bacterial growth kinetics. Later on, during that lecture, we studied regarding the Monod growth process, et cetera. So, we will continue further.

In the previous lecture, we studied that there are three aspects during the overall decay of the oxidation of the organic matter in the reactor. So, first is that the substrate continuously decreases with increase in time, the bacteria first it starts multiplying itself and later on during the endogenous respiration is again decays back and oxygen continuously is required for the oxidation of the substrate and for the growth of the bacteria. So, today, we will try to understand the decay process in much more detail.

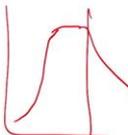
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- ❑ For accuracy, only the biodegradable fraction of the biomass should be considered since there is also an inert, non-biodegradable organic fraction, not subject to bacterial decay. But for simplicity, let's consider here the total VSS.
- ❑ The decay rate can be expressed as a first-order reaction:

$$\frac{dX}{dt} = -K_d \cdot X$$

Where K_d = endogenous respiration coefficient, or bacterial decay coefficient (d^{-1})



The decay rate can be expressed as a first-order reaction:

$$\frac{dX}{dt} = -K_d \cdot X$$

The relationship that we discussed earlier with respect to growth, that was more on the growth phase. So, now, when substrate actually decreases, we found that the bacteria which

are in the system, they start using their own cellular materials for their own survival, and thus we have endogenous metabolism state which is reached and this implies that the part of cellular matter itself is getting destroyed by means of some of the mechanisms active in the endogenous respiration state and this is done. So, if we subtract this endogenous decay or respiration stage, we can obtain the net growth rate et-cetera.

So, for accuracy only the biodegradable fraction of the biomass should be considered since there is also an inert or non-biodegradable or refractory organic fraction, which is not decayed naturally and which is not subjected to bacterial decay, but for the simplicity, we considered that that total VSS which is present in the reactor, that is the only biodegradable force and the volatile suspended solid which is there, it is considered to be the one which can decay.

Now, the decay rate in the can be expressed as first order reaction. So, if we can depending upon x , that dx/dt can be a function of x . So we have minus this K_d which is the called as endogenous respiration coefficient. And remember, this section is for the not the growth phase we are trying to model is for this phase where that decrease is happening, where the death rate is more as compared to the growth rate. So, this is the stage for which we are trying to model this particular curve.

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- For typical domestic sewage, K_d varies in the following ranges:
 - ❖ **Aerobic treatment**
 $K_d = 0.04$ to 0.10 mgVSS/mgVSS.d (base: BOD_5) (Metcalf & Eddy, 1991; von Sperling, 1997)
or
 $K_d = 0.05$ to 0.12 mgVSS/mgVSS.d (base: COD) (EPA, 1993; Orhon and Artan, 1994)
 - ❖ **Anaerobic treatment**
 $K_d = 0.02$ mgVSS/mgVSS.d (base:COD) (Lettinga et al., 1996)

So under that condition, for typical domestic sewage, the K_d value varies in the range of point 0.04 to 0.1 milligram VSS per milligram VSS per day based upon the BOD_5 and similarly the K_d value may be 0.05 to 0.12 depending upon the COD value. For anaerobic treatment, the

value is further lower 0.02 milligram VSS per milligram VSS per day, and this is based upon the COD references have given different values of K_d .

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Net bacterial growth

The net bacterial growth is obtained by :

$$\frac{dX}{dt} = \mu \cdot X - K_d \cdot X$$

or

$$\frac{dX}{dT} = \mu_{max} \cdot \frac{S}{K_s + S} \cdot X - K_d \cdot X$$

$\frac{dx}{dt} = \mu X$

$\mu = \mu_{max} \frac{S}{K_s + S}$

The net bacterial growth is obtained by:

$$\frac{dX}{dt} = \mu \cdot X - K_d \cdot X$$

$$\frac{dX}{dT} = \mu_{max} \cdot \frac{S}{K_s + S} \cdot X - K_d \cdot X$$

Now, the net bacterial growth rate can be obtained by using this equation. So, earlier we had written that for the growth phase $\mu \times dX/dt$ is equal to $\mu \times X$, but in that decay phase we are writing that dx/dt is equal to minus $K_d X$. So if we combine them together, we can write this particular equation for net bacterial growth. Now, in the previous lecture, we also gave that μ equal to $\mu_{max} S$ upon K_S plus S because the growth is a function of substrate concentration. So, overall, the equation can be written like this combining together both the growth equation as well as the decay equation.

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Production of biological solids

Gross solids production

❖ Bacterial growth, that is, biomass production can also be expressed as a function of the substrate used. The greater the substrate assimilation, the greater the bacterial growth rate. This relation can be expressed as:

Growth rate = Y (Substrate removal rate)

$$\frac{dX}{dt} = Y \frac{dS}{dt}$$

where,

- ✓ X=concentration of microorganisms, SS or VSS (g/m³)
- ✓ Y=yield coefficient, or the coefficient of biomass production; biomass (SS or VSS) produced per unit mass of substrate removed (BOD or COD) (g/g)
- ✓ S=concentration of BOD₅ or COD in the reactor (g/m³)
- t=time (d)

Handwritten notes:
 $Y = \frac{\text{Biomass production rate}}{\text{Substrate removal rate}} = \frac{dX/dt}{dS/dt}$

Growth rate = Y (Substrate removal rate)

$$\frac{dX}{dt} = Y \frac{dS}{dt}$$

Now, this leads to that there will be certain production of biological solids during the substrate removal. So, what is the gross solid production? So, bacterial growth that is the biomass production can be expressed as a function of the substrate used also. So, and this is like defining with respect to yield coefficient. So, the yield coefficient is defined and we have used the concept of yield coefficient earlier also during the stoichiometry overall growth stoichiometry equations. So, yield coefficient is the ratio of biomass production divided by the substrate removal rate.

So, it may be it may be defined as the y is equal to biomass production rate divided by the substrate removal rate. So, this can be written and for this overall thus we can write this is equal to dX/dt and this is dS/dt. So, if we use this equation, we can write that dX/dt is equal to Y dS/dt. So, this is there. So, this equation can be used. So, in this equation that from this we can see that the greater is the substrate assimilation that is the whatever if the substrate utilization rate higher greater is the bacterial growth rate.

So, higher will be the bacterial growth rate so, this relationship can be expressed by this and already X is the concentration of microorganism, which is generally, we will be using VSS and as is the concentration in terms of BOD or COD, so, gram per liter t is the time so, this is the equation.

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Net solids production

- Previous equation expresses the gross bacterial growth without taking into consideration the reduction of the biomass due to endogenous respiration.
- When including the endogenous respiration, the net solids production becomes:

$$\frac{dX}{dt} = Y \frac{dS}{dt} - K_d \cdot X$$

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Now, previous equation expresses the gross bacterial growth without taking into consideration the reduction of the biomass due to endogenous respiration. So, when including endogenous respiration, the equation can be written as this. So, now we have the net solid production becomes like this. So, we have two different equations now, we have written till now for dX/dt and so, going further in a wastewater treatment plant, there is a substrate also which is getting utilized.

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Substrate removal rate

- In a wastewater treatment system, it is also important to quantify the rate at which the substrate is removed.
- The greater the rate, the lower the required volume for the reactor (for a certain concentration of the substrate) or the greater the efficiency of the process (for a certain volume of the reactor).
- Rearranging equation of growth rate, the substrate removal rate can be expressed as:

$$\frac{dS}{dt} = \frac{1}{Y} \frac{dX}{dt}$$

The substrate removal rate can be expressed as:

$$\frac{dS}{dt} = \frac{1}{Y} \cdot \frac{dX}{dt}$$

So, we should understand what is the substrate removal rate, which was here defined as dS by dt . So, we can viceversa write using this equation that dS/dt is equal to 1 by Y into dX/dt , so, the greater is the rate lower is the required volume. So, if the substrate removal rate which is the actually the pollutants or the organic matter which is present in the water, if its rate of removal is good or very high, that means the volume which will be required for the reactor, which will be lower, so, greater is the rate, lower is the required volume.

So, in a way we want to have the dS/dt to be high and if dS/dt is to be high, the dX/dt also has to be high. So, this is there and this equation is only the rearrange equation of the earlier equation.

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□ The substrate removal is associated with the gross biomass growth. Substituting $\frac{dX}{dt}$ for $\mu \cdot X$, we obtain:

$$\frac{dS}{dt} = \frac{\mu}{Y} \cdot X$$

or

$$\frac{dS}{dt} = \mu_{max} \cdot \frac{S}{K_S + S} \cdot \frac{X}{Y}$$

$\frac{dX}{dt} = \mu X$
 $\mu = \mu_{max} \frac{S}{K_S + S}$

We obtain:

$$\frac{dS}{dt} = \frac{\mu}{Y} \cdot X$$

$$\frac{dS}{dt} = \mu_{max} \cdot \frac{S}{K_S + S} \cdot \frac{X}{Y}$$

Now, the substrate removal rate is associated with the gross biomass growth also. So, in place of dX/dt , we can substitute dX/dt for μX . So, earlier we had studied that this dX/dt is equal to μX . So, this dX/dt was μX . So, in place that is why we have written that dX/dt is equal to μX is replaced here and if we replace like this and further replace μ is equal to μ_{max} the Monod equation S upon K_S plus S . So, the equation becomes like this. So, we have this

particular equation. Now, this is the equation which can be used for finding out the substrate removal layer.

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Treatment Kinetics

The term μ_{max}/Y in the below equation can be replaced by k which denotes the maximum substrate utilization rate per unit of time.

$$\frac{dS}{dt} = \mu_{max} \cdot \frac{S}{K_S + S} \cdot \frac{X}{Y}$$

$$\frac{dS}{dt} = kX \frac{S}{K_S + S}$$

Typically, for sewage, $k = 8.0$ per day on BOD_u or COD basis.

Two boundary cases can be identified for which simple approximations can be developed.




10

The term μ_{max}/Y in the below equation can be replaced by k which denotes the maximum substrate utilization rate per unit of time.

$$\frac{dS}{dt} = \mu_{max} \cdot \frac{S}{K_S + S} \cdot \frac{X}{Y}$$

$$\frac{dS}{dt} = kX \frac{S}{K_S + S}$$

Going further we will try to understand that treatment can it takes in a reactor. So, already we have the term μ_{max}/Y this μ_{max}/Y which is there in the equation can be replaced by another term which is called as K . So, this K is defined for various value so, can we replace by K which denote the maximum substrate utilization rate per unit time. So, this μ_{max}/Y is defined and we have μ term which is called as K which is the maximum substrate utilization rate and its typical value is around $K = 8$ per day on BOD, ultimate BOD or COD basis. So, this is there.

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□ When $S \gg K_S$

Then $\frac{dS}{dt} = k \cdot X$ ✓

Here, the removal is independent of the substrate concentration, namely, a zero-order reaction. The removal rate depends only on the mass of microbial solids present in the system.

□ When $S \ll K_S$

$$\frac{dS}{dt} \cong kX \frac{S}{K_S} = k' \times S$$

= (a first order reaction) ✓

where $k' = k/K_S =$ the specific substrate utilization rate, $(\text{mg/l})^{-1}(\text{t})^{-1}$ ✓

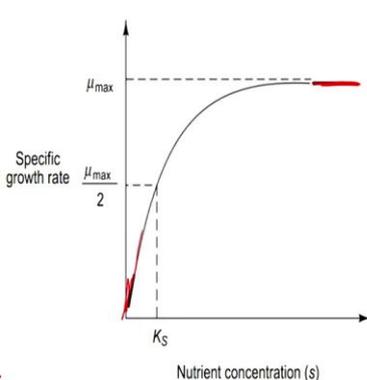


Fig. Typical specific growth rate and limiting nutrient concentration curve

(Source: Arcevala S.J. and Asolekar S.R., "Wastewater Treatment for Pollution Control and Reuse", 3rd Ed., Tata McGraw Hill, 2007. 11)

When $S \gg K_S$

$$\frac{dS}{dt} = k \cdot X$$

When $S \ll K_S$

$$\frac{dS}{dt} \cong kX \frac{S}{K_S} = k' \times S$$

Now, this K value depending upon whether S is greater than K_S or what is the value relationship between S and K_S the substrate removal rate will vary. So, when S is very very much greater than K_S so, under that condition dS/dt becomes kX here thus the removal is independent upon the substrate concentration and it is zero order reaction and the removal rate is only dependent upon the mass of microbial solids present in the system. So, this is there. So, we have dS/dt which is fastest and it is dependent upon the X value.

When the concentration is much lower than the K_S value. So, under that condition this equation will be reduced. So, will be K_S will be at there in the denominator and then we can write K by K_S as a new type of constant which is called at the specific substrate utilization rate and its unit is a milligram per liter per unit time and time per raise to per unit time.

So, this is inverse of both, so, we have 1 upon milligram per liter and per unit time, so, this is the unit which is there for K dash. So, this can be used further and here we can see this is a first order reaction. So, we have a first order reaction under one condition and we have zero-order reaction under one condition when the substrate utilization rate is fastest.

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Table. Typical values of constants in treating waste

Waste	Basis	Yield Y	K_d day^{-1}	μ_{max} day^{-1}	K_s mg/l	k day^{-1}	k'	Author
Domestic	BOD μ	0.57 ✓	0.052	0.67	54	-	0.0234	Handa
Domestic	COD	0.40	0.09	~3.2	60	8.0		Sherrard
Domestic	BOD $_2$	0.37	0.098 ✓		45	8.35		Hashimoto
Domestic*	BOD $_3$	0.73 ✓	0.075	✓		✓	0.017-	
					✓	✓	0.03	Eckenfelder
Chemical							✓	
Petrochem	BOD $_3$	0.31-	0.05- ✓				0.003-	
		0.72	0.18				0.012	Eckenfelder

From lab experiments, the data can be plotted to give the values of the coefficients and constants involved for domestic sewage and a few other wastewaters.

(Source: Arceivala S.J. and Asolekar S.R., "Wastewater Treatment for Pollution Control and Reuse", 3rd Ed., Tata McGraw Hill, 2007. 12)

The typical values of constants in that treating waste are written here for different like depending upon the type of basis in various authors have reported various values of Y , the yield coefficients so, we can see the yield coefficient is varying from around 0.3 to 0.7 or 0.75, this is the highest value. The K_d value the endogenous decomposition rate is from 0.05 to 0.098 or 0.1. So, this is the highest value which is reported similarly, μ_{max} value, K_s value K value and K' value are reported in the literature in different books, et cetera.

(Refer Slide Time: 12:43)

Table. Overall BOD removal rates, K for different sewage treatment processes (Arceivala)

Process	Likely SS mg/l	Overall K per day (20°C)	
		Filtered effluent basis	Unfiltered effluent basis
Extended aeration	4000-5000 ✓	20-30	20-25
Facultative aerated lagoon	50-150 ✓	0.6-0.8	0.3-0.5
Fully aerobic lagoon	100-350 ✓	1.0-1.5	0.4-0.5
Waste stabilization pond	- ✓	-	0.1-0.15

(Source: Arceivala S.J. and Asolekar S.R., "Wastewater Treatment for Pollution Control and Reuse", 3rd Ed., Tata McGraw Hill, 2007. 13)

Now, going further overall BOD removal rates for different sewage treatment processes this is given in the book of Arceivala. So, the different processes like extended aeration facultative aerated lagoon process fully aerobic lagoon process et cetera. The values are

overall K values at 20 degrees centigrade depending upon the filtered effluent basis or unfiltered effluent basis they are given. So, different values of K et cetera are reported depending upon the SS concentration which is in the wastewater. So, depending upon this these values are reported and they can be used.

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Handling of solids

The processes listed in the above table are of the 'suspended growth' type in which three different ways of handling solids can be identified, namely facultative, flow-through, and flow-through with recycle.

Note: the incoming flow is Q , the influent substrate is S_0 and the outgoing substrate is S . The mass, of solids, X , in each case is different.

□ In facultative systems, the solids are uncontrolled, as it were. They may settle or go out with the effluent. The power input level may be zero (as in the case of waste stabilization ponds) or low and therefore not enough to keep all solids in suspension.

Fig. Wastewater treatment systems with different arrangements for handling solids produced in the system. (a) Facultative, (b) Flow-through without recycling, (c) Flow-through with recycling

[Source: Acevalva S.J. and Azolekar S.R., "Wastewater Treatment for Pollution Control and Reuse", 3rd Ed., Tata McGraw Hill, 2007. 14

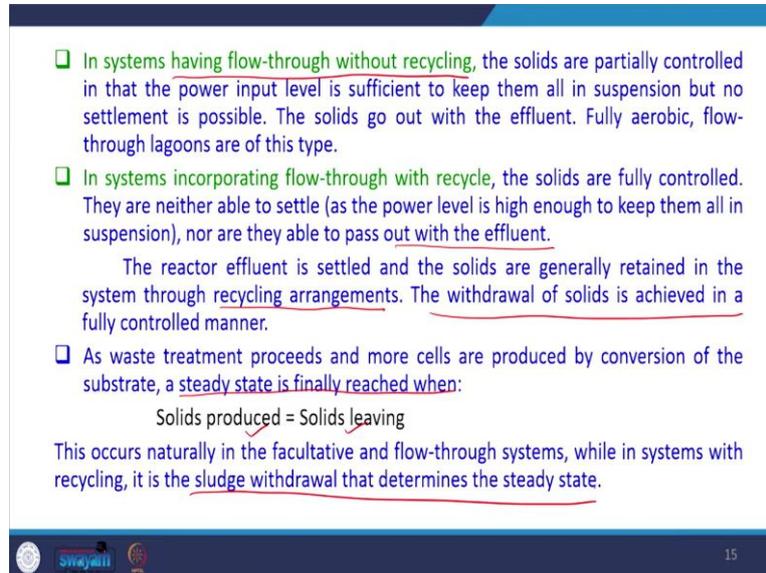
Now, during the treatment biological treatment process a lot of solids get generated because we have the growth of a microbial biomass and the process listed for suspended growth type there are we have lot of like this reactor which is their aerator and we have a which is having volume V and solid concentration is X_1 and here the inlet is having a flow rate of Q and the initial substrate concentration is S_0 and then we are finding that there will be certain conditions in the facultative process the concentration of bio-solids will be much lower than in the reactor. Whereas, in the case of flow through without recycling reactor, this X_2 concentration will be same as X_2 here.

In the third case, when recycling is held the X_3 this concentration is much lower than the X_3 . So, the inlet we have the solid is not going to the biomass solid is not going and actually it is being wasted via this sludge process. So, here the sludge is being wasted and some amount of sludge is recycled back so as to maintain the solid concentration at the limit of X_3 . So, we have three systems.

So, in the facultative system the solids are uncontrolled and they may settle, they may go out with their effluent, the power input level may be 0 as in the case of waste stabilization pond are very low that means, we have no power from outside and therefore not enough to keep all

solids in the suspension. So, some amount of solids they settle down and the concentration which is going outside is lower than the X_1 .

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- In systems having flow-through without recycling, the solids are partially controlled in that the power input level is sufficient to keep them all in suspension but no settlement is possible. The solids go out with the effluent. Fully aerobic, flow-through lagoons are of this type.
- In systems incorporating flow-through with recycle, the solids are fully controlled. They are neither able to settle (as the power level is high enough to keep them all in suspension), nor are they able to pass out with the effluent.
The reactor effluent is settled and the solids are generally retained in the system through recycling arrangements. The withdrawal of solids is achieved in a fully controlled manner.
- As waste treatment proceeds and more cells are produced by conversion of the substrate, a steady state is finally reached when:
Solids produced = Solids leaving

This occurs naturally in the facultative and flow-through systems, while in systems with recycling, it is the sludge withdrawal that determines the steady state.

In the second case in which the systems is in the system having flow through without recycling, the solids are partially controlled in that the power input level is sufficient to keep them in the suspension, but no settlement is possible that mean the solids go out with the effluent and fully aerobic flow through lagoons are of this type. So, here we can see the concentration is same as X_2 because no settling is happening.

In the third case, where the system incorporating flow through with recycle, the solids are fully controlled, they are neither being able to settle they are not allowed to settle because we have high power input and also they are not allowed to pass out with the effluent. The reactor effluent is settled after the treatment and the solids are generally retained in the system through recycling arrangement and the excess solids are withdrawn or they are wasted. So, as a waste treatment process its and more cells are produced by conversion of substrate.

A steady state is finally reached when the solids produce are equal to solids leaving this occurs naturally in the facultative and flow through systems while in the systems with recycling, it is the sludge withdrawal that determines the steady state. So, this happens differently.

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Sludge age and hydraulic retention time

The hydraulic retention time is the same for each of the three above methods of handling solids, namely:

$$t = V/Q$$

But the time for which the cells remain in the system can be quite different. The sludge age or the mean cell residence time, θ_c , can be estimated as follows:

$$\theta_c(\text{days}) = \frac{\text{Mass of solids in system}}{\text{Mass of solids leaving system/day}} = \frac{x \cdot V}{x' \cdot Q}$$

where x and x' are concentration terms, with x' being the solids withdrawn from the system or escaping out with the effluent, or both, as the case may be.



The hydraulic retention time $t=V/Q$

The sludge age or the mean cell residence time, θ_c , can be estimated as follows:

$$\theta_c(\text{days}) = \frac{\text{Mass of solids in system}}{\text{Mass of solids leaving system/day}} = \frac{x \cdot V}{x' \cdot Q}$$

Now, what is the different amount of sludge age, how it is defined, what is the age of sludge which is there in the reactor? Similarly, with respect to what is the hydraulic detention time of the wastewater inside the reactor, these are very important terms with respect to design of any system. So, the hydraulic retention time is the same for each of the three above methods and it was t is equal to V by Q .

So, this is how, but the time for which cells remains in the system can be different. So, the sludge age or the mean cell residence time θ_c . So, this is estimated using this particular definition. So, we should try to remember that definition as compared to the formula. So, the sludge age is defined as the ratio of mass of solid in the system divided by mass of solid leaving the system per day.

So, this is how we define the sludge age. We can use this formula for finding out for some particular cases like if the system volume is given the mass of solid in the system is x into V where x is the concentration of the solid inside the reactor. Now, x' is the concentration of solid leaving the system and if Q is the flow rate, so, we can find out $x' \cdot Q$. So, this will be the mass of solid leaving system per day. So, we can use this formula for calculating the sludge age or mean cell residence time.

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- In flow-through systems, $x' = x$ and, hence, $t = \theta_c$. In flow-through systems with recycling such as extended aeration systems, $x' < x$. Hence $\theta_c > t$.
- In extended aeration systems, for example, the hydraulic time, t , may be around 1-2 days while θ_c may range from 20-30 days owing to recycling.
- In facultative aerated lagoons also, $\theta_c > t$ though it is not possible to accurately estimate θ_c since the solids settled at the bottom are not measurable.

$\theta_c \geq t$

Handling of solids

The processes listed in the above table are of the 'suspended growth' type in which three different ways of handling solids can be identified, namely facultative, flow-through, and flow-through with recycle.

Note: the incoming flow is Q , the influent substrate is S_0 and the outgoing substrate is S . The mass of solids, X , in each case is different.

- In facultative systems, the solids are uncontrolled, as it were. They may settle or go out with the effluent. The power input level may be zero (as in the case of waste stabilization ponds) or low and therefore not enough to keep all solids in suspension.

Fig. Wastewater treatment systems with different arrangements for handling solids produced in the system. (a) Facultative, (b) Flow-through without recycling, (c) Flow-through with recycling

In flow through systems where like the x dash is equal to x the solid concentration like here we saw that the x dash which is leaving is same as this. So, in this flow through system hence θ_c is equal to this the hydraulic retention time is same as sludge age.

In flow through system with recycling such as extended aeration, the concentration in the stream which is leaving the system is much less of the solids and thus the sludge age is much greater than the hydraulic detention time. In extended aeration system, for example, the hydraulic time may be around 1 to 2 days, while the θ_c may range from 20 to 30 days owing to recycling.

In facultative aerated lagoons also that θ_c value is much greater than t . So, that means, overall, the mean cell residence time can be equal to or greater than the hydraulic HRT. So,

this is very important to remember. And there is another term which is very important for any other system where the bacterial growth is happening.

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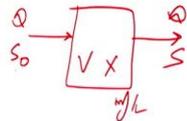
Food/Micro-organisms Ratio

The food/micro-organism (F/M) ratio gives the substrate removal rate per unit of solids in a system. It can be computed from:

$$\frac{F}{M} = \frac{(S_0 - S)}{X \cdot t}$$

This ratio should be stated along with its basis.

It is not possible to express F/M ratios for facultative aerated lagoons and stabilization ponds as a part of the microbial solids (M) settle at the bottom.



18

The food/micro-organism (F/M) ratio:

$$\frac{F}{M} = \frac{S_0 - S}{X \cdot t}$$

And that is called food to microorganism ratio. The food to microorganism ratio gives the substrate removal rate per unit solid in the system and it can be computed from this. So, food to microorganism ratio this is being defined here. So, the food which is in the reactor system, like the initial system, which was given where volume was this and the concentration inside the biomass was this and the flow was coming like this at concentration is zero and it was leaving after treatment at S.

So, in this case, the S₀ minus S, so, this is the its unit will be in milligram per liter something like this, with respect to solid or BOD, COD et cetera and the mass of the solid inside their system is already the concentration is x so, again this is in milligram per liter. And so, this is X by t, the ratio should be detailed along with this basis and F/M ratio for facultative aerated lagoons and the stabilization pond as a part of microbial solid settle at the bottom. So, we try to find out by F/M ratio. So, this is this.

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Build-up of solids in the system

The net solid production rate can be written as:

$$\left(\frac{\Delta X}{\Delta t}\right) = Y \cdot \left(\frac{S_0 - S}{t}\right) - K_d X$$

$\frac{dx}{dt} = \frac{S_0 - S}{t}$

At steady state, $Q_c = \frac{X}{\frac{\Delta X}{\Delta t}} \Rightarrow \frac{dx}{dt} = \frac{1}{Q_c}$

Hence, $\frac{1}{Q_c} = Y \cdot \frac{S_0 - S}{X \cdot t} - K_d$

Or $\frac{1}{Q_c} = Y \cdot \frac{F}{M} - K_d$

Solving for X, we get: $X = \frac{Y(S_0 - S)}{(1 + K_d Q_c)} \cdot \frac{Q_c}{t}$

The net solid production rate can be written as:

$$\left(\frac{\Delta X}{\Delta t}\right) = Y \cdot \left(\frac{S_0 - S}{t}\right) - K_d X$$

At steady state,

$$Q_c = \frac{X}{\frac{\Delta X}{\Delta t}}$$

$$\frac{1}{Q_c} = Y \cdot \frac{S_0 - S}{X \cdot t} - K_d$$

$$\frac{1}{Q_c} = Y \cdot \frac{F}{M} - K_d$$

Solving for X, we get:

$$X = \frac{Y(S_0 - S)}{(1 + K_d Q_c)} \cdot \frac{Q_c}{t}$$

The net solid production can be using, because we have endogenous decay phase also and this overall can be written like this that delta X/delta t is equal to the growth rate phase because we have the substrate utilization rate is given by ds by dt. So, in place of ds/dt we are writing S₀ minus S upon t where t is the HRT and why is the yield coefficient, already defined and since, there is a decay also so, we can write like this.

So, at the steady state when the condition is at the steady state we can easily solve this equation. So, overall the theta C, theta C can be written as X upon dX/dt as per definition it

was given like this is per definition mass of solid in the system divided by mass of solids leaving the system per day.

So, we have θ_c is equal to mass of solid in the system and mass of solid leaving the system per day. So, this is θ_c is equal to X upon dX/dt . So, using this equation, we can write that $dX/dt = 1$ upon X and this dX/dt can be written as 1 upon θ_c . So, in this equation if we divide by X , so, this equation can be if we divide by X , so, X will come here and Y by X . So, we have Y by X minus K_d and in place of 1 upon X dX/dt we are writing 1 upon θ_c . So, this is cell means cell residence time and this S_0 minus S upon t divided by X is actually the food to microorganism ratio.

So, we have a equation where we can write with respect to a sludge age food to microorganism ratio and K_d . So, this is the equation and this equation can be easily solved for other form also with respect to X , also we can solve, but here right now, we can concentrate this equation or this equation can be used further.

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- The previous equation is simplified for flow-through systems since $\theta_c = t$.
- In recycling systems, the term (θ_c/t) has a multiplier effect on X .
- Hence, extended aeration systems have a much higher value of X than flow-through lagoons.
- Substituting, $t = V/Q$ and writing in concentration terms, the equation becomes:

$$X \cdot V = \frac{YQ\theta_c(S_0 - S)}{(1 + K_d\theta_c)}$$

Substituting, $t = V/Q$ and writing in concentration terms, the equation becomes:

$$X \cdot V = \frac{YQ\theta_c(S_0 - S)}{(1 + K_d\theta_c)}$$

So, we will try to solve some equation. The previous equation can be simplified for flow-through system since, the θ_c is equal to t and in recycling system that term θ_c by t has a multiplication factor of X . Hence, the extended aeration system have a much higher

value of X, than the flow through lagoons. Substituting the value of t which is the hydraulic retention time is equal to V by Q.

So, there is another equation which can be written if we solve we will be getting this equation YQ, where Q is the flow rate, theta C is the meantime residence time S₀ minus S₁ plus K_d theta C. So, we can easily get this equation also for one relationship where the theta C value is related to with respect to concentration of X volume of the reactor, the substrate utilization rate, etcetera.

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Substrate removal efficiency

From equation at steady state, one can write,

$$\frac{1}{\theta_c} = \frac{Y(dS/dt)}{X} - K_d$$

$$\frac{1}{\theta_c} = Y \cdot k \left[\frac{S}{K_S + S} \right] - K_d$$

Solving for S,

$$S = \frac{K_S(1 + K_d\theta_c)}{\theta_c(Y \cdot k - K_d) - 1}$$

$$S = \frac{(F/M) \cdot K_S}{k - (F/M)}$$

$$S = \frac{1}{k'Y} \left[\frac{1}{\theta_c} + K_d \right]$$

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$$S = \frac{1}{k'Y} \left[\frac{1}{\theta_c} + K_d \right]$$

Build-up of solids in the system

The net solid production rate can be written as:

$$\left(\frac{\Delta X}{\Delta t}\right) = Y \cdot \left(\frac{S_0 - S}{t}\right) - K_d' X$$

$$\frac{dS}{dt} = \frac{S_0 - S}{t}$$

At steady state,

$$\theta_c = \frac{X}{\frac{\Delta X}{\Delta t}} \Rightarrow \frac{dX}{X dt} = \frac{1}{\theta_c}$$

Hence, $\frac{1}{X} \frac{dX}{dt} = \frac{1}{\theta_c} = Y \frac{S_0 - S}{X \cdot t} - K_d$

Or $\frac{1}{\theta_c} = Y \cdot \frac{F}{M} - K_d$

Solving for X, we get: $X = \frac{Y(S_0 - S) \theta_c}{(1 + K_d \theta_c) \cdot t}$

And overall, the substrate removal efficiency the previous equation which was written here this particular equation can also be written like this. So, this is the same equation which was given and here the dS/dt actually upon x has been written in terms of k_s plus S . So, it was earlier given and if you solve for S this particular equation will get any of these three equations easily and solving we can get this equation.

So, any of these two equations can be used for finding out the substrate removal efficiency or the final substrate concentration outside the treatment system. So, we can use any of these equation. So, we will be using these equations a little bit for solving some of the problems.

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Production of Biological Solids

Problem. Calculate the biological solids production in a treatment system, assuming steady state.

Data:

- Reactor volume: $V = 9,000 \text{ m}^3$ ✓
- Hydraulic detention time: $t = 3 \text{ d}$ ✓
- Influent substrate (total BOD_5): $S_0 = 350 \text{ mg/L}$ ✓
- Effluent substrate (soluble BOD_5): $S = 9.1 \text{ mg/L}$ ✓
- Biomass in the reactor (VSS): $X_v = 173.3 \text{ mg/L}$ ✓

Coefficients of the model:

- ✓ • Yield coefficient: $Y = 0.6 \text{ mgVSS/mg BOD}_5$
- ✓ • Endogenous respiration coefficient: $K_d = 0.06 \text{ d}^{-1}$

Swayam Spherling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

So, in this question the first question it is given that calculate the biological solid production in a treatment system assuming that the operation is at a steady state and some data is given here and the coefficients so, the reactor volume is 9000 meter cube, the hydraulic detention time is three day the system has to be operated. The influent substrate concentration is 350 milligram per liter. And it is found that in that after treatment the substrate concentration is only 9.1 milligram per liter. The biomass in the reactor VSS has been found out experimentally and it is found that it is 173.3 milligram per liter.

Now, for solving we are using that the yield coefficient is taken from the literature and it is 0.6 milligram VSS per milligram BOD_5 and the endogenous respiration coefficient is 0.06 per day. So, this is the value which is given. Now, we have to use this all data to find out the biological solid production in the treatment system. So, what is the net amount of solid which is getting produced per day that we have to find out.

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Solution:
 Assuming finite time conditions within the steady-state hypothesis,
 Equation $\frac{dX}{dt} = Y \frac{dS}{dt} - K_d \times X$ can be rewritten as:

$$\frac{\Delta X_v}{\Delta t} = 0.6 \frac{\text{gVSS}}{\text{gBOD}_5} \times (350 - 9.1) \frac{\text{gBOD}_5}{\text{m}^3} \times \frac{1}{3.0\text{d}} - 0.06 \frac{\text{gVSS}}{\text{gVSS} \times \text{d}} \times 173.3 \frac{\text{gVSS}}{\text{m}^3}$$

$$\frac{\Delta X_v}{\Delta t} = 68.2 \text{ g/m}^3 \times \text{d} - 10.4 \text{ g/m}^3 \times \text{d} = 57.4 \text{ g/m}^3 \times \text{d} = 0.058 \text{ kg/m}^3 \times \text{d}$$

Since the reactor volume is 9,000 m³, the global net production is:

$$0.058 \text{ kg/m}^3 \times \text{d} \times 9,000 \text{ m}^3 = 522 \text{ kgVSS/d}$$

Swajathi Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

$$\frac{dX}{dt} = Y \frac{dS}{dt} - K_d \times X$$

$$\frac{\Delta X_v}{\Delta t} = 0.6 \frac{\text{gVSS}}{\text{gBOD}_5} \times (350 - 9.1) \frac{\text{gBOD}_5}{\text{m}^3} \times \frac{1}{3.0\text{d}} - 0.06 \frac{\text{gVSS}}{\text{gVSS} \times \text{d}} \times 173.3 \frac{\text{gVSS}}{\text{m}^3}$$

$$\frac{\Delta X_v}{\Delta t} = 68.2 \text{ g/m}^3 \times \text{d} - 10.4 \text{ g/m}^3 \times \text{d} = 57.4 \text{ g/m}^3 \times \text{d} = 0.058 \text{ kg/m}^3 \times \text{d}$$

The global net production is:

$$0.058 \text{ kg/m}^3 \times \text{d} \times 9,000 \text{ m}^3 = 522 \text{ kgVSS/d}$$

Now, assuming that this is finite time condition with the steady state conditions we are assuming so, equation this equation can be has to be used. Now, this equation can further be written as we have Y which is given 0.6 gram VSS per gram BOD₅ dS/dt. So, dS actually by dt is from 350 to 9.1 gram BOD₅ per meter cube and in place of dt we are writing the hydraulic detention time which is 3 day, 3 day is written and K_d is already given 0.06. And the X value is 173.3.

So, if we solve it, and we can see that this unit is going off, similarly, so, we get gram per meter cube per day, which is the unit here, then similarly, for the second case also this goes up. So, we have gram per unit day per unit meter cube. So, this is the overall if you find it was 57.4 or 0.58 kg per meter cube per day. So, this is the amount of net production of solid per day. So, this is per meter cube, since the reactor volume is 9000 meter cube. So, if you

multiply by 9000 the global net production is 522 kg VSS per day this is what we obtain with respect to the net production of solid per day.

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Therefore, the net production of biological solids in the system (expressed as VSS) as a function of the substrate utilisation is 522 kgVSS per day. In the calculations above, it can be seen that $68.2 \text{ g/m}^3 \times \text{d}$ is the gross production and $10.4 \text{ g/m}^3 \times \text{d}$ is the destruction by endogenous respiration.

In this problem, the net production is approximately 85% of the gross production. In this problem, numbers with decimals have been used only to clarify the calculations. In most practical applications, round figures are more frequently used for representing BOD and other variables.

  Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

Therefore, the net production of solid biological solid in the system is 522 kg VSS per day and in the calculation above it is can be seen that 68.2 gram per meter cube is the gross production and 10.4 is the destruction of due to the endogenous respiration. So, in this problem the net production is approximately 85 percent of the gross production. So, we can go further.

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Hydraulic Detention Time and Solids Retention Time

Problem. Calculate the hydraulic detention time and the sludge age in the sewage treatment system described (without a settling tank and solids recirculation). The main relevant data are:

Reactor volume: $V = 9,000 \text{ m}^3$ ✓

Input and output variables:

- Influent flow: $Q = 3,000 \text{ m}^3/\text{d}$ ✓
- Influent substrate (BOD₅ total): $S_0 = 350 \text{ mg/L}$
- Effluent substrate (BOD₅ soluble): $S = 9.1 \text{ mg/L}$ }

Model coefficients:

- Maximum specific growth rate: $\mu_{\text{max}} = 3.0 \text{ d}^{-1}$ ✓
- Half-saturation coefficient: $K_s = 60 \text{ mg/L}$ ✓
- Endogenous respiration coefficient: $K_d = 0.06 \text{ d}^{-1}$ ✓

  Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

Some other problems also we can solve for the same question, it is being asked that calculate the hydraulic detention time and the sludge age in the sewage treatment system without the

settling tank and solid recirculation, the volume is this the input variables are Q is equal to 3000 meter cube per day the data given. The solid reduction the substrate reduction is from 350 to 9.1.

The maximum specific growth rate is given 3 per day and the half saturation coefficient is 60 and endogenous decay respiration coefficient is 0.6 per day. So, if we have to find out we have to find out the hydraulic detention time.

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Solution:

a) Hydraulic detention time,
 Since $t = V/Q = 9000 \text{ m}^3/3000 \text{ (m}^3/\text{d)} = 3.0 \text{ d}$

b) Sludge age,
 The value of μ is given by Equation

$$\mu = \mu_{\max} \times \frac{S}{K_s + S} = 3 \times \frac{9.1}{60 + 9.1} = 0.395 \text{ d}^{-1}$$

The sludge age is given by following Equation:

$$\theta_c = \frac{1}{\mu - K_d} = \frac{1}{0.395 - 0.06} = 3 \text{ d}$$

As expected, in the present example $t = \theta_c$, since the system has no solids recirculation.

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 Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

The value of μ is given by Equation

$$\mu = \mu_{\max} \times \frac{S}{K_s + S} = 3 \times \frac{9.1}{60 + 9.1} = 0.395 \text{ d}^{-1}$$

The sludge age is given by following Equation:

$$\theta_c = \frac{1}{\mu - K_d} = \frac{1}{0.395 - 0.06} = 3 \text{ d}$$

So, in this case, because volume is given and the flow rate is given it is very easy, 3 and which was the earlier one. The value of μ can be calculated using this particular equation μ_{\max} is equal to this and it is 0.395 per day and the sludge age can be calculated this equation was derived earlier. So, using this it is coming as 3 day. So, as expected in the present example, that 3 day it is same the system has no solids recirculation, because.

(Refer Slide Time: 30:09)

Cell Wash-out Time

Problem. Calculate the suspended solids concentration to be reached, under steady-state conditions. The relevant data are:

- Influent substrate (total BOD₅): S₀ = 350 mg/L
- Effluent substrate (soluble BOD₅): S = 9.1 mg/L
- Hydraulic detention time: t = 3.0 days
- Yield coefficient: Y = 0.6 mgVSS/mgBOD₅
- Endogenous coefficient: K_d = 0.06 d⁻¹

Solution:
Since,

$$X_v = \frac{Y \times (S_0 - S)}{1 + K_d \times t} = \frac{0.6 \times (350 - 9.1) \text{ mg/L}}{1 + 0.06 \text{ d}^{-1} \times 3 \text{ d}} = 173.3 \text{ mg/L}$$

Swajani Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

$$X_v = \frac{Y \times (S_0 - S)}{1 + K_d \times t} = \frac{0.6 \times (350 - 9.1) \text{ mg/L}}{1 + 0.06 \text{ d}^{-1} \times 3 \text{ d}} = 173.3 \text{ mg/L}$$

Now going further, we can we calculate the suspended solid concentration to be reached under steady state condition. The data is same as earlier. Most of the data are same. So, for finding out this we can calculate from the yield coefficient and all these values are given that is 173.3 which was given actually in the question in the first part. So, this was given but we can calculate the same using this value. So, we can see easily that 173.3 is correct value which was found out or given in the problem earlier the cell wash-out time.

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Effluent Substrate

Problem. Calculate the soluble effluent BOD concentration, after steady-state conditions have been reached in the system. Since the system has no solids recirculation, the sludge age is equal to the hydraulic detention time. The relevant data for this example are:

- Sludge age (equal to the hydraulic detention time): $\theta_c = t = 3.0 \text{ d}$ (according with Problem 8)
- Maximum specific growth rate: $\mu_{\max} = 3.0 \text{ d}^{-1}$
- Half-saturation coefficient: K_s = 60 mg/L
- Endogenous respiration coefficient: K_d = 0.06 d⁻¹

Solution:
Since,

$$S = \frac{K_s \times [(1/\theta_c) + K_d]}{\mu_{\max} - [(1/\theta_c) + K_d]} = \frac{60 \times [(1/3) + 0.06]}{3.0 - [(1/3) + 0.06]} = 9.1 \text{ mg/L}$$

Swajani Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

$$S = \frac{K_s \times [(1/\theta_c) + K_d]}{\mu_{\max} - [(1/\theta_c) + K_d]} = \frac{60 \times [(1/3) + 0.06]}{3.0 - [(1/3) + 0.06]} = 9.1 \text{ mg/L}$$

Similarly, the effluence or state which was given as 9.1 we can change the values and we can solve it to under this condition and the equation that we have derived earlier we can use the equation and all the values if we are using the same value as in the previous questions, the answer will come out to be 9.1 milligram per liter.

So, this was given earlier but we can calculate depending upon the sludge age, the hydraulic detention time and all other nu max KS values et cetera. So, we can see easily that we can solve any problem if the system is at a steady state, we know the parameters and then we can solve.

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Loading Rates on Biological Reactors

Problem. Calculate the reactor volume of an extended aeration activated sludge system, given that:

- $U = 0.12 \text{ kgBOD}_5/\text{kgVSS}\cdot\text{d}$ (adopted)
- $Q = 5000 \text{ m}^3/\text{d}$ (design data)
- $S_0 = 340 \text{ mg/L}$ (design data)
- $S = 5 \text{ mg/L}$ (desired)
- $X_v = 3500 \text{ mg/L}$ (adopted)

Solution:
Since,

$$V = \frac{Q \times (S_0 - S)}{X_v \times U} = \frac{5000 \text{ m}^3/\text{d} \times (340 - 5) \text{ BOD}_5/\text{m}^3}{3500 \text{ gVSS}/\text{m}^3 \times 0.12 \text{ kgBOD}_5/\text{kgVSS}\cdot\text{d}} = 3988 \text{ m}^3$$

Springer, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

$$V = \frac{Q \times (S_0 - S)}{X_v \times U} = \frac{5000 \text{ m}^3/\text{d} \times (340 - 5) \text{ BOD}_5/\text{m}^3}{3500 \text{ gVSS}/\text{m}^3 \times 0.12 \text{ kgBOD}_5/\text{kgVSS}\cdot\text{d}} = 3988 \text{ m}^3$$

Similarly, the loading rate can also be calculated for the same data. Here the data a little bit changed as compared to earlier and here the U is given this is then the Q value flow rate is different, the substrate is changing from 350 to 5 milligram per liter whereas, the X₃ value is this under this condition, this is the volume of the reactor that will come out.

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References

- ❑ Davide Dionisi. Biological wastewater treatment processes: mass and heat balances. CRC Press, 2017.
- ❑ Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment
- ❑ Marcos von Sperling, Basic Principles of Wastewater Treatment. IWA Publishing, 2007.
- ❑ G L Karia, R.A. Christian, Wastewater treatment: Concepts and design approach. PHI Learning Pvt. Ltd., 2022.
- ❑ Arceivala S.J. and Asolekar S.R., “Wastewater Treatment for Pollution Control and Reuse”, 3rd Ed., Tata McGraw Hill, 2007..

30

So, with this we have come to the end of this lecture. We use these references for preparation of this slide. You can refer to these for further studying. Thank you very much.