

Biological Process Design for Wastewater Treatment
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Lecture - 13
Bacterial Growth Kinetics - I

Welcome to this NPTEL online certification course on Biological Process Design for Wastewater Treatment. So, today we are going to learn regarding the bacterial growth process and its kinetics. So, in today's lecture, we will start with the first studying the biological oxidation of carbonaceous matter and how it progresses. So, already we have studied in previous lectures little bit regarding the process of the formation of different biomasses in particular the bacterial growth process. So, we will continue with the same.

So, we can easily understand that during the aerobic reactions for the stabilization of organic matter, in a closed system like in a bottle such as the BOD which was used in the BOD test.

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Time progress of the biological oxidation of the carbonaceous matter

- Neglecting intermediate mechanisms, it can be said that the aerobic reactions for the stabilization of the organic matter occur, in a closed system (such as the bottle used in the BOD test), in a sequence in which the two following main mechanisms are predominant (Eckenfelder, 1980):
- **Initial stage: predominance of synthesis (anabolism):** At the beginning, the organic matter present in the wastewater is used by the microorganisms for their metabolic activities of growth and energy conversion, therefore prevailing the activities related to synthesis. This phase results in oxygen consumption and in an increase in the microorganisms population and can be represented by the generic equation (Hanisch, 1980):
$$8CH_2O + NH_3 + 3O_2 \rightarrow C_5H_7NO_2 + 3CO_2 + 6H_2O + Energy$$

Here, CH_2O represents the organic matter and $C_5H_7NO_2$ represents the cellular matter.

This phase results in oxygen consumption and in an increase in the microorganisms population and can be represented by the generic equation (Hanisch, 1980):

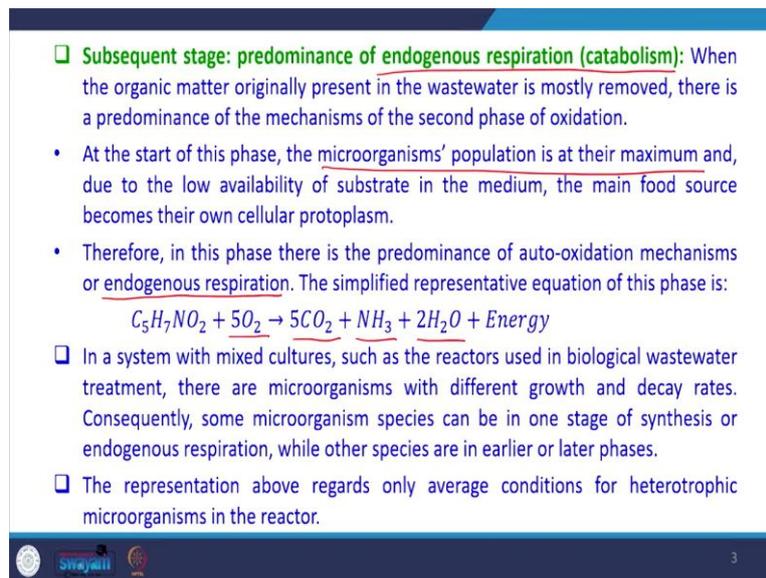


The sequence in which the mechanism of biological oxidation occurs follows two steps. First step or the initial stage includes the predominance of synthesis or anabolic reaction. So, in this reaction, the organic matter which is present in the wastewater is used by the microorganisms for their metabolic activities of growth and energy conversion. Therefore,

the prevailing activities related to synthesis these are the first step towards the activity of the synthesis.

And this phase results in oxygen consumption like the one reaction which is shown here, and then increase in the population of the microorganism which is shown here. And certainly during the reaction, we have CO₂ which is getting emitted and we obtain energy during the process. So, the oxidation takes place of the organic matter and we get energy out of this process as well as some amount of microorganisms get formed.

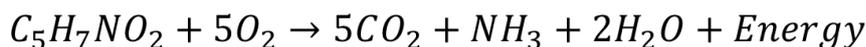
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□ **Subsequent stage: predominance of endogenous respiration (catabolism):** When the organic matter originally present in the wastewater is mostly removed, there is a predominance of the mechanisms of the second phase of oxidation.

- At the start of this phase, the microorganisms' population is at their maximum and, due to the low availability of substrate in the medium, the main food source becomes their own cellular protoplasm.
- Therefore, in this phase there is the predominance of auto-oxidation mechanisms or endogenous respiration. The simplified representative equation of this phase is:
$$C_5H_7NO_2 + 5O_2 \rightarrow 5CO_2 + NH_3 + 2H_2O + Energy$$
- In a system with mixed cultures, such as the reactors used in biological wastewater treatment, there are microorganisms with different growth and decay rates. Consequently, some microorganism species can be in one stage of synthesis or endogenous respiration, while other species are in earlier or later phases.
- The representation above regards only average conditions for heterotrophic microorganisms in the reactor.

The simplified representative equation of this phase is:



In the subsequent stage, the second stage, we have endogenous respiration or catabolic, which happens. So, the when organic matter or eternally present in the wastewater gets mostly removed, then there is a predominance of the second phase of oxidation. So, in this particular stage, the microorganisms' population is at their maximum and due to the low availability of substrate because all the organic matter has already degraded, the main food source becomes the cellular protoplasm itself. Therefore, in this phase, there is a predominance of auto oxidation mechanism or endogenous respiration.

So, this in during this process, the microorganisms themselves actually they oxidize, and then we have CO₂, ammonia and H₂O which gets released along with the energy in a system with mixed culture such as reactors used in the biological wastewater treatment. There are microorganisms with different growth and decay rates and therefore, some microorganism

species can be in one stage of synthesis, whereas the other may be in the second stage or endogenous respiration stage.

So, this is possible that they may have different growth phases, because we have we are using mix culture and the representation which has been given here, it is only the average condition of heterotrophic microorganisms in the reactor.

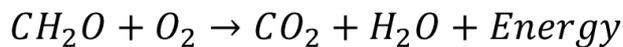
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- The total oxygen consumed in both phases is defined as the ultimate oxygen demand (BOD_u). The addition of equations ~~55~~ and ~~56~~ leads to the simplified equation for the oxidation of the organic matter (identical to the general equation of aerobic respiration):
$$\checkmark CH_2O + O_2 \rightarrow CO_2 + H_2O + Energy$$
- The value of the theoretical oxygen demand (ThOD) based on the stoichiometric relations of Equation ~~57~~ differs, in a certain way, from what is found for the ultimate demand, being, in reality, a little higher.

WHY?

The living, as well as the dead bacteria, serve as food for higher organisms. In each transformation, new oxidation occurs, but in the general balance, a certain fraction of the organic matter, resistant to biological attack, remains. This fraction is responsible for the deviation between the values of the ThOD and BOD_u (Sawyer and Mc Carty, 1978).



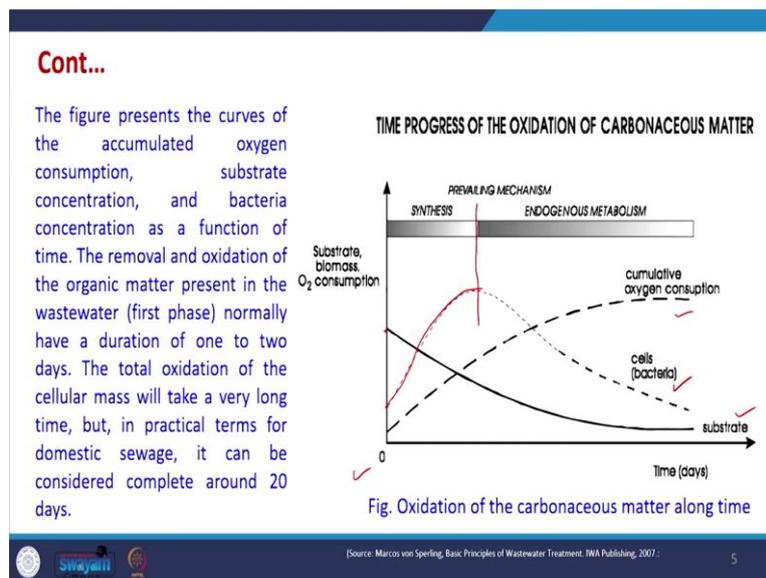
The total oxygen consumed in both phases, like in the first phase anabolism and catabolism it can be defined as the ultimate oxygen demand. So, we have already we have studied the BOD_u which is the total oxygen consumed in the both phases. The addition of previous two equations leads to the simplified equation of this form. So, we can combine the previous equation 1 and 2 to write the simplified equation for the organic matter and the value of theoretical oxygen demand, already we have studied regarding the theoretical oxygen demand.

So, if we calculate the theoretical oxygen demand for this particular reaction, and based upon the stoichiometry of this reaction will be finding that this will differ from the BOD_u value. So, why this is happening and this is so because the living as well as the dead bacteria, which actually serve as food for the higher organisms during the transformation are in the new oxidation reaction that occur a certain fraction of these organic matter, they are always

resistive to the biological attack, so they do not oxidize and as such they remain in the water itself.

So, this fraction actually, which does not get oxidized causes the deviation from the theoretical oxygen demand value and the BOD value, so, ThOD value will be higher, but BOD value ultimately in the water will be lower and that can be identified by the various tests et cetera. And then further calculation though this is there.

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Now, if we actually see that, what is the progress of oxidation of carbonaceous matter, we can represent the curve using this type of figure and here there are the growth is given with respect to three phase three different things, but when is that how the cells or bacteria are increasing. So, we can see that they are increasing like this and so, here we have two section, one is a synthesis section, where the maximum maxima has been reached and after the maxima has been reached we have endogenous metabolic or endogenous respiration which happens.

So, now, these the bacteria themselves are being utilized as a source of energy for the other bacteria which are there in the system. Now, substrate will certainly get a substrate or the organic matter, it decreases with the increase in time and it is continuously decreasing. The oxygen consumption is continuously increasing and it will maximize to a certain value which is BOD_u. So, that we studied. So, this is the total oxidation of carbonaceous matter along with the time and the time progress of the oxidation of the carbonaceous matter. So, this can be represented by this figure.

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Principles of Bacterial Growth

Synthesis phase ✓: As we know, heterotrophic organisms use organic matter as a form of condensed energy for their various metabolic processes, including growth and reproduction.

With the use of oxygen or another electron acceptor, these organisms oxidize the organic matter, with the production of more cellular matter (growth and reproduction) and energy release. This is the synthesis phase.

Endogenous Respiration ✓: If the substrate available in the medium starts to become scarce, such as in sewage treatment, in which organic matter is progressively removed, the microorganisms need to find other organic matter or condensed energy sources.

The main substrate directly available for the cell to use is its own cellular protoplasm. In this stage, the balance is negative (reduction of cellular matter or bacterial concentration in the medium), characterizing the endogenous respiration stage.

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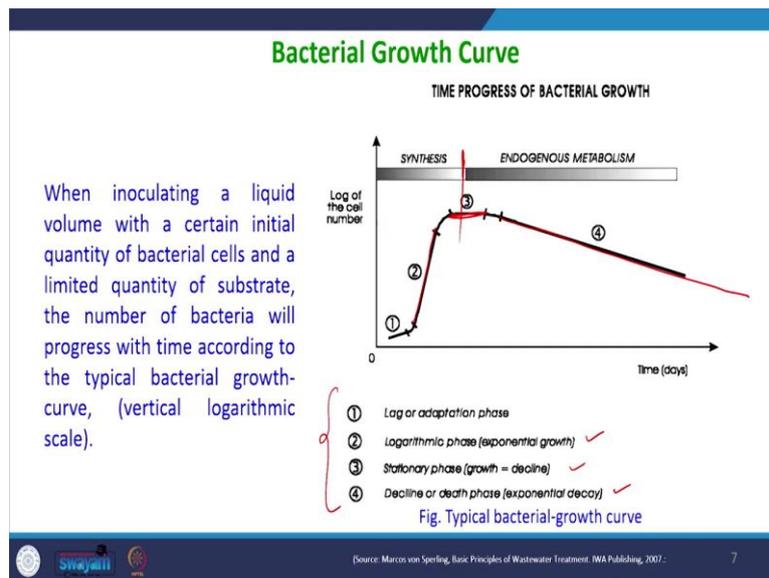
Now, going further already since, we have studied will be concentrating on the bacterial growth phase now first and the synthesis how we can represent the bacterial growth et cetera. So, that will be studying in this part of today's lecture. The synthesis phase, we have two phases we have studied, the synthesis phase and the endogenous respiration phase. The heterotopic microorganisms they use organic matter as a form of condensed energy for their various metabolic processes, including growth and reproduction.

So, with the use of oxygen and or any other electron acceptor, these organisms they oxidize the organic matter with the production of more cellular matters. So, that means, we have growth and production of the microorganism and energy release. So, this is the synthesis phase we have already studied.

During the endogenous respiration phase, if the substrate is available in the medium, which will become less because we have already peaked and the synthesis phase is virtually over. So, under those conditions the substrate which is available in the medium that becomes scarce. So, under that condition the organic matter is already removed and the microorganism need to find organic matter from other sources.

So, the main substrate which is available for the cell to use under this condition is their own cellular protoplasm. So, they start eating themselves. So, in this stage the balance is negative because reduction of cellular matter is happening or the bacterial concentration in the medium is decreasing. So, this is called the endogenous respiration stage. So, this is there.

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Overall, the bacterial growth curve which was earlier given also it can be represented like this. So, we have the synthesis phase up to the maxima is occurring and then we have endogenous metabolism which is occurring. Now, we can divide the bacterial growth curve into different phases, and these phases are depicted here are listed here.

So, these are one the first phase is called lag or adaptation phase, then we have logarithmic phase, where the exponential growth happens, then we have the stationary phase or the anyway growth and decline phase both are equal. So, we have virtually the stationary phase and then finally the endogenous respiration is stage where the bacteria is start decreasing continuously with time.

So, this is the bacterial growth curve which is we can see and observe if we do some wastewater treatment or growth of bacteria inside a batch reactor. So, this will be the curve that we will be obtaining. Now, different phases are defined here.

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- **Lag Phase:** It is a period for enzymatic adaptation of the bacteria to the new substrate supplied (von Sperling, 1983a). This phase can be reduced in the case of typical domestic sewage, in which the bacteria have already acquired the necessary enzymatic equipment.
- **Experimental-growth phase:** Here, the cells divide themselves at a constant rate. Plotted on a logarithmic scale, the number of cells grows linearly, justifying the alternative designation of the logarithmic phase.

There is an excess of the substrate in the medium, allowing the growth rate to reach its maximum, with the only limitation by the microorganisms' capacity to process the substrate. Along with the maximum growth rate, there is also the maximum substrate removal rate.
- **Stationary phase:** In this phase, the food in the medium starts to be scarce, and the bacterial growth rate is equal to the death rate. So, the number of cells is maintained temporarily constant.

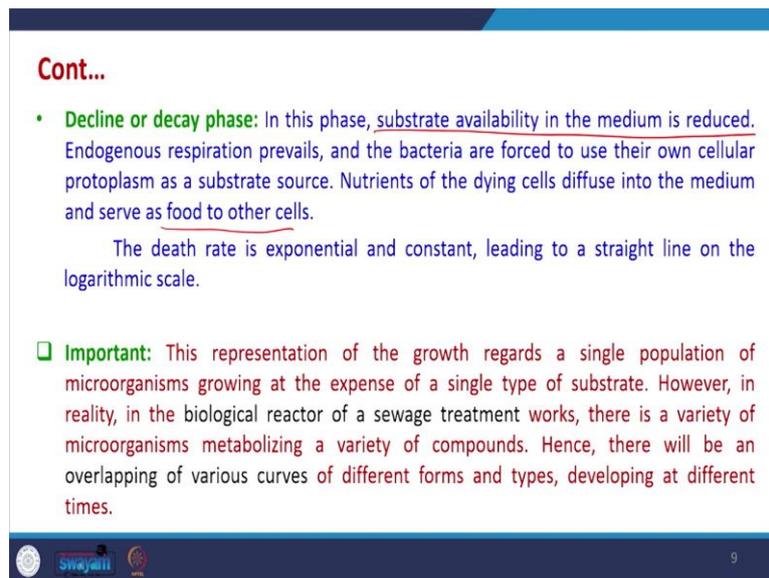
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So, the lag phase actually it is the period during which the enzymatic adaptation of the bacteria to the new substrate happens. So, this phase can be reduced in the case of typical domestic sewage in which the bacteria has already acquired the necessary enzymatic equipment. So, this lag phase is with respect to adaptation of the bacteria in the new conditions with respect to substrate with respect to temperature et cetera. After the initial lag phase, we have experimental growth phase.

So, here the cells divide themselves at a constant rate and we have on if you plot this on the logarithmic scale the number of cells grow linearly. So, thus is exponential growth phase and as there is an excess of substrate that means, the organic matter which is present in the medium is very high. So, thus the growth rate reaches its maximum and with the only limitation by the microorganisms capacity to process the substrate. So, depending upon that how quickly microorganism process the substrate, it will be there. So, we have an abundance of organic matter or substrate present in the system during this phase.

Along with the maximum growth phase, there is also the maximum substrate removal rate also because the microorganisms are growing as well as the concentration of the organic matter is decreasing the fast as possible. Now, after this exponential growth phase we have a stationary phase. So, in this phase the food in the medium starts to decrease and it starts to become scarce and the bacterial growth rate becomes equal to that death rate. And so, the number of cells in this phase virtually remain constant and after the stationary phase, we have a decline or decay phase.

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- **Decline or decay phase:** In this phase, substrate availability in the medium is reduced. Endogenous respiration prevails, and the bacteria are forced to use their own cellular protoplasm as a substrate source. Nutrients of the dying cells diffuse into the medium and serve as food to other cells.
The death rate is exponential and constant, leading to a straight line on the logarithmic scale.
- **Important:** This representation of the growth regards a single population of microorganisms growing at the expense of a single type of substrate. However, in reality, in the biological reactor of a sewage treatment works, there is a variety of microorganisms metabolizing a variety of compounds. Hence, there will be an overlapping of various curves of different forms and types, developing at different times.

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So, in this phase the substrate availability in the medium is reduced and endogenous respiration prevails and the bacteria are forced to use their own cellular protoplasm as a substrate source. Now, in this condition the bacteria start using their own cellular protoplasm as organic matter to get the energy which is required for their sustenance. So, nutrients of the dying cells diffuse into the medium and serve as a fuel to the other cells. And that death rate is exponential and is constant and leading to a straight line on the logarithmic scale.

The importance of this whole phases is that it represents the growth regarding a single population of microorganism growing at the expense of single type of substrate. However, in reality, the biological reactor of a sewage treatment or wastewater treatment, there are a variety of microorganisms, which are metabolizing a number of compounds.

So, different types of organic bitters are present different types of microorganism are present. Hence, there will be overlapping of various curves of different forms and types and thus they will be developing at a different time. So, then the growth the overall curve may be different as compared to the one and we will be having a more stable this the stationary phase will be more there in the actual condition.

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Observed tendencies to use these concepts in sewage treatment plants

- ❑ **Very high loading systems**
 - ❖ In the exponential growth phase, the substrate availability is high. This indicates that the concentration of the substrate (e.g. BOD) in the effluent will also be somewhat high.
 - ❖ Thus, the majority of sewage treatment systems do not operate in this phase.
- ❑ **High loading systems**
 - ❖ Examples: conventional activated sludge, high rate trickling filters
 - ❖ The concentration of substrate in the effluent is lower, but the cellular mass has a high organic fraction, requiring the separate stabilization of the excess sludge.
 - ❖ Due to the high load, the volume required for the reactor is smaller than in the low-loading systems.

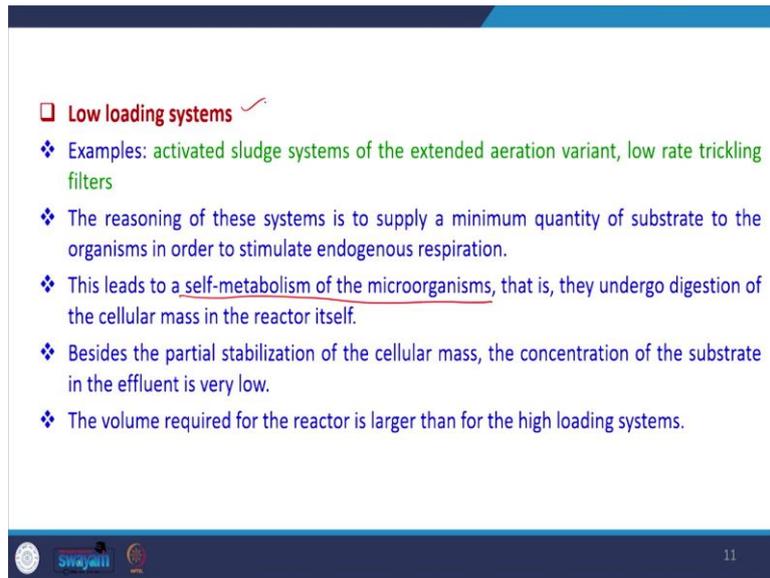
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Now, what are the observed tendencies to use these concepts in the sewage treatment plant? So, how to what are the different how to use this curve in the sewage treatment plant this is a little bit discussed here. So, if very high loading systems are there, that means, that when the substrate is very high, so, in the exponential growth phase the substrate availability is high. So, this indicates that the concentration of substrate example like BOD in the effluent will also be somewhat high. And there is the majority of sewage systems do not operate in this phase. So, very high loading systems are not very common, because BOD is generally average.

Now, in the second case, we have high loading system in place of very high now, we have high loading system example, the conventional activated sludge process, high rate tickling filter that we are going to study later on in detail. The concentration of substrate in the effluent is lower, but the cellular mass has a high organic fraction requiring the separate stabilization of the excess sludge.

So, we have to condition the excess sludge. We have to separate out the excess sludge, so that we can maintain a particular cellular mass in the activated sludge process or high rate trickling filter due to the high load the volume required for the reactor is smaller than the low loading system that are possible.

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Low loading systems

- ❖ Examples: activated sludge systems of the extended aeration variant, low rate trickling filters
- ❖ The reasoning of these systems is to supply a minimum quantity of substrate to the organisms in order to stimulate endogenous respiration.
- ❖ This leads to a self-metabolism of the microorganisms, that is, they undergo digestion of the cellular mass in the reactor itself.
- ❖ Besides the partial stabilization of the cellular mass, the concentration of the substrate in the effluent is very low.
- ❖ The volume required for the reactor is larger than for the high loading systems.

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Now, there could be low loading systems where BOD is less. So, for this, the activated sludge system of the extended aeration variant is used or low rate trickling filters are used. The reasoning for this system is to supply a minimum quantity of substrate to the organism in order to stimulate the endogenous respiration. This leads to cell metabolism of the microorganism that is they undergo digestion of the cellular mass in the reactor itself. And besides the partial stabilization of the cellular mass, the concentration of the substrate in the effluent is also low.

So, under this low loading system, the system is operated under a condition where some amount of endogenous respiration is happening. So, we do not have to that much care of the excess sludge otherwise in a high loading system, we have to take care of the excess sludge because the system is still operational under the exponential growth phase and we have to separate out the excess sludge continuously.

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Low loading systems

- Examples: activated sludge systems of the extended aeration variant, low rate trickling filters
- The reasoning of these systems is to supply a minimum quantity of substrate to the organisms in order to stimulate endogenous respiration.
- This leads to a self-metabolism of the microorganisms, that is, they undergo digestion of the cellular mass in the reactor itself.
- Besides the partial stabilization of the cellular mass, the concentration of the substrate in the effluent is very low.
- The volume required for the reactor is larger than for the high loading systems.

Thus, in the low loading system, the volume required for the reactor is larger than for the high loading system, but we do not have to take care that much of the excess sludge so, this is the good thing. And now, we are going further and we will try to understand the kinetics of the bacterial growth, now, we have studied the phases.

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Kinetics of bacterial growth

Specific gross bacterial growth

- The bacterial growth can be expressed as a function of the bacteria concentration at a given time in the reactor.
- The **net** growth rate is equal to the gross growth rate minus the bacterial decay rate.
- The **growth rate of a bacterial population** is a function of its number, mass, or concentration at a given time. It is mathematically expressed as:

$$\frac{dX}{dt} = \mu \cdot X$$

where X = concentration of the microorganisms in the reactor, Suspended solid (SS) or Volatile suspended solid (VSS) (g/m³)
 μ = specific growth rate (d⁻¹)
 t = time (d)

Handwritten notes: $\frac{dx}{dt} \propto X$

It is mathematically expressed as:

$$\frac{dX}{dt} = \mu \cdot X$$

Now, we will try to understand the kinetics of the bacterial growth. Now, one term is called a specific gross bacterial growth. So, the bacterial growth can be expressed as a function of

bacteria concentration at a given time in the reactor. The net growth rate is equal to the gross growth rate minus the bacterial decay that is possible. So, the growth rate of a bacterial population is a function of its number, mass or concentration at a given time. So, that we can write if we are assuming that x is the concentration of microorganism in the reactor and which is like suspended solid or the volatile suspended solids.

So, they actually this SS or VSS they represent the concentration of the microorganism in the reactor. So, already we have studied regarding SS, VSS earlier during the wastewater characterization. So, if this is the x so, the growth rate of x that with the dx/dt as per this will be proportional to the concentration itself and the proportionality constant is called a specific growth rate. So, this is there. The time at t where t is the time in days or any other appropriate unit which can be used.

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$$X = X_0 e^{-kt}$$

$$\log X = \log X_0 - kt$$

- ❑ Previous equation, when integrated, assumes an exponential form, which, when plotted on a logarithmic scale, results in a straight line. This logarithmic phase is shown in the figure of a *typical bacterial-growth curve* previously seen.
- ❑ The growth rate is for growth without limitation of the substrate. However, bacterial growth is a function of the availability of the substrate in the medium.
- ❑ When the substrate is present at a low concentration, the growth rate is proportionally low. In sewage treatment, the carbonaceous matter is usually the limiting growth factor.

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Going further in this previous equation when if we integrate this equation and it will assume an exponential form that will be having an equation which is like x equal to if we assume something $x e^{-kt}$ so, this will be something of this form $x_0 e^{-kt}$. So, this will be this form and if we write the log form of this so, this will become a straight line. So, like if $\log x$ is equal to $\log x_0 - kt$. So, this is there. The typical bacterial growth curve it will be, the growth rate is for without limitation of the substrate, however the bacterial growth as a function is a function of availability of substrate in the medium.

So, we should not use this particular equation because the x the growth rate of x is actually a function of substrate also. So, when the substrate is present at a low concentration, the growth rate is proportional low and in sewage treatment the carbonaceous matter is usually the

limiting growth factor. So, how much amount of organic matter is present or not present depending upon that the growth happens.

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- The specific growth rate μ must be therefore expressed as a function of the substrate concentration. **Monod** presented this relation according to the following empirical formula:

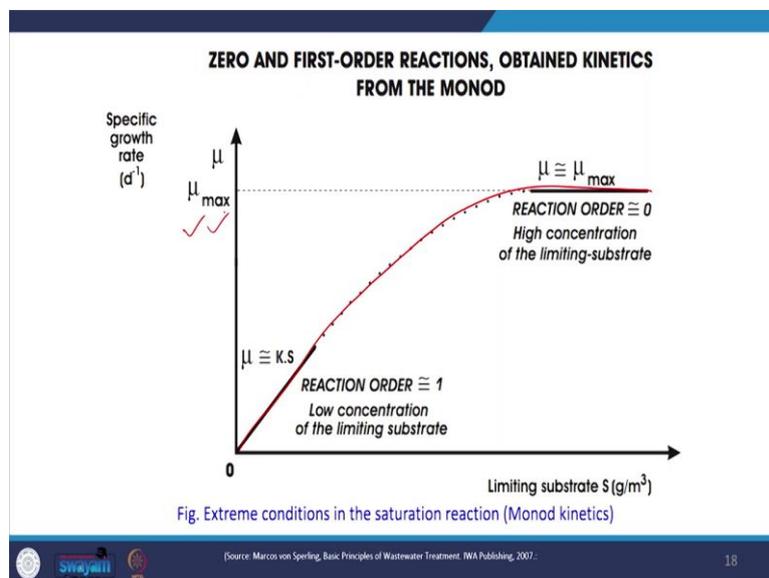
$$\mu = \mu_{max} \cdot \frac{S}{K_S + S}$$

where μ_{max} = maximum specific growth rate (d^{-1})
 S = concentration of the limiting substrate or nutrient (g/m^3)
 K_S = half-saturation coefficient, which is defined as the substrate concentration for which $\mu = \frac{\mu_{max}}{2}$ (g/m^3)

- Graphical representation of above equation is shown on the next slide.

Monod presented this relation according to the following empirical formula:

$$\mu = \mu_{max} \cdot \frac{S}{K_S + S}$$



So, under this condition, the specific growth rate μ must be therefore, expressed as a function of the substrate concentration. So, this was the idea that a μ should be function of substrate concentration. So, Monod presented this relationship according to the following empirical equation that μ is equal to $\mu_{max} \cdot \frac{S}{K_S + S}$.

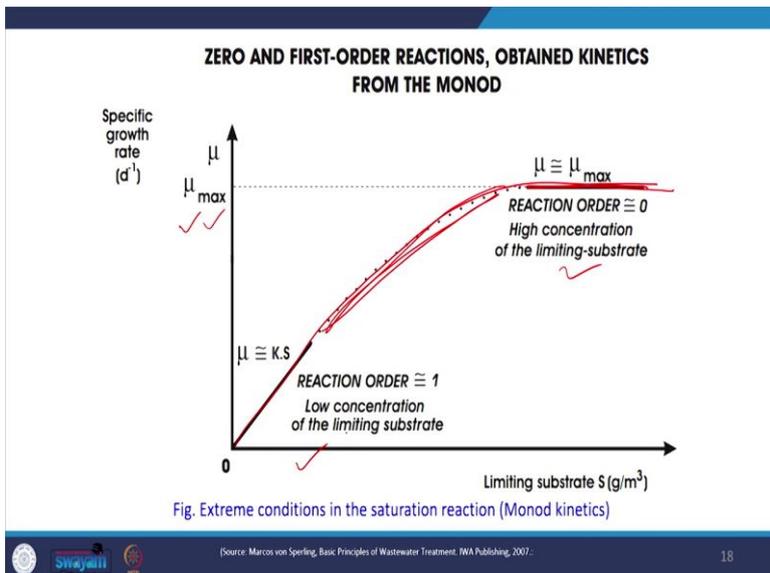
So, here, μ_{max} is the maximum specific growth rate, this is and its unit is the per unit time here like per unit day, then S is the concentration of limiting substrate or nutrient gram per meter cube. K_s is the half saturation coefficient, which is defined as the substrate concentration for which μ is equal to μ_{max} and this equation can be plotted like this, the one which is shown here. So, later on so, this if you plot μ this can be represented by this equation where the μ is continuously increasing, and it goes to μ_{max} . So, this is there. Now, we will try to understand that how this growth happens.

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Kinetics represented by the Monod equation S

- ❑ In the case of a substrate removal reaction, when its concentration is still high and not limiting, the global removal rate approaches the zero-order kinetics.
- ❑ With the consumption of the substrate, the reaction starts to decrease, characterizing a transition or mixed-order region.
- ❑ When the substrate concentration is very low, the reaction rate starts to be limited by its low availability in the medium. In these conditions, the reaction kinetics approach first order.
- ❑ These two situations occur depending on the relative values of S and K_s , as described further.

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So, the kinetics represented by Monod equation. So, we will be trying to discuss this Monod equation now, in the case of highs in the case where the substrate removal reaction is happening and when the concentration is still high and it is not limiting that means S value is

very high and it is not reacting under that condition the global removal rate approaches the zero-order kinetics. So, in this case it is not dependent upon the substrate. So, we have this is the condition where high concentration of limiting substrate is there the reaction this growth will always be at the μ_{max} .

So, this is the condition when the conjunction of the with the consumption of the substrate the reaction starts to decrease and characterizing a transition or mixed order regime that is represented here in this condition in between condition. So, this is like mixed order regime. After that when the substrate concentration becomes low the reaction order starts to be limited by the low availability of the medium.

So, in this condition the reaction kinetics approaches the first order kinetics which is shown here in this figure, we can see the reaction order becomes less equal to one and low and this is true for low concentration of limiting substrate and these any of these condition may happen depending upon the relative values of S and K_S which was defined earlier. Now, what is the relative their different conditions already we see.

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❖ **Relative substrate concentration: HIGH**

$S \gg K_S$ ✓ Reaction approximately zero order
✓ Growth rate μ independent of S

- When the substrate concentration is much higher than the value of K_S , K_S can be neglected and the equation is reduced to:
$$\mu = \mu_{max}$$
- The reaction follows **zero-order kinetics**, in which the reaction rate is independent of the substrate concentration.
- In domestic sewage treatment, this situation tends to occur at the head of a plug-flow reactor, where the substrate concentration is still high.

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So, when the relative substrate concentration is high. So, S is much, much greater than K_S . So, under that condition μ becomes equal to μ_{max} and this reaction follows zero-order kinetics in which the reaction rate is independent upon of the substrate concentration. So, this will happen more in the domestic sewage treatment. And this situation tends to occur at the head of a plug flow reactor where the substrate concentration is still high. So, we will in the next few lectures we will be studying regarding plug flow reactor CSTR et cetera.

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❖ **Relative substrate concentration: LOW**

$S \ll K_s$ ✓ Reaction approximately first order
 ✓ Growth rate μ is dependent on S (directly proportional)

- When the substrate concentration is much lower than the value of K_s , S can be neglected and the equation is reduced to:

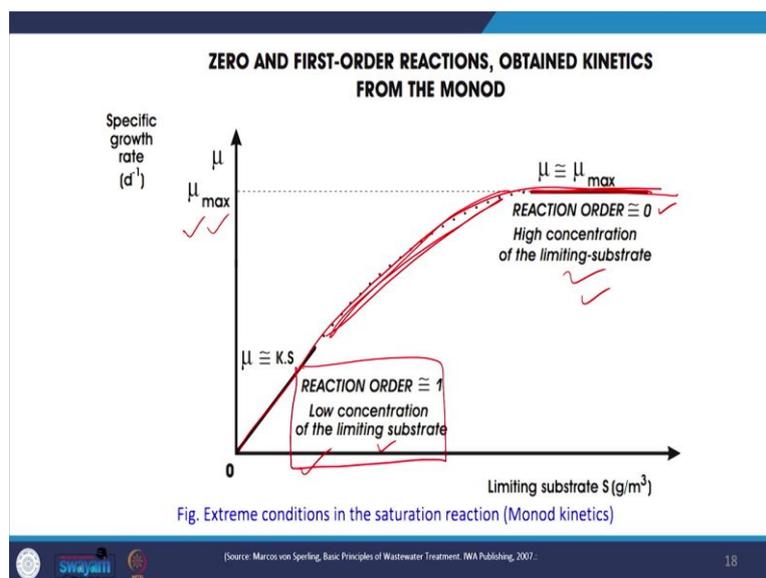
$$\mu = \mu_{max} \cdot \frac{S}{K_s}$$
- As μ_{max} and K_s are constants, the term (μ_{max}/K_s) is also a constant, and can be substituted by a new constant K . Consequently, Equation 8 is reduced to:

$$\mu = K \cdot S$$
- In this situation, the growth rate is proportional to the substrate concentration. The reaction follows **first-order kinetics**.
- This situation is typical in the treatment of domestic sewage in a complete-mix reactor in which the substrate concentration in the medium is low due to the requirements of low values in the effluent.

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When the substrate concentration is much lower than the value of K_s :

$$\mu = \mu_{max} \cdot \frac{S}{K_s}$$



Now, when the relative substrate concentration is low that means, S is much much less than K_s . So, under that condition the equation becomes μ is equal to $\mu_{max} S$ upon K_s and now, we can write μ_{max} upon K_s as another constant K and does the equation actually, it changes to μ is equal to $K S$ that means, μ now, the growth is function of substrate, the growth rate is proportional to substrate concentration and the reaction follows first order kinetics and this is this will happen in the treatment of domestic sewage in a complete mix reactor and generally the activities like processes et cetera are like a complete-mix reactor.

So, the growth rate in the complete-mix reactor follows first order kinetics and it will be the true for the activated sludge process et cetera. So, this is the overall curve is the one we have a nu is equal to mu x reaction order is 0 and when the low concentration of limiting substrate is there, the reaction order is one and this is the condition. So, this is the growth.

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□ In the laboratory, the curve of specific growth rate as a function of the concentration of limiting substrate (or a transformation of it) can be constructed, and the values of K_s and μ_{max} can be extracted. In domestic sewage treatment, values of K_s and μ_{max} in the following ranges have been reported:

Aerobic treatment (Metcalf & Eddy, 1991):

- $\mu_{max} = 1.2 \text{ to } 6 \text{ d}^{-1}$ ✓
- ✓ $K_s = 25 \text{ to } 100 \text{ mg BOD}_5/\text{l}$ or $K_s = 15 \text{ to } 70 \text{ mg COD/l}$

Anaerobic treatment (van Haandel and Lettinga, 1994; Chernicharo, 1997):

- ✓ $\mu_{max} = 2.0 \text{ d}^{-1}$ (acidogenic organisms)
- $\mu_{max} = 0.4 \text{ d}^{-1}$ (methanogenic organisms; combined biomass)
- $K_s \approx 200 \text{ mg COD/l}$ (acidogenic organisms)
- $K_s \approx 50 \text{ mg COD/l}$ (methanogenic organisms)

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Now, in the laboratory the curve of specific growth rate as a function of concentration of limiting substrate can be constructed and the value of K_S and μ_{max} can be extracted. So, in domestic sewage treatment values of K_S and μ_{max} in the following ranges have been reported in the literature like in Metcalf Eddy for aerobic treatment that μ_{max} value has been reported in the range of 1.2 to 6 per unit day.

Then similarly that K_S value is from 25 to 100 milligram BOD 5 per liter or K_S in terms of COD is 15 to a 70 milligram COD per liter. For anaerobic treatment that μ_{max} value have been reported in the range of 0.4 to 2 depending upon whether acidogenic organisms are present or methanogenic organisms are present.

Similarly, the K_S values are in the range of 50 milligrams COD per liter to 200 milligram COD per liter depending again on the acidogenic or methanogenic organisms are present or not. So, there are different types of, there are certain limitations of Monod equation as well as the advantages are also there.

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Limitation of Monod equation

- Monod equation is not based on theoretical principles, it is essentially empirical.
- This was derived for a single organism metabolizing a single substrate. However, in wastewater treatment, there is a *multiple population* assimilating a *multiple substrate*.

Advantages of Monod equation

- Its structure permits the representation in a continuous form of the range of variation between the extremes of lack and abundance of nutrients in the medium.
- Therefore, depending on the value of S, the Monod equation can represent approximately the kinetics of zero and first orders, as well as the transition between them.

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So, Monod equation the limitation is that the Monod equation is not based upon the theoretical principles, it is essentially empirical in nature and this was derived for a single organism metabolizing under a single substrate. However, in wastewater treatment, there are a number of multiple population assimilating in a multiple substrate. So, thus we have this is the major deterrent in the use of Monod equation for the wastewater treatment.

Now, advantage is that the Monod equation structure permits the representation of continuous form of the range of variation between extremes of lack and abundance of nutrients in the medium. So, we can easily establish that and therefore, depending upon the value of S in terms of BOD or COD, the Monod equation can represent approximately the kinetics of zero and first order as well as the transition between them. So, it is very easy to represent the kinetics and the transition in between them between zero and first order using the Monod equation.

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Example

Express μ as a function of μ_{\max} for the following conditions:

- a) Domestic sewage; $S = 300$ mg/L (adopt $K_s = 40$ mg/L) ✓
- b) Domestic sewage; $S = 10$ mg/L (adopt $K_s = 40$ mg/L) ✓
- c) Glucose; $S = 10$ mg/L (adopt $K_s = 0.2$ mg/L)

Solution: a) Domestic sewage ($S = 300$ mg/L)

From Equation,

$$\mu = \mu_{\max} \times \frac{S}{K_s + S} = \mu_{\max} \times \frac{300}{40 + 300} = 0.88\mu_{\max}$$

Hence, $\mu = 0.88\mu_{\max}$ ✓

In these conditions, in which S is large in comparison with K_s , the growth rate μ is close to μ_{\max} . There is a great availability of the limiting nutrient and the population presents a high growth rate. The reaction is approximately zero order.

Swayam Spering, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

From Equation,

$$\mu = \mu_{\max} \times \frac{S}{K_s + S} = \mu_{\max} \times \frac{300}{40 + 300} = 0.88\mu_{\max}$$

Will take one example before closing this section. So, here it is being asked to express the growth rate as a function of μ_{\max} or the following condition when the domestic sewage the substrate concentration is S is 300 milligram per liter. And in another condition for the substrate is 10 milligram per liter, and it is being suggested that to use the K_s value of 40 milligram per liter for both the cases and in the third case, the S value is assumed to be 10 milligram per liter. And the K_s value is suggested to be 0.2 milligram per liter, so, for first case, if we write the μ value will be $0.88\mu_{\max}$, so, we are just substituting the value.

So, in this case, we are finding the value to be 0.88 of the μ_{\max} so, that means, the growth rate is around the maximum value in this condition in which S is large in comparison with K_s we can see the S is much larger than the K_s , the growth rate is close to μ_{\max} . This is that means that there is a greater availability of the nutrient and the population present is in a high growth rate phase, that reaction is appropriately of zero-order. Now, this condition will not be very frequent in that statement of domestic sewage and occurs only in some cases.

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This situation is not very frequent in the treatment of domestic sewage and occurs at the head of a plug-flow reactor, where the substrate concentration is still high.

b) Domestic sewage ($S = 10 \text{ mg/L}$)

$$\mu = \mu_{\max} \times \frac{S}{K_s + S} = \mu_{\max} \times \frac{10}{40 + 10} = 0.20\mu_{\max}$$

Thus, $\mu = 0.20 \mu_{\max}$

As S is small in comparison with K_s , the growth rate is much lower than μ_{\max} . This indicates that there is not much availability of the limiting nutrient in the medium. This situation is typical in a complete-mix reactor treating domestic sewage, in which the substrate is completely homogenised and is present at a concentration equal to the effluent one.

   Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

Domestic sewage ($S = 10 \text{ mg/L}$)

$$\mu = \mu_{\max} \times \frac{S}{K_s + S} = \mu_{\max} \times \frac{10}{40 + 10} = 0.20\mu_{\max}$$

Now, for the second case, when the S is 10 milligram per liter, it is 0.2, then μ is found to be $0.2 \mu_{\max}$ that means as the S is small in comparison with K_s the growth rate is much lower than μ_{\max} and this indicates that there is not much availability of the limiting nutrient in the medium. So, this is there and this will be possible in the complete-mix reactor like activated sludge processor, et cetera.

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c) Glucose ($S = 10 \text{ mg/L}$)

$$\mu = \mu_{\max} \times \frac{S}{K_s + S} = \mu_{\max} \times \frac{10}{0.2 + 10} = 0.98\mu_{\max}$$

Hence, $\mu = 0.98 \mu_{\max}$

The concentration of S is the same as in item b, but since in the case of glucose K_s is very low (high affinity), the denominator of the expression is practically equal to S and, as a result, μ is almost equal to μ_{\max} . Consequently, there is a high availability of the substrate and the growth rate is very close to the maximum.

   Sperling, M.V., 2007. Biological Wastewater Treatment Series Volume Two Basic Principles of Wastewater Treatment.

Glucose (S = 10 mg/L)

$$\mu = \mu_{\max} \times \frac{S}{K_s + S} = \mu_{\max} \times \frac{10}{0.2 + 10} = 0.98\mu_{\max}$$

For the c case, when the S is 10 milligram per liter and a KS is only 0.2 so, again we are finding nu is equal 0.98 nu max. So, again the though the concentration of S is the same as in the item b but since in this case glucose for the glucose KS is very low and it has high, the bacteria have very high affinity. The denominator of the expression is practically equal to S itself and in this case the nu is almost equal to nu max. So, consequently there is a high variability of substrate and the growth rate is very close to the maximum. So, this is there.

So, today we have learned regarding little bit of the growth kinetics and the Monod equations and how to use the Monod equation under different conditions of substrate concentrations and weather and we also learned that what is the impact of substrate concentration on the growth curve, as well as the value of KS. So, with this we will end today's lecture. Thank you very much.