Interfacial Engineering

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Lecture-07

Young-Laplace equation; Tutorial

Surface tension, pressure difference, equlibrium shape; curvature effect

Welcome back.

So, in today's video lecture, we will look at what is known as the Young-Laplace equation. So, in last class, we looked at the derivation of Young-Dupre using which we obtained a relationship between contact angle and the surface energies right so this equation Young-Dupre was very useful I mean we saw that it is I mean Young-Dupre will be very useful when you want to you know assess the nature of the surface whether it is hydrophilic or hydrophobic using the parameter called you know contact angle, three-phase contact angle.
But in today's video lecture, we will look at Young-Laplace equation, right. So, I mean and we will look at you know how it can be, how the governing equation can be obtained, right. So, in general Young-Laplace equation is used to understand you know a predict the equilibrium shape of curvature right.
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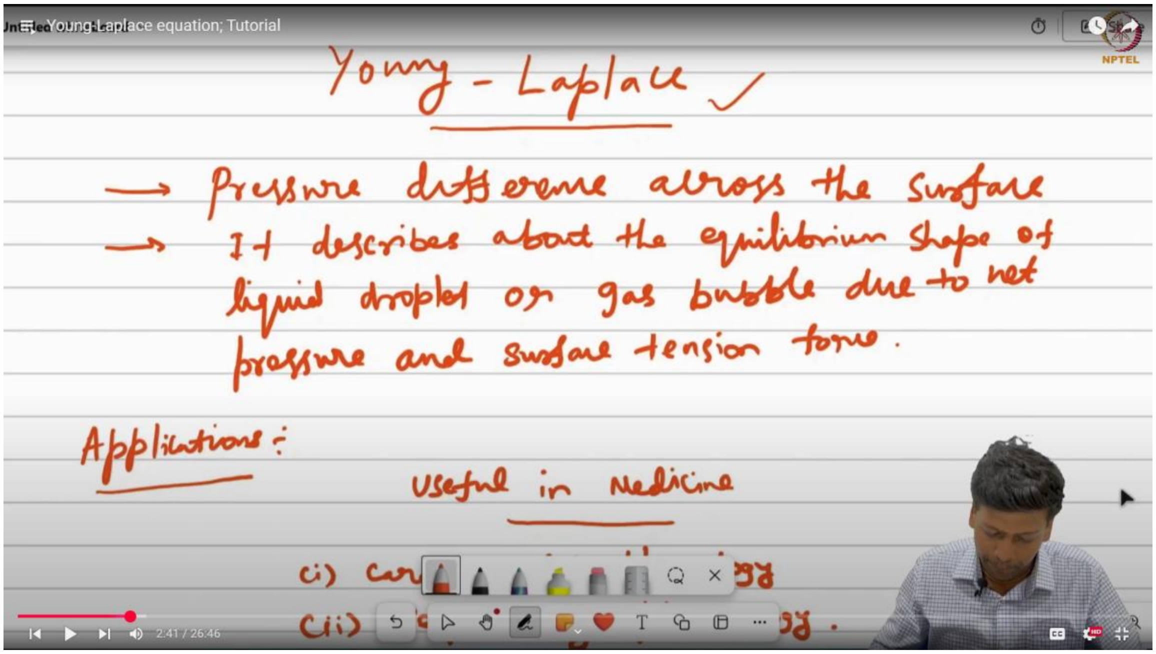

We know that whenever we consider curvature there will be the pressure difference across the interface right and there is also additional force that comes into picture that is nothing but surface tension force as a result of pressure as well as surface tension force at equilibrium depending on the you know the forces that are acting one would get a definite I mean equilibrium shape right and so we will look at in today's video lecture how we can use this the force I mean force I mean use this force balance equation and you know to arrive at a generalized or general Laplace pressure equation okay right let's begin right yeah so as I explained just now so the pressure difference across the surface is what we are going to look at when we deal with in Laplace equation so In short, Young-Laplace equation is used to describe the equilibrium shape of the liquid droplet or the gas bubble due to net pressure and surface tension force acting at the interface. So, one can list many applications when it comes to Young-Laplace, but I have listed a few based on the application in Medicine, where the concept of Young-Laplace, the principle of Young-Laplace equation or principle of Young-Laplace is applied. in the field of cardiovascular physiology or respiratory physiology. You can refer to any textbook for the application in detail.

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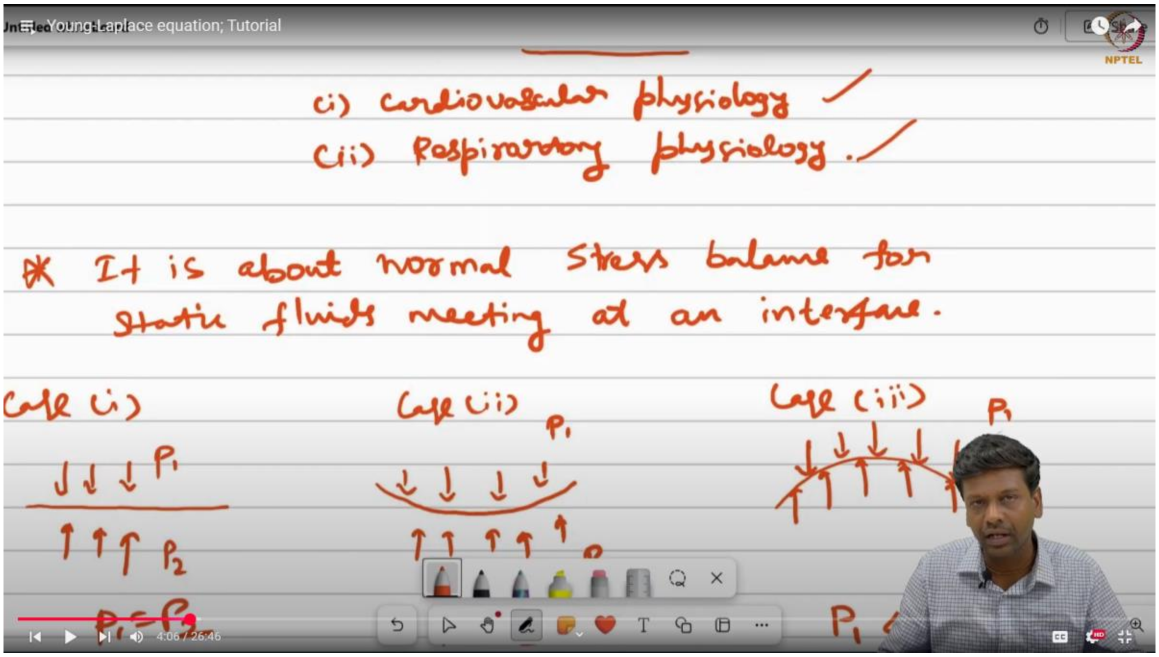

But right now what we will do is, we will try to understand how we can actually obtain a simple relationship between pressure and the surface tension. Right. So let us look at a different scenario as we have, you know, schematically shown here. Let's say we have case one scenario where you deal with case one. I mean, situation like case one.

and situation like case 2 and case 3. So in these cases, if you look at the pressure that is acting on the surface, let's say for example case 1, you see that here in this case the pressure you know acting on both sides are equal in such scenario one would expect flat interface right because the pressure effect is actually eliminated because both the side the pressure acting on both sides are equal. right so so we consider this is mostly applicable in flat interface case whereas you consider another scenario where we experience you know curvature okay we deal with the curvature where in this scenario we expect that the pressure you know acting at the top is greater than pressure acting you know, upward, right? On the other side, we have a scenario where we might deal with, let's say, geometry curvatures similar to case 3, where the pressure acting downward may be less than pressure acting upward, okay. But not only pressure force that are acting on these surfaces, there is another force that is also, you know, existing, right, that is nothing but gamma, which is nothing but surface tension force, okay, which is acting at the interface, all right.
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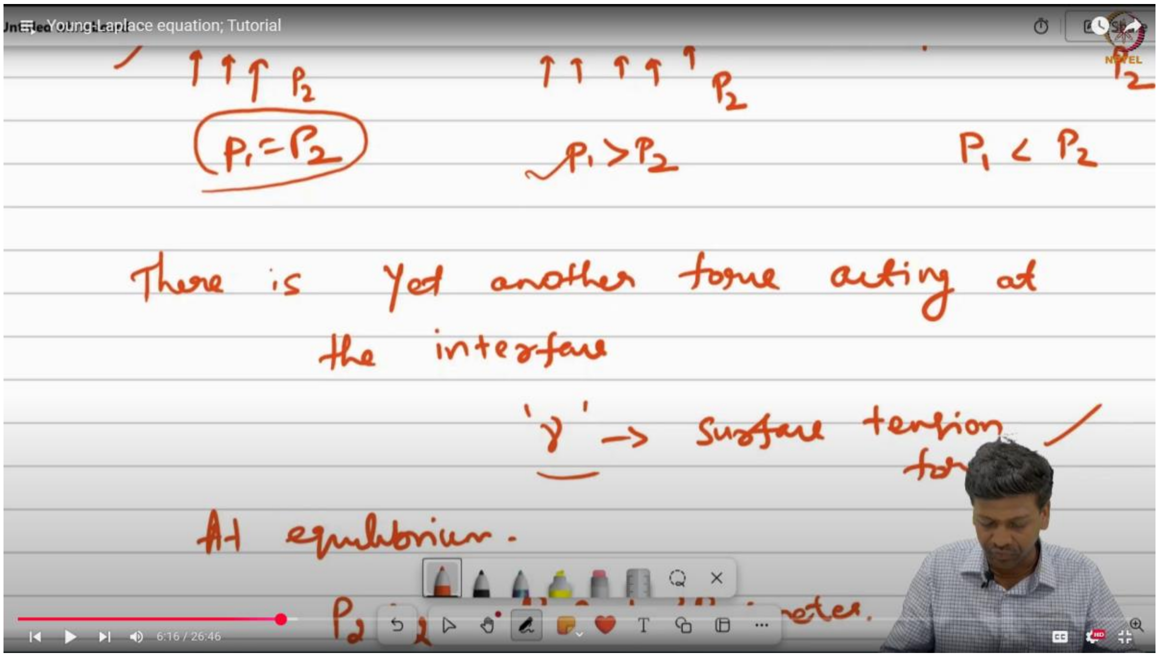

Now, if you were to, you know, perform a simple force balance, okay, on these kind of, on these curvatures, you know, or various geometries, you would understand that it is simply nothing but, you know, you also have to include gamma, which is nothing but, you know outside pressure and the area corresponding to given area and inside pressure acting on area which is you know acting downward and then you also have surface tension force multiplied by perimeter so this you know if you perform simple force balance you would achieve a equation of this kind Now, we know that whenever we deal with a case where the surface or interface is considered, we know that work is being done by the system because surface tension usually pulls the surface down because of the surface tension effect, right? So you consider that as.

I mean in such case we say that when we consider that work is being done by a system. Apart from that you also have to consider the pressure form that is acting from both sides. So it could be pressure work done on the system as it is work done by the system. So in such scenario you will have work done due to delta peak and work done due to gamma. So we know simply the work done is nothing but

$$W=\gamma dA$$

Okay. And this work then can also be expressed in terms of the, you know, change in pressure and change in volume, which is nothing but $\Delta P*dV$. If this is our way to be our, you know, governing equation, then how we can get a relationship between pressure and gamma in a more compact form, right?"

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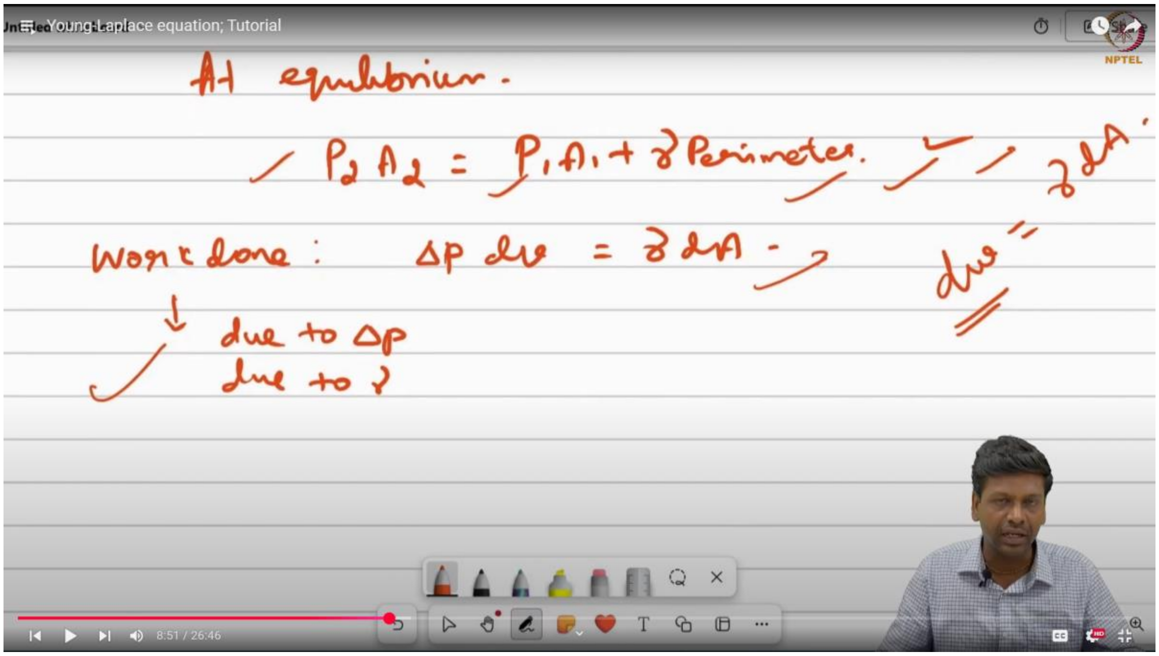

This is a differential equation, but we need to obtain equation, general equation, okay, in a more compact form, right? So what we will do is, we'll start with a simple, you know, you know, since we are going to deal with a general equation, generally, I mean, for a general case, we'll start with curvature, okay, where we will deal with, you know, where, uh, we, we deal with two, uh, you know, two, uh, major axis, right. Predominantly because, if I, if we talk about spherical geometry where, you know, the, the radius, right. I mean radius is, I mean, both major axis is, of same radius, right.

Whereas if you talk about a cylinder, there is a major axis and there is a minor axis. Okay. So, since depending on the geometry, one would have to choose a suitable, you know, equation. So, for that, what we can consider is we can take a curvature, right, which has got two, you know, primary axes, right. So, with respect to the center of the axis, one would get two primary axes, right, okay.

So, this we will take for an example, and if I name this, you know, a geometry, a rectangle-like geometry, okay, this is an infinitely small, you know, element, okay, control element. So, we can also say that it is nothing but a rectangular geometry. But it has got two major axes, primary axes, right.

So, in the first case, if I name this as AOB, this triangle will have angle α\alphaα, whereas the other triangle will have angle β, okay, right. The length and breadth, so the AB, okay, the distance from A and B is x, and B and C is y, right. Then the area here is simply

$$A=x*y$$

Now, this is at time t=0, right. Now, let's say that after some time, we impose a small perturbance, right. Like you consider that there is a pressure force as well as a surface tension force acting on this surface, which will make the surface deviate from the existing one, right.
So, there will be a change in area, right, and there will be a displacement basically, a deformation or displacement will occur, okay. So, if you consider that after some time t, time t equals some time t, so what will happen is, let us say, I mean, either you will get, you know, a positive curvature or negative curvature, but let's understand that the curvature that we get is, you know, let's say bigger than this, okay, yeah. So, I'm trying to just draw a similar geometry, somewhat bigger than this, okay. So, in this case, we have a triangle.

Okay, I mean, this is just a triangle, I'm sorry, yeah. In this case, we get a triangle. A similar triangle that is A′ dash, if I call this as A′ dash B′ dash C′ dash and D′, this will be a four dash, okay.
So, we said in this case what we assume is that when there is a change in the area or, you know, there is displacement with respect to the curvature because of the pressure force and surface tension force, what we try to propose is that they increase or they decrease, but they proportionately change, right.
Which means that the angle in which, you know, they form is the same as the existing one, right. They said this is also α\alphaα, this is also β. So, this angle and this angle are the same. This β and this β are the same, but they proportionately vary such that, you know, only the distance between A′ dash and O′ dash changed a little bit, right, right.
If I call this as R1, A2, AO, if I call AO the distance between AO as R1 and the distance between O and C as R2, then since it is proportionately increasing or decreasing, I would call that as the distance between A′ dash and O′ dash, I would call it as

$$R1+dz$$

whereas the distance between O′ dash and C′ dash would be

$$R2+dz$$

okay.

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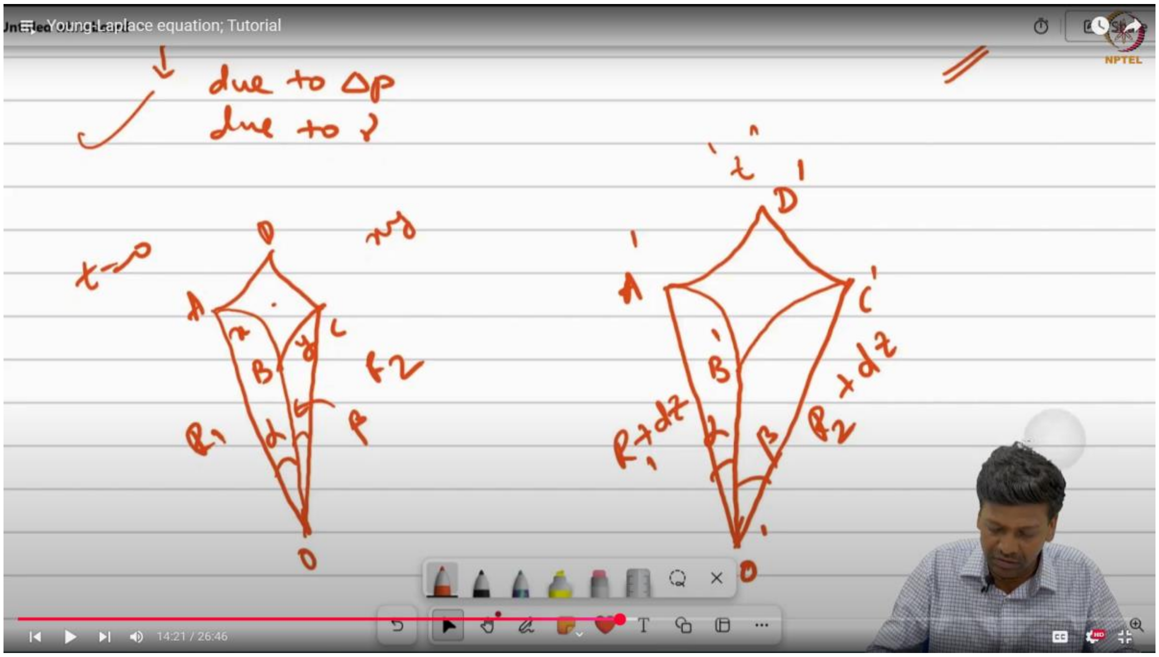

So the assumption here is the angle doesn't change, but only they proportionately vary, right. So in this scenario, we can, you know, have the following propositions.

Like, for example, let's say I know sin α in this case will be:

$$\sin \alpha = x / R_1$$

Okay. In this case, sin α will be:

$$\sin \alpha = (x + dx) / (R_1 + dz)$$

Since sin α is the same, that is the proposition we set in this case.

In such a scenario, you will have:

$$(x + dx) / (R_1 + dz) = x / R_1$$

Similarly, if you say sin β = sin β at both instances, right, then you can say that:

$$\sin \beta = y / R_2$$

and

$$\sin \beta = (y + dy) / (R_2 + dz)$$

So this is the proposition that, you know, we follow in this case, right.

So simply what one would get is:

$$(x + dx) / (R_1 + dz) = x / R_1$$
$$(y + dy) / (R_2 + dz) = y / R_2$$

This is what, I mean, this is the relation we obtained from these propositions.

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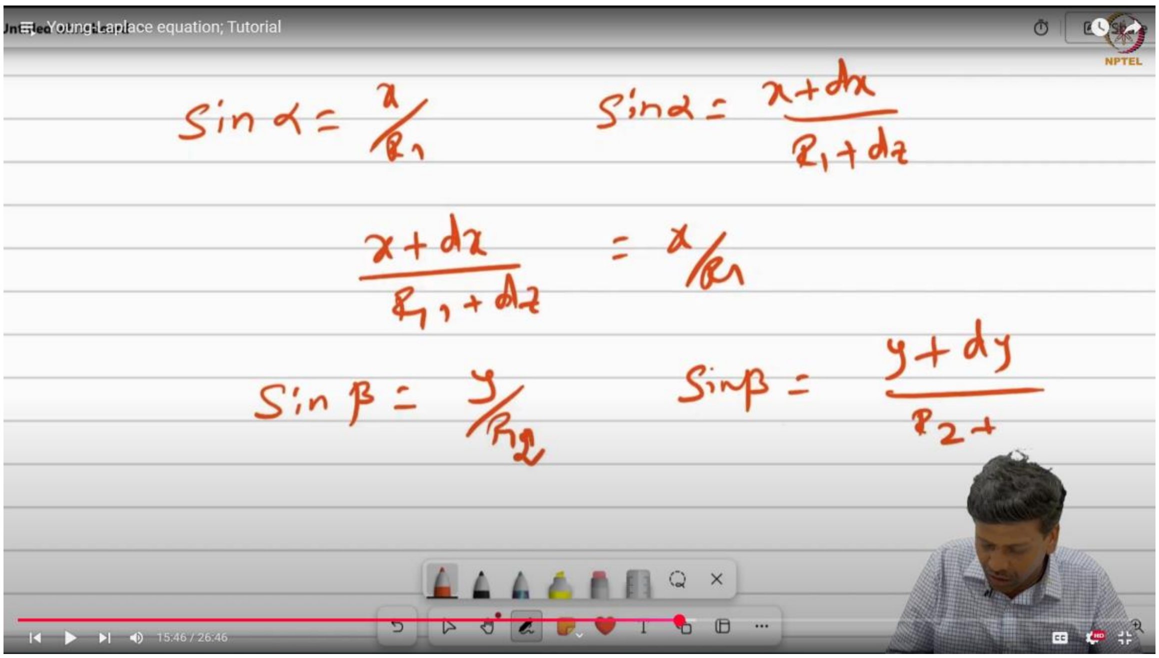

Now we go back to the equation that we initially started with, that is nothing but:

$$\Delta P \, dV = \gamma \, dA$$

This is our governing equation.

Now we know what dA is, you know, change in area. We know change in area, you know, because it changed from the curvature—I mean, it changed from this geometry to this geometry, right?

So then dA will be simply nothing but:

$$dA = (x + dx)(y + dy) - xy$$

Right. So if I simplify this further, this will be simply:

$$dA = xy + x\,dy + y\,dx + dx\,dy - xy$$

So I can cancel xy and xy. Because dx dy, dx itself is very small, dx dy will be negligibly small. So we can actually ignore this one.

Then what we will be left with is:

$$dA = x\,dy + y\,dx$$

Okay. Now we know ΔP is nothing. ΔP dV can be written as:

$$\Delta P\,dV = xy\,dz$$

Okay. In such a case, this will be simply nothing but:

$$\gamma\,(x\,dy + y\,dx)$$

Okay. Right. So, ΔP here is nothing but:

$$\Delta P = \gamma\,(dy / y\,dz)$$

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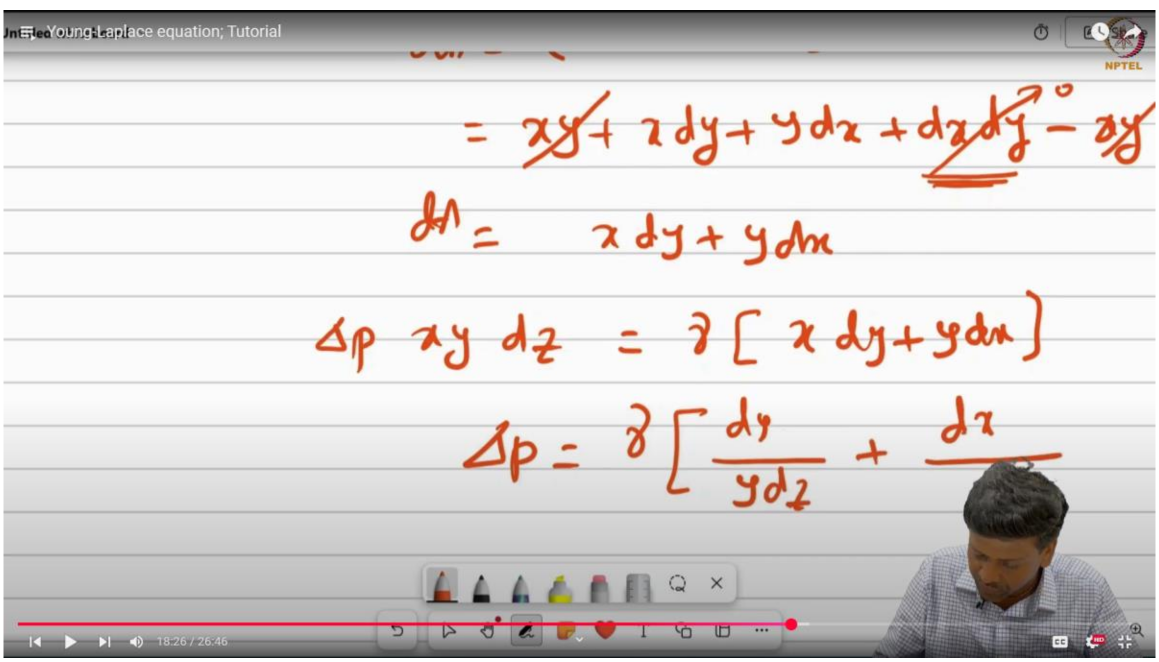


Okay. Plus dx / x dz.

Now, we are left with one step, that is, we need to know what y dz and x dz are, right? Then we will be able to get the general equation, Young–Laplace equation.

So, for that, what we do, we resort to the proposition that we set, that is:

$$(x + dx) / (R1 + dz) = x / R1, \text{ all right?}$$

And,

$$(y + dy) / (R2 + dz) = y / R2$$

So this is what we have.

So if I simply simplify this, I would get:

$$(x + dx)\, R1 = x\, R1 + x\, dz$$

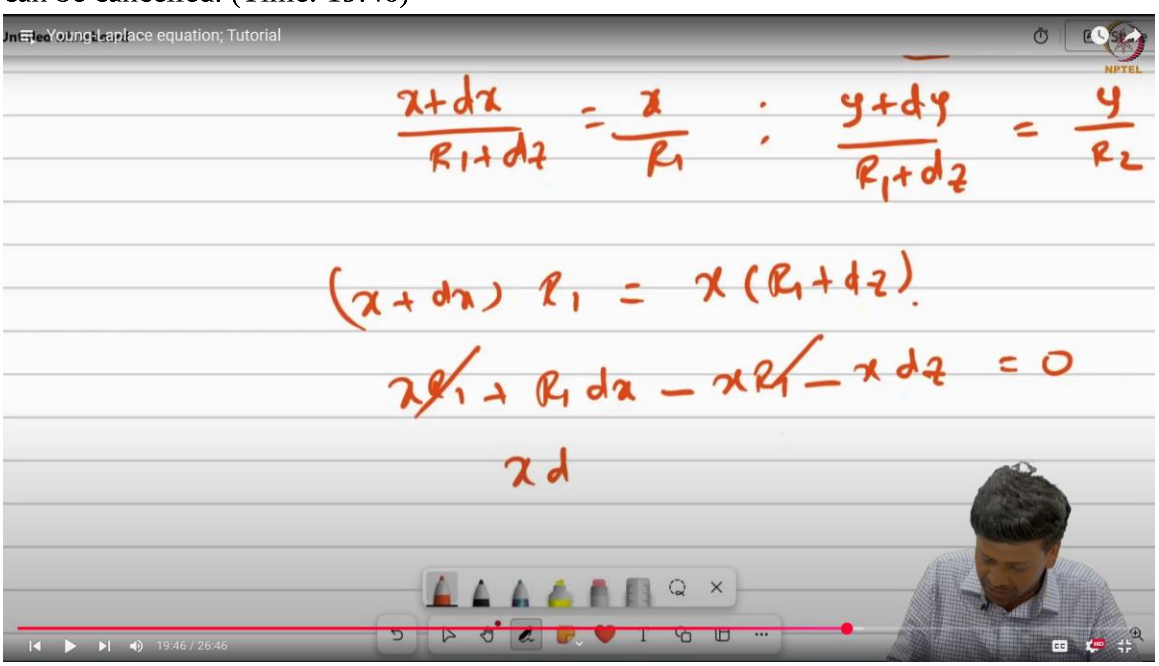
Okay.

So similarly, if I just try to simplify this, I would get:

$$R1\, dx - x\, dz = 0$$

$$x\, R1,\ x\, R1$$

can be cancelled. (Time: 19:46)



x dz = R1 dx.

Okay.

Similarly, if you do for other cases, you will get $y\,dz = R2\,dy$.

So if I substitute back, then I would get:

$\Delta P = \gamma\,(dy / y\,dz + dx / x\,dz)$

Okay.

So $dy / y\,dz$ is $R2\,dy$, plus $dx / x\,dz$ is $R1\,dx$.

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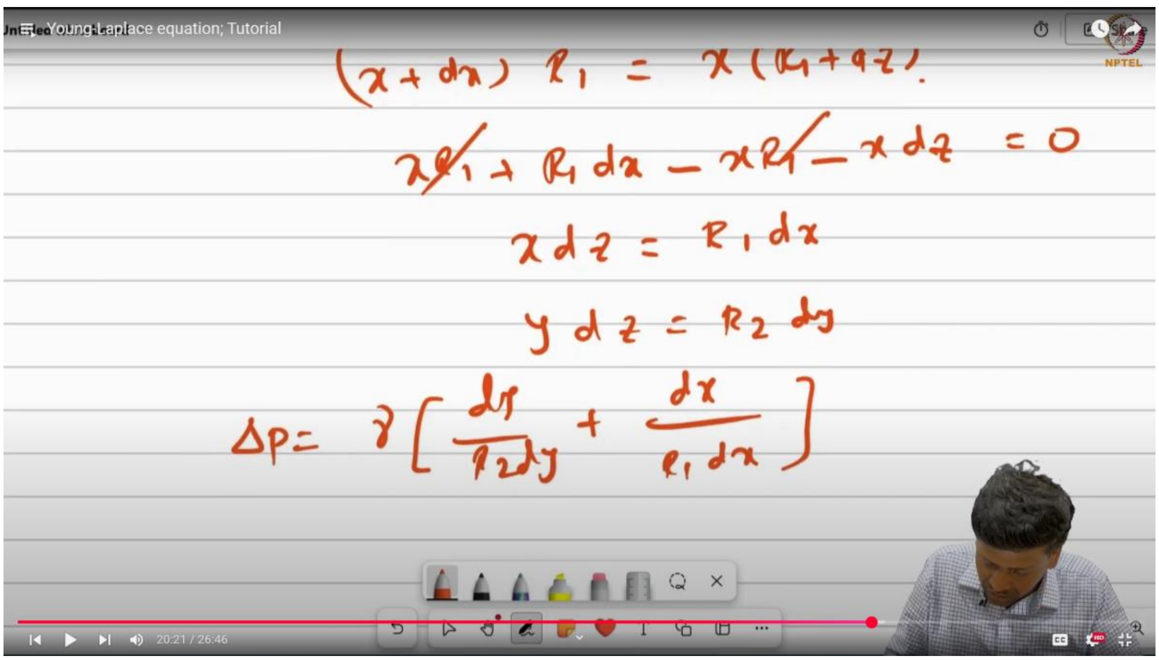

Okay.

So then simply I will have ΔP, which is nothing but $\gamma\,(1 / R1 + 1 / R2)$ itself.

So this is our compact Young–Laplace equation in the general case.

Let's consider limiting cases.

Let's say we deal with cylindrical geometry.

In such a scenario, you know, the major axis will be infinitely larger than the minor axis.

So mathematically, we can say that R1 $\rightarrow \infty$.

In such a scenario, we will get $\Delta P = \gamma / R2$.

Right, this is for the case of a cylinder.

Now, if you deal with spherical geometry, okay, in such a scenario, we know that R1 = R2 = R, so we can simply say $\Delta P = 2\gamma / R$.

Okay, in the case of a flat surface or flat geometry, okay, since both axes tend to infinity, okay, you can say that $\Delta P = 0$.

Okay, so depending on the situation, depending on the geometry that we deal with, one would obtain, one would get a different compact equation.

Okay, this is nothing but the Young–Laplace equation.

All right, so now what we will do, we will move on to the tutorial part today.

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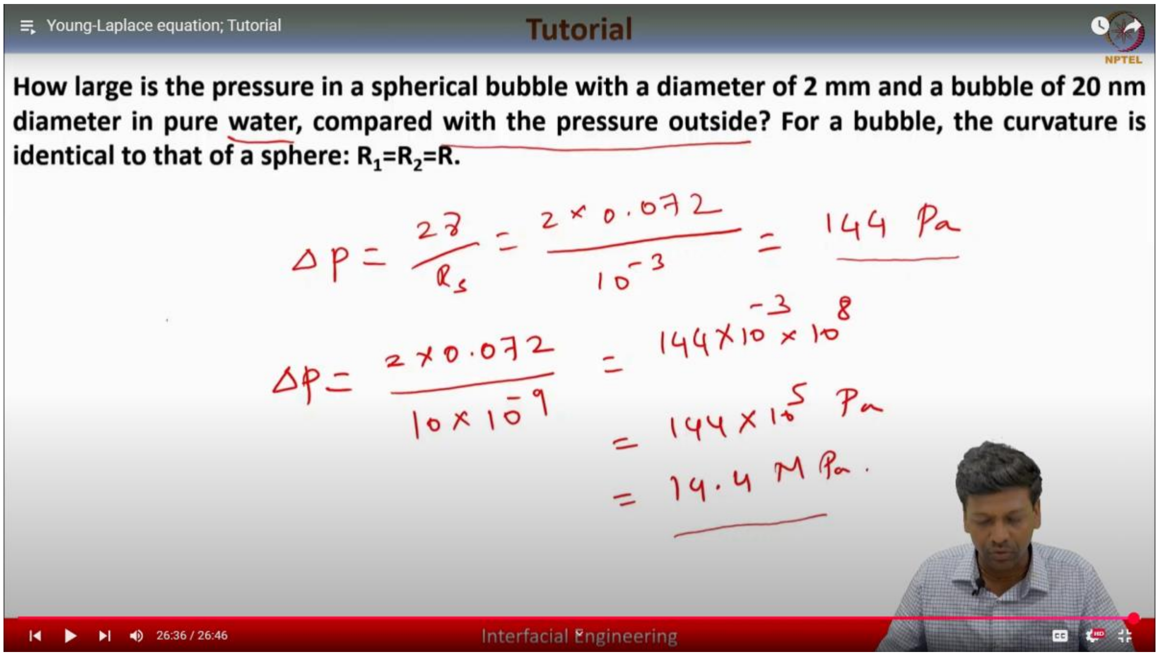

So here, we have given, you know, the problem where we are asked to find out the pressure or compare the pressure outside, okay, so in two cases: one is we have got, you know, the spherical bubble.

In the first case, the diameter of the spherical bubble is 2 mm, sorry, 2 mm, which is a macro bubble, and in the other case, the diameter of the bubble is 20 nanometers, okay, so nanobubble.

So, in these two scenarios, okay, what is the, you know, Laplace pressure? That is what we have to calculate in this.

So, we know a spherical geometry, curvature, okay, we can state it in the compact equation, that is,

$$\Delta P = (2\gamma) / Rs$$

which is nothing but

$$\Delta P = 2 \times \gamma$$

In this case, since it is for water (pure water), the air-water interface interfacial tension, right, in this case will be 72 millinewtons (mN) per meter, so one would take 0.072 Newton per meter, divided by the radius.

The radius of the curvature is given, which is 2 mm. The diameter is 2 mm, so you can take the radius as 1 mm, so which is nothing but $10^{(-3)}$ m.

So, in this case, one would get, you know,

$$\Delta P = 2 \times 0.072 / (1 \times 10^{(-3)})$$

which is simply nothing but 144 Pascal.

Right. So, you can say the pressure outside, compared with the pressure outside, maybe you can, one step further, you can say that the pressure will also be 144 Pascal plus the pressure inside, right?

Okay, you can also say in this way or you can keep it simply as $\Delta P = 144$ Pascal.

What about the other case, which is a nanobubble or the bubble which is 20 nanometers in diameter?

So, in this case, the ΔP is simply nothing but

$$\Delta P = (2 \times 0.072) / (10 \times 10^{-9})$$

which gives $\Delta P = 144 \times 10^{-3} / 10^{-8}$

This would be simply

$$\Delta P = 144 \times 10^{5} \text{ Pascal}$$

or it will be simply 14.4 MPa (Mega Pascal).

Okay, see the difference: in the case of 2 mm diameter, you can see that it is simply 144 Pascal, whereas in the case of a 20 nanometer diameter bubble, again, the pressure inside is, you know, so much larger right compared to the macro bubble, which is 14.4 MPa.

Okay, so this is purely due to the curvature effect.

Okay, so thanks to the Young–Laplace equation, using which we can be able to calculate what is the, you know, the compact pressure, what is the pressure inside the bubble or the droplet.

So, we will stop here. I think this is very clear to you.
So, we stop here we will see from the next lecture. Thank you.