

Fluid Mechanics
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Lecture - 29
Characterization of particles 2

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Particulate matter atoms/molecules
• solid particles
• Liquid Drops? } Deformability
• Gas Bubbles }
○ → ○



So, in the previous class we kind of had a introduction to what we are going to say in the rest of the course right in terms of fluid particle mechanics. So, one of the thing that came out of the discussions last week right, was that if you consider a particulate matter we said you know a particulate matter could be either in the form of solids or solid particles or liquid drops or gas bubbles right that is what we said right depending upon. So, you have we said that you know there is a dispersed phase and a continuous phase and you know your continuous phase could be a gas or a liquid ok

And your dispersed phase could be either particles liquids or gas right in that sense your particulate matter when we are going to discuss in the course, it could be solid particles liquid drops and gas bubbles right. Now, what do you think is that if you want to distinguish the three can you think of one parameter which is kind of differentiates solid particles from the other two.

So, when we say that you know we are dealing with fluid particle systems or particle filled systems and I am working with solids you know filled in a fluid or liquid or gas bubbles

in a fluid. What do you think would be one important parameter that you would have to you know worry about, when you are dealing with liquids and gases compared to solid particles, deformability of the particles right. Because when you are working with you know solid particles I mean if you work you know under certain conditions you can assume that you know the solid particles will continue to remain as you know rigid, hard you know they retain their size, shape you know during the processing operation, but whereas, if you consider liquid droplets or gas bubbles it could so happen that you know they could deform right.

What I mean by that? You know you could be working with like say a spherical liquid droplets to begin with you know, but; however, during processing operation it can become elongated right. So, such deformability you know of the particulate matter you would have to worry when you are dealing with you know liquid droplets or gas bubbles.

Now, for any of these particulate matter what is the constituent of each of these particulate matter right, everything is basically made up of atoms or molecules right. So, when we have atoms and molecules that constitute this particulate matter you should know a little bit about something called as inter atomic sorry atomic or inter molecular forces right, all of you know about this.

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Inter atomic / Inter molecular
forces
 $U = U_A + U_R$

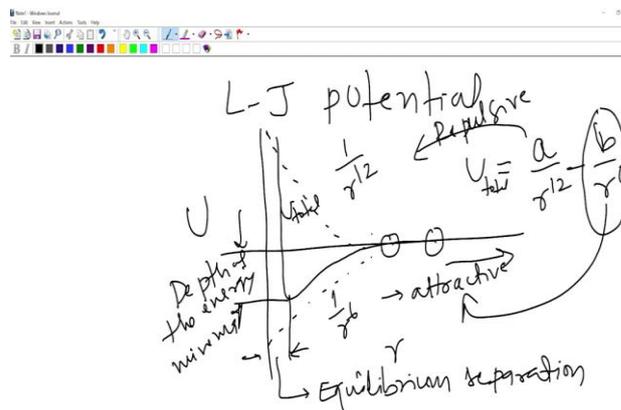


So, you know a little bit about inter molecular forces inter atomic forces, I guess in the previous semester in thermo I think there was a section on inter molecular forces right all

of you know what it is ok. So, the only thing that is important is that you know. So, whatever the atoms and the molecules that make up these particles we should know that you know there is something called as a inter molecular or inter particle forces between them. And in general, your inter molecular force you know is typically a summation of some attractive forces plus some repulsive forces right ok. And there could be different contributions to the total you know interaction potential between the atoms and molecules ok.

You would have heard of what are the things that contribute to this intermolecular forces that you heard of atomic bonding there is something called as ionic bonding, there is something called as a covalent bonds and metallic bonds Van-der-Waals forces right. So, all of these things contribute to the inter atomic or inter molecular forces.

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And one of the very famous examples of inter molecular forces is something called as a L J potential, I do not know how many of you have heard of L J have you heard of this Lennard-Jones potential right. So, typically if you were to have you know total potential which is powered as a function of distance r it turns out that there is going to be a repulsive force and this repulsive force goes as 1 over r power 12 and there is an attractive force that goes as 1 over r power 6 ok.

So, in this case your total potential U is 1 over you know r power 12 minus 1 over r power 6 and negative because it is a attractive force ok; negative is attractive force right. So,

maybe just let me just do that right here. So, that is your, you know attractive force and 1 over 12 it is a repulsive force right.

$$U_{total} = \frac{a}{r^{12}} - \frac{b}{r^6}$$

And if I were to sum of these two typically your plot would look like this something like this it will be something like that. And that is a total potential that is U in this case you if you want to write as U total or just U and what can you say from this plot I can say something about what is the depth of the minima of the energy minima.

The depth of the energy minima will tell you something about how strongly are the atoms or the molecules are bound together in a given material right that is what the depth of the minima represents, and that minima position this is what is called as it tells us something about the equilibrium position right. On an average what is the typical distance between the atoms of the molecules that constitute a given particle that is your equilibrium separation.

Only thing is that the functional form of this interaction potential would definitely depend on the type of particles that you are dealing with right. Typically people think about Lennard Jones potential when you are thinking about like say molecules like methane you know argon and things like that, but if you move to solid material then you would have to worry about you know other kinds of functional form of the interaction potential.

So, in the case of the Lennard Jones is 1 over you know r power 12 there is typically a constant a and constant b, and that is constant a and b depends on the type of material that you are dealing with. And so the attractive contribution to Lennard Jones potential comes from the Van-der-Waals forces right and the repulsive contribution basically comes from the fact that you know every atom or a molecule has a electron cloud right. And you know when two particles come close enough the repulsion between the electron cloud is what gives rise to the repulsive forces yeah, any questions?

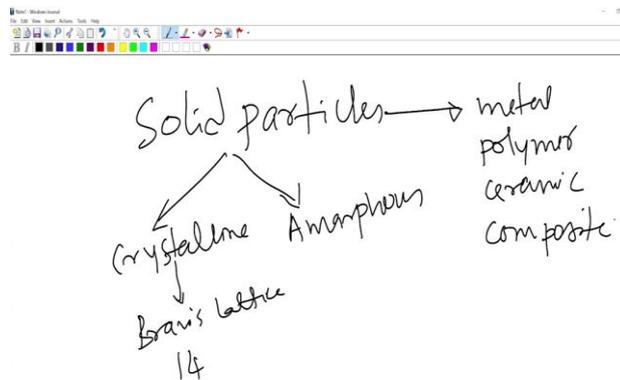
Now, if I look at this plot, can I say something about the range of interactions, can I say something about long range interactions and short range interactions, which one of them is long range and which is short range in this case, what I mean by that is, what I mean by long range and short range is say there now I have two atoms or molecules you know I

have held them together say they are you know I start bringing them close enough. Now, if the distance between the atoms or molecules is very large in this case if I look at this particular plot I can clearly say that you know any distance above you know what I have marked there; there is no interaction right, because the interaction is essentially 0 right.

The moment I start bringing them together, you can see that the particles start feeling the attraction first right you know this the curve that is below this horizontal it starts becoming nonzero of first right ok. Therefore, I can say that in this case attractive forces are long range compared to the repulsive forces right. So, whenever people talk about you know long range or short range forces everything depends on what kind of interaction forces you are working with ok.

In this case Van-der-Waals are long range compared to you know the electrostatic interactions, but it would depend on you know the kind of particle system that you want to work with.

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Now, specifically if you come to solid particles right; solid particles people may be working with particulate solids which be crystalline or amorphous right ok. And again crystalline you know all of you have done I am sure some basic material science where you will know that you know crystalline material can exist in something called as Bravais lattice right have you heard of this ok; Bravais lattice ok.

So, there are 14 Bravais lattices right you know cubic, hexagonal, trigonal, monoclinic and things like that. And of course, in the crystalline material there is an ordered arrangement of you know atoms or molecules that make up the solid, but of course, in the case of amorphous materials you know there is no long range order right. So, yeah; so and of course, you know you can think about when you think of a solid particles your solid particles could be made up of a metal or a polymer or a ceramic particle or it could be some kind of a composite as well right.

Now, so this is a little bit of basics about you know what the particles are made up of right. Now, is this really important when you are trying to think about fluid particle mechanics? It may or may not, it depends on you know what kind of fluid particle operations you are trying to do.

For example, if I am like say in powder technology, you people worry about somebody mentioned about making particles by reducing the size right. If I have a large particle and if I am reducing its the size of the particle you know by some means maybe I am hammering it or you know some means then I would have to worry about you know what would be the modulus of the material you know whether because you know a certain type of materials are much easier to break compared to the other.

So, you know then you know all these, you know some information about the intermolecular forces inter you know inter atomic forces you know what is the toughness of the particle, you know all these information's may become important. But; however, when I am worrying about you know like say there is a solid spherical particle of radius r you know and there is a fluid going around it really does not matter whether the particular is you know crystalline or you know on amorphous right.

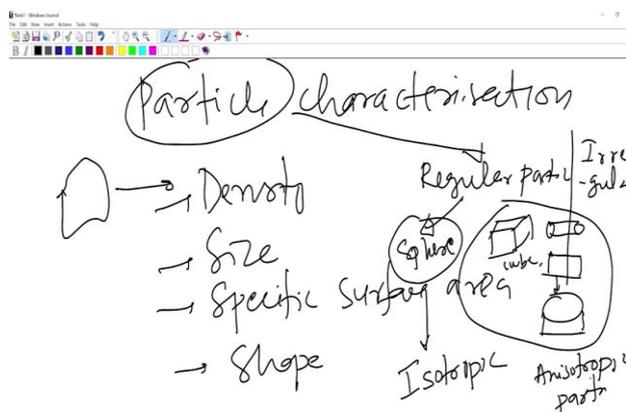
So, therefore, having a basic idea about the solid particles what they are made up of whether the bonding between the atoms that make up the particle is it ionic you know crystalline, is it ionic covalent or metallic. So, all these information could be useful in some cases and again having an idea about the Bravais lattice could be useful in some applications. Just to give you an example people make up particles by a particular say in reaction right.

So, in reactions when you want to make particle, what happen initially you have a nuclei that is formed right. Now, the nuclei grow and then it becomes a bigger particle in the end

right. Now, the way they the next atoms comes and attached to this nuclei, it would again depend on the specific sort of you know the you know the orientation of different crystalline molecules that are in the particle and stuff like that.

And you know there are a lot of people who talk about adsorption of some things onto a particle surface and there are cases where people have kind of identified that you know some molecules attached to one one one plane of the particle you know compared to the other planes and things like that. So, therefore, the having details you know having a basic idea about you know the material science of particles becomes important in some field ok, but in some other field you know you may be happy with thinking about the characterization of materials in terms of knowing their size, density, shape and things like that ok.

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So, let me just wind up at this point and then we will just go to you know some basics about particle characterization ok. So, if I have like say a particle, now I want to characterize these particles. So, what kind of properties do you think you know I would like to characterize these particles for and I have just drawn you know some object, but let it need not be this right it could be any you know any particle right I mean I have something like that. So, I would like to characterize these particles. What do you think are the properties that I should worry about, yeah what is that?

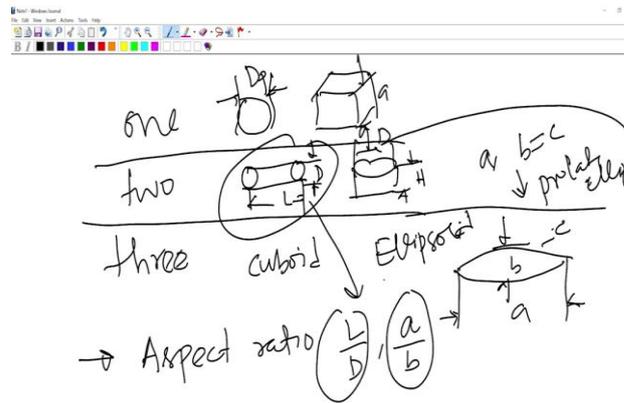
Yeah of course, I mean of course, density is one you can think of course, size of the particle itself right ok. So, basic calculation that I can think about you know I should know what is a, the size of the particle, I should know the density. You can also think about something called as a specific surface area and of course, you can think about you know characterizing the shape as well right, because you know you may be working with particles are different yeah.

So, when I say size we will come back to that ok. So, let us say that I have at the particles in general right you take any particle in general, it can be a regular particle or a irregular particle right. What do I mean by regular particle you could have like say a sphere right or you can have like say a cube like particle or like say a cylinder or like say a disc like particle right you know. So, maybe yeah that is the side view the top view is going to be something like this right that is your disc like particles right you.

So, when people work with particles, the particles can be something called as a regular particles, I mean when I mean by regular particles you know we know something about the shape of the particles right. So, again in the regular particles I can say that spherical particle is an example of something called as a isotropic particle ok. All the other things that I have done here the example of what is called as a anisotropic particles right because if you take a sphere right no matter you know from which side I look it will look exactly similar right.

Whereas if you look at other shapes like you know cylinders, discs like particles, say ellipsoids and stuff like that you know they do not look the same in from different directions. So, you can think about regular particles being a spherical and non spherical.

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And again depending upon the kind of particle that you are going to deal with you may need either one dimension or two dimension or three dimension for a characterizing the particles right. If I take a like see a spherical particle all I need to know is what is the diameter of the particle DP as long as I mentioned one dimension the size of the particle is no right.

And the same is also true for a cube right if you take a perfect cube all the size is like say if the side length is known all I need is one dimension for characterizing in the particle. But if I can you think of examples where I need two dimensions to characterize the particles.

Example would be a cylinder right. So, I should know what is the length of the particle and I should know also what is the diameter of the particle right. So, similar could be even if you take a disc right; if you take a disc again you should know what is this height and what is the diameter or the radius right any examples where you should specify all three dimensions for characterizing the particle.

Student: Cuboid.

Cuboid right, if we example could be cuboid wherein you know you have to have all the three dimensions specified. Again a general class of ellipsoid also need a three dimensions right, you could have ellipse which can have three different dimensions if you take

something that looks like that. So, that is your dimension a right, if this is b, there is also another dimension right the c, when a is; if this is a and there is another dimension c here right.

If you have a case where a and b is equal to c there is an example of something called as a prolate ellipsoid right ok. If it is a prolate ellipsoid then of course, you need a only two dimensions because the two dimensions are the same and the length is different that is an example of a prolate ellipsoid.

But; however, if you take a general case of ellipsoid you would have to identify three dimensions you have to specify it for it to be completely characterized ok. When people talk about a particles that are not spheres that is non spherical particle one of the parameter that people always talk about if you go up to literature something as an aspect ratio ok. So, in the case of like say a cylinder here your aspect ratio is your L by D right, that is the length of the rod divided by the diameter right.

And of course, if you take up a prolate ellipsoid again its going to be either a by b or you know a by c right. So, people think about you know a non dimensional length non dimensional way of expressing dimensions and in the case of elongated particles aspect ratio is something that people use ok.

$$\textit{Aspect ratio: } \frac{L}{D}, \frac{a}{b}$$

So, now, when you work with regular particles; I mean, like say if somebody comes and tells you that hey you know tell me what is the dimension of this particle right. I mean, for me I can quickly take up a scale and I can measure it you know I can specifically say, what is the size of the particle right.

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Now; however, if you are working with like say some arbitrary shape particle say that you know you have you know some shape like that, and if I want to calculate what is the dimension of the particle. What do you think one could do? I have an arbitrary shape particle, I would like to characterize this particle in terms of the dimension ok, what do you think one could do, any thoughts?

So, let us say that you know this is an image of a particle that I obtained by using like say a microscope all of you know what is a microscope right. So, what is typically done is you take some say solid base, you spread particles in the surface I take this and I put on around a microscope right. There is a IPs right there is an IP, so you basically go and focus on the particle surface.

Whenever, I capture an image what I actually get is something called as a projected area right. So, what the microscope does is, it measures the projected area of the particle right of course, you know microscope is a, you know you can only do a 2D imaging right. If I have a spherical particle of course, if I look it from the top it will appear, so a circle right.

Now, whenever you have a image that I acquired using a microscope and if I have some way of doing something called as a image analysis ok; that means, I have some way of you know characterizing you know my the particle in the image right. So, what people do is when you have a irregular object like this, one of the dimension that people talk about is what is called as a equivalent sorry equivalent circle diameter. What you do is I have an

image of an object I have some way of measuring the area and what I do is I find out what is the area of the circle that has exactly the same area of the projected particle you know area that I have taken ok.

If I have a way of doing that from that I can basically back calculate what is the area of the circle whose area is equal to the area of the particle under consideration ok, that will give me what is called as a equivalent circle diameter. This is one way of you know characterizing the size of an irregular you know object, there are other methods. In one of the method what is basically done is if this is your area what you do is you basically draw a bisector ok. A draw bisector such that the area on one side of the bisector and the area on the other side are the same. And I say that the length of this bisector is the dimension of the object, this is in literature is called as a Martin's diameter ok.

So, what you do is you take an object you start; you basically find out a bisector and as long as the area on the two sides of the bisector of the same. You find out what is the length of that bisector which basically divides the particle into two areas and the dimension corresponding to that is the diameter of the particle.

There is also something called as a Feret's diameter. In this what is basically done is what you do is you basically construct two tangents ok, what you do is you construct two tangents ok. Now, when you construct two parallel tangents if these two parallel tangents exactly and come pass the particle that you have the distance between these two tangents is what is called as a Feret's diameter.

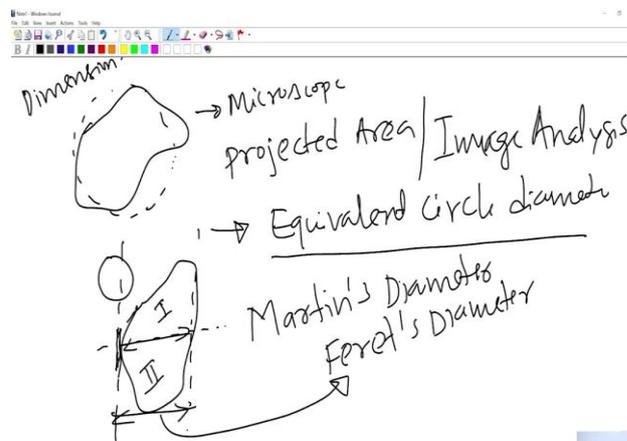
Again, these are the guidelines in the literature for thinking about you know how does one go about finding the dimensions, but what is the dimension that I have to use for a particular application, it depends on several factors. It would depend on you know what kind of access do I have for a particular technique. For example, I am able to discuss about the equivalent circle diameter, Martin's diameter, Feret diameter because there is a microscope. I am able to capture the image, this is a projected area projected image of the particle and there is a access to an image analysis and I can easily calculate the area of the you know the projected area and I can equate that to πr^2 right and I have a way of back calculating what is the radius right.

So, because I had access to microscopy and image analysis I could think about equivalent diameter, Martin diameter and Feret's diameter ok, if you want to do it you know with a

scale and a pencil is very hard right you cannot do it ok. So, therefore, the kind of dimension that I am going to associated with a non spherical or a sorry irregular particle depends on what is the access that I have for a particular technique for measuring the dimension plus it will also depend on what am I going to do with this dimension ok, what is it; what is the use of this and what to what use I am going to put it depends on that.

But these are some guidelines that people use in the literature and you should be you know aware of this. Now, there is yeah somebody has a question. Ok. Yeah. Yeah.

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Yeah, there will be a different line yes I completely agree with you. So, what are you saying is. So, in this case I have drawn a horizontal line right, in this case there is a horizontal line that basically bisects right. I could have had a vertical line, I could have it in any angle as I said it depends on you know this is one. So, if I were to do it what I would do it that maybe the best description would be you draw tangents at different orientations, you get a dimension for every orientation, you get that you know that diameter that bisects, the area into two equal you know area halves and take an average of that that I think that would be the best representation of a Martin diameter ok. You understand what I am trying to say.

So, instead of drawing one line you could think about doing multiple lines right as long as you know the area on both the sides are equal I am going to get a dimension. So, you could think about it enough the average martin diameter could be more in diameter 1 plus 2 plus

3 divided by 4. So, you have to figure out you know what is the best dimension that you would like to extract out of such analysis and then you know and it as I said, it depends on you know what application you are going to put this dimension into in your calculations yeah ok.

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Sphericity

$\phi_s = \frac{(S_p/V_p)_{\text{spherical particles}}}{(S_p/V_p)_{\text{particle under consideration whose nominal size is } D_p}}$

$= \frac{(S_p)_{\text{sph}}}{(S_p)_{\text{particle, } D_p}}$



Now, so the other way of kind of working around you know irregular shaped particles are part of they are not spherical is by exploiting a quantity called sphericity. This is typically defined by

$$\phi_s = \frac{\left(\frac{S_p}{V_p}\right)_{\text{spherical particle, } D_p}}{\left(\frac{S_p}{V_p}\right)_{\text{particle under consideration whose nominal size is } D_p}}$$

That is your I will explain a little bit I see some strange faces, but you know yeah I am going explain this a little bit ok. So, now, all you have to do is say that I have an arbitrary object like this ok, what you do is you say that the volume of the particles are under consideration right. So, let me put it this way ok.

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Handwritten notes on a slide showing a cube with side length a . The derivation for the surface area to volume ratio ϕ_s is as follows:

$$\phi_s = \frac{(S_p/V_p)_{\text{SPHERE}}}{(S_p/V_p)_{\text{CUBE}}} = \frac{(6/D_p)}{(6a^2/a^3)}$$

Using the volume of a sphere $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$, the ratio for a sphere is:

$$\phi_s = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$$

For a cube, the ratio is:

$$\phi_s = \frac{6/a^2}{6/a^3} = \frac{6}{a^2} \cdot \frac{a^3}{6} = a$$

Equating the two ratios to find the equivalent diameter D_p of a sphere:

$$\frac{3}{r} = a \implies r = \frac{3}{a}$$

Substituting $r = \frac{3}{a}$ into the sphere volume formula:

$$V = \frac{4}{3}\pi \left(\frac{3}{a}\right)^3 = \frac{4}{3}\pi \frac{27}{a^3} = 4\pi \frac{9}{a^3}$$

The final result for the cube's ratio is:

$$\phi_s = 0.806$$

Let us do an example and I will come back, let us say that I have a cube say that the dimensions of the cube is a , I would like to find the what is the a ϕ S for this cube ok. So,

$$\phi_s = \frac{\left(\frac{S_p}{V_p}\right)_{\text{SPHERE}}}{\left(\frac{S_p}{V_p}\right)_{\text{CUBE}}}$$

$$a^3 = \frac{4}{3}\pi r^3$$

$$\phi_s = \frac{\left(\frac{S_p}{V_p}\right)_{\text{SPHERE}}}{\left(\frac{S_p}{V_p}\right)_{\text{CUBE}}} = \frac{6/D_p}{(6a^2/a^3)} = 0.806$$

Now, let us go back to the definition ok. So, when you are extracting the size by equating the volume right when you are extracting the size by equating the volume your V_p here and V_p here is exactly same right, what are you doing is you are taking up a sphere and you are saying volume of the sphere is equal to the volume of the particle in the concentrations there.

There therefore, your ϕ_s definition can also be the surface area of a sphere divided by surface area of particle under consideration of course, whose normal size is D_p right. So, you can either express it as S_p by V_p of a spherical particle divided by S_p by V_p of a

particle of whatever arbitrary shape that you are considering as long as your diameter nominal diameter is the same as a sphere or you can also express it in terms of the surface areas itself ok. What you do, but no, but of course, when you want to get the equivalent diameter you would have to equate the volumes, volume with the sphere is equal to the volume of the particle under consideration you get the nominal size of the sphere and of course, you have to use that for your calculation ok.

Now, so this number that we got right 0.806 does it make sense for you? Because we know that what do you think will be the value of ϕS it cannot, it will be always less than 1 right because we know that you know for a given material sphere occupies the least surface area right ok. Therefore, this is always going to be less than one of course, if you take a spherical particle your ϕS is going to be one because the surface area exactly the same, but any deviation from spherical shape is going to lead you to a value which is less than 1, the more closer to 0 it is; that means, it is highly non spherical ok.

That means, if I take a very very long needle like particle for example, maybe this pen. If I take this and if I calculate a sphericity of this and if I compare the sphericity to or something like this, but a shorter length of course, the sphericity works the shorter one is going to be larger compared to the longer needle right.

So, therefore, this sphericity kind of tells you something about how do I think about kind of considering the shape effect just by you know. So, you will look at lot of expressions which are kind of developed for spherical particles, but the deviation from the spherical shape is kind of accounted for by incorporating is sphericity into the calculations ok. So, in that context this is an important parameter that you will come across you know maybe in some calculations ok.

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Size
shape
Specific Surface Area

$$A_{SP} = \frac{S_A}{m_p} \left(\frac{m^2}{kg} \right)$$

$$= \frac{4\pi R^2}{\rho \cdot \frac{4}{3}\pi R^3} = \frac{3}{\rho R}$$

Now, we also said; so, therefore, so the size is important and we also said shape is also important we have discuss some aspects of shape, what about specific surface area? It is typically you know defined by ASP; specific surface area, it is basically defined as the surface area of a particle divided by the mass of the particle ok. It typically has units of you know sorry it has units of meter square per kg right.

If you take a particle a spherical particle for example, your $4\pi R^2$ is your surface area right if you take mass of the particle I can express because density is equal to mass by volume ok, I can express mass in terms of density the particle into volume of the particle right, yeah. So, therefore, I can write this as $4\pi R^2$ divided by ρ of the particle into $4/3\pi R^3$ right therefore, this becomes 3 divided by ρ times R right.

$$A_{SP} = \frac{S_A}{m_p} \left(\frac{m^2}{kg} \right)$$

$$A_{SP} = \frac{4\pi R^2}{\rho V_p} = \frac{4\pi R^2}{\rho \frac{4}{3}\pi R^3} = \frac{3}{\rho R}$$

So, this specific surface area is an important quantity because when people are working with like say any surface reactions or anything that involves surfaces, what would be your freely interest is what is the total surface here that is available for any process ok. For example, if you look at surface reactions or adsorption and you know things like that what

is really important is given a certain mass of material, what is the total surface area available ok.

So, if you especially if you look at particulate samples which are smaller in dimensions like nano particles and you know and colloidal particles and things like that one of the parameter for which all the particles would be characterized is something called as specific surface area, which is the surface area of the particle divided by the mass of the particle ok. And of course, as you would see it would depend on the size of the particle, it would depend on you know the shape and you know parameters like that.

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The slide is titled "Density of the particle densitometry". It contains the following elements:

- A diagram showing a beaker labeled "Dispersion" and a container labeled "powder". An arrow points from the powder to the dispersion, with the text "Solution densitometry" and "Stable Dispersion" written above it. Another arrow points from the dispersion to the text "Ideal Mixing Rule".
- Equations for mass fractions:

$$\frac{M_p}{M_D} = X$$

$$\frac{M_S}{M_D} + \frac{M_P}{M_D} = 1$$

$$\frac{M_S}{M_D} \cdot \frac{M_S}{\rho_D} = 1 - X$$
- The Ideal Mixing Rule equation:

$$\frac{D}{\rho_D} = \frac{M_S}{\rho_S} + \frac{M_P}{\rho_P}$$



The last thing that I wanted to quickly brush up on is talking about density of the particle. So, how does one measure density of the particle? So, I will give, I have a like say either a liquid that contains some particles or say I have some powder sample ok, it could be a dispersion or it could be a powder sample and I would like to measure what is the density of the particles you know that I have in the system.

So, what do you think is the best way of doing this, some thoughts? So, this there is a and there is a technique called; there is a technique called solution densitometry ok, what is done is this ok. Say that I have prepared dispersion say that I could have a powder sample or I could have a dispersion given to me if I have a way of making up something called as a stable dispersion. What I mean by stable dispersion is that you know I have a dispersion, if I take samples from different you know locations you know within the container they

should have same number of particles and they should exist as individual particles and things like that ok.

So, if I have something like that, if I am able to prepare a stable dispersion what I can do is I can use something called as an ideal mixing rule. And I can say that volume of the dispersion that I have prepared is equal to volume of the particles plus volume of the solvent or your dispersing medium if you want to call it as. Of course, this is if your particles are not reacting and things like that right as long as you have inert particles in the fluid this is going to be true ok.

Now, so the; so what I can do is I can manipulate this equation ok; I can manipulate this one of the things that I can do is I am going to divide this by the mass of the dispersion everywhere right. Now, what I am going to do is I am going to multiply this V_S by M_D term right sorry this is M_S right sorry M_D . I can the second term I can multiply it by mass of the solvent divided by mass of the solvent I can do that right and this term I can multiply by mass of the particle divided by mass of the particle yeah.

$$\frac{V_D}{M_D} = \frac{V_P}{M_D} \frac{M_P}{M_P} + \frac{V_S}{M_D} \frac{M_S}{M_S}$$

Now, if I look at volume of the particle divided by the mass of the particle, this is $1/\rho$ of particle right multiplied by, so you have mass of the particle divided by mass of the dispersion. Similarly, this term is going to be $1/\rho$ of solvent multiplied by mass of the solvent divided by mass of the dispersion. Similarly, this is going to be $1/\rho$ of dispersion right I can do that.

$$\frac{1}{\rho_D} = \frac{1}{\rho_P} \frac{M_P}{M_D} + \frac{1}{\rho_S} \frac{M_S}{M_D}$$

$$\frac{M_P}{M_D} = X; \frac{M_S}{M_D} = 1 - X$$

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$$\frac{1}{\rho_D} = \frac{1}{\rho_P} X + \frac{1}{\rho_S} (1-X)$$

$$\frac{1}{\rho_D} = \frac{1}{\rho_S} + \left(\frac{1}{\rho_P} - \frac{1}{\rho_S} \right) X$$



Therefore,

$$\frac{1}{\rho_D} = \frac{1}{\rho_P} X + \frac{1}{\rho_S} (1 - X)$$

$$\frac{1}{\rho_D} = \frac{1}{\rho_S} + \left(\frac{1}{\rho_P} - \frac{1}{\rho_S} \right) X$$

What this tells you is that if I have a way of measuring; if I have a way of measuring $1/\rho_D$ that is a dispersion density as a function of mass fraction, if I have a way of doing that you would get a straight line where your intercept is going to be $1/\rho$ of solvent and your slope is going to be $1/\rho_P$ minus $1/\rho$ of S. Therefore, just by measuring the intercept and the slope I am I can basically calculate what is the density of the particle ok.

This is a simple solution densitometry root what is being done is there you basically preparer or series of dispersions 5 or 6 samples, I know what is the mass fraction of particles in the dispersion I go and measure the density of each of these stable dispersions and I plot $1/\rho$ of dispersion as a function of mass fraction just by measuring the slope on the intercept I can actually calculate what is the density ok.

So, basically in today's class what we have done is we kind of talked about basics of particulate matter we said that; you know the all the particles whether you are going to

deal with solid particles, liquid droplets or gas bubbles they all are made up of atoms and molecules there is something called as a intermolecular forces which holds these particles together in these particles ok.

We talked about one functional form of interaction potential, L and J potential. So, you should know a little bit about what is the strength of attraction or the you know the depth of the energy well and what is the equilibrium separation distance between the particles, then we talked about regular particles, irregular particles some way of characterizing them. And we also talked about sphericity how do we think about you know thinking in terms of characterizing irregular shaped particles for sizes and things like that ok. So, with that maybe I will stop today tomorrow we will continue with some other aspects of fluid particle mechanics, so.