Process Control - Design, Analysis and Assessment
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Process Modeling

Welcome to the lecture on process modelling. This is the fifth lecture in this course.

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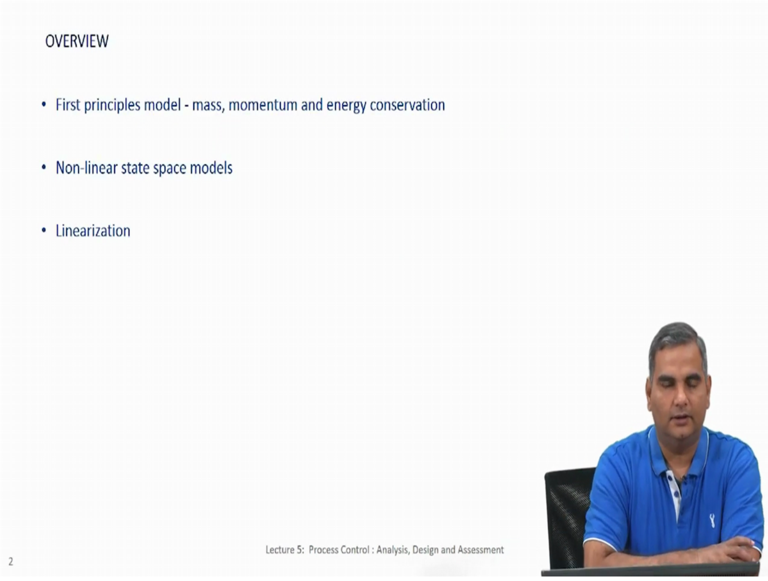

In this lecture I will talk about how you build these first principle models and to be used with control. Then introduce the notion of state space models which we will discuss in more detail in the classes after this. And as I have been talking before we typically assume in the undergrad process control that we teach, the underlying processes are linear.

However most real engineering processes are nonlinear, particularly chemical engineering processes. So to be able to apply many of the techniques that we describe in this course you need to linearize the model so that we get it into the standard linear forms. So I will teach how we do this also.

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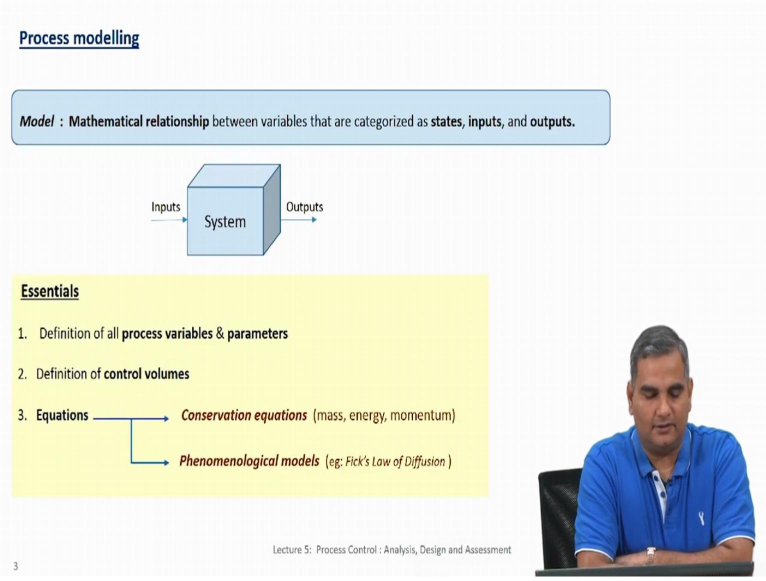

And as we have discussed before, the aim of a modelling exercise is to model the effect of inputs on the outputs in the system. The reason why we want to do this is only if we understand the effect of inputs on the outputs we will be able to figure out how to manipulate the input so that we control the outputs to the values that we want them to take. So this model is critical for control.

So when we talk about a model, basically a model is a set of mathematical equations and we think about how we are going to write these equations. So to do that and to model a system there are certain steps that you take and we will describe this in detail now. The very first thing that you do is you define all the variables of interest that you think should be there in the model or participate in the mathematical equations. These are variables that may change over time.

These are variables that reflect what is happening in the process and so on. And if you take any process there are also what are called these design parameters or parameters of the system which do not change with time. However, they also participate in the model equation. So it is critical that we define these parameters that are relevant for the system also, okay. Let us say we have done all of this and then we have a set of process variable definitions and parameter definitions.

Then what does really modelling mean? How does one start writing these equations that relate these variables and parameters? So model essentially means that you are going to write certain conservation equations and the typical conservation equations that one writes are mass conservation which is very important in chemical engineering and energy conservation and we also write momentum conservation, not as much, maybe in chemical engineering problems. But these are the three main conservation equations we write.

Now basically these say that mass cannot be created arbitrarily and energy cannot be created arbitrarily. If you want a real life example this is something like doing a bank balance, right? So you have certain amount of money in the bank account and then you put in some money, take out some money. So you can reconcile what is happening with the money in your bank account.

Similarly, you can reconcile what is happening to the mass in the process and also you can reconcile what is happening to the energy in the process and you can also reconcile what is happening to momentum in the process. Now this momentum balance equation is very critical and important for mechanical and aeronautical systems and so on.

And it is also used in chemical engineering but if you see most of the models that people talk about, very simple assumptions are made to kind of get rid of momentum balance and then people look at largely mass and energy balances. And we will see in the one example that we are going to look at in this lecture. We are going to talk about only mass balance and we are going to make some assumptions which will make us not write the other two balances. Now are these the only equations that we write?

It turns out that these equations might not be enough to get us enough equations to describe how the process variables and parameters interact with each other. So we also write what are called phenomenological models.

These are models which have been validated and used in process modeling and these models are based on observations and scientists coming up with phenomenological explanation for this observation. One such model that many of us will be very familiar with is the Fick's Law of diffusion. There are similar phenomenological models or equivalent phenomenological models in heat transfer, momentum transfer and so on.

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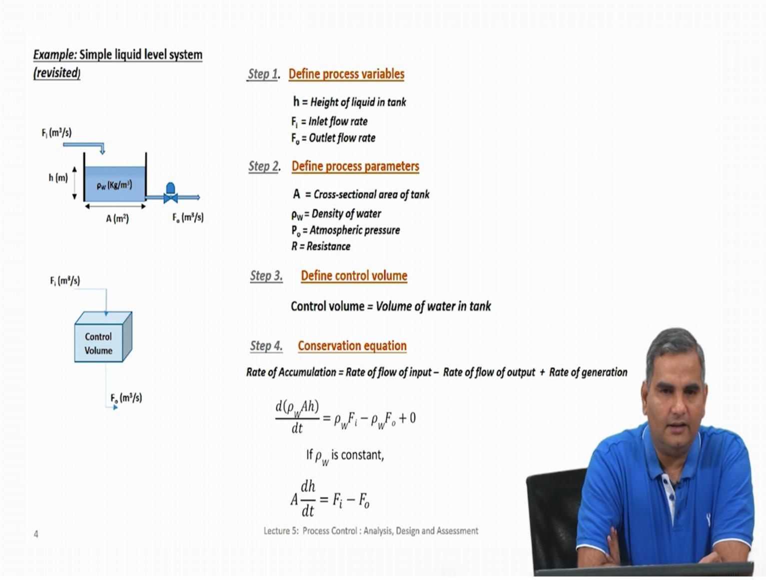

Now when we write these model equations we also have another notion of a control volume which was there in the previous slide and I will explain the control volume with this example here. So you have to write these model equations for control volume and typically when you have a complicated system you can come up with different control volumes for which you write these balance equations.

And it is a bit of a skill to identify appropriate control volumes for which you are going to write the balance equations or the conservation equations and so on. And depending on the kind of control volumes you have chosen the model will look slightly different in its form and structure and all that. So control volume really is closed volume something like this.

So the key thing is you have to come back to the starting place and then you kind of make sure that everything that comes and goes out of the control volume is all accounted for. So you do not make any mistakes in terms of leaving out things that come into the control volume and things going out of the control volume. Similar to what I talked about in terms of the bank balance you just want to make sure that whenever there is a credit, you account for all the credits and whenever there are debits, you account for all the debits.

So if you leave out something then the balance will not be alright. So a similar principal works here. So you have to make sure that everything comes in is accounted for and everything that goes out is also accounted for. And as I said before, it is important to identify the correct control volume. It is important to basically not leave out anything that comes in or goes out of the control volume. So let us take this example that we have been talking about for a while now which is basically this tank of water.

And let us assume that our objective is to be able to control the height of the liquid in the tank. And in this particular case I have a figure which shows that we are going to manipulate the flow out of the tank F not to control this. So just to give you an idea of what you might be varying and this picture also shows that the flow into the control tank F i is something that can change over time.

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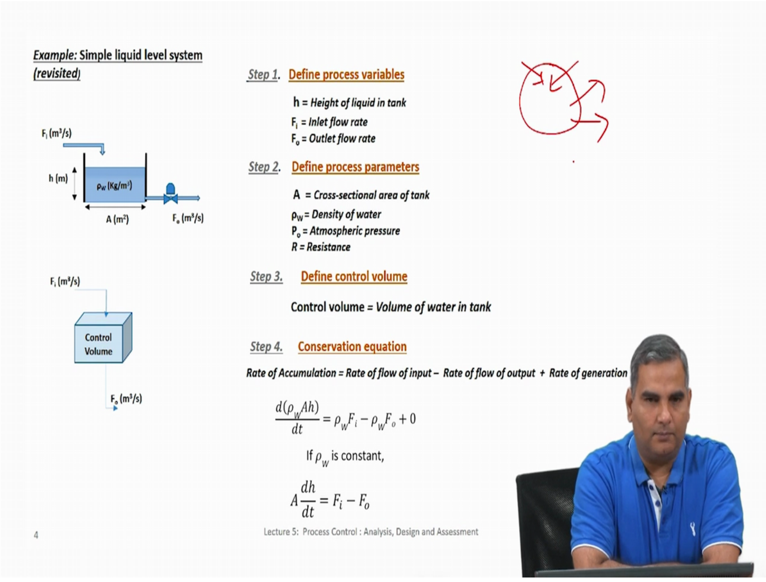

So when we look at process like this the very first step as I said before is to define the process variable. So typically if you have done a lot of modeling, you will quickly understand the variables that you need to define and you will be able to progress very smoothly. If you are doing the modeling exercise for the first few times then you might not be able to write down all the process variables of interest right away.

However, this is something that you should not worry about because you start enumerating the variables that you think are important. And as you go to writing these equations you will see that if you have left out some variables you can add them at that time. So in this case let us do a first cut on the process variables. I might say the height of the liquid in the tank h is an important process variable. So I will give h as a variable of interest.

Similarly, I know what comes in is important so I have F i which is inlet flow rate and F not is the outlet flow rate. So these are the process variables which is where I have to write these model equations. For now, we see that there are three variables that are there. So I need to write enough equations to be able to do computations of all of these three. It does not mean that I need three equations because in this case I might say F i is a variable that I have no control over.

It is something that comes from processes before the tank. So since I do not have any control valve I cannot control what F i is. So I might not be able to come up with an equation to describe F i because F i has to be given as a value to my model because it is exogenous to the model. It is outside the battery limits of the model. So if that is the case then you would see that you will be looking for two equations h and F not and we will hope that F i comes from some other place to be able to solve this model.

Now I also mentioned that there might be other parameters which should not change. However, they will become a part of the equations. So let us look at what are the parameters that might be of interest. So in this case we might have the cross sectional area of tank A as a parameter of interest. Density of water if we assume it is not varying, we can have a parameter definition rho w.

Let us just add p not to see if that is going to come into picture or not. And then I am also going to define another variable if this is not obvious here. The area of the tank is obvious, density of water is obvious, atmospheric pressure is obvious and so on. So I am going to define something called R which is the resistance and you will see how we will use this as we go along in the modeling approach.

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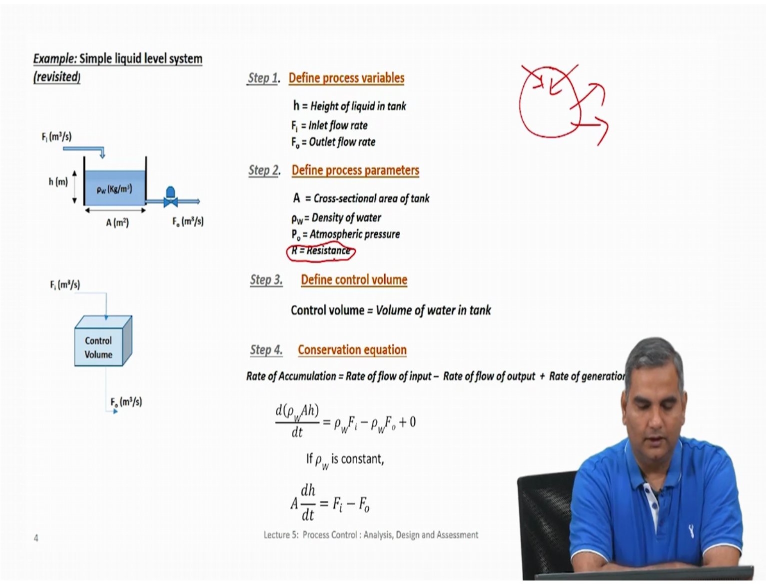

So at this point we have defined variables and parameters that we think are important and that we think are really complete for us to finish this modeling exercise. And as I described at the beginning of this slide, we need to look at defining a control volume. So in this case let us define a control volume which is the water in the tank itself. And let us write a balance equation for this.

So since we did not define any temperature and since we did not define velocities and so on basically we are making very simple assumptions about energy and momentum and hence we are actually not going to write those conservation equations. We are simply going to write mass conservation equation which is all that we need as far as this control problem is concerned with the current set of assumptions that we are making.

Now conservation equation will be of the form, rate of accumulation equal to rate of flow of input minus rate of flow of output plus rate of generation. So this is irrespective of what conservation equation you are writing for or what phenomena for which you are writing the conservation equation.

So if it is a mass conservation you will say rate of mass accumulation equal to rate of mass flow of input minus rate of mass flow of output plus rate of mass generation and so on. So similarly if you are going to do for energy you will write this for energy, you will write for momentum and so on.

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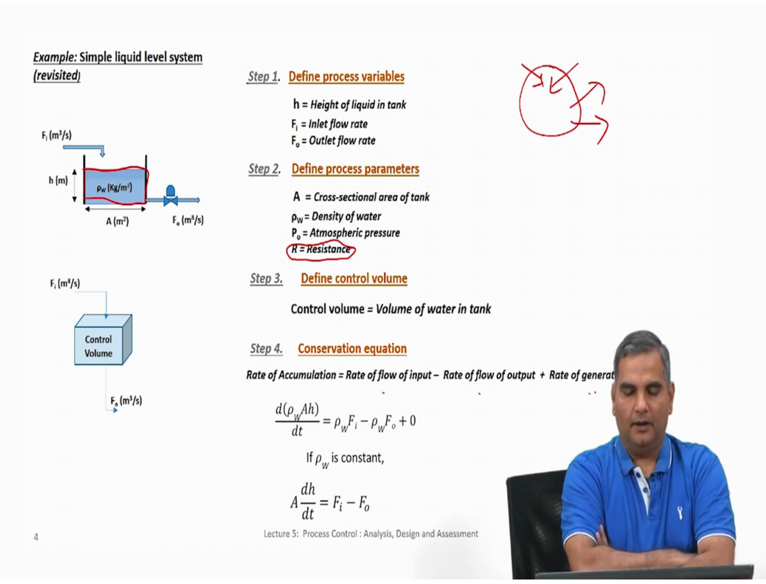

Now it is important to understand that at this point you also have to really worry about the units that you are going to use for each of these variables. And in this example we have height in metres and F i and F not are flow rates so I have those as metre cube per second. And density is in kilogram per metre cube, area is in metre square and so on. Now when we look at this equation since we talked about rate we will have units of time in the denominator. And let us look at each of these terms here.

If I say rate of flow of mass then I know the flow rate is F i and if I multiply that by the density of water I will get a mass flow rate in. So let us check if that is true. F i is metre cube per second and rho is kilograms per metre cube and this cancels, I get kilograms per second.

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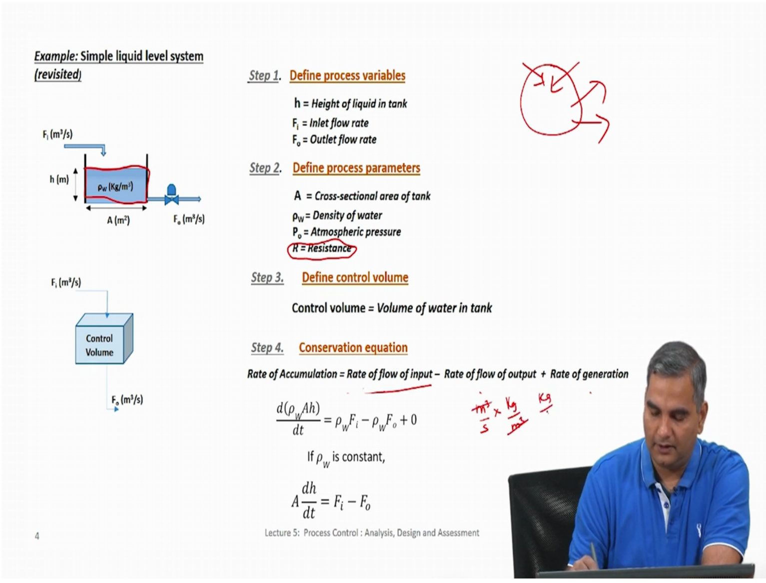

Similarly, the rate of flow out I will get as rho w F not and since there is no mass generation here we are going to have 0 for the rate of generation. So that is the right answer. On the left hand side what we have is rate of accumulation which basically means that I have to come up with some term which is kilogram per second and if you notice the control volume we are basically saying whatever is the liquid in the tank, how does it change with time? So the differential of the total amount divided by time.

So if you look at this the total amount is rho w A h. A is metre square, h is metre, so you will get a metre cube and rho w is kilogram per metre cube so the numerator will be simply kilogram. And since we have d t in the bottom we have kilograms per second. So this is the rate of accumulation. Now if you assume this rho w is a constant you can pull that out and then cancel rho w on both sides and then you have an equation of the form A d h by d t is F i minus F not.

Notice that while this looks like a volume balance equation it is really not a volume balance. We started with a mass balance and then because of the assumption that rho w is a constant and we can take it out and cancel it, we have this equation right here.

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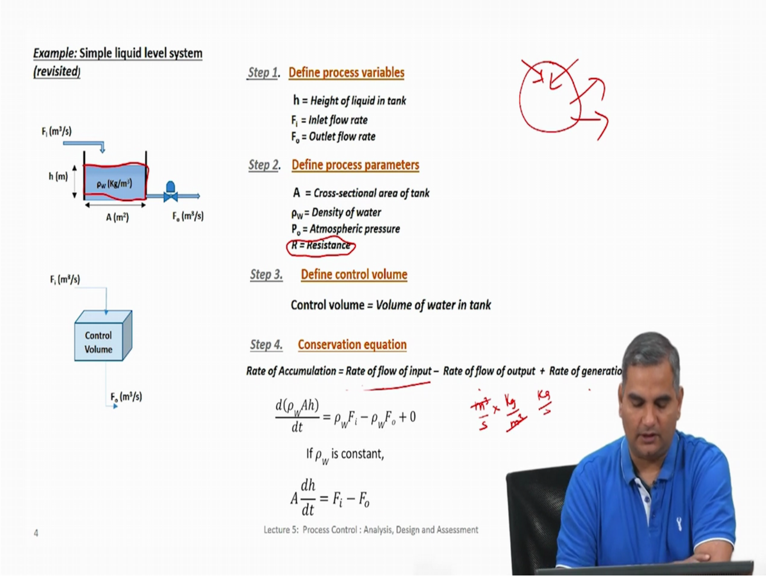

Now if you look at the previous slide as I mentioned before we had three variables h, F i, F not and I said F i is outside the battery limit so the value for F i has to come out from somewhere and the model equations are not going to be able to generate a value for F i. However the model equations should generate a value for both h and F not. We have only one equation here, the mass conservation equation which is A d h by d t is F i minus F not. So we are looking for two equations for the two variables but we have now currently one equation.

Now you cannot get the other equation through energy or momentum balance because you have to define more variables and make this problem more complicated for those equations. So even if you get more equations from other conservation there will be more variables that you will have to define which will consume those equations. For example, if you wanted to write energy balance then you will have to bring in temperature and that becomes a variable and whatever energy balance equation you write will have to be used to calculate temperature.

So at this point we ask the question what do we do and how do we get the next equation? This is where the second part of modeling that we talked about which is phenomenological phenomena based equation that we write. So in this case if you look at it from experience and from physics that we know that if that is a certain height of liquid in the tank, then the pressure at the bottom is going to be different based on the height of liquid in the tank.

And since there is an atmospheric pressure in the outlet of this pipe and the pressure at the bottom is going to be a function of the height of liquid in the tank, we know that the pressure difference is also going to be a function of the height of the liquid. And we know the flow rate is going to be related to the pressure difference and pressure difference is related to height.

So phenomenologically we are going to say the outlet flow is going to be a function of height. And now it is a question of what phenomenological model we use. And in this case you could use a model of this form which is F not is R root h. This model form has its bases in physics but basically what we are going to say is this is a phenomenological model here that I am going to use.

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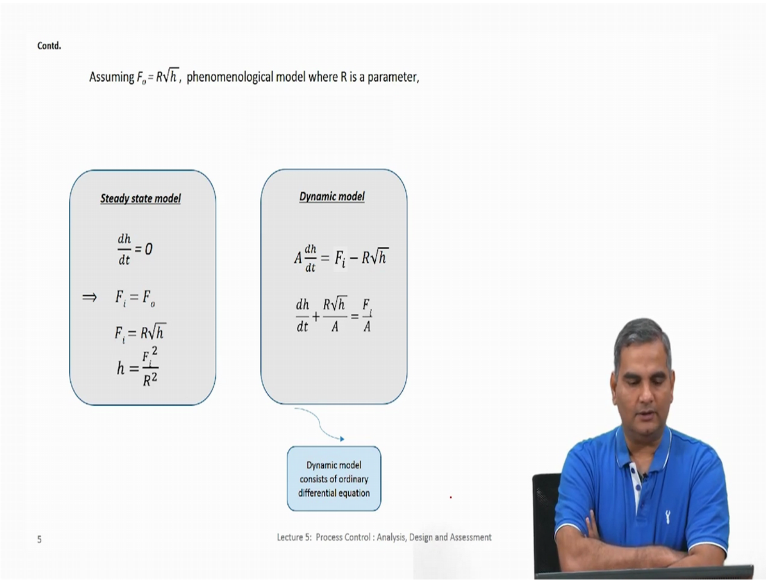

And once I have this if you notice, I had an original differential equation. Now I have the secondary equation which relates F not and h. So I have two equations and two variables. I can simplify this by substituting for F not as R root h into this equation and I get this A d h by d t is F i minus R root h. This is now what is called the dynamic model. So we started with the variables F i, F not and h. So if you give me F i value then I can compute h from this and anytime I compute h I can use this equation to compute F not.

So basically this model is consistent. This model will do whatever we are expecting the model to do. So I could write this equation as A d h by d t is F i minus R root h. Or in another standard form I can write d h by d t, get all the h terms to one side plus R root h by A is F i by A. So if you notice now you will see something interesting. So what we are saying is the output, the height of the liquid in the tank in some dynamic fashion is going to be a function of this input F i, right?

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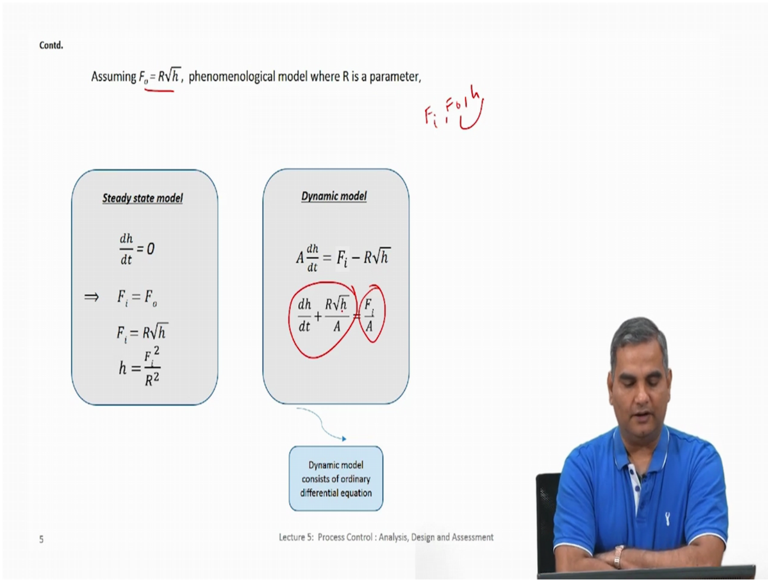

So whenever I change the inlet flow rate then I can use this equation to figure out how the height will change and once I figure out how the height will change I can also figure out how the outlet flow from the tank will change using that equation there which is F not is R root h. So this is called the dynamic model. Now if you assume that the height is at steady state, that is height is not varying with time then you can say is d h by d t is 0 here in this case.

And once you have d h by d t is 0 you will know that F i has to be equal to F not, only then d h by d t can be 0. This is from the previous equation here. If this is 0, F i is F not. And you know F not is R root h, right? So if you replace F not by F i you will have F i is R root h. So h is equal to F i squared by R squared. So basically this is a steady state model which says if you change the inlet flow rate to a new value F i after the dynamic settle down things change and d h by d t is not varying any more then the height will settle down at F i squared by R squared is what this basically says.

Now if you notice the previous equation you will notice that I have a first order differential term, however I have a nonlinear term. So if this were just h then I would call this as a linear model because it is root h, I do not have a linear model. But as we discussed before lots of this control ideas use this notion that the process is linear. So somehow we have to linearize this nonlinear model. So the idea of linearization is the following.

So what you are going to do is you are going to use some simple Taylor series approximation to do the linearization. I just want to mention here that the nonlinear equations, while they are difficult to handle, it is not as if people do not use direct non linear models in control. There is this whole field of nonlinear control where nonlinear models are used. But as far as most of the undergraduate control courses are concerned and most of the implementations in the industry are concerned, people assume the models linear.

And from a teaching view point you can see why this might be easy for us to kind of teach if it is a linear system. But from an industrial viewpoint the reason why the notion of linearity works is because whenever we run a process we expect the process to run around its steady state.

And typically if you have good control then you are controlling your process quiet close to its steady state. And hence since you are operating in a small region, linearize approximation in a small region would be a very good model for the process. That is the rational way. Though most of the industrial implementations assume linearity for nonlinear process, they still work quite nicely.

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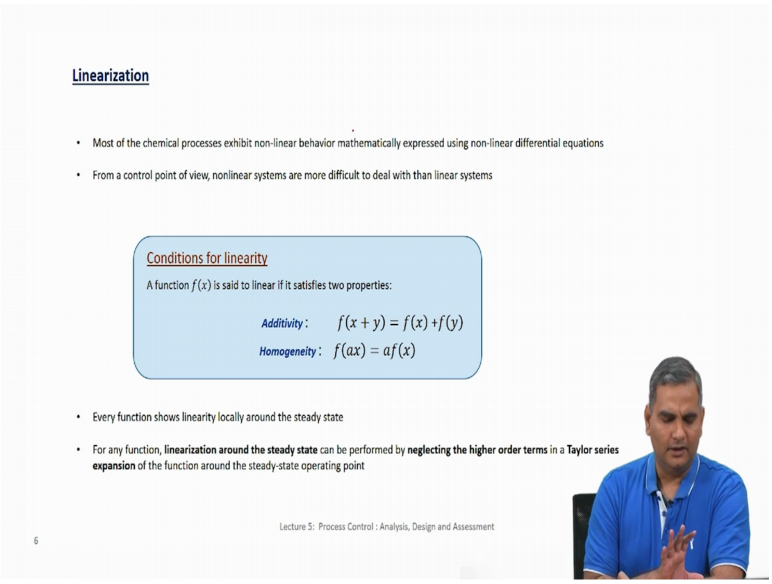

Now just a quick mathematical definition of when we call function f of x is linear when it satisfies these two properties. One is called additivity which is if I take two variables and then I calculate the function of sum of those two variables then that should be equal to the individual sum of the functions when they are evaluated at these separately. So f of x plus y is f of x plus f of y.

And the other condition is called homogeneity which is when I have a variable and I do scalar multiple of the variable and evaluate the function f of a x and that has to be equal to the scalar multiple times the function value. So if function satisfies these two, we call this function linear.

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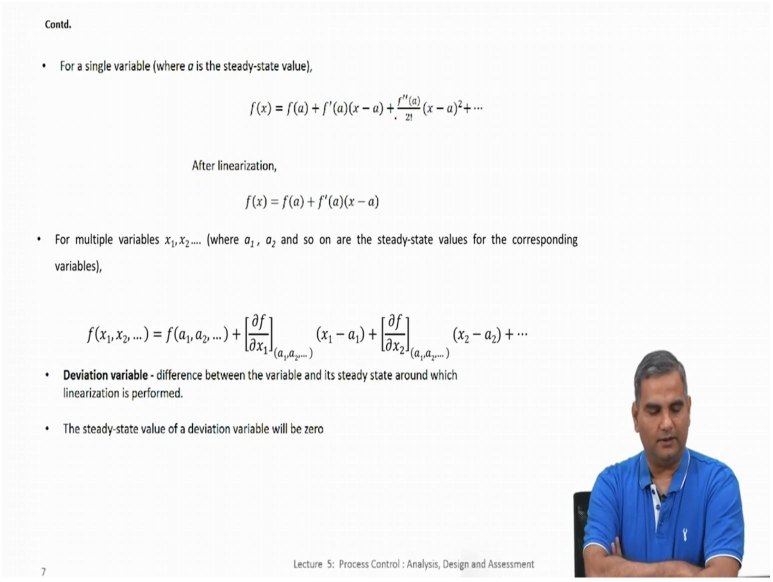

Now this is probably quite well known for most of you. When we have a function f of x and let us say we want to linearize this function so whenever we try to linearize a function we linearize that function around an operating point. So we also have to choose an operating point. So in this case let us assume if I choose a as the operating point then I can approximate this function as f of x equal to f of a plus f prime a which is the differential of f with respect to x evaluated at a times x minus a.

This is the second derivative of f evaluated at a by 2 factorial, x minus a whole squared and so on. So this is called the Taylor series approximation. And for linearization what we do this you first retain these two terms then I approximate this function as f of a plus f prime a, x minus a and then makes this function f of x a linear function because you see that you just have a linear term here.

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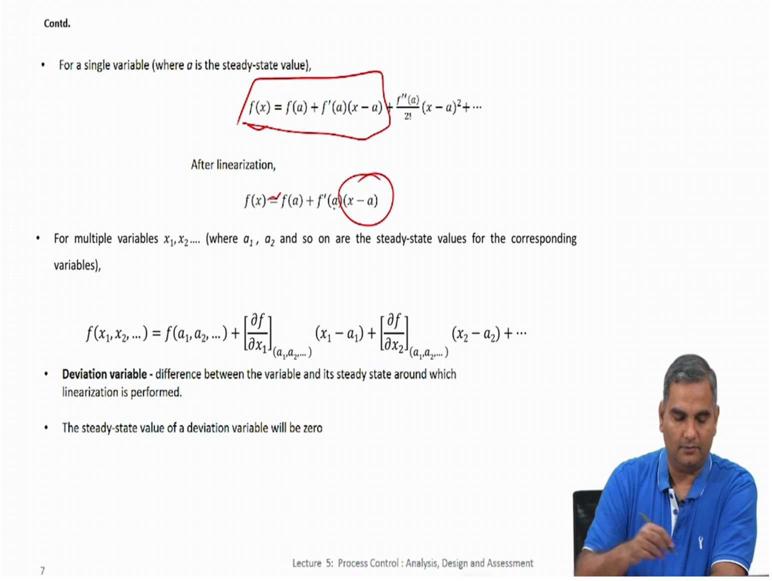

Now the same thing can be done for multivariate functions. I have the formula here but we will not describe this here. We will describe this in much more detail later when we go to the multivariate control systems. We also define from control viewpoint something called a deviation variable. What it means is suppose I have a variable x and I have a steady state for x, x steady state, okay, on which I am linearizing the function.

Then instead of describing everything in terms of x, if I describe everything in terms of x minus x steady state then this variable is called the deviation variable. That is what is the variable value which is over and above the steady state value, right? So how much has its deviated from the steady state? So that is why we call this as a deviation variable.

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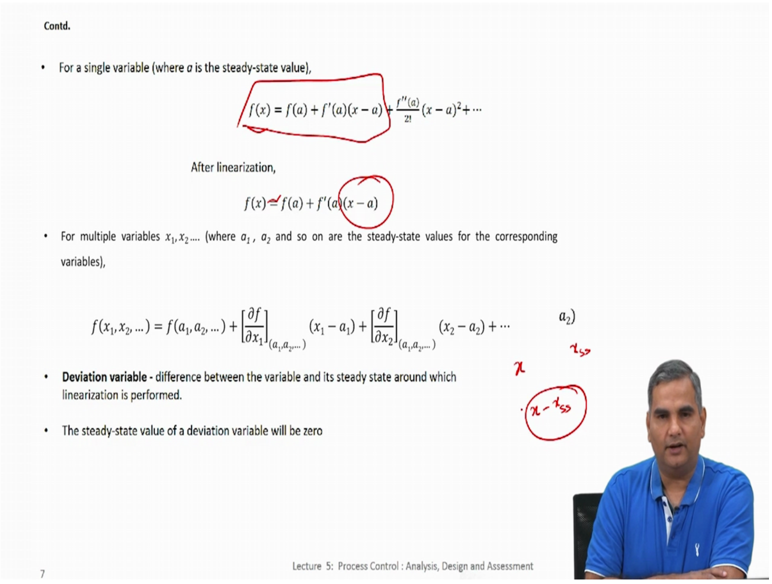

From a control viewpoint writing the equations in terms of deviation variables is very useful because it simplifies the equation tremendously. So let us go back to the liquid system example and then see how this works out. So I have these equations A d h by d t is F i minus R root h where F not is R root h and we got this nonlinear differential equation.

So let us look at what happens in steady state because we said we are always going to linearize around an operating point which is a steady state and we are already seen this at the steady state d h by d t is 0 which basically says if I say F i is at a steady state this goes to 0. So the right hand side goes to 0. We can take A to the other side which would mean F i minus R root h is 0. So at steady state let us call this F i s s, F i steady state value is R root h steady state.

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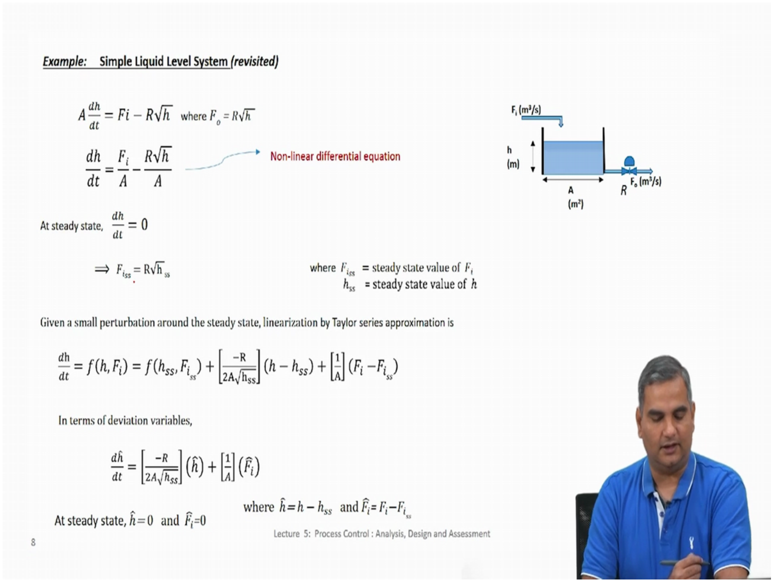

So these are the two steady state values. Now what you can do is you can basically take this function here root h and then linearize that using Taylor series approximation. So the first derivative of root h will be 1 over 2 root h and since we are going to do Taylor series approximation we need the first derivative. So if you do a very simple Taylor series approximation of this function here, the first function F I remains the same because that is already in a linear form.

Now this root h is the only nonlinear term so you notice that I get a root h in the denominator here so this comes out of the Taylor series approximation. And once I do this Taylor series approximation of this equation, I could subtract this equation from that equation and write everything in deviation variable form. And when you write these things in the deviation variable form and you evaluate the first derivative at the steady state value or the operating value you will come up with the equation of this form.

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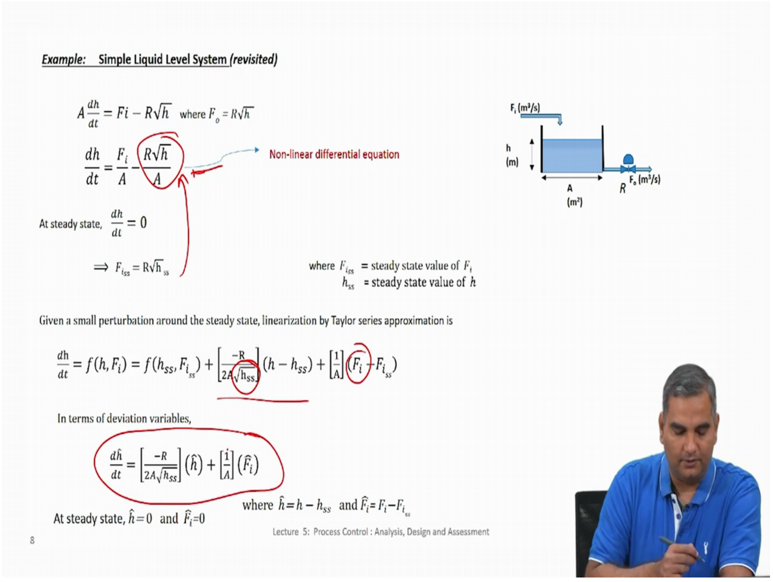

So this is a very simple algebra that you should try and do yourself. Basically the idea is simply write the Taylor series expansion of this (fo) term and evaluate the first derivative at h s s because that is the steady state operating point at which we are going to evaluate this.

And what should you then do is you have that equation which is your linearized equation. Then subtract this equation from the other one, right? Then you will get a particular form which will be something like this here and now you can notice that this is a fixed value and this is a fixed value and this is a linear term and this is a linear term.

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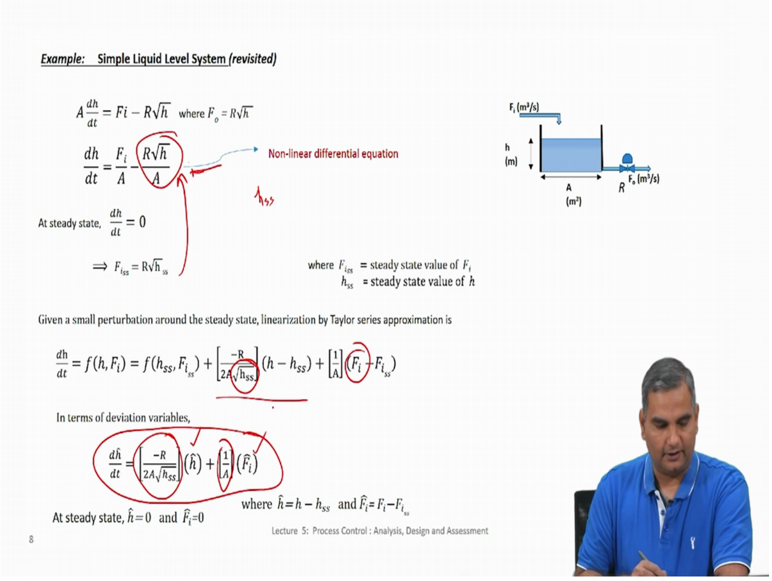

So we started with the nonlinear equation and we have come up with a linear equation at the end. So in this lecture we looked at process modelling. How it is a question of defining the correct process variables of interest, defining parameters of interest and then writing the conservation equations and the phenomenological models. Make sure that you have enough equations and variables and make sure that you write these conservation equations for control volumes.

And then once you have a nonlinear model, do Taylor series approximation to get it into linear form, write the equation in terms of deviation variables and then you will have a final form where I have a linear differential equation model that relates the inputs and outputs. So we will pick up from here in the next lecture and start using these models in control. Thank you.