Process Control: Design, Analysis and Assessment
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Stability Analysis-Various methods – Part 3

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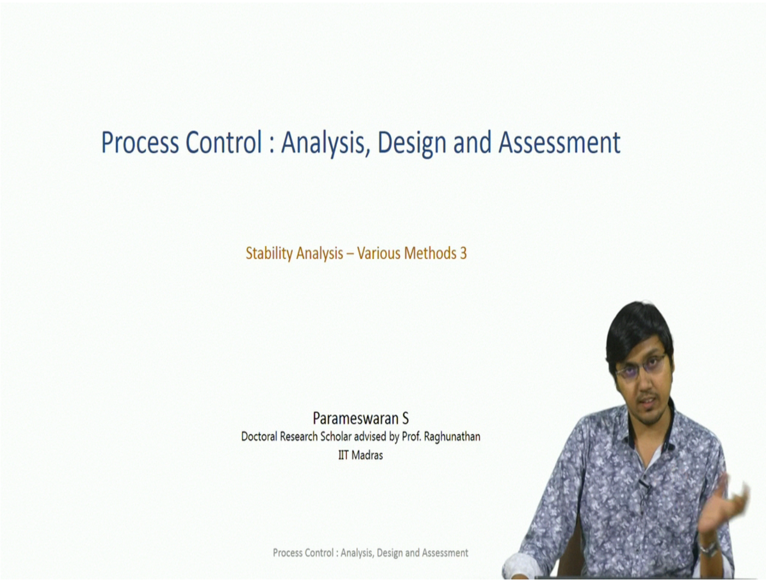

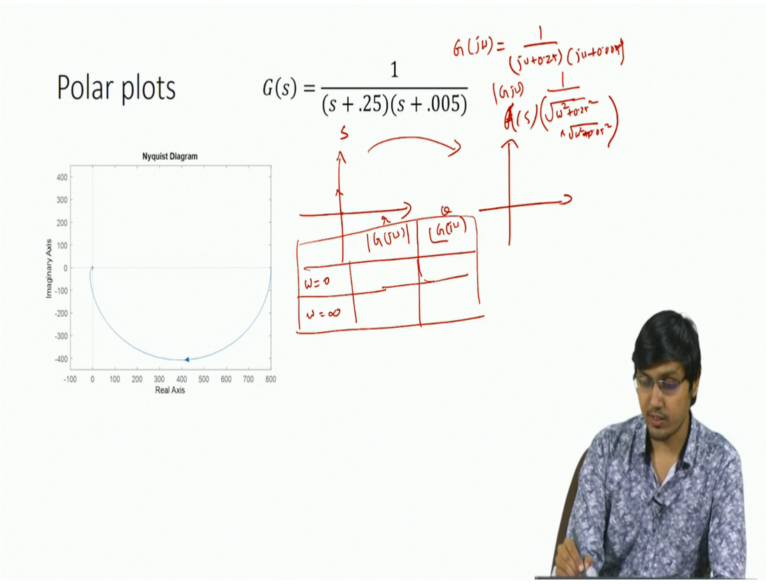

In this lecture we are going to continue that looking into the stability analysis like different methodologies to the stability analysis, so basically we are going to focus on something, like this has already been thought the lectures done by Sir, so and we are also looking out how to draw this plots etc, from how to draw polar plots? What is the polar plot and how the polar plots can lead to disability criteria plot etc, so that is what we are going to look into the structure.

Starting with what is the polar plot? Basically like if you can remember from the lecture Sir has taken, so we have S plain and we have this FS plain where we mapped every point the S plain to the FS plain, where this is like here this is a G of S plain, it is a G of S plain here, so basically we map this, so one way to understand how this mapping work is to represent things in polar coordinate, say this is basically a complex plain, a complex number can be represented both in rectangular coordinate and system as well as the polar coordinate system.

So basically, now we will start looking how to get these things, so one thing we ask is like okay, we will start at this origin point okay and then we will increase the value of omega and then we will see what happens basically, that is have we are going to, so basically we will start omega equal to 0, and then see what happens and you will see omega equal to infinity what happens?

So that is what we are going to see what happens, we will not go into the details of how these things, but this is to get intuition of how these plots come, basically we are expecting you to do Matlab get all this plots but still like we will look into like how this plot comes from a very, just forgetting some intuition what exactly this, how we can easily get this figure S is what we are trying to focus, so now when we have omega equal to 0, his angular frequency equal to 0, and angular frequency is equal to infinity.

Basically, we are again interested in two things right, one is the magnitude mod of G omega and other is the angle, angular of the G omega, so when we said this is the, in polar coordinate or this is the R and this is the theta right, so that is what we are trying to find out and then once we know this we can actually approximate this plot, see this plot can be drawn in a more accurate fashion by considering few more points, but still like for are now will stick on this idea to, this is just to get intuition of how we have to start doing all this things that is what we are trying to do.

So basically what we are going to do? We are going to have the complex number G of J omega which is nothing but 1 by J omega +0.25 and J omega +0.005 right, so 0.005 basically like if you confine the magnitude of G of J omega then what it will become, it will be nothing but 1 by root of omega square +0.25 square in denominator in the root of omega square +0.05 square right, so that is what you get in the mod of J omega.

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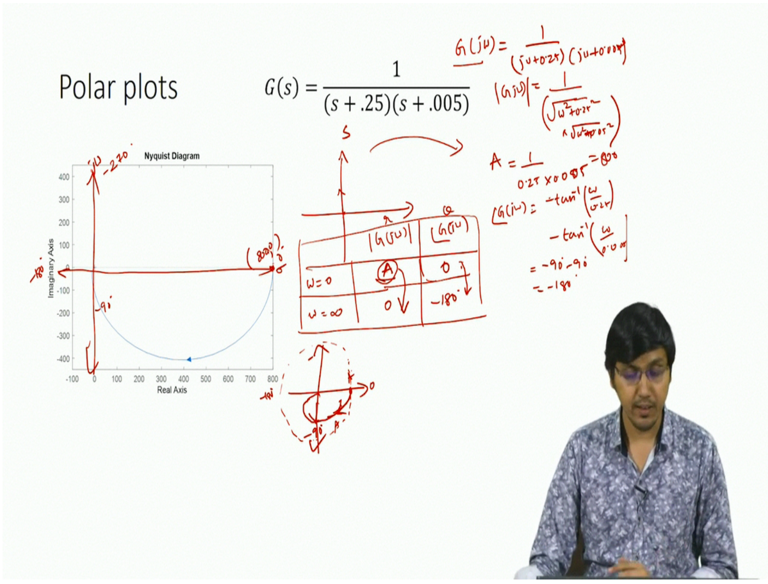

So now will substitute the values of omega equal to 0 and see what happens, when omega is equal to 0 what it becomes is it simplifies to 1 by 0.25 into 0.05 right, so that is what the 0.005, so that is what it simplifies to, so and this value will substitute here, so 1 by 0.25 by 0.005 is what the value will let say, split as call it as A for now as what you will get here and what is the angle?

Clearly when omega equal to 0, this is a positive constant that is a real number and it is a positive constant, so the angle is 0, so basically here the angle is 0 right, so just see if you just substitute the in G of J omega, omega equal to 0, the J term vanished, so basically there is no imaginary term in that, so what you have is 1 by 0.25 into 0.005, which is just the real number and a positive number, so positive number means the angle is 0, so that is what we have here.

And when we say omega equal to infinity, what we have omega equal to infinity in sense the same mode of G of J omega we take it is nothing but 1 by root of omega square +0.25 square plus root of omega square +0.05 square and the substitute omega equal to infinity, basically what happens it becomes like 1 by infinity from which is nothing but 0, so the magnitude is 0, then the frequency is very high, infinity and now if you substitute for angle, let us see what happens for angle.

Angle of G of J omega is nothing but what? Tan inverse, minus tan inverse omega by 0.25 minus, so because it is a pole, it is a denominator, so we are the minus tan inverse, now minus tan inverse omega by 0.005 right, so if you substitute omega equal to infinity what happens tan inverse infinity is 5 by 2, so 5 by 2 is 90º, so -90 for this term and -90 for this term, so basically it becomes -180º, so when omega is equal to infinity this become -180º, so basically what we have done is we have taken the extremes, and when angular frequency is 0 what happens and when the angular frequency is infinitely what happens, what happens in sense of the value of the magnitude and the phase that is what we are not in this table and with this table we will try to draw the G of S plain and see how it behaves okay.

So basically if you can remember from your schooling or something how to represent the complex numbers, we basically tell that this is the imaginary axis J omega axis and this is the real axis right, so this is the real axis and this is the imaginary axis and this is 0° okay and we will, we take anticlockwise as positive and clockwise as negative, let is now look at clockwise basically, so this is like -90° and this side is -180° and this side is -270° right, this is what we look, so in clockwise its use to conventions that we take anticlockwise as positive and clockwise as negative, so here we have taken clockwise as direction, we count in a clockwise direction, so we have put the minus sign, so -90°, -180°and -270°, so this is what we have studied.

So now that is plot all these points and then see how this graph will come, so that is what we should get the same graph that we have got using Matlab, so basically what we can do, when omega equal to 0, so we start to omega equal to 0, there is a point omega equal to 0, and this A value is 800, if you can find this value I think it will turn to 800 that is what I can see from the graph but you can calculate by itself and see if it is close to 800, so and this one will start at 800, so on the real line, this is the real number right, A is real number, it had no imaginary term.

So basically we are marking at that point or real line, so this is 800 and the angle is 0°, so this is the 0° line, 0° so basically you can think, you can even forget angles and then say that is no imaginary part and it is a real number, positive real numbers I am just giving mark 800, 0, where 0, there is no imaginary part, so anything you can either use concept of angle and then plot this point or you can just use the coefficient of J is 0, so there is no imaginary, so this is 800, 0 point, so anyway this is the point that we will get when omega equal to 0.

And then when omega increases basically what we can see is, the angle this is the content anything that we are interested into seeing in okay, the magnitude decreases to 0 and the angle actually changes from 0 to -180°, so basically if you can just plot here and then this look, we started somewhere here, there we had a some value of magnitude and the angle was

0 and what happens to words when omega gives on increasing, what happens the magnitude keeps on decreasing, so if you see here, if you can draw a circle like this, so this is A, this is A, this is A, everywhere circulars of a same radius right, but what happens as when omega increases, at omega infinity is 0.

So basically when omega increases they should decrease, so this decreases and then the angle also changing from 0 to -180°, so this is -180°, so basically it comes under, I remove all this, now this is better, so magnitude decreases and also the angle is also it changes to -180°, so to words going to 0, the angle becomes close to -180°, so this is how you draw the qualitative plot, so when you mark this arrow to say that okay, omega is increasing or it changing from 0 to infinity in this way, so this is what you mark it here.

So this is what is called the polar plot, so why it is called polar plot because it is drawn using this polar representation, polar representation magnitude as well as the angle etc, so this is the easy way of doing and some interesting things to be more accurate what you can do is, you can try to see at what point it crosses the imaginary axis, so if I ask you like at what point it will cross the imaginary axis, basically what you can do, you can take this term and then you can say this terms real part must be 0 right.

So if on the imaginary axis real part is 0, so you can equate this term in real part is equal to 0, and then find what is the frequency at which the real part is 0 and then find this point and do things, so all the things can be done getting more accurate representation of how it look like but this is the, we are just trying to looking how the qualitative in a quick way of something without using Matlab, so how you draw the figure and it is called a polar plot, so we will go through one more examples that it makes things clear, but this is the idea, the best way to start with omega equal to 0, and omega equal to infinity mark the extremes and then use the magnitude and angle to the plot and that plot is called the polar plot okay.

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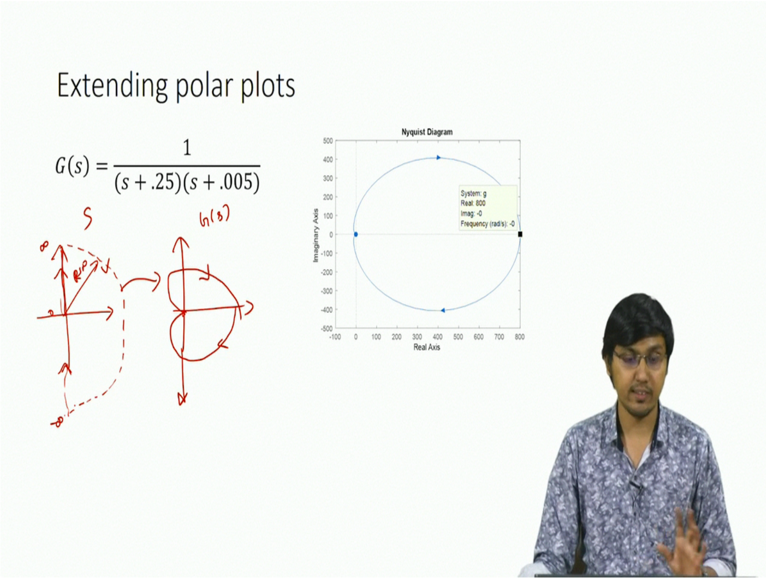

So, yes and so what this polar plot actually, but we, very okay, I am not still not going into the detail of these but I just value for sake of inclusion, so when the take from the S plain to the G of S plain, so what is polar plot says is when I the omega from 0 to infinities what it says right, so now and I go around like this when I change the, when I connect go around the S plain by a big orbit and then can I still continue to do the close control like this from omega minus infinity to 0 can I complete the cycle.

So what basically happens in G of S figure is, suppose we have add this figure. Okay, what basically will happen is these for the portion, for a completion of this what happens is basically this becomes the mirror image of that, so, yes, this has to be the mirror image of it, yes, so it should become a mirror image of it, so basically when intuitively feel like okay, this and this line or just like minus, 0 to infinity omega changes and this is minus infinity to 0 omega changes.

So basically the angular alone changes every omega and then this at R equal to, this is like a infinity radius, when you were actually covering the entire S plain then you would infinity radius and that actually boils to your point here, so we will, let us not go into the detail, but what you need to take from this slide is like okay, I drawn the polar plot, if you want to extended for 0, not only 0 to infinity, but also from the entire S plain, if I want to go in a corn tune in clockwise direction and then actually come back, the Nyquist plot what happens basically is it is the polar plot with its mirror image, so that is the thing you already remember.

It is a polar plot with the mirror image, so we draw the polar plot and draw the leakage of it and what we get is the Nyquist diagram okay, that is something that we have to take care of it and you can just substitute and see if you are really interested curious to know like what exactly happened, so how this we can mirror the image, it is not very tedious calculation things, but you can let go and see like this, if you take a infinity radius in S plain and what happens when you are taken infinity radius, that is boils down to a point or something, so that is what you will get it.

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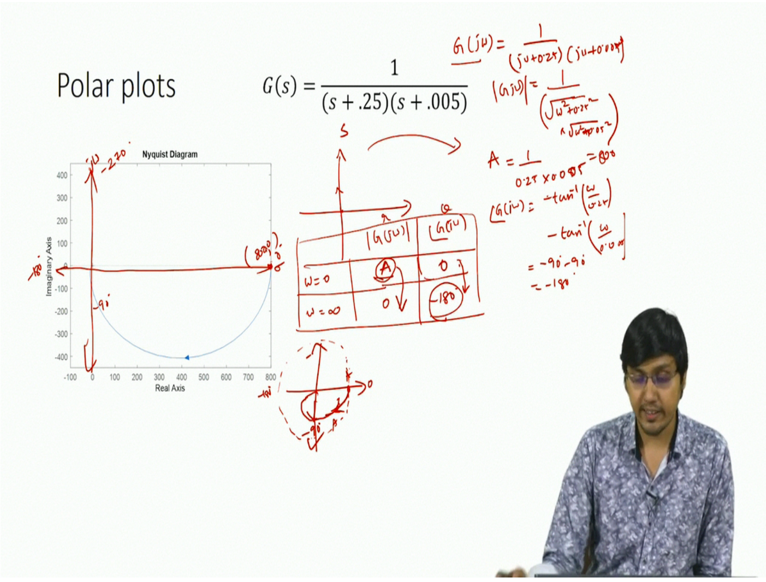

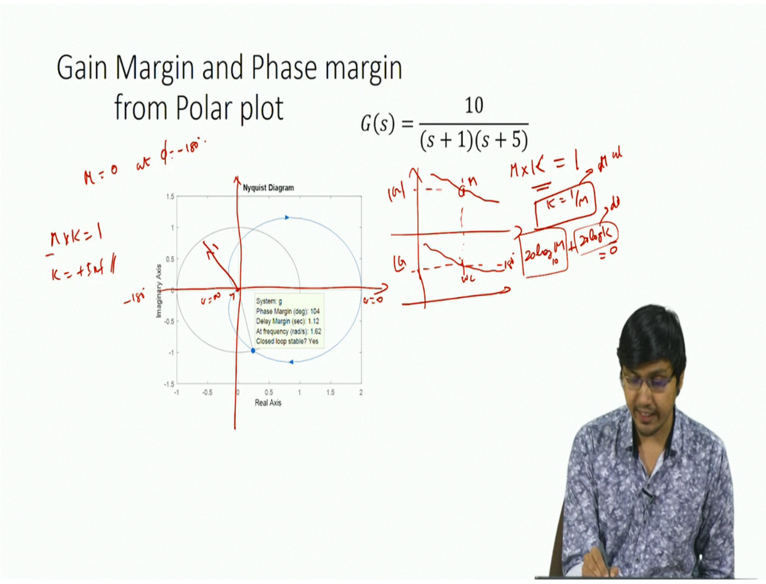

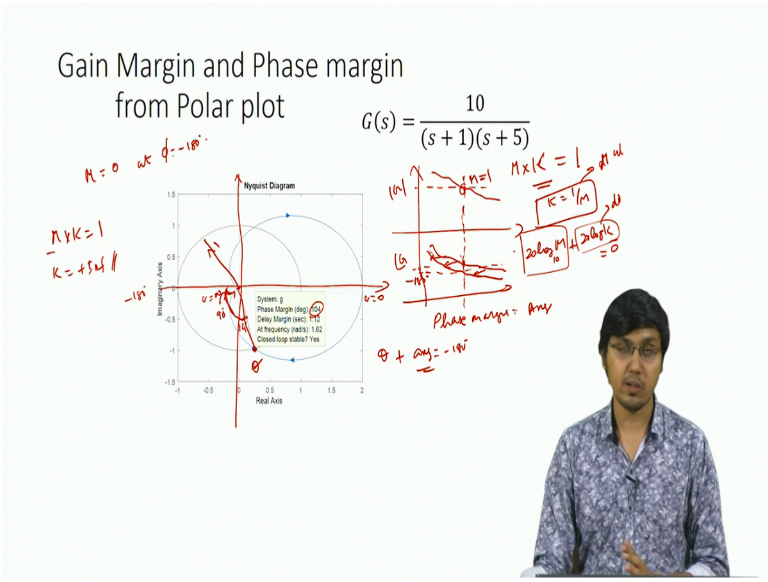

So and okay when we did this Bode plot etc like we were talking about this gain margin, phase margin etc right, so interest might be okay, from this polar plot or from this Nyquist diagram I still should be able to have a way to figure out what is this gain margin, what is this phase margin etc, so but the thing is like it is very simple, like the same concept, the same idea that we are using Bode plot the same thing we are going to apply here, so if you can recollect what exactly we said when about the gain margin and phase margin in Bode plot.

What we say was, we add a magnitude plot and we add angle plot and then we said that the angle was decreasing and then the magnitude was also decreasing and at the angle of -180° is when we get a critical frequency because that is the thing that we get the in phase, sine waves and all those things, so basically that is the critical frequency we get, so this is the omega C and we actually mark, we actually see what is the amplitude of, what is the magnitude of G of J omega at that point and we said if this amplitude was to say some M value, what is the multiplicative gain that is M into say K, what is the multiplicative gain K that we can have.

So that multiband multiplying it will become one, so K is nothing but 1 by M, so that is what we add it multiplicative gain margin or a decibels what we said, what is the additional decibels or because you need to take log of it, it becomes like addition, so 20 log M the base 10+ 20 log K become 0 because log of 1 is 0, so then we say okay if I know the same value what is the gain value, what is the decibels, that is what, it can have to add to that we get 0 here, so that is what we say gain margin in decibels, this is the gain margin in decibels and this is the gain margin in multiplicative form. Okay.

So and again like why still idea was the -180° right, -180° what was the frequency -180° and what was the magnitude at that frequency at the critical frequency, the frequency at which the face work was -180° it is call the critical frequency, so at that what was the magnitude, that is what relates to the gain margin, so the same concept we apply here, so when we say this is the -180°, so you just need the polar plot for finding this gain margin and phase margin, so the mirror image is not really required for this, so but anyway like we have mirror image, it is this, forget for now.

So this is the real axis and the imaginary axis, so and this is the polar plot, the bottom one is the polar plot, so this is omega equal to 0, and this is the omega equal to infinity point, so this is the omega infinity point, so basically what happens you draw circle of radius 1 okay, that is first thing you do and the next thing what you can do is like here at -180°, so at -180° what was the gain, at -180° the gain is 0 right, so it is going to, the gain is going to 0, the magnitude is going to 0 basically M is going to 0 at, whenever you get the phase is equal to -180°.

So basically it means that you can have infinite gain right, so basically if you wanted to say some M into K must be equal to 1 and then now M is equal to 0 because at -180°, so if you can go back to the table what we have drawn, we said the -180° is got at the gain of 0 right, so here M is 0 here, so here basically when M is 0, K is 1 by 0, which means it is tending to 0, so basically it is like a infinity value, very high-value, you can say very high-value is what you see here, so this system is, from gain margin prospective is as a infinite gain margin, so you can add any gain to system still the system will be stable, that is what you can say from here and then with regards to the phase margin.

So what is the phase margin? Phase margin is like when I have a magnitude of 1, so now I shift the question and ask okay, I have a magnitude of 1 here and that I have phase plot here, so what is the shift I have to do in this phase plot okay, so this is the -180° line, somewhere I draw this here, so what is the shift I have to do, to this phase plot, so I shift like this, by what angle it has to shift so that the system will become unstable, so it means that at which M was 1 was equal to -180°, so that is what, so at magnitude equal to 1 what is the angle I have to decrease or what is the log I have to introduce, so that at my angle it becomes -180°, so that angle is called the phase margin right.

So if you can remember phase margin, what is the angle and I have to add, so that my final angle and the magnitude of 1 is becoming -180°, so that is what we said phase margin, so the same thing, what we can do is? We can draw a circle of magnitude 1, so this is the actual of magnitude 1, so at this point we see what is the phase, so this phase is somewhere between like, this is 90°, so and this is like little more than 90°, so it is 14° here, so we can add 104° basically to well the system will be stable, to make it a unstable we have to add 104°.

So that is what it says phase margin is 104°, so that is how you calculate the phase margin, so to phase margin we again same rule will be take the magnitude of 1, we fix the magnitude of 1 and that find the angle and that we say I what angle we have to add, so that the angle becomes 900°, that is what, so here the angle is somewhat say minus, here, what is the angle, where is the theta and then you say -180°, this theta plus what angle you have to add, so that is becomes 900°, so that is what you added and find the value of angle here, so this is how you calculate the phase margin, gain energy in the polar plot or Nyquist plot, so that is what number.

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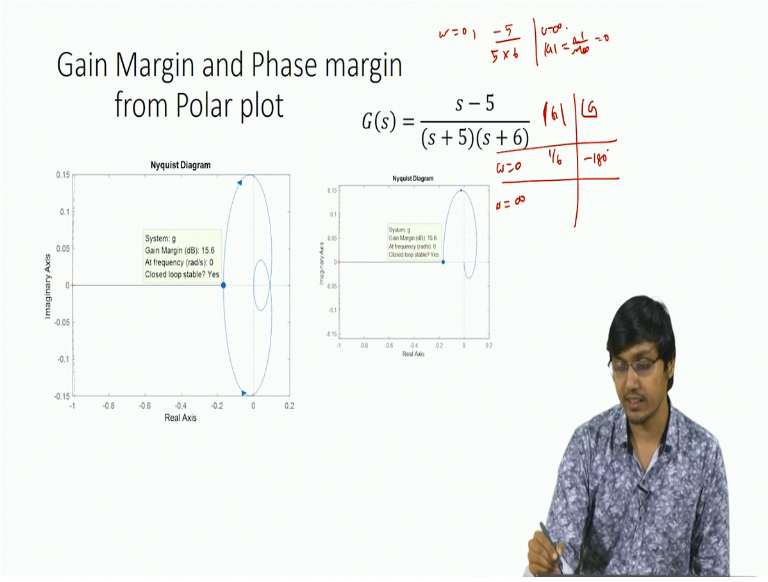

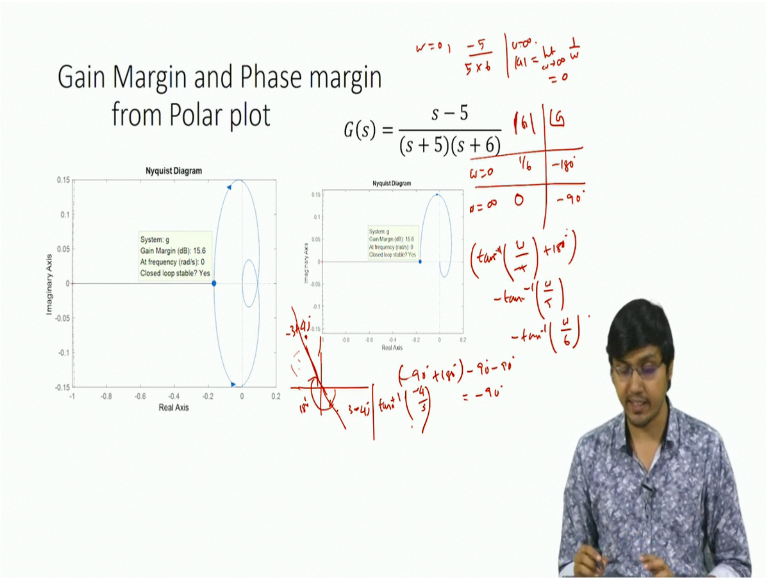

So just do it once it is a very simple concept, if you understand the Bode thing, the same concept will apply here, so basically you ask only two questions, you ask 1st question like -180° we call it as a critical frequency and at that critical frequency what is the magnitude? And by what K that I have multiply this magnitude so that I get gain of 1, so this is the gain margin and what is phase margin? We take the magnitude as 1 and then see what is the angle? And then we say by what angle you have to by what phase last we have to produce so that it becomes -180°, so that the phase margin, so that the same you do here, so that is the pretty much same thing.

Here another example here, this is the polar plot and this is the Nyquist diagram, so if you can see here, if you draw the polar plot and draw the Nyquist plot like this is become like very easy to draw right, so this is like okay, we just draw it basically, so maybe, this may be a good thing to draw because this looks very interesting shape right, so now we will say magnitude of again omega equal to 0, what happens omega equal to infinity, what happens to magnitude and then angle, so this are the two things that we have to draw here.

So I would omega equal to 0 what happens this will become -5, so what omega equal to 0, this becomes -5 by 5 into 6, so this becomes like 1 by 6 basically, so this is 1 x 6 and what is the angle, angle is the negative number, real number, so it is -180°, so remember negative number is the phase log of -180° okay, phase of -180°, a log of 180°, so at omega infinity what happens? If we substitute this basically what happens is this becomes like, you can take the magnitude as root of omega square +5 square by omega square, root of omega square +5 square into root of omega square +6 square that basically will give, if you substitute limit G of J omega to infinity, what basically happens is, it will come after form of 1 by infinity limit omega tends to infinity by omega form which nothing but 0.

It will come as some limit omega equal to infinity 1 by omega form which will become 0, so this is effectively 0 here and then angle, so omega equal to infinity you have magnitude is 0 and if you see the angle basically what you have is, we have tan inverse, omega by minus phi, minus tan inverse omega by phi, minus tan inverse omega by 6 magnitude substitute of omega and remember like this is a complex number form and if you form to find a angle here like if you could draw this, basically it is that this is in this quadrant right, minus phi, omega is have in this quadrant, when omega is instances to infinity, so basically whatever angle you get you have do, basically you will get angles somewhere here, this and this are symmetric when you take the tan inverse.

So basically what you to do, you have to +180° here, so that you shift it back to this quadrant, so if you could refer to the complex number, things from school textbook you will understand like why we need to shift this angle here, so basically and then we substitute omega equal to infinity, so there what you have is like basically we have -90° because tan inverse minus infinity is -90°, +180° and then -90° and -90°, so what it turns out to be is nothing but -90°, so this is what you get as -90° here.

So this thing, quadrant thing you just recorrect from complex number, how to find the angle of a complex number, so basically what you can imagine is if you something like -3 + 4J and, sorry 3+ minus 4J here and -3 plus 4J here, so you take the tan inverse, if you want to find angle, basically will get that inverse of -4 by 3 right here and this is nothing but tan inverse of 4 by -3, so both are like the angle wise they are the same thing, but you can really see that this is really an acute angle and this is nothing but just opposite to this or basically this plus 180° is what will give this angle, so that is why this additional 180° we are adding it here.

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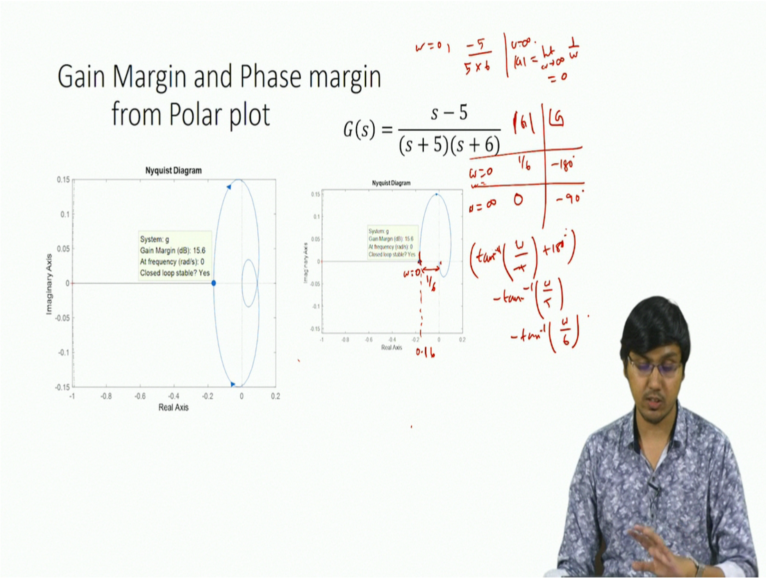

So basically we are now done with all the things that we are, so we are done with all the things that we want, so now we will go and draw the plot and see whether it works fine are not, so now with this what we have is at omega equal to 0. Okay, the magnitude is nothing but 1 x 6 which coming her own like say 0.16 something, so this is what you is starting point, the amplitude here, so this is nothing but 1 x 6. Okay, so and then what we have is like as an 1 omega increases, so when omega increases what happens angle also increases from -180° to -90° and then the magnitude also keeps.

So here actually it is little more interesting because you need to have more points here because you have 0 and 2 poles, so if you take more points here, substitute more points like middle values of omega here you can basically find that this graph shows that the amplitude is increasing at first and then a decreasing and then it goes to -90°, so at the endpoint you can see it is -90°, so basically what you have is, this is what the polar plot you have and then you take the mirror image of the polar plot and you will get the Nyquist plot, so that is what one will have it here.

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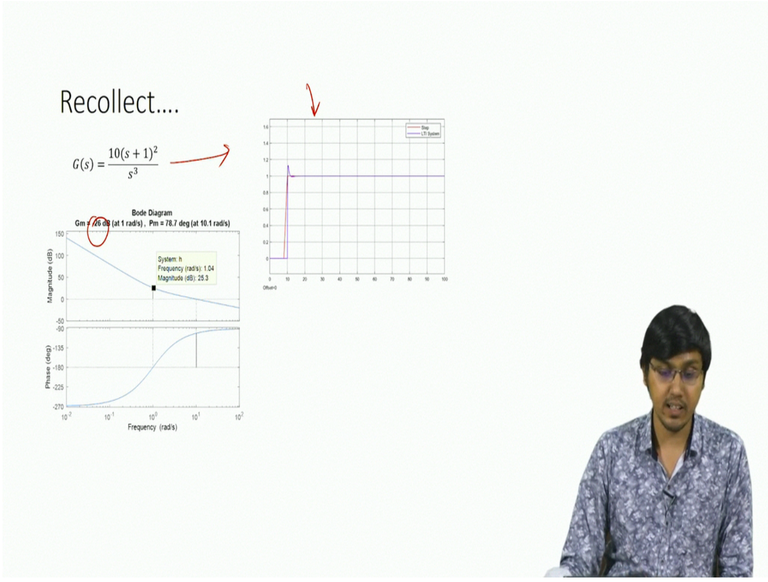

So in the previous lecture we asked this question like Bode plot was not able to capture, whether this system is stable or not, but this system was degrees stable system, but Bode plot said this was having a negative gain margin and then it was probably we felt it was not stable but still with experiments we found that we put a controller and then we prove that it is for a stable system, so that we know can I run my plot, my Nyquist stability criteria help us with this. Okay, so that is what we are trying to ask the question.

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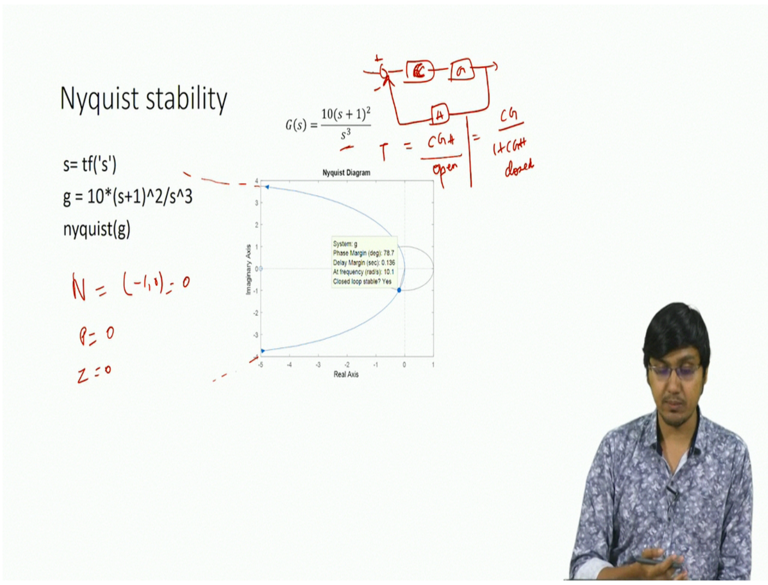

So now we draw the Nyquist diagram, similarly construct the polar plot and then construct the measurement of the polar plot without you get the Nyquist diagram and then what you need to do basically, you need to just find out some values right, you have to first find, number of clockwise encirclements about the point -1, 0 right, so this is the first thing that you have to find and then like even remember you are trying the Nyquist plot for GC which is like open loop transits function.

So what we call as open loop transits function, if you have loop like this would, you have G, controller C and that you have process G and then you have feedback H, so this is what you have, so basically what you draw, for which you draw the Nyquist plot is nothing but C into G into H, so this is the transits function, say you call it, some T, this is what for you will or the Nyquist plot, this is all the open loop transits function, but the closed loop transits function nothing but CG by 1 +CGH, so this is the closed loop transits function.

So remember this is open loop transits function, this is the closed loop transits function. Okay, so now coming back to our question, number of clockwise encirclements, clearly this is not encircling anything, so this is not a circular type, this is not encircling anything this is just going like this and this is just going like this, so definitely number of clockwise encirclements is 0 and number of poles on the right-hand after S plain is also like P is also 0, so you get there is also 0, which means clearly this is metastable that is what is also says the system is stable.

You can also would use Matlab to know that system is, you can just ask whether the system is stable or not, you can ask for stability margins, just click on Nyquist diagram and then see the options, play with this and Matlab and we will get this would.

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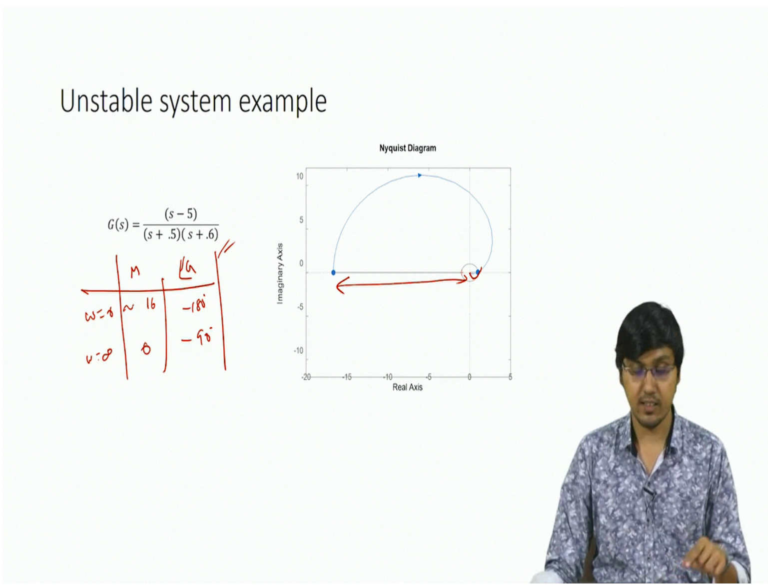

So in unstable, let us go for unstable systems. Okay, let us take this system and then we saw the polar plot, so this is the polar plot and then you can try to draw this polar plot, I am not going to try or workout this, but you can just draw this, basically what you need to do is, you do just put omega equal to 0, omega equal to infinity and just check whether the endpoints are matching etc.

See this is not 0, this comes down and then goes back, so it is -90°, so it goes -90° and magnitude goes to 0, and here the magnitude is somewhat, what it is, somewhat approximately 16 or something. I am not sure, so that is the magnitude you have an that, that is the angle you have, so this is what you will find from this, but you can just express this in terms of magnitude and then angle and then just substitute omega equal to 0, and omega equal to infinity and then see what happens.

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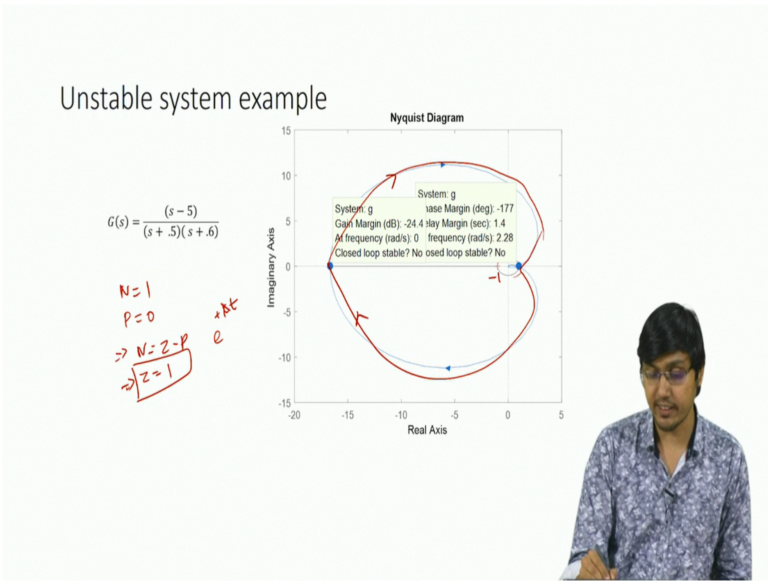

So basically you get this polar plot and then you take the mirror image of the polar plot you will get the Nyquist diagram, so now -1+ J 0 point is here, there is a clockwise encirclement, this is the big thing right, if you can see the big thing, big thing is a clockwise encirclement, so N equal to 1 okay, N equal to 1 is a clockwise encirclement and there are no poles on the right inside of S plain of the open loop transits function, so P equal to 0, ends what you get, you have this equation N equal to Z minus P right

So when N equal to 1 and P equal to 0, you get Z equal to 1 which means that in the closed loop transits function, there is a pole, one pole on the right and side of the S plain which made the system is unstable, so what it means if there is a pole on 800 S plain, what it means basically is you get E power some positive constant into T, say lambda T and that this is going to get as time increases this is going to, amplitude going to get increase in so much, that is what we are going to get, so this is the one example of unstable systems, so remember this is the encirclement that you have, the big one, so this is the encirclement that you have, this is the clockwise encirclement and this is not encirclement yes.

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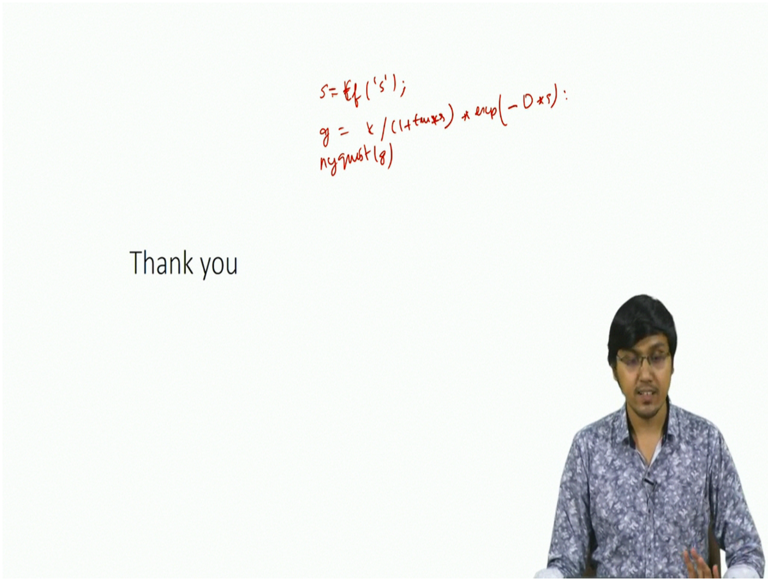

So yes, we are coming to the end of this Nyquist stability and things, but like you can we are just done some very few examples, so we may actually use Matlab to get more examples and then just create some random transits functions, create a system with time delay, so basically in Matlab if you want to give time rate systems, if you want to give time rate systems in Matlab basically what you need to do is, you have to put S equal to transits, so this is just to tell that you use this variable for transits function and then you say some transits function G equal to K by 1+ K Tao S into exponential of minus D into S.

So if you can do this and that you put Nyquist here, so if you can plot Nyquist graph and then see that should various values of K and Tao and D, and then see what actually happens when the system go unstable, what are the effects at that time on stability, so these are all something that it will be interesting for you to yourself try and learn and that will be like many things more interesting, so with this we are coming to the end of the lecture on Nyquist stability analyses. Thank you.