

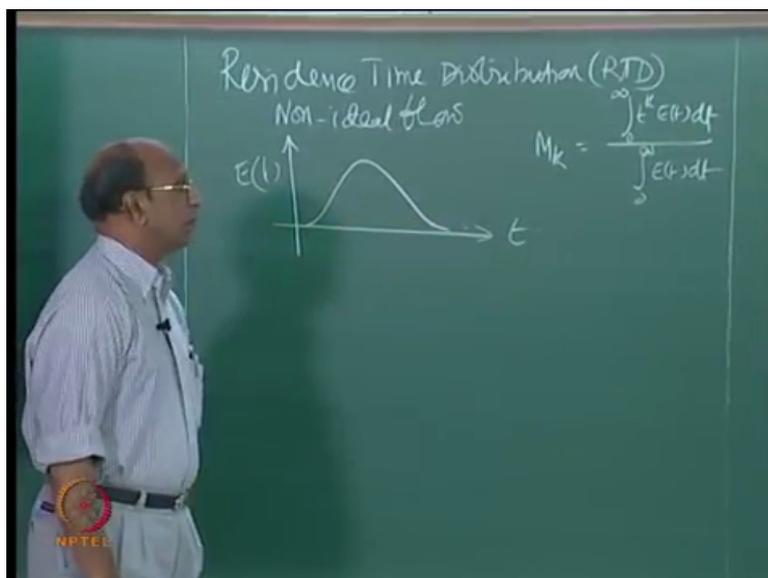
Chemical Reaction Engineering 1 (Homogeneous Reactors)
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Lecture 53
RTD for Various Reactors Continued Part 2

Okay we start now. Yes, so once we understood this distribution $E(t)$ versus t , yes something like this, we have to now talk about you know the mean and variance of this. You know for any distribution you have area under the curve, okay, and also the mean. Mean is given by the first moment, correct no, moment first. Yes and also you have the variance, okay, second moment. So those things also will come here. The reason is using this \bar{t} and also you know variance we can find out the non ideal parameters.

What are the non ideal parameters? For example axial mixing, that is one parameter. How much axial mixing is there quantification? How do you quantify axial mixing? You have to quantify axial mixing only in terms of dispersion number, okay. Then dispersion number we can calculate using this you know variance and first moment, second moment and all that.

And most of the time we will not go for f curve but we will go for the e curve, this kind of distribution, right? So the general definition of this distribution is that, okay, M_k , k th moment is 0 to infinity, t to the power of k , $E(t) dt$ divided by 0 to infinity, $E(t) dt$, okay. That is the general one, good okay.

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So if I want to take 0th moment what do we do? K equal to 0, M_0 will be what?

Students: 1.

Professor: 1. So what is the meaning of that? What does 0th moment gives? Okay.

Student: Initial.

Professor: What is initial? What is first moment?

Student: Mean.

Professor: First moment is mean, okay. Second moment is?

Student: Variance.

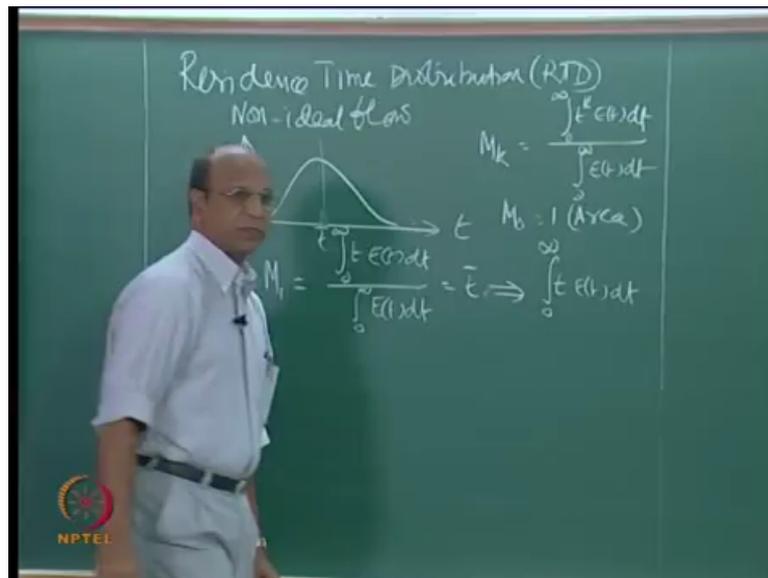
Professor: Variance. But 0th moment is?

Student: Area.

Professor: Area under the curve. So M_0 when we put here 1, what is the total area under the curve? We have normalized this. That is why you have C divided by $\int_0^{\infty} C dt$ and all that is there, okay. So that area under the curve, this area M_0 is equal to 1 area, okay. That is 1. So M_1 is $\int_0^{\infty} t E t dt$, $\int_0^{\infty} E t dt$, right? That is \bar{t} and, okay, M_1 is nothing but for us it is \bar{t} , right?

It is nothing but \bar{t} for us yes because this average will be somewhere here depending on the shape of the curve so this is \bar{t} , right? And this also can be written as, what is $\int_0^{\infty} E t dt$? 1. So that is why this is also equal into $\int_0^{\infty} t E t dt$, right? Okay good.

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So now I think the difference between variance and what is the other one no? Standard deviation, okay. Yes, what is this one? Sigma square is M_2 minus M_1 square.

Student: (())(04:52)

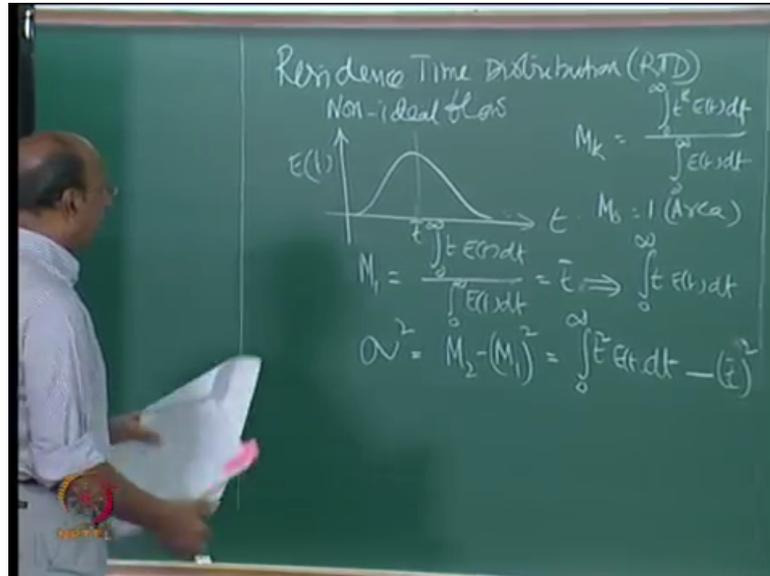
Professor: That is standard equation, okay. Yes, so this will be for us M_2 is of course anyway the same thing so this is please see M_2 means K equal to 2 so t square and $E(t) dt$ divided by this one that is 1. That is why I am not writing that but I will simply write here 0 to infinity t square $E(t) dt$ minus \bar{t} square, correct no? Yes, \bar{t} square or otherwise of course I have to put this one and then put whole square. So these beyond this normally we do not go.

Third moment generally we do not go, right? And there are some cases where people have gone to third moment also and most of the time for single parameter models using this and using this we can estimate for example d by uL values. D by uL is quantifying mixing. And also tanks in series, right? I have 10 tanks in series. I will conduct RTD experiment and then I will find out whether these 10 tanks are behaving really as 10 tanks or not because ideal tanks 10 must behave as 10 tanks, okay.

So and if it is non-ideal that may be equal into 8 tanks equivalent. That you can get again from RTD you have to have that curve and then you find out variance of that curve and you know very beautiful expression there is one, very simple expression in chemical engineering. Sigma theta square is equal to $1/N$. Heard of it Swami? Never. Then RTD it comes. Those who have got I think some good RTD exposure in their B Tech and M Tech they know this. And also there is another equation also.

Sigma theta square is equal to 2 d by u L where sigma theta square equal to, you know that is dimensionless variance, okay. So those things we have to find out, okay good. So this is one and there is another one.

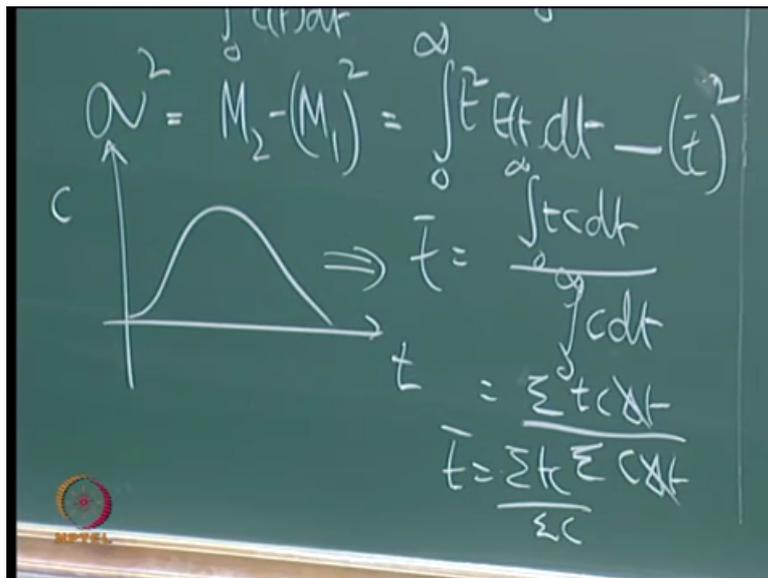
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Levenspiel simplifies these two in terms of even concentrations. So that means if I have simply c versus t, will the shape changes? Shape is exactly same but only thing is scaling will change, okay. So that is why distribution will not change. That means again my sigma square and also that M 1 you know t bar that will not change. How do I find out t bar from here? So yes for this one t bar will be integral 0 to infinity t c t d t divided by, no this is not 1 so I cannot ignore that. That is not 1.

So this also can be written as sigma of t c delta t divided by sigma of c delta t. If you take equal intervals of delta t then this also will get cancel. So this will be simply sigma t c by sigma c equal to t bar. This is what is the problem he has also solved as an example in his book Levenspiel book, right?

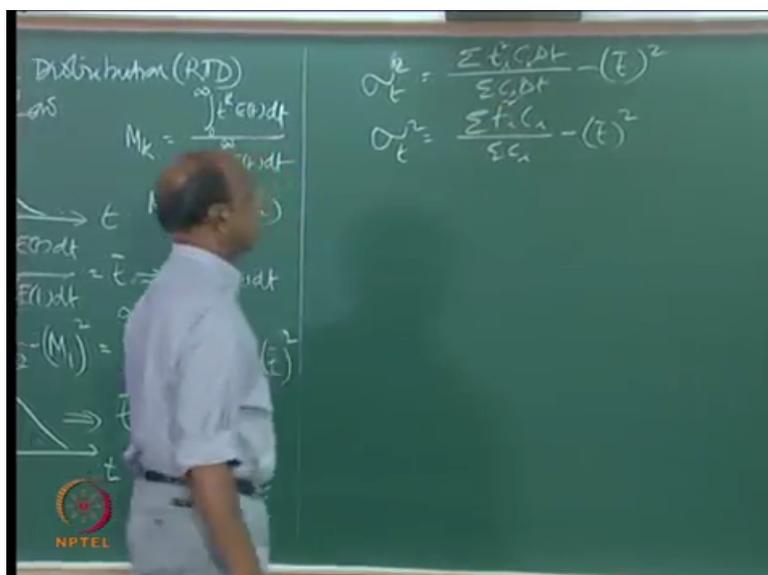
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So these things are a must for us because for every distribution you are going to find out what will be you know \bar{t} and σ^2 . These two are a must for us, okay. Good. So this is the one and like this you have I think even using same thing for this distribution σ^2 also you can calculate as, okay, I will write here to differentiate, okay. This \bar{t} is okay. Yes this is \bar{t}^2 equal to we have I am giving the final expression divided by σ^2 of what? $\int c \Delta t$ minus \bar{t} square.

If Δt is again if you are taking same maybe 1 minute 1 minute and you should take as small as possible. If you take bigger one then you will get more error, okay. Yes depending on the data. So this also can be written as $\sigma^2 = \int t^2 c_i dt_i - (\bar{t})^2$.

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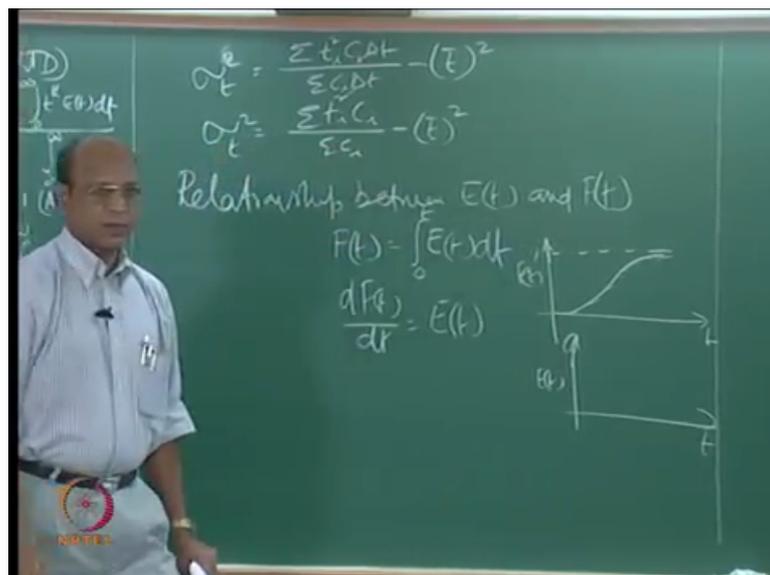
So these are the equations what we have and most of the time in the examination also we will be asking this, right? Yes, so from the data we will give concentration versus time data and these distributions we are talking about only for this kind of increasing and decreasing. F curve also has some distributions which are valid both and that we are not doing that, okay. Good. That is fine.

So now I think this is enough for us as far as this is concerned and there are other things now which also we need before going further to develop the equations that is, okay. Direct delta function we have to do one thing and the other thing is, okay. Let me first do dimension, okay. The relationship between E t and F t. It is obvious relationship E t and F t, okay. So we know that F t equal to 0 to t, there is no infinity here, E t d t.

So I told you E t will you know as pulse input will give you E t and step input will give you F t. But once you conduct let us say E t that means pulse input can you convert that into F t? Easy, that is straight forward. The rivers reverse also. That means you conduct F t that is step input experiment and then try to find out E t. How do you do that? Differentiate this. This is d F t by d t equal to E t.

What is the meaning of this? Meaning is I have F t curve versus t, yes like this. Yes what do I do? Yes, you have to find slopes at various points, okay. So when you take this slopes at various points, right, so what kind of shape? E t I will get here, E t versus t.

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Initially what is the slope? Almost 0. And at sometime it goes to increasing and afterwards again decreasing. So what shape you get? Exact like this. Initially it is 0 slope and then

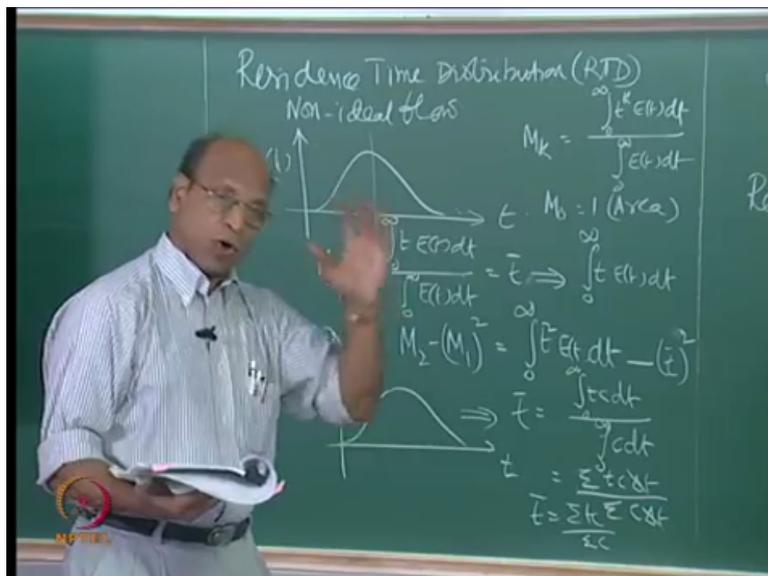
slowly it is increasing, reaches maximum and then afterwards comes down. You know this slope is 0 no? So yes we will get like this one I think. So you should get the same thing, okay. So that means one is possible to see the other one that is all, okay. Good.

So the other one which I have to tell you now is because all these things are required for us, okay. I think later I can tell because all chemical engineers are very famous for dimensionless distributions, dimensionless parameters. So that also I have to write because E t we are talking. What are the units of E t?

Student: Timings.

Professor: Timing. So I will try to convert that into E theta where there are no dimensions. That will come a little bit later. But I think before that let us complete with t everything and there is one more thing which we have to discuss here, it is direct delta function. Direct delta function which is normally shown as like this, infinity, this is time, this is what ever that function, okay. Yes, please take this.

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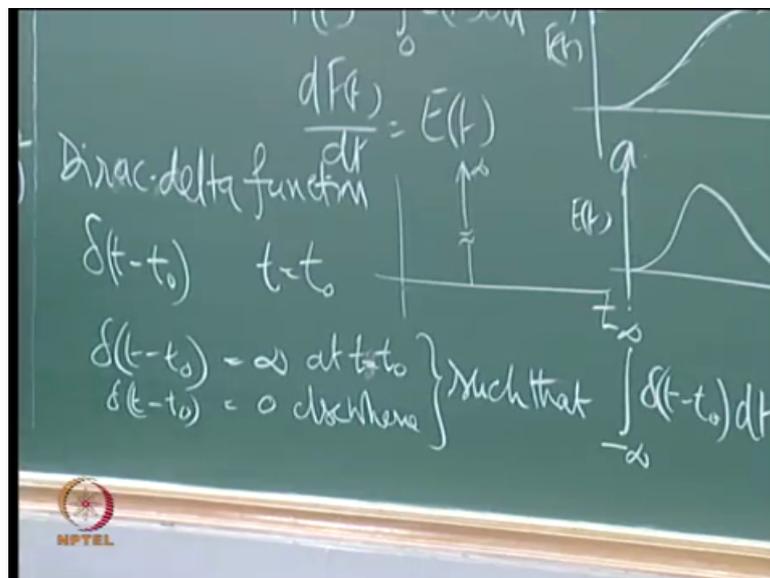
Direct delta function is defined, this is required I think this is why Levenspiel book you know many people like it because he knows that none of us are capable of remembering all this so that is why he gives in his books also these definitions again so that we do not lose, okay. We are not lost, okay. Direct delta function is defined to deal with discontinuous functions in calculus, okay. Good.

Thus, please write, delta t minus is our representing. T minus t not is a distribution curve which is 0 everywhere except t equal to t not where it is infinite right, okay. Next line I think you know these things are required because in an ideal PFR what is the output you are getting? Ideal PFR. E t is defined only for ideal PFR in terms of direct delta function. That is why and using that also from RTD we can calculate volume and also conversion.

Given volume conversion, given conversion volume from RTD side, okay. So that is why some idea about this direct delta functions are required. Good. So next line you just write. The area under the curve is unity and width of the pulse is 0. In symbols you can write this delta t minus t not equal to infinity at t equal to t not and delta t minus t not equal to 0 elsewhere, okay, such that minus infinity to plus infinity delta t minus t not d t equal to 1.

You are able to see my handwriting or becoming worse and worse? That is the area under the curve. Minus infinity to plus infinity but that function is valid only at t equal to t not. But only thing is it should be somewhere between minus infinity to plus infinity that t not must be there. That is all that is the requirement, okay.

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So that is one. You know if you really see mathematics and history of mathematics these functions are defined to solve some physical problems. Otherwise it is not just for fun they have defined. So they started solving you know particularly how universe started and the mathematics they started deriving. Under some conditions they have to define some functions. All these functions are like that.

So that is why most of the people talking about astronomy and also you know this universe how it started and all that there must be an excellent mathematician. Otherwise they cannot. And those are the people who defined many new functions. And of course later in engineering these functions also are used to represent you know direct delta function, impulse function all that. So that is why if you read the history that is why you know we do not have good syllabus I say. So the good syllabus should also contain stories in chemical engineering.

Yes really, stories in mathematics, stories in physics. Stories in physics and mathematics are thousands, okay. So how this idea started and all that, how chemical engineering started I have a story. I will tell some time later you know in the department or outside I think you know. I have to give that story also because otherwise people are forgetting about that stories. What is the starting point? The real starting point for chemical engineering is the industries, sulphuric acid industry, calcium carbonate industry.

You know washing soda. Washing soda is sodium carbonate. Yes, those industries which are a must at that time. I am talking about 1600s 1700s 1800s. And you know actually I felt also very bad about the way we are now teaching, the way we are behaving as faculty also that we never tell you anything about industry, correct no?

Just happily coming and then writing some very nice beautiful equations, right, and then you nod your head as if you understood and I will take it that as if you understood and then exam is conducted. Various chemical industry, this is what is the problem with people who are doing simulations. Without knowing rotameter they simulate rotameter, okay. I mean simplest example I am telling. So that is why you simulate. Simulate is required once you have thorough physical concepts, then simulation, then understanding automatically comes.

Not the other way. First simulate and then you know without knowing why you are simulating and then trying to find out something out of it. And you know when I say that all of you will be thinking that this man does not know simulation that is why he is telling, okay. I mean there is logical conclusion, okay. All of you will be telling or otherwise they may be thinking that this guy is against with that simulator so that is why I think he is you know. All this idiotic things you know mind always goes in wrong direction only.

If I tell you do not go in this direction your curiosity will increase. Why did he tell you know this go? He asked me not to go in that direction. So you finally go only in that direction, okay. And if I ask you, okay go this direction you do not care. Do not go means you know really it

is the thing. So that is why all these functions have really beautiful stories and those stories are unfortunately we are missing it. I do not know whether schools also they are not able to tell.

And this planet lags wonderful teachers who knows the stories more. History is very important I say. You may not care because you also do not know history. You know only your father or maximum his father beyond that who is his father where they are stayed, I think 90 percent of this generation do not know that, correct no? Because now the single families and mother in law will not accept father in law, father in law will not accept daughter in law, daughter in law all in laws only, okay.

One in law I mean actually you know I like the best word in English is this brother in law, sister in law, daughter in law, okay. When I have daughter in law you know what is the real meaning of that? In law she is my daughter. When you have brother in law, in law he is your brother, okay. That means as per law is concerned he or she you no correspondingly daughter or sister or brother. It is a wonderful expression you know. In other languages we do not have that kind of beauty.

Only that English language only this word only I really like it, okay. But in Telugu we will call if you have brother what I think yes, Anna if he is elder brother, okay. I think Tamil also Anna, Tambi. That is all. But I think you know whoever it is brother. Brother is universal. And if you go to some other family you are still brother but in law, okay. If your daughter goes to some other family she is still daughter there but in law. It is wonderful.

So like that there are many things if you are thinking you know this fellow does not have any other work. Really I do not have any other work. I do not have really any other work, thinking like this only. And I do not know how to pay tax and I do not know how to go, in fact I cannot even manage TT if I do not have a seat in the train. I do not know how to ask him you know can you take money and then give. And I do not know. I am afraid to ask. So that is why travel I think either standing or sitting whatever is available that is all.

I do not know how to manage. And now people will go with that pant and then something hanging here, something hanging here, correct no? So, all white and then they just simply go that hundred rupees take and give me seat. We are not able to do that. I think you know I am a very useless fellow. That is why to tell you the secret always my wife scolds me because I do

not know what to do. And I do not know even to go and manage with this gas fellow. Now they started giving only limited gas and all that. So you know like that.

So that is why I started thinking I am enjoying it I do not know, okay. Okay good. So that is why these functions also it is not simply mathematical functions without any physics. To solve some problems only all these functions how come into picture, okay, many functions. Good. So then we will have some more properties, okay. Please write this. The useful property of this function is $\int_a^b f(t) dt = F(b) - F(a)$, who said that? Dinesh, how do you know?

Student: Already did it.

Professor: Already did it where, which subject?

Student: CRE only.

Professor: Oh CRE you did it already, okay, f t not. Okay, but only thing is what is the requirement Dinesh?

Student: (0)(25:01)

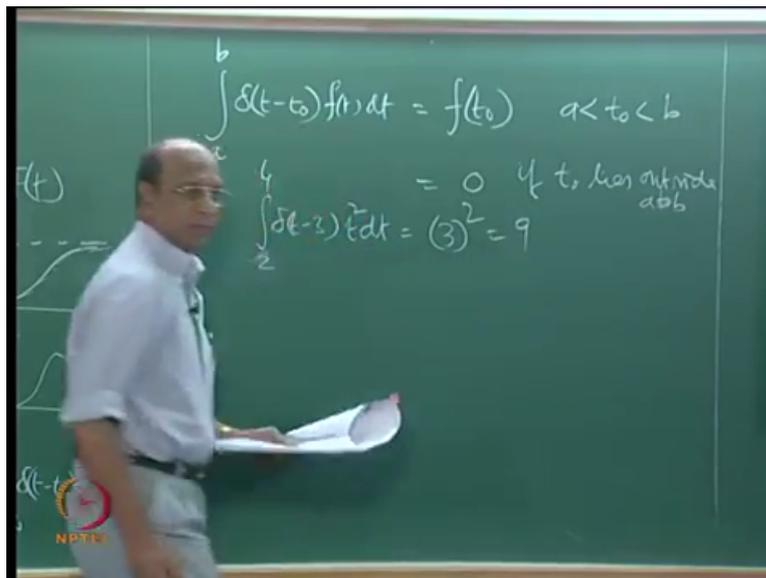
Professor: Requirement is?

Student: T not should lie in between.

Professor: Yes, exactly, t not must lie between those two integrals and yes limits. That is very important otherwise somewhere else you cannot do anything. So otherwise it is 0. If t not lies outside a to b not a minus b, okay. So that is one. So now I will just give you I think all of you may be easily getting this. So to calculate $\int_2^4 (2t - 3t^2) dt$ what is the value? Alga is very seriously thinking.

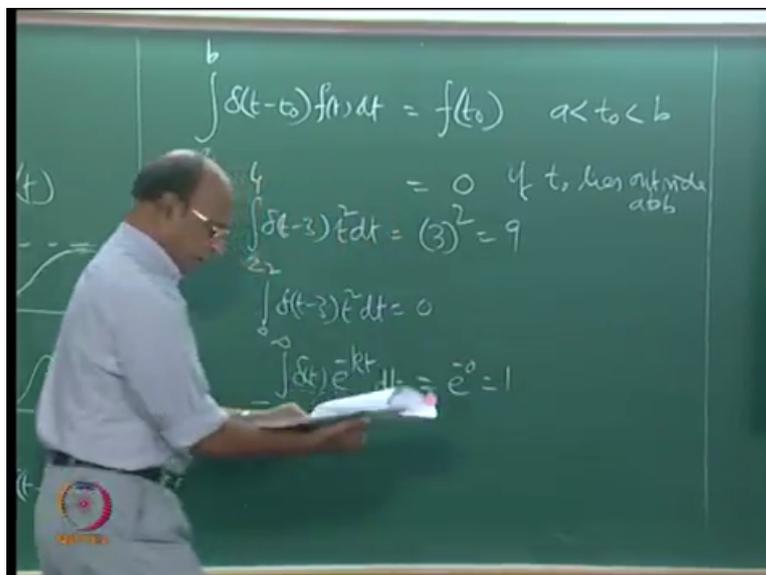
Alga I am waiting for you. 9, why 9? What is the function corresponding to this? This f t it is t^2 , right? So it is inside the interval t minus 3 between 2 and 4, right? So this is t not, this is 3, this is f t, this is t^2 , then what this must be? Good.

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So, one more example $\int_0^2 \delta(t-3) t^2 dt = 0$. Because I think that is okay. Yes now one more. One more is this. $\int_{-\infty}^{+\infty} \delta(t) e^{-kt} dt$. Why 1? That is t minus 0, okay. So this will be e power minus 0 so that is 1. So this is e power minus 0 equal to 1, okay.

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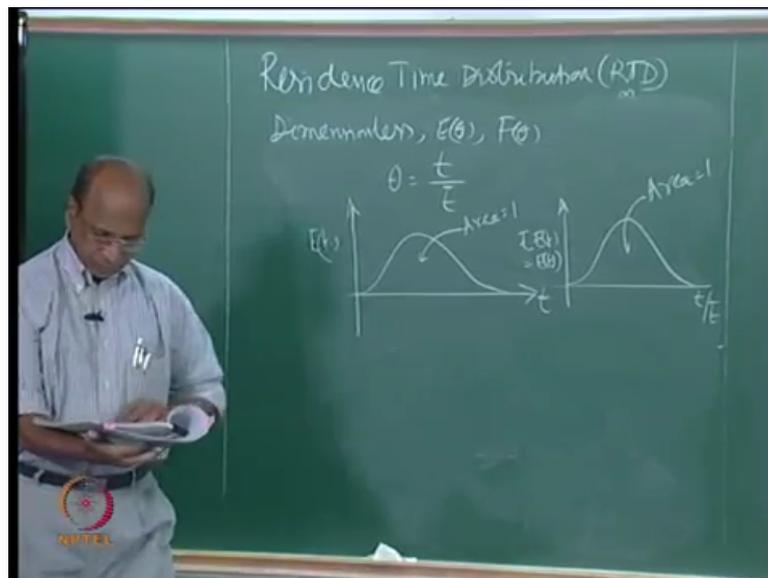
So like that you have all the functions. That is good. I think this is one and yes all other things are fine. So these things are also required for us after sometime you know when you are actually solving the problems. And now let us see the dimensionless $E t f t$, $E \theta$, $f \theta$, etc. So, next one is dimensionless $E t f t$ or $E \theta$ and $f \theta$. So we have to make $E t$ and $f t$ dimensionless so we call them $E \theta$ and $f \theta$. So first we define.

Student: (0)(28:03)

Professor: Everything we will write dimensionless. You will write here E theta f theta. So that means we would like to express that in terms of time also dimensionless, okay. So that is why, okay. So the first definition is theta equal to t by logically t bar, okay. Yes, so that is the one. So now can you make $E t$ into E theta? There are many ways of making. I think I will stop asking I will tell you. So I have here $E t$ versus t , right? So this shape maybe general shape is like this. Good. So what is the area Abdul? 1.

So now first thing is I have to change the t into theta. So that is why I divide this scale by t by t bar, okay. So, now I will leave $E t$ as it is. Now what will be the area under the curve? The same thing if I divide by t bar what will be the area under the curve? 1 by t bar. But my requirement is area equal to 1 . So what should I do? Multiply by t bar. So that is why you have here t bar $E t$ which is nothing but E theta. So then again you get almost same shape. Area equal to 1 , okay.

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So that is E theta. E theta now does not have units. Yes, you can also do this you know mathematically that is 0 to infinity $E t d t$. How do you convert that into E theta? Again t bar funda, t equal to $d t$ because t definition I know. $d t$ equal to t bar. So this is 0 to infinity, yes, this is $E t$, t bar theta, right? No, t bar d theta. So this is now E theta so this is equal to 0 to infinity E theta d theta, okay. Rashmi, no problems?

Okay, so now if someone was telling that when I write $f(t)$, $f(t)$ is dimensionless, okay. But we want to convert that into $f(\theta)$, right? Yes so $f(t) dt = f(\theta) d\theta$, please remember $t = E \theta$. Can you convert this one into $f(\theta)$? Again same funda I say. Substitute for $t dt$.

Student: () (31:16)

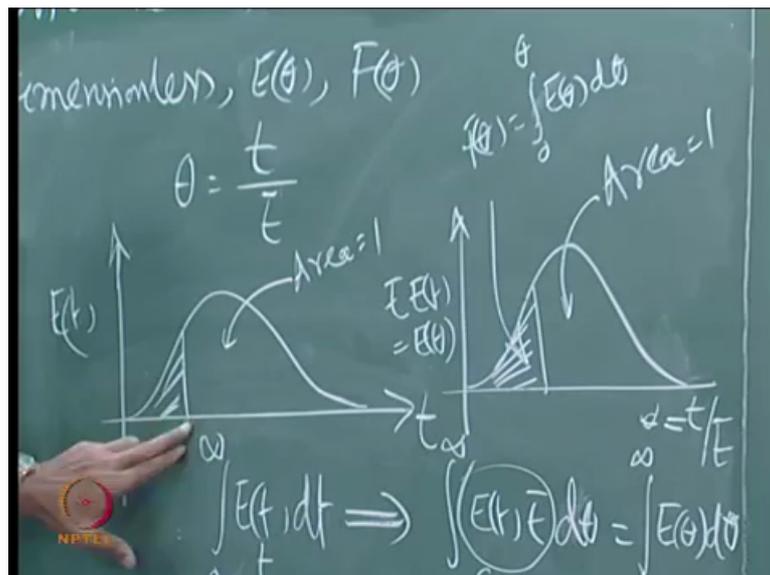
Professor: What is that?

Student: () (31:27)

Professor: You cannot have that. You have $d\theta$ that should be corresponding to θ otherwise Rashmi will get angry. That is why what she said is right. That is why entire class not educated no. Otherwise we do not know I think what limit you are putting if she has not asked that question. We should have accepted, I also should have not mentioned, things would have gone. So our ignorance would have been there all the time, okay. It is θ , 0 to θ . So this will be 0 to θ $E \theta d\theta$ which is nothing but $f(\theta)$, right?

So $f(t) dt = f(\theta) d\theta$ that is all. $f(t) dt = f(\theta) d\theta$ yes. You will get that I think because you see here we have $E \theta$, θ , right? This is equal to θ , right, t by t bar. So what is $f(\theta)$? This must be this area equal to $f(\theta) d\theta = \int_0^\theta E \theta d\theta$, correct no. So this area again is exactly same as this area, area wise.

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Because but this is θ value and this is t value that is all. So that is why $f(t) dt = f(\theta) d\theta$, right? Good. So why we are doing that is when you are actually writing models for example axial dispersion model or CSTR with dead space, CSTR with bypass or CSTR with bypass

and dead space both. When you are actually doing all that so many times we will derive the equation in (33:27) of E_{θ} or f_{θ} which is dimensionless.

By the way why do we use dimensionless equations in chemical engineering? I think not only in chemical engineering, in engineering. Civil engineering also they use but I think chemical engineers use lot of dimensionless equations because we have mass transfer, heat transfer and also fluid flow and also reaction engineering. For example (33:48) column number is a dimensionless number.

Student: (33:55)

Professor: What analogy?

Student: (33:58)

Professor: Not every time but I think yes except in friction factor only has that relationship. That is you know $f/2$ equal to, first let me I will come back to you. Yes $f/2$ equal to?

Student: Standard number.

Professor: You heard of j_d , j_h ? Nowadays many teachers are not telling j_d and j_h . There is no funda, okay. Yes (34:30) and all that yes. So j_d equal to j_h equal to, j_h take care of heat, j_d takes care of diffusion mass and $f/2$ takes care of, yes that is a beautiful analogy between mass, momentum and heat. That is one but not that is mainly. Mainly it is only to take care of the dimensions because I can convert always my system into 0 to 1, okay, or in terms of 1, 2, 5, 10 like that units.

So that is why we have so many unit you know dimensionless modelling equations where I do not have to talk you know, what is exact length? You see when I am talking about length of the plug flow I do not have to tell whether it is 10 feet or 20 feet or 3 metres or 10 metres. I can only simply say dimensionless length is point 5. So whatever length you put there if it is 5 metres it will be 2 point 5 metres. If it is 5 kilometres if it is 2 point 5 kilometres.

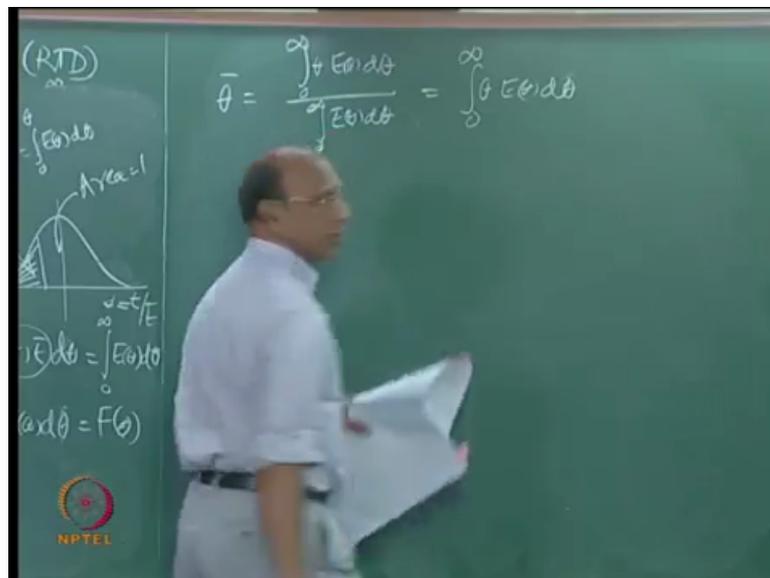
So that is one of the main advantages in dealing with dimensionless parameters. But you have to choose correctly, what are the parameters you have to use so that you will get logical dimensionless quantities? That is one of the main important things in chemical engineering why we do that. So maybe some of you in your projects also do that. Make a dimensionless

we say in the beginning and then solve that in terms of differential equations or whatever. Good. Okay good.

So this is the one and so now when I have this kind of distribution E theta versus theta, this also will have variance and also it will have yes I think you know theta bar, okay. So what is now equation for theta bar here for this case? That is like t bar. Here I have theta bar that maybe 1, okay. But what is equation for that? You know already so that is why I am telling.

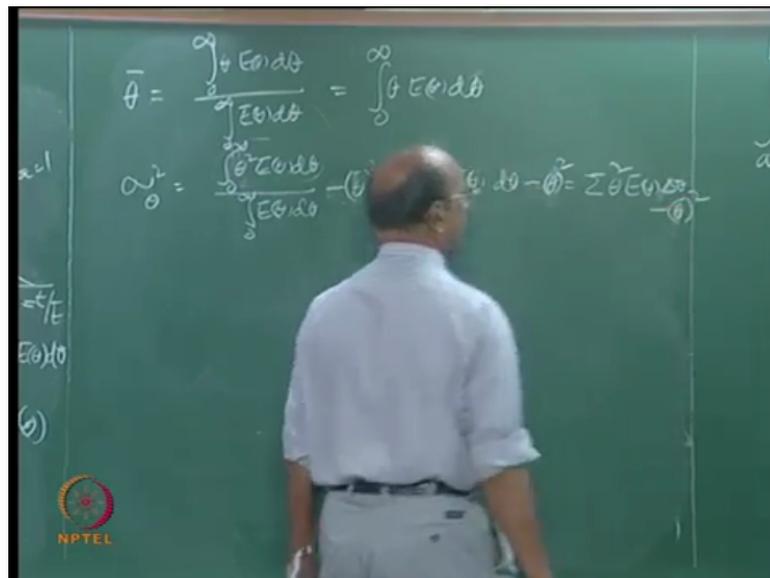
It is exactly like the definition $\int_0^{\infty} \theta E(\theta) d\theta$ divided by $\int_0^{\infty} E(\theta) d\theta$ which is equal to $(\bar{\theta})$ (36:38). So this is one so this becomes 0 to, okay.

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Now $\int_0^{\infty} \theta^2 E(\theta) d\theta$ equal to same thing $\int_0^{\infty} \theta^2 E(\theta) d\theta$ which is minus 1, okay. Minus 1 or you know this theta equal to, right? Okay, this is 1 square is alone. So this is also equal to you can take only this because this equal to 1 so $\int_0^{\infty} \theta^2 E(\theta) d\theta$ minus 1 which is also equal to $\int_0^{\infty} \theta^2 E(\theta) d\theta$. This theta bar equal to 1. Otherwise theta bar square you can write, okay. No problem. This is also theta bar square, okay, theta bar square.

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Now what you have to do is that when you conduct the experiment and you will have first c versus t data. So that can be converted into $E t$, then it also can be converted into $E \theta$. If someone gives you directly $E \theta$ versus θ data and ask you what is sigma theta square, let us say sigma theta square after doing all this I will get point 1 example, okay. So from my tanks in series model this sigma theta square equal to $1/N$ where N equal to number of tanks.

Now from the data I conducted the experiments on that setup and then I found out sigma theta square equal to point 1. So how many number of tanks we have? 10. That is what is the advantage. Point 1, N equal to 10. That is the answer. That means you know that the tanks in series model can also be used for plug flow. Theoretically if you have infinity you will get plug flow, right?

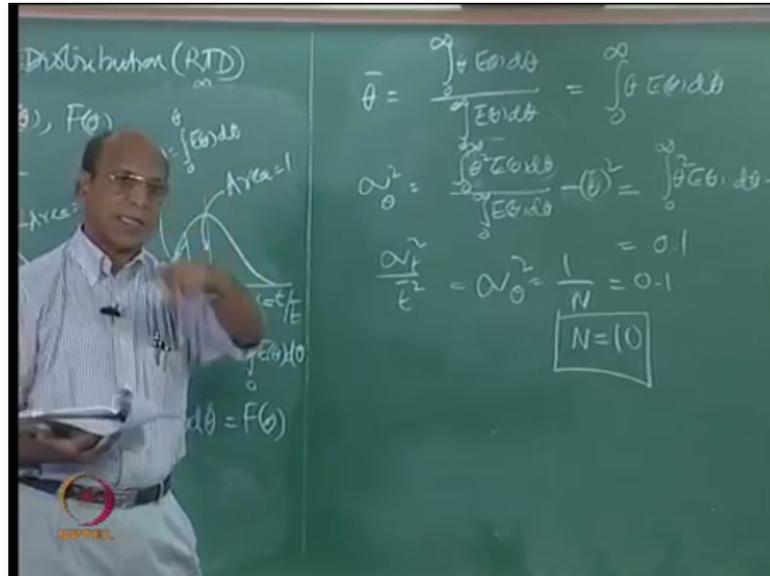
But still you may not have that kind of infinite number of tanks so you conduct the experiments on supposed to be plug flow reactor, it is not plug flow, you can now convert that into equivalent number of how many tanks. Why? Because if I know number of tanks 10, if I have first order reaction, right, what is the equation for conversion?

Student: $1 - 1/(1 + k\tau)$.

Professor: $(1 - 1/(1 + k\tau))^{10}$ equal to $1 - 1/(1 + k\tau)$ yes tau to the power of, this is 10 tanks I say, 10. So then I can calculate very easily the actual conversion that is coming from the system. I have the packed bed, right? But you know I do not want to go to dispersion model and then do it because I think tanks in series model also can represent the flow inside through some number. I do not know what is the number so I will conduct RTD experiment.

I will get E theta versus theta curve, c versus t and all that convert this, right? And in fact you do not have to go even for that. So this is also nothing but sigma t square by t bar square. That is the definition of sigma theta square.

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So even concentration versus time curve we can calculate sigma t square, t bar square, that is nothing but sigma theta square which is let us say point 1 what you got from the experiment. So now use that to calculate what will be the number of tanks in series and from there I know if it is first order reaction because first order is easy to remember that is why I am telling. So first order equation is c_A by c_A not equal to 1 by $1 + k \tau$ to the power of N . That tau is for each tank, tau i to the power of N .

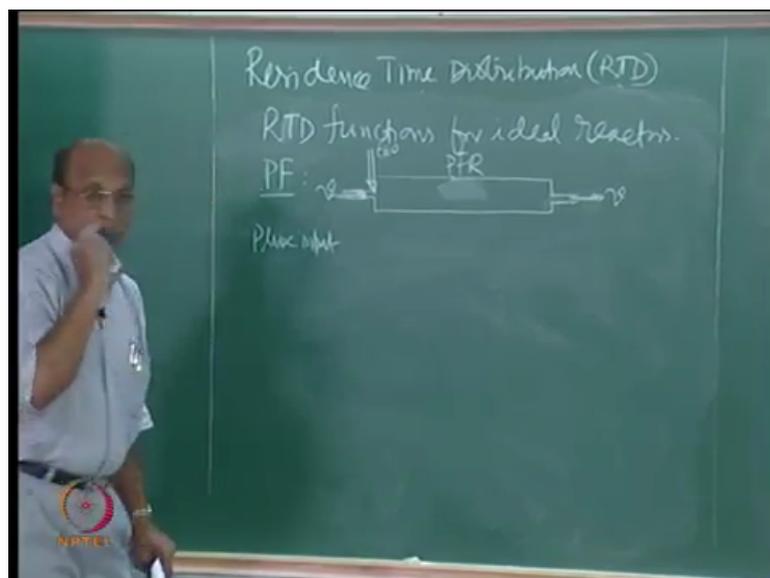
N I know 10, now I can calculate c_A by c_A not which is nothing but $1 - x_A$. So conversion I can calculate. That is the reason why we are interested in this sigma theta square, sigma t and all that. Similarly d by $u L$, okay, axial mixing I am trying to find out in terms of d by $u L$. Now d by $u L$ may be point 1. If I have that much d by $u L$ how do I calculate conversion? That is not that easy the way 1 by $1 + k \tau$ to the power of N . There is a more complicated equation which I will tell you later.

So that is the equation which you have to use for calculating conversion. That is the connection between RTD and conversion equations, right? So first find out whether the reactor is ideal. If it is not ideal you do not know what is the axial mixing. Conduct the RTD experiment. Find out what is the dispersion and to find out what is the d by $u L$ you need this equation, right?

So once you know this equation then you can go back estimate what is d by $u L$, go to that conversion equation, that conversion equation has d by $u L$ because you are assuming it is axial mixing model. Substitute there and then calculate. That will come a little bit later but now itself I am giving you the introduction, okay. Good. So these are the things now. So what is left now is let us now derive the RTD functions for the ideal reactors.

Ideal reactors we have mixed flow and plug flow, so these two, okay. RTD functions for ideal reactors, ideal reactors first let us take PF plug flow reactor, okay. Plug flow reactor, this is now ideal plug flow reactor. Yes good. So we have here volumetric flow rate v , volumetric flow rate v coming out. So yes this is PFR. Okay let me write here PFR. First let me conduct the pulse input experiment. So pulse input is a direct delta function which is simply entering here. This is at time t equal to 0, okay.

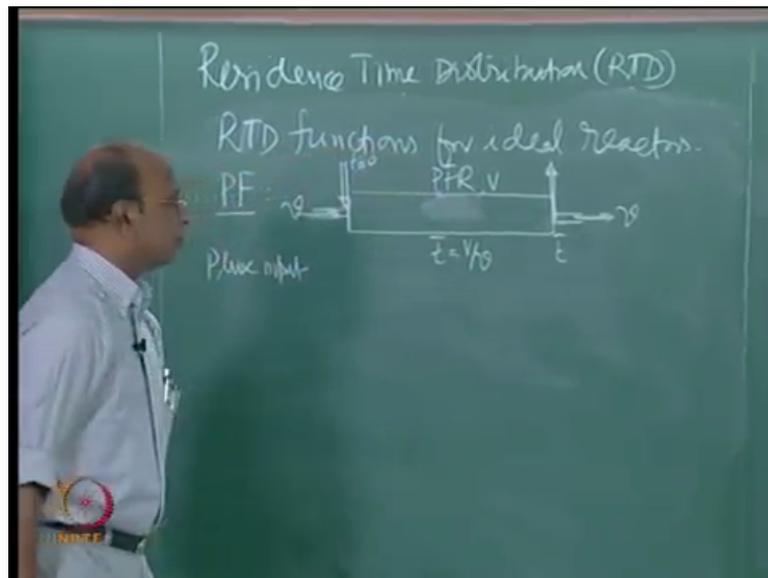
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How do I inject that? Because I have to very carefully take without any disturbance some syringe or something and then just inject which will go and then sit down throughout the cross section that means your disc. That is what is the ideal one and in reality it is not so easy, okay. So that is why you have to have without any disturbance simply this ideal pulse that is 0 thickness and infinite height and area under the curve equal to 1.

The same pulse you have to just inject at time t equal to 0 and now here yes okay this is t bar equal to volume by volumetric flow rate. This is volume V so at this point it just comes out just for representation this is at t bar because it is ideal pulse disc, okay.

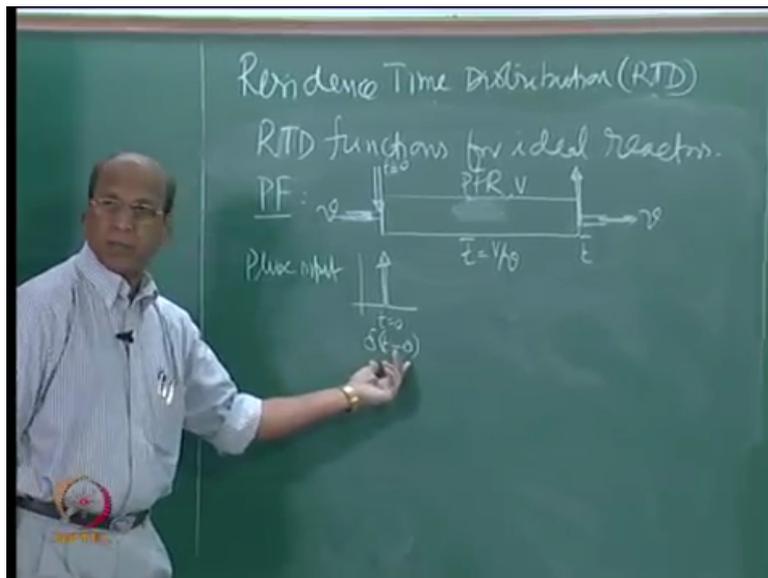
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Disc you cannot imagine as a pulse because you know thickness should be infinity and all that so there is a very thin colour slide just inside, okay. Nanometre thick disc which is centred and we know that because it is ideal without any disturbance that nice beautiful very thin red colour will be moving without any disturbance till here and that also exactly t bar equal to volume by volumetric time it will just come out.

If that equal to 10 minutes or 10 seconds it will exactly come out at 10 seconds. Now how do I represent this as an equation because there is no derivation here for this, right? So here the direct delta function is entering something like this but this is t equal to 0. How do I write an equation for this? This is $\delta(t - 0)$. If I can call this one as $E(t)$ this at 0 time, $E(t)$ normally we have to only write here, okay. $E(t)$ only we have to write there but that is why I think I will simply mention that the mathematical function is $\delta(t - 0)$. What is the meaning of this Abhijeet?

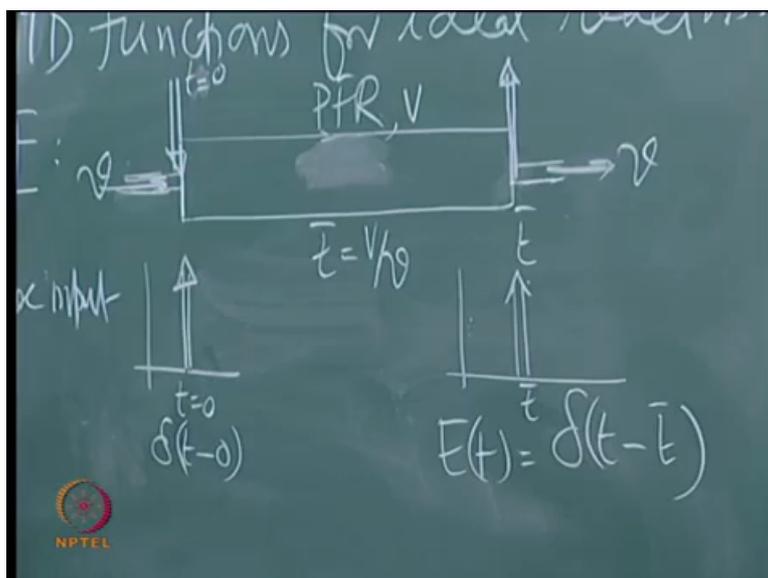
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Student: Pulse input.

Professor: It is a pulse input definition valid only at time t equal to 0. So now what is the equation here? So the same output that is here it comes out at t bar. This we can call now as E t because that is the exit age distribution function. Actually it is the distribution function but only mathematical definition is thickness equal to 0, correct and height equal to infinity and area under the curve equal to 1. So that is why here we have E t at the exit, δt minus t bar. So what you have to remember here is this is valid only at t equal to t bar and then all other times it is 0, right?

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So now I can imagine the same thing with theta. Theta equal to t by t bar. So now how do I write that here? This is delta theta minus 0, right? And here E theta equal to?

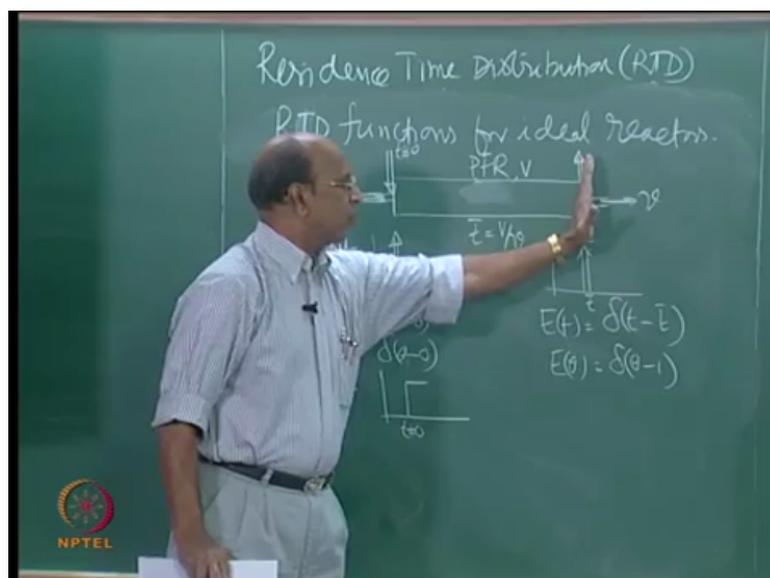
Student: () (47:34)

Professor: Excellent, theta minus 1, right? So this is delta theta minus 1 because we know in this case t by t bar equal to 1, right? Yes because all the material is coming exactly and this function is valid only at theta equal to 1. And all other times what is E theta? 0, right? Yes, so that is the one. And of course E theta now definition is that direct delta function where it has got area equal to 1, thickness equal to 0, height equal to infinity. So that is why this is very easy for us to discuss, no problem at all.

So next one is step input. For step input how do I represent step input in pictorially? Here I have normally we put like this no? This is at t equal to 0. And now what will happen? So now I switched over from white fluid to red fluid, right? So then actually if you have ideal plug flow doing that experiment will be really beautiful and if you like colours, okay.

White fluid and red fluid and like piston the white fluid will be simply displayed until this point and then finally here you will have only red fluid and white fluid you will not have any trace of it, right?

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Yes, so when I see at t equal to t bar by 2 this one 50 percent all this side will be red, all this side will be beautiful white, right? So after sometime if you go to point 75 then this will be red and that will be white. So that is why we call piston flow also. Plug flow, piston flow that

name has come because of that. It just simply pushes like a piston. You know no you would have seen how the piston is moving in some missionary.

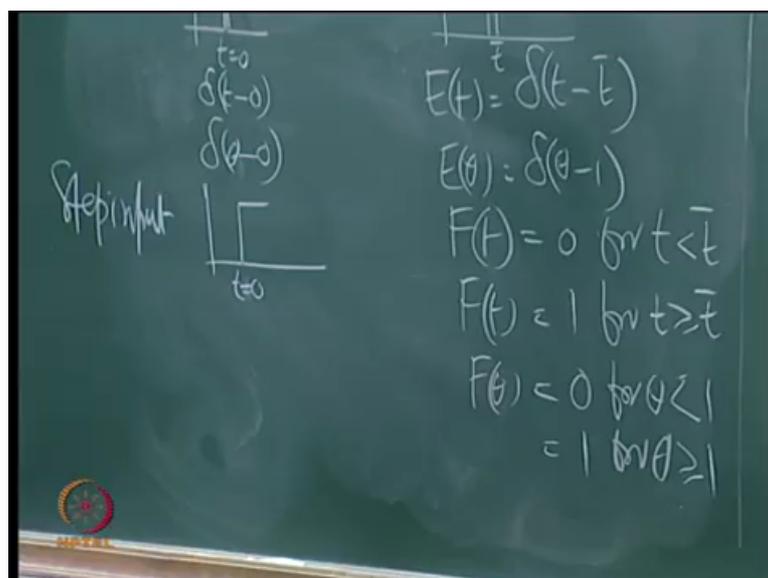
So there it beautifully goes forward and very smoothly without any sound and all that, particularly old rail engines, okay. I think new rail engines also have the piston I think. All engines have that, okay. So that is what piston flow and the piston ending point must be exactly like flat velocity profile then only it is moving like this. Of course we are only talking one forward direction so it moves like a piston.

Exactly that is what is happening here. Exactly 50 percent of the time I have a red piston simply pushing the white piston outside, okay. So now what is the equation we can write here? If I am writing in terms of time $F(t)$, yes what is $F(t)$? Okay, I will give the proof $F(t)$. If t less than \bar{t} what is $F(t)$?

Student: 0.

Professor: 0, so this is $F(t)$ equal to 0 for t less than \bar{t} . So now $F(t)$ equal to 1 for t greater than equal to \bar{t} . Similarly now $F(\theta)$, what is $F(\theta)$ Swami? 0 or less than 1 and the other one is 1 for θ . That is all the equations. It is very simple, okay. But only thing is understanding delta function.

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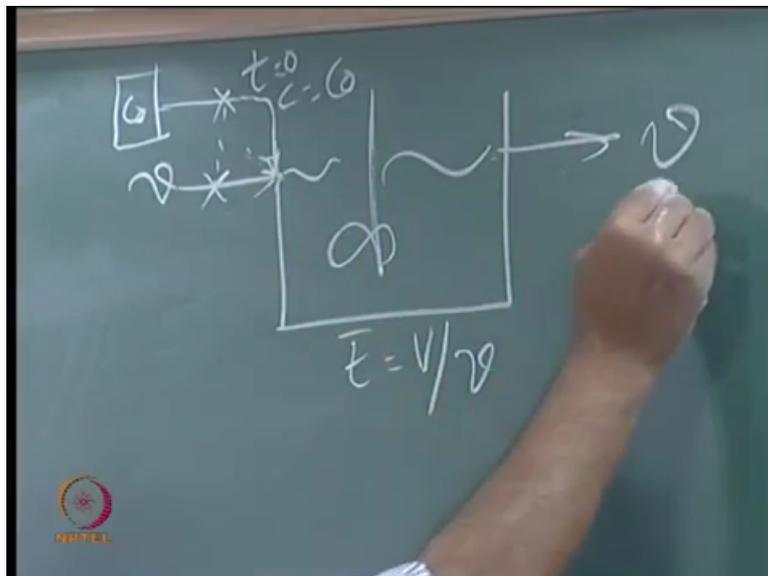


Now let us take mixed flow MF, okay. So conducting experiment for MF also is very easy, okay. So let me take step input here and write the step input equation and I will ask you to derive for pulse input, okay. You have to write the material balance and then not

differentiating you know $e F$ you will have an equation so if you differentiate that equation you will get also $E t$, not that. You have to write the material balance for tracer and then only derive. It is not that easy, okay.

So that is why I am telling you but I am taking this also both are equally complicated or both are equally simple so that is why $F t$ let me take that is step input. So this is volumetric flow rate, volumetric flow rate. So this is here I have tank here with c not. Yes, so both are anyway connected here. So once this is opened that is closed, this is opened this is closed. So at time t equal to 0, c equal to c not it is entering and immediately after that you will see some concentration coming out, right? Good.

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So now after some time t it is a step input. That means continuously tracer is entering whereas that is the difference between this and the other one. So when pulse input only at time t equal to 0 you add 10 grams of tracer suddenly here. Anywhere you can add because if it is ideal perfect mixing everywhere it will be mixed instantaneously, right? Yes, so then onwards no more tracer input but here in step input, okay, step input I have to write, okay.

For step input we have continuously entering so now let us write tracer balance input equal to output. This is universal equation, plus accumulation plus reaction, okay. What are the terms I can remove there?

Student: Reaction.

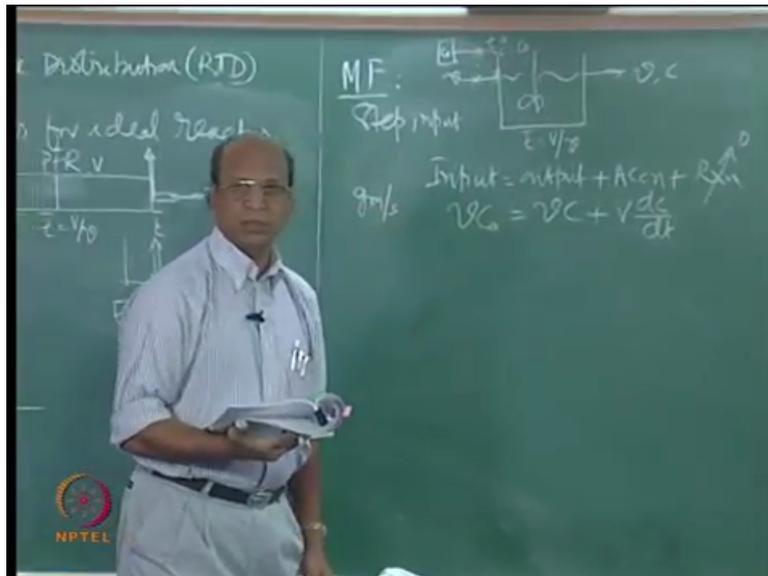
Professor: (())(54:16)

Student: Reaction.

Professor: Yes, only reaction but why not accumulation? As far as flow is concerned it is steady but as far as tracer is concerned it is unsteady. Tracer is unsteady state. It will become steady state once the entire thing is removed then finally you have only $c = 0$, okay. So that is why what you are writing for the tracer. Tracer is unsteady state. That is the reason, okay. So now this is 0. What is input in terms of grams per time? Let us say grams per second.

This is v into c not continuously entering, correct no? What is coming out after time equal to 0? So v into c continuously coming out and what is accumulation? V into $d c$ by $d t$. We are writing simply c because there is no A and all that, there is no reaction. We are not talking about reaction of certain compound, okay. Good. So now this is what we have to solve.

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So it is a first order differential equation and it needs at least one boundary condition. What is that logical boundary condition you take?

Student: () (55:46)

Professor: Yes, $B C, c$ equal to 0 in the outlet only, okay. Yes and those people who do not believe this you try with at t equal to 0 as initial condition and c equal to c not. You will not get a solution. That is why mathematics and sometimes physics will not accept, okay. Yes, because mathematics and physics needs a chemistry between them, okay. So that is why, okay. Good.

So now I think I will ask you to solve this, right? Or one more step shall I give? You can solve it no? Excellent, very good. Yes someone already solved here? Yes, c by c not equal to $1 - e^{-t/\tau}$?

Student: Minus t by τ .

Professor: Yes, minus t by τ or here we are using \bar{t} . That is what and by definition of c by c not what is that? $F t$, because we already proved that for $F t$. So this is also equal to $F t$, good. Yes, so now what is $F \theta$? We know that $F t$ equal to $F \theta$. So this is $F \theta$ equal to $1 - e^{-\theta}$, okay. From this you can calculate what is $E \theta$. What is $E \theta$? $E \theta$ equal to, Rahul?

Student: E power minus θ .

Professor: E power minus θ , okay. Yes, and I feel you know this is one of the simplest model equations in chemical engineering. It is representing a model. What is that model? Mixed flow reactor, okay. RTD function. So you see so beautiful very simple, need not be always complicated equation when you say mathematical model, right? This is also a mathematical model for CSTR or mixed flow reactor, good.

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The image shows a chalkboard with handwritten mathematical derivations for a CSTR model. At the top left, it says "Step input" with a small diagram of a step function. Below that, the mass balance equation is written as $g_m/s \quad Input = output + Accn + R_{xn}$. The next line is the differential equation $vC_0 = vC + v \frac{dc}{dt}$. Below that, the boundary condition is given as "BC $t=0, C=0$ ". The concentration response is derived as $F(t) = \frac{C}{C_0} = 1 - e^{-t/\tau}$. The cumulative distribution function is given as $F(\theta) = 1 - e^{-\theta}$. Finally, the residence time distribution function is derived as $E(\theta) = \frac{dF(\theta)}{d\theta} = e^{-\theta}$. An NPTEL logo is visible in the bottom left corner of the chalkboard image.

Yes now so if I plot this you will get this kind of response here like you know this one, okay. Here also I should have plotted for $F t$. For $F t$ you should have got this is θ equal to 1. So this is $F \theta$, this is 1, okay. And this also of course t also you will get at t equal to \bar{t} that also equal to, this is how you have to get the response from the outlet.

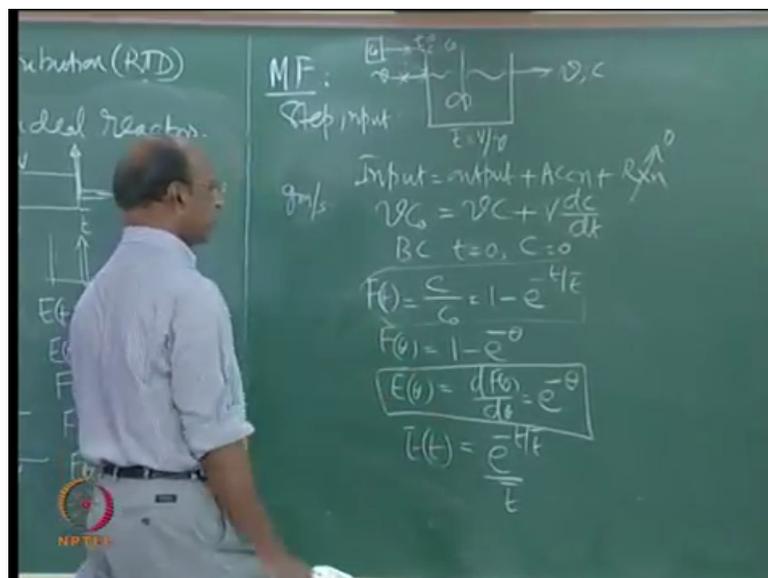
Similarly here how do I plot this response $F(t)$? Because now I am plotting this equation $F(t)$, okay. $E(t)$, what is $E(t)$? Accept or you do not accept? $E(t)$ will be $e^{-t/\tau}$. This is what is $E(t)$. I think Sushmita looks so tired. Most of you look tired except me.

Student: (0)(59:31)

Professor: Yes, but you know that is not excuse. I have one more thing because the moment I saw slight consideration immediately I think you know yes and all. No this is one last part is there. That also takes quick time. See 12th I can take the class and 13th is holiday (unfo) unfortunately. For me unfortunately because I do not have any other work except this. So then again 14th we do not have class. So I have to take only class on 15th then.

So that is why I am trying to finish as quickly as possible next week itself, okay, otherwise maximum Monday, right? So that is the reason why I am just pulling and today I taught I think whatever I thought I have to tell because once you know all this then you will have a nice way of finding out like doctor. I will give some curves, you have to tell me answer. What is the disease with this particular equipment? That is how it is only diagnostics, right? Okay, that will come later. So now let us plot this one.

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$F(t)$ when I am plotting versus t only so it is an exponential curve $1 - e^{-t/\tau}$ so it will be something like this, okay. Abdul, when it reaches infinity? When $F(t)$ reaches 1?

Student: Infinity.

Professor: Answer I already told. He has question paper answer before you know examination, okay. Yes so when t equal to infinity only then it becomes 0 then you have $F t$ equal to 1, right? Okay good. Now I will ask you another question. Within 1 t equal to t bar what is $F t$?

Student: 63 point 2 percent.

Professor: He seriously started calculating, okay. Good. So this is t bar by t bar so this may not be exactly scale but anyway point 632, okay. So what is the meaning of that? What is i funda, what is decay, what is initial rate? Totally I think trigonometry is always high brain I think. It is not easy to understand. Tell me.

Student: (01:02:30)

Professor: So?

Student: (01:02:35)

Professor: What rate you are talking? I am simple fellow, my question is very simple. What is the meaning of that point 632 at t bar?

Student: (01:02:52)

Professor: That point 63 is $F t$, not time.

Student: (01:03:05)

Professor: What is the definition of $F t$ I say? What is the definition of $F t$?

Student: Fraction of material that has stayed.

Professor: Fraction of material which has stayed at time?

Student: 0 t .

Professor: T that means what is the straight forward meaning is 63 percent of the material is spending only one mean residence time, correct no? So the remaining 30 percent what is happening?

Student: (01:03:35)

Professor: More than that, okay. Yes it is taking you know. So that is the one of the reasons again because of the residence time distribution and whereas what is this \bar{t} ? How much material come in a plug flow reactor? What is $F t$ for a plug flow reactor, ideal plug flow?

Student: 1.

Professor: 1, that means all the molecules are exactly spending same time within that \bar{t} and the conversion in each and every particle, I am repeating many time same thing, okay. So each and every particle is exactly converted so the final average conversion also is same for all the packets or particles, okay, in a plug flow reactor. Whereas here this 63 percent is coming in the first 1 mean residence time, not only that, initially something is coming.

And afterwards some time some more material is coming. After some time some more material is coming, right? So that means I can now calculate what is the fraction of material coming between time t plus Δt or otherwise between 0th minute and first minute. 0th minute and first minute what is the material that is coming? How do you calculate that? Okay, this is $E t$ versus t . How do I plot this? Actually I have to plot, okay. $E t$ versus t how do I plot? Where it has to start decreasing?

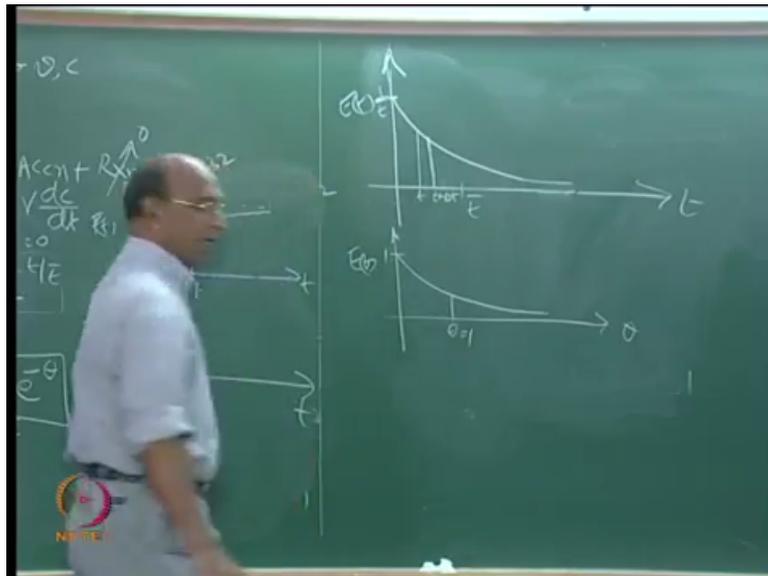
Student: 1 by \bar{t} .

Professor: That is very important. So at t equal to 0 this is nothing but 1 by \bar{t} and then exponential decay. And Abdul, what time it reaches 0?

Student: Infinite.

Professor: Infinite, okay. Now I can calculate from here the \bar{t} is somewhere here, okay. What is the fraction of material between some time t and t plus Δt ? Yes, okay this is what is the advantage of plotting this one as $E \theta$ versus θ . Yes, now what is the starting point for $E \theta$ versus θ ? So one starts and then goes like this. So this is θ equal to 1, okay.

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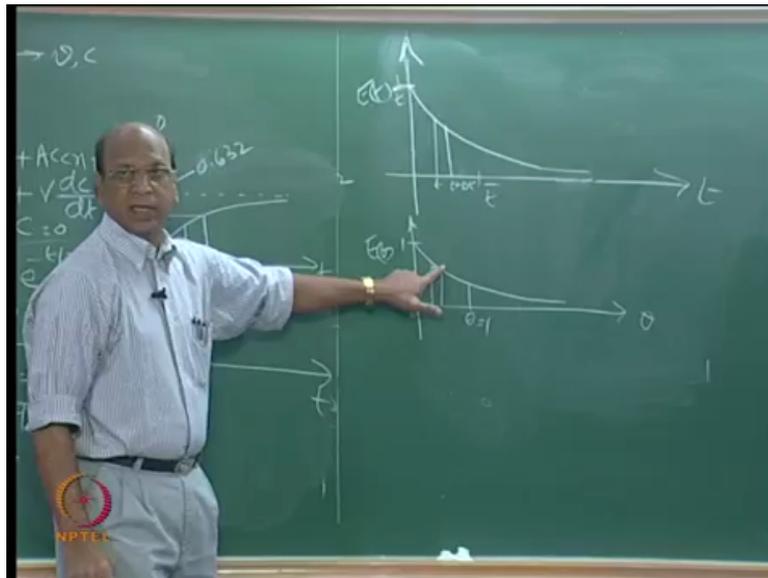


Now can you tell me between theta equal to point 2 to point 3 what is the fraction coming out? You have equation I say, $E \theta$. $E \theta d \theta$. What did I say? Between point 2 and point 3 so the average time is what? Theta equal to point 25. Delta theta equal to you have calculator. You have to calculate $e^{-0.25}$ exactly into point 1.

Student: (01:07:47)

Professor: Point not 77 you got? That is the percentage of material that is coming between this time 10 minutes, okay. So now I do not know whether you are appreciating you know why we have to make dimensionless (01:08:11). So I do not have to mention now any timing, okay. So theta equal to point 1 to point 2 that is point 1 interval and anyway the average I have to take because that means we are only trying to find out something like this. That is why that average you know this equal to this, okay.

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So this value is around how much you said, 7 percent, okay. Okay, 7 point 8 percent. Almost 8 percent of the material is spending between point 2 to point 3 theta, okay. So like that I think we can calculate at whatever fraction and correspondingly we can calculate what is the conversion in that and take the average of all that, then you will get the average conversion from that, okay. Good. So now the next one is laminar flow. Ideal reactor is PF and also MF.

I think to divert you I think I will dictate something for laminar flow. Laminar flow I think Levenspiel has done soon beautifully, okay. Please take this. When a vessel is long enough then the dispersion or tanks in series model well describes its flow behaviour. But how long is long enough? This is where words you know always we use long enough, okay. But how long is long enough, okay? Then next, for packed bed and for turbulent flow in pipes just about any vessel length is long enough.

However for laminar flow pipes we may be in other flow regimes in particular that of the pure convection model, okay. Let us first look at the extremes for laminar flow, okay. Good, 1, I think you can put 1. Say that if the tube is long enough, we are talking about only laminar flow now, then the molecular diffusion in the lateral direction will have enough time to distort the parabolic velocity profile so that the dispersion model applies.

Now second below, next point. If the tube is short enough and the flow rate is high then the molecular diffusion has not enough time to act. So all we need to consider as causing a spread in residence time of fluid is the velocity profile. We are in pure convection regime. Third point is, if flow is so slow that the main moment of fluid is by molecular diffusion not by

bulk flow then we enter the pure diffusion regime. We rarely meet this situation in chemical engineering. Good.

I think next para you write. Gases are likely to be in dispersion regime not in the pure convection regime. Liquids can well be in one regime or other. Very viscous liquids such as polymers are likely to be in the pure convection regime. Yes, next para.

The pure convection model assumes that each element of fluid slides past its neighbour with no interaction by molecular diffusion. Thus this spread in residence time is caused only by velocity variations. This is in fact beautiful information I am sure none of you would have seen this, okay. Yes, I do not know at this point of time you may not appreciate in the class but I think you know this is wonderful information.

What is entire thing is told is that we always assume that we have beautiful parabolic velocity profile. You know the life of this beautiful parabolic velocity profile is only in seconds. If it is gases immediately after the formation of that profile even at low flow rates molecules starts moving up and down radially, laterally and also you know axially, all that. So that means it is no more a parabolic profile. If you do not have parabolic profile you do not have laminar flow.

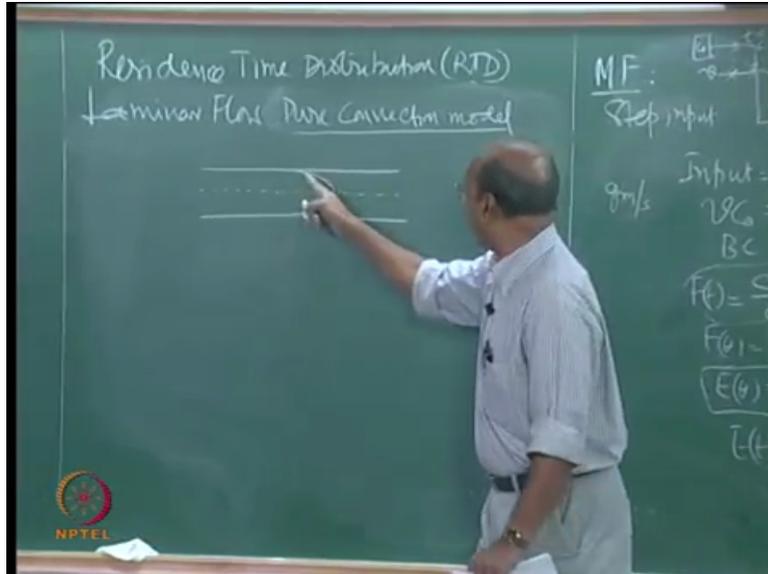
That is why laminar flow is called pure convection model because there is no molecular movement. Once that laminar flow starts like the basic definition everyone tells in fluid mechanics class is imagine that sliding of pack, you know pack of cards. That is what we say. That is also told here. So that means without any disturbance with the card below it is simply sliding without any disturbance. That will happen only for high viscous fluids just for some time.

So when you have that kind of clear definition for laminar which is called also pure convection model, the residence time distribution is only because of profile. Otherwise if there is a molecular movement so that also will disturb this profile and your residence time distribution will be disturbed. That is why this is a very restricted case which is called pure convection model. This I think you would have not heard. Anyone heard of this? I do not think anyone of you have heard of this.

That pure convection model is only possible with (pa) parabolic velocity profile and when we are talking this one you know in polymer this comes as a reactor. In polymer processing laminar flow reactors can be used. So that is why we need to know what kind of residence

time distributions we get and those residence times are purely by velocity variations if you have parabolic velocity profile like this, right?

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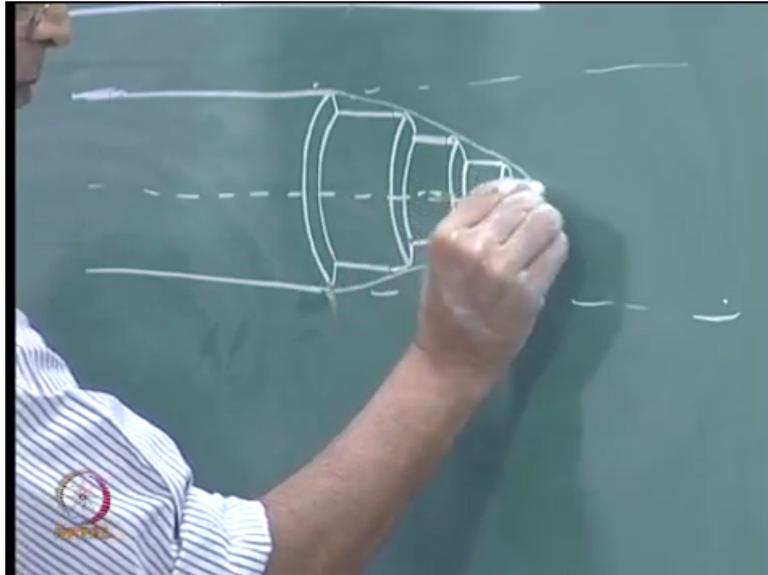


That is why for gases it is not possible and for liquids with less viscosity and all that also that will be disturbed very quickly, right? And what happens is if you allow that for some more time, if you have sufficient length, you can use normal dispersion model because by the time molecules arrange themselves it is flow, it is convection. Yes, it is not pure convection. By convective flow material is moving and molecules are getting dispersed because molecules are moving up and down laterally, radially and all that.

So you will get this kind of normal distribution, right? So how do we now develop an RTD equation $E(t)$ for this one $E(t)$ and $F(t)$? Shoji, what can we do? Can we change the 5 minutes? They will also relax, okay. Yes, because let me complete this I think you know again. Before that I think we can also draw another figure which will nicely represent the same thing. You can draw this figure.

That is actually the meaning of laminar flow. It is like one pipe inside, over that another pipe, over that another pipe and all of them are moving like that without disturbing the other pipes. That is what we say the pack of cards, okay. And I think this is very important information. None of you definitely would have known this kind of information till now. And the centre is somewhere here.

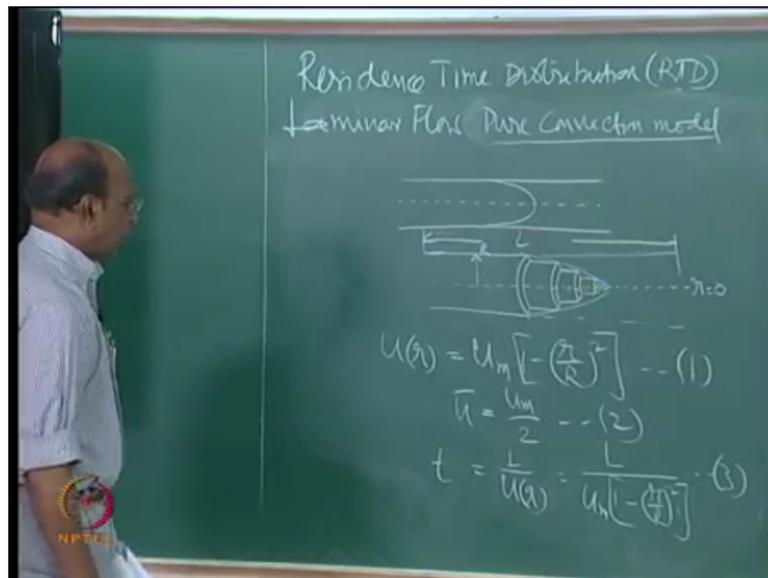
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So that is what is this profile meaning, parabolic profile, okay. So the material which is there on the tip of this will go much faster, okay. Yes, so now when you want to derive an equation for this we need some information about fluid mechanics. So this total length is L , okay. Shoji you can continue? Okay yes, changed already, okay. So this is L and of course this is r equal to 0 . This is total, okay. This is capital R . Yes, so we have an equation and all of us know about this is that we have u r equal to u m maximum velocity, yes 1 minus r by R whole square.

This is equation 1. Let me put the equations for this. So we also know u bar average velocity equal to u m by 2 . That is equation 2, okay. These things are from fluid mechanics. And we also know that t time is nothing but length by u r , correct no, time length by velocity. And this is also equal to L by, u r is this, u m 1 minus r by R whole square, correct. Yes, this is equation number 3. Yes.

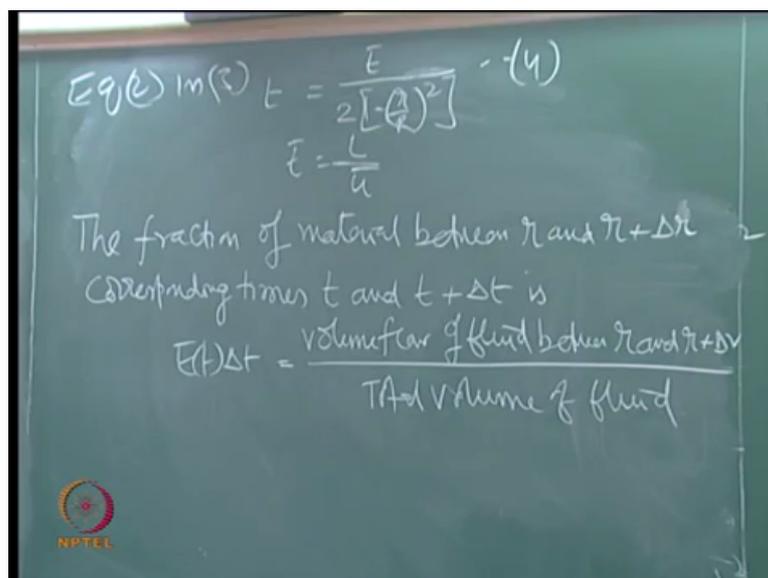
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Now substituting this equation 2 in 3 what you get is t equal to t bar 2 1 minus r by R whole square. Yes, I mean of course where t bar equal to mean residence time. Yes, how t bar is defined? This is equation 4. Yes, so t bar equal to L by u bar. That is the equation. So now this important thing what we have to understand. Afterwards it is only mathematics.

The fraction of material between r and r plus Δr corresponding to times t and t plus Δt is $E(t) \Delta t$ given by volume flow of fluid between r and r plus Δr divided by total volume of fluid, okay. Let me explain this. The definition of $E(t) \Delta t$ we know. That is the fraction of material spending a time between t and t plus Δt .

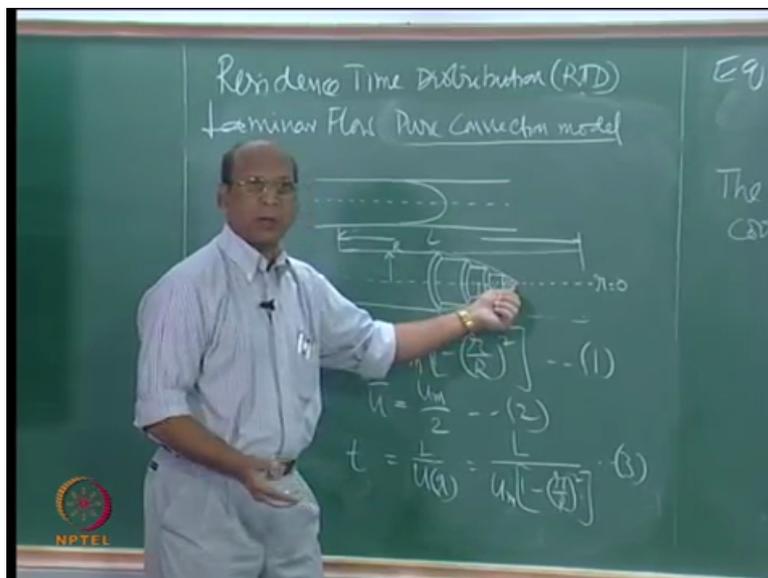
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Now this diagram will give you a nice picture there because when I have some r here from centre, this is r and maybe slightly Δr is somewhere here, okay. So you can also imagine this r and Δr is a small pipe with a thickness of Δr , right? So, depending on its position here that is coming. You know if you take this one as Δr that is coming the fastest, right?

So, this also corresponding to some time that means when this is coming earlier so that is also corresponding to sometime earlier sometime t . That means each concentric pipe which is coming at different r will come here at different times. This one will come in the shortest time. What is that? T bar by 2.

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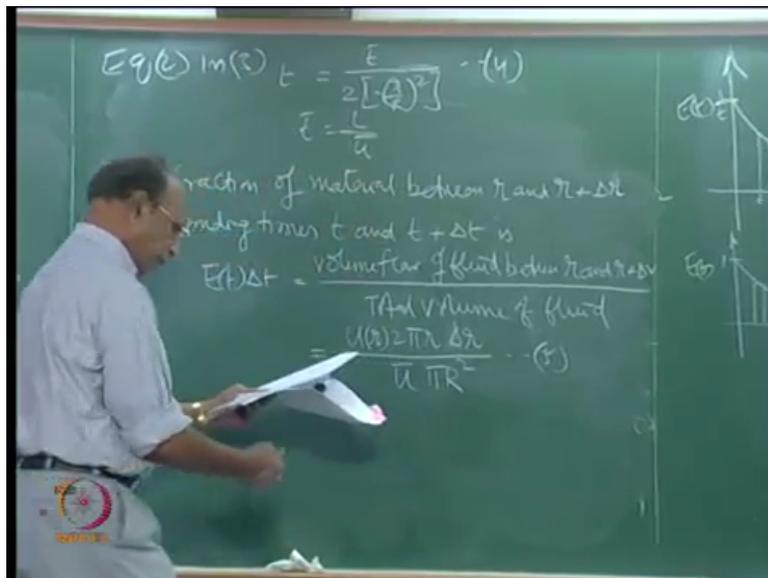
Somewhere middle if I take that will take some other time. And somewhere only small Δr if I take near the wall that takes lot of time, right? So that is what is the simple definition here. That means there is a relationship between r and t and r plus Δr and t plus Δt . So in this flow when I am imagining that these are all the concentric tubes, this tube and minus this tube so that is the fluid.

This is one tube and around that we have another imaginary pipe and that volume of that one will come in a different time corresponding to its own velocity. Why? Velocity is changing along the radius, correct no? Different velocities at different radial positions. That is why they spend different times. So that is what is this and that means if I take one annulus that is the volume of fluid between r and r plus Δr which is automatically connected with some t and Δt .

How do I get that t and r connection we will deal later. This is the basic definition, okay. So now if I write the equation for this, this $E \Delta t$ will be. What is volume flow of fluid between r and $r + \Delta r$? If you remember your fluid mechanics earlier it will be $u \cdot 2\pi r \Delta r$, okay, or Δr , right? I can also write Δr .

So divided by what is the total? Total flow rate is simply $u \cdot \pi r^2$, correct no? That is the volumetric flow rate. Cross sectional area multiplied by this u . This is u , right? Okay. So this is equation number 5, okay.

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Of course $\pi \cdot \pi$ can get cancelled. Good. So now another thing that is left to understand is this equation 4 actually we have already the relation. The equation 4 relates r and corresponding time, correct no? Yes, r and corresponding time. So now you can differentiate equation 4 with respect to t so to get $\frac{dr}{dt}$, okay. I think you know with respect to t I do not have to write, okay.

$\frac{dr}{dt}$ or $\frac{dt}{dr}$ also we can write. So differentiate equation 4 like this. $\frac{dt}{dr}$ equal to, can someone differentiate quickly? $\frac{dt}{dr}$ by $\frac{dr}{dr}$. $\frac{dr}{dr}$ you can bring that side. At least one person if you differentiates and tell me. Rashmi, quickly. Yes tell me. Sorry (())(01:25:00).

Student: $\frac{dt}{dr} = \frac{1}{r}$ minus r by R .

Professor: $\frac{dt}{dr} = \frac{1}{2r}$.

Student: $\frac{dt}{dr} = \frac{1}{r}$ minus r .

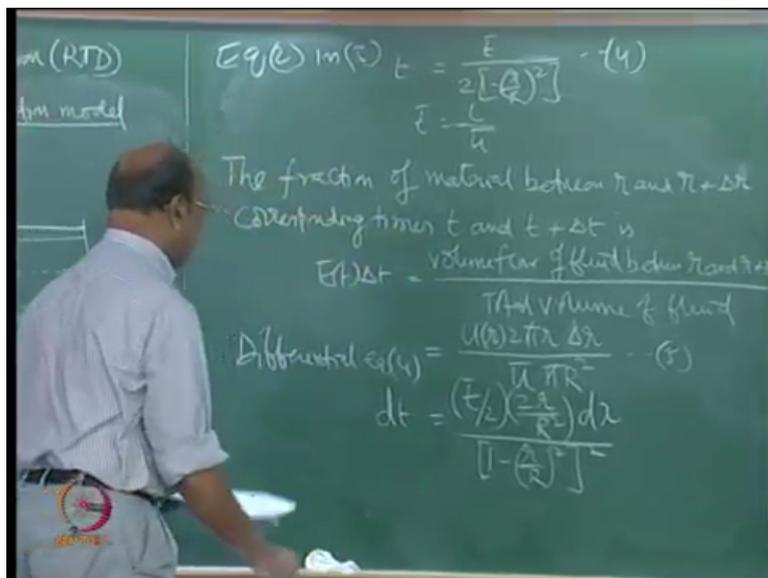
Professor: Okay, t bar?

Student: R by r into r by R.

Student: T bar r by r square 1 by 1 minus r by r whole square.

Professor: Excellent, that is correct thing. So I mean many people would have not tried I know because you are tired so what you get for d t is now I am just writing this one t bar by 2 so that you know later for me it is little bit easier. Actually what he said is right. And also I am writing 2 r by r square, correct no? What he said is this 2 this 2 will get cancelled, okay. That is what you said. And then d r divided by 1 minus r by R square and whole square. That is the one. That is d t, okay.

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Yes, and from equation 4 what is t by t bar? From equation 4, 2 t bar by 2 equal to 1 minus r b R whole square, correct? Right, 2 t bar by t equal to this is one. This is equation.

Student: T bar by 2 t.

Professor: T bar by?

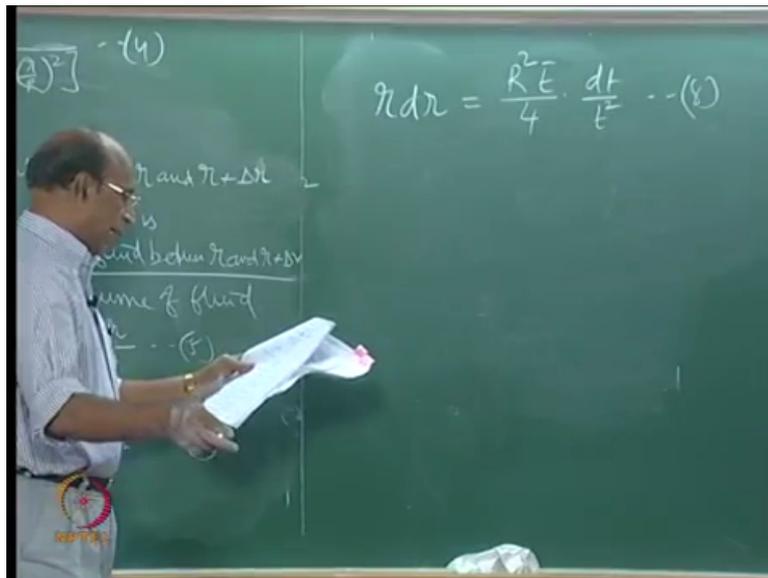
Student: 2 t.

Professor: Yes, t bar by 2 t. Yes, you are right, equal to this 1 minus r by R square so this is equation 7, okay. So using now equations 7 and also 6 and substitute here in this equation. Oh this is 6 yes, okay. E t delta t you have to use. Yes I am just thinking about your timing you

know whether you are able to take otherwise I have, okay. I think let me do once we start doing that, okay. Okay please write this.

Substituting equation 7 in 6 and rearranging what you get is $r dr$ or Δr , okay. I think let me write dr only. Δr , dr same so I think here also I will write dr only. $R dr$ equal to capital R square t bar by 4 into dt by t square. So can someone check? This is equation number 8, correct.

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So now another thing is now substitute equation 8 in 5 in this equation. Substituting equation 8 in 5 what you get is $E t dt$ equal to u by u bar, t bar by 2. So this is equation number 9, okay. Okay so we also have equation from 1 and 2. From equation 1 and 2 what you have is $u r$ by u bar equal to 2 into 1 minus r by R whole square. So this is equation 10. Good. So now 7 and 10 can I combine what do I get? $U r$ by u bar equal to t bar by t .

So from equation 10 and 7 what you get is $u r$ by u bar equal to t bar by t , yes. So this is only just for converting one from the other, okay. So this is equation 11. Can you tell me substituting 11 in 9? Yes, so Ramakrishna what did you get?

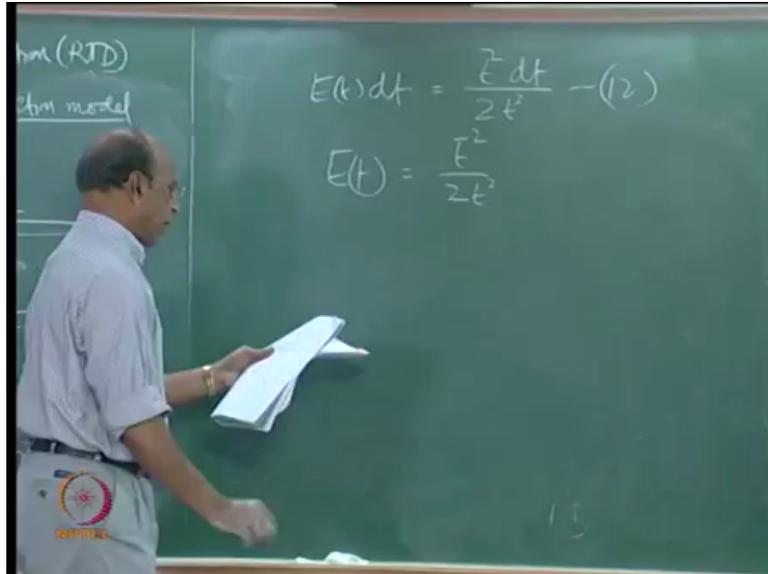
Student: (())(01:30:07)

Professor: $E t dt$ equal to t bar square dt by 2 t cube. This is equation 12. Now what is $E t$?

Student: T bar square by 2 t cube.

Professor: T bar square by 2 t cube. So this is the equation for laminar flow E t exit stage distribution function.

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And yes so this is equation 13 but I think before writing 13 there, is there any limitation for this? When it is valid? When it is not valid? When is this valid? If I answer that I think that, okay you are still alive. It is valid only between 0 to t bar.

Student: (())(01:31:07)

Professor: Yes, you know you start seeing the fluid only after t bar by 2 because you know from fluid mechanics the average velocity is twice of the maximum velocity, right? Correct no?

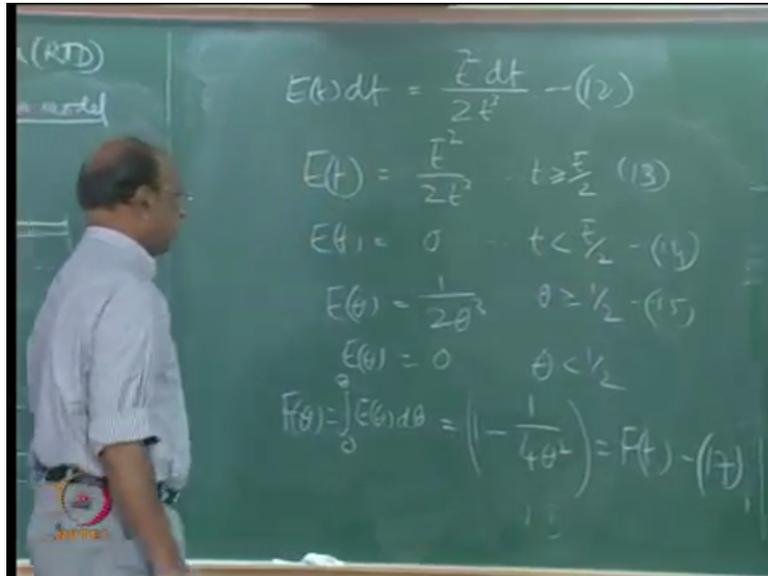
Student: No, half the maximum.

Professor: Half the maximum velocity yes, okay. U bar equal to half the maximum velocity so that is why t bar by 2 is the first time what you see the tracer coming out, okay. So that is why the limitation here is this is valid only for t greater than or equal to t bar by 2. And E t will be 0 for t less than t bar by 2, okay. Now tell me E theta. E theta is again one of the very beautiful equations. this is 14. 1 by, excellent, 1 by 2 theta cube. This is another equation which I like.

The first equation is E theta equal to e power minus theta for mixed flow reactor, okay. So this will be good. So this will be again validity theta greater than or equal to half or 1 by 2. So this is equation 15 and E theta equal to 0 for theta less than 1, okay. So we can also integrate

this and then try to put the limits because F theta equal to integral 0 to theta E theta d theta. So this will be $1 - \frac{1}{4\theta^2}$. This will be F theta which is also equal to F t. So this equation is 16. That is equal to 16, okay.

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So maybe at this point of time you are not able to absorb but the derivation is not at all difficult, okay. But only algebra comes after that definition of writing volume of the fluid between r plus Δr by total volume that is all. Afterwards it is only mathematical manipulation where you can easily find out. But only I just want to ask you how do I draw this E theta versus t or E theta versus theta.

Yes, you are very good yes. It is not 0, okay. Yes, it is valid only from half to theta, okay, upper limit. Then only you get that otherwise you do not get that, okay. Good. So how do I plot this E theta versus theta? Where does it start?

Student: (01:24:21)

Professor: Theta direction it starts with half, yes. So till what point I have to go? What is this starting point?

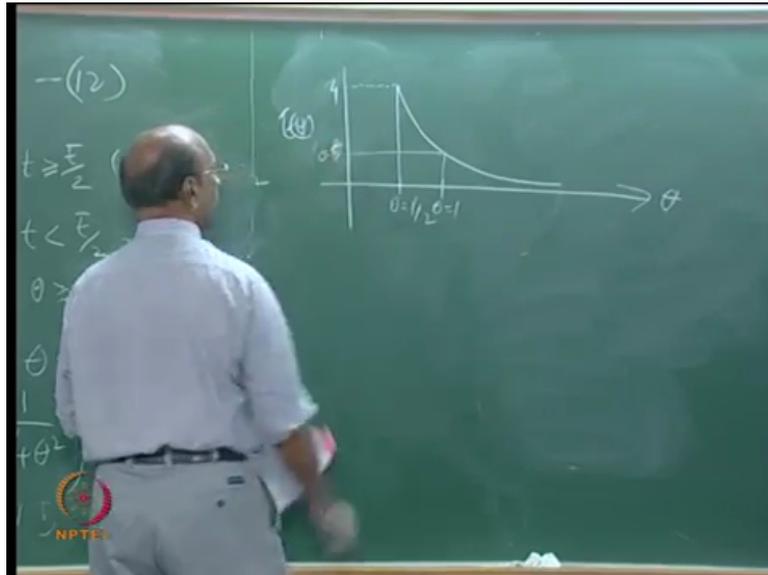
Student: 4.

Professor: Excellent. I think still some people are alive, okay. This is 4. Yes then afterwards how does that go? Decreases as 1 by x cube. Yes, 1 by x square, 1 by x cube, 1 by x we have no. 1 by x cube no 1 by theta cube. So this falls very fast and goes like this. This is not

exponential. Why I am drawing now is that this is not exponential. That is also decreasing but that mixed flow is decreasing exponentially down.

E power is simply minus theta. But this is 1 by 2 theta cube, okay. So that is why it comes out here. So somewhere here you will have theta equal to 1. What is the value of E theta at theta equal to 1? Yes somewhere here point 5, okay.

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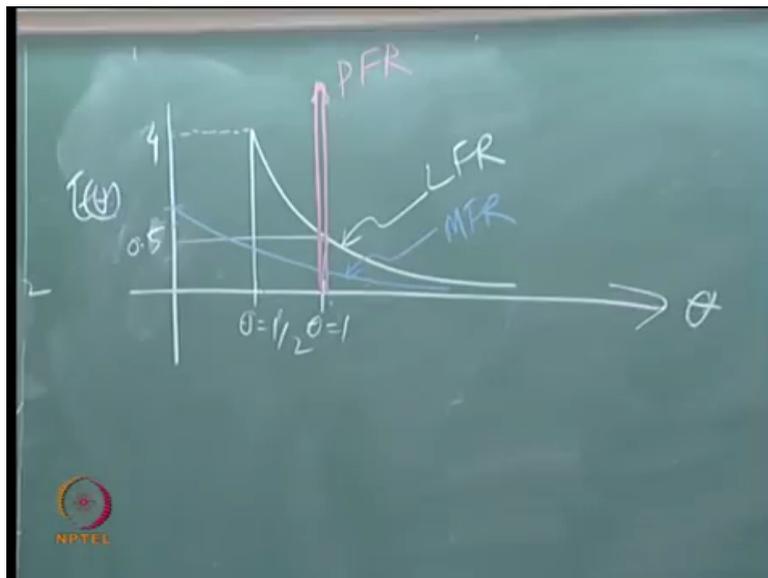


So now you see and the same thing if I just show you like for plug flow and the same graph I just want to plot plug flow, right? E theta how do I plot? What Swami? Still this fellow is not leaving. Yes, so E theta for plug flow. Theta equal to 1. This is for PF, okay, at theta equal to 1. And for mixed flow.?

Student: E theta equal to 1 exponential (0)(01:36:50)

Professor: Yes, at theta equal to 1 somewhere here it goes exponential decay. So this is MFR, that is PFR, okay, and this is. See all three ideal things are there, okay. Yes, but only one thing is left I think what I thought today doing.

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You know now that diagnostics maybe I think we will do in the next class because I think maybe it is too much for you. This also I do not know how many people have absorbed that but God knows it may be in third quiz.