

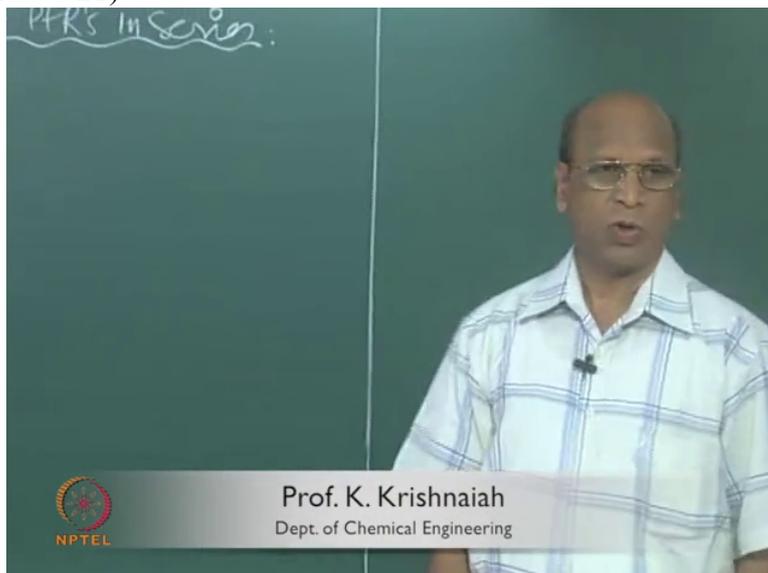
**Chemical Reaction Engineering 1 (Homogeneous Reactors)**  
**Professor R. Krishnaiah**  
**Department of Chemical Engineering**  
**Indian Institute of Technology Madras**  
**Lecture No 27**  
**PFR and MFR in series**

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I think we will start now? Yeah, in the last class what I have done was I just gave only discussion and analysis of this plug flow reactors in series. But I thought now

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we have to also give that expression, right, that equation for number of P F Rs in series, and when do you use this P F Rs in series? That also we discussed 0:00:35.4 do you remember?

(Professor – student conversation starts)

Student: 0:00:36.9

Professor: Yeah, when we have to increase conversion, you can add one more P F R,

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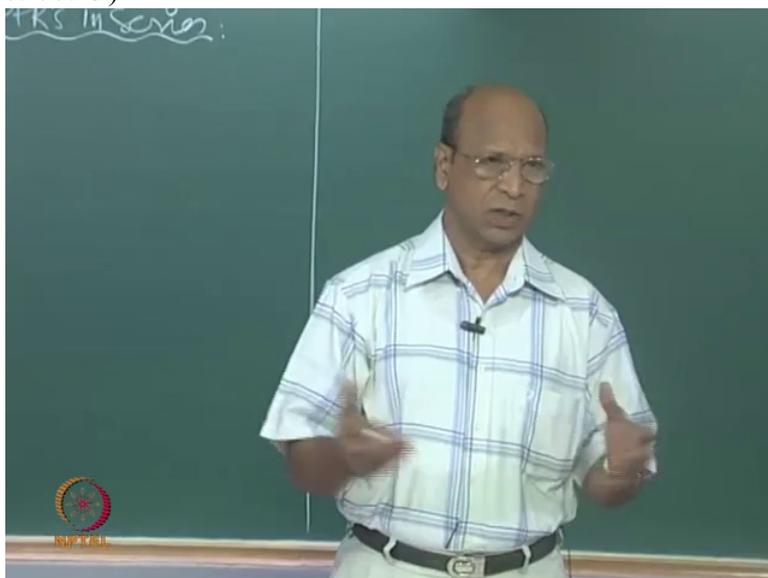


the reason is that you are increasing the residence time. So then naturally conversion will increase, Ok.

(Professor – student conversation ends)

So on the other hand if you have parallel

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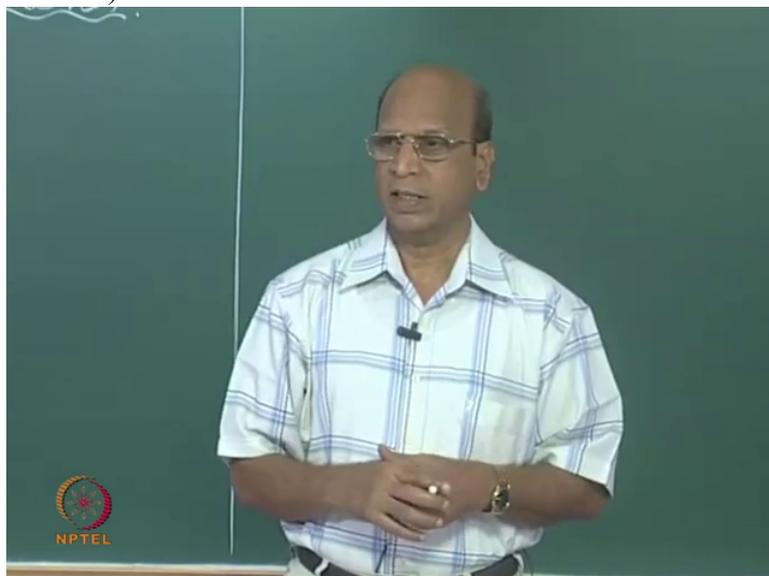
equation, means parallel P F Rs, then you add another P F R if you want to increase the production rates, Ok. So that is what, I mean same thing with even mixed flow reactors. If you put one

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more reactor in series with may be 2 or 3 already there, then your conversion will increase. But parallely you put another M F R then you will get

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production rate increasing, but condition there is maintain conversion same when you have either plug flow or mixed flow in parallel, in parallel, Ok. What are the equations you have to use? You have to use the same equations of what you have already

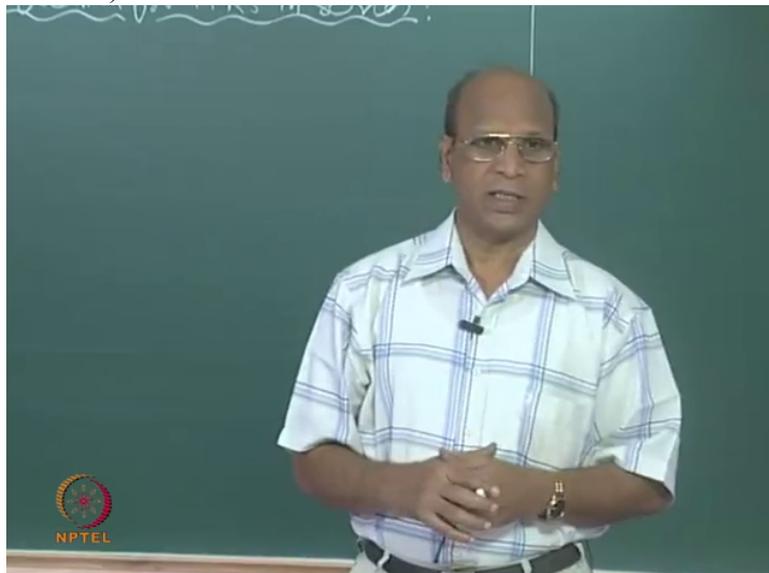
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derived.

Mixed flow we have derived, series also, right, yeah but mixed flow series

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only first order we have done. The other order I left to you which are more difficult, algebraically more difficult, otherwise it is not that difficult at all, just algebraically.

So that is why, again I am repeating. Many, many, many times I am repeating in this class that unless you work, you cannot remember, you cannot do anything in the examination. At that time do not blame me and do not blame C R E. Ok.

It is now just depending on you. I think you have got what is the basic concept in reaction engineering and what is plug flow and all that, those concepts are with you. But only thing now you have to see is your work, from your side you have to work hard. Otherwise you cannot. You cannot score marks also.

That anyway you will see this Friday Ok, twenty first. I am not threatening you but you know, if you are able to do that will be easy for you. If you are able to do, able to spend sufficient time in solving the problems.

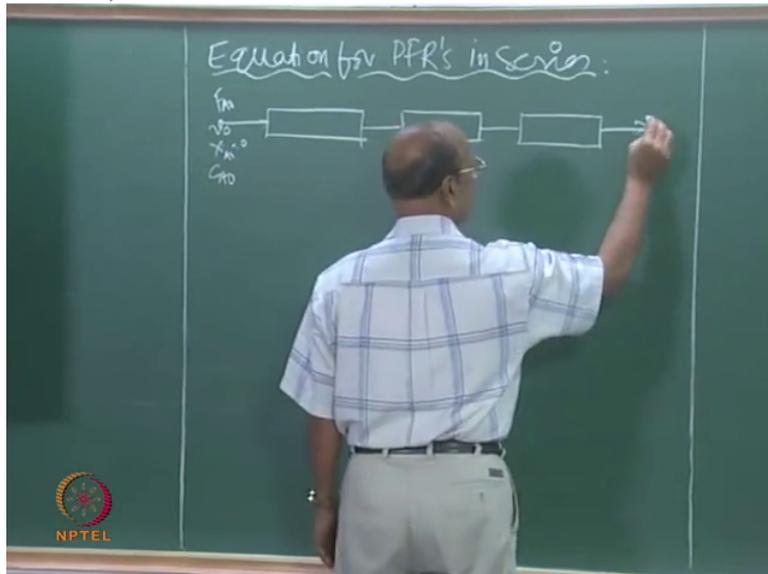
But if you think that mental solving is Ok, so that means reading the problem, Ok A is given, B is given, and Ok, volume is given, conversion I will calculate. Because all the data given there. You do not actually put on the paper. So then you are going to lose.

At that time the first person blamed is me. Oh my God, this guy gives very difficult questions. They are not difficult questions. Only thing is you have not done your duty properly, right? Ok. So that is why I thought again I will start this P F R in series to actually derive the equation. It is not difficult. It is same thing but only thing you may have some doubts, see something like this.

I have one piece of P F R and then I have another piece and then we have another piece. Let us say only three are there. So here I have  $F_A \text{ naught}$ , and  $v \text{ naught}$ ,  $X_A \text{ naught}$  equal to zero etc and  $C_A \text{ naught}$ .

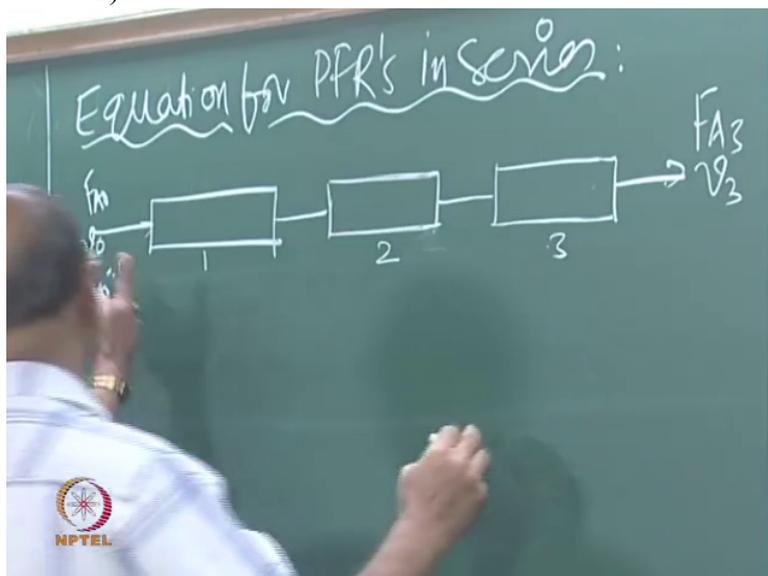
And at this end I have,

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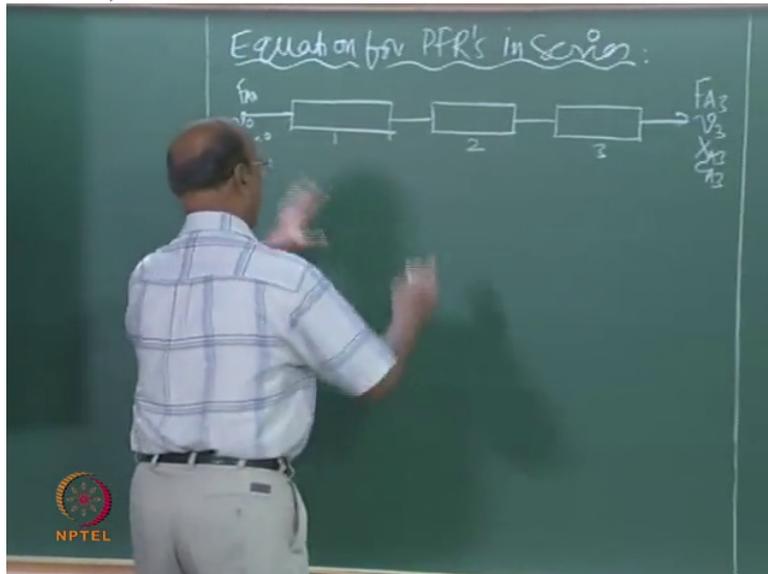
Ok if I say that I have, this is 1,2, 3, here I have  $F A_3$ ,  $v_3$ , for constant density systems it may be same

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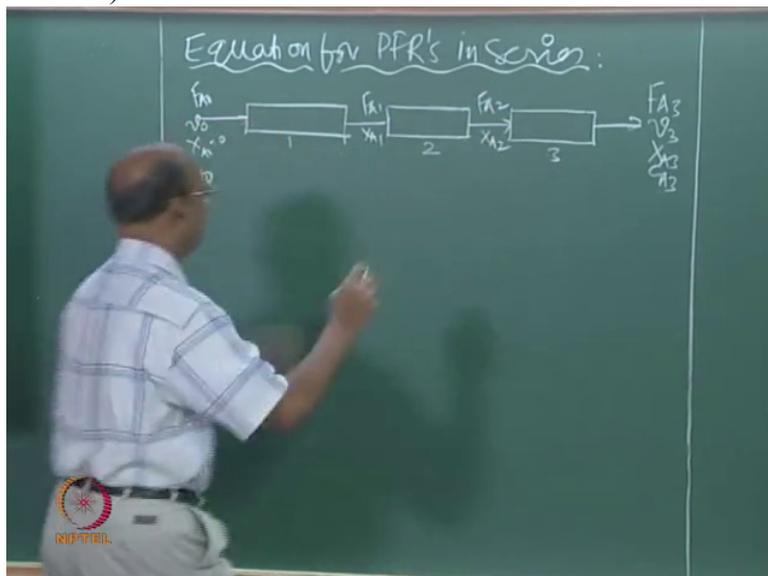
Ok and also  $X A_3$  and  $C A_3$ . Now in between if

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I write this, this will be  $F_{A1}$ ,  $X_{A1}$ ,  $F_{A2}$ ,  $X_{A2}$ . These are the inputs, that is all.

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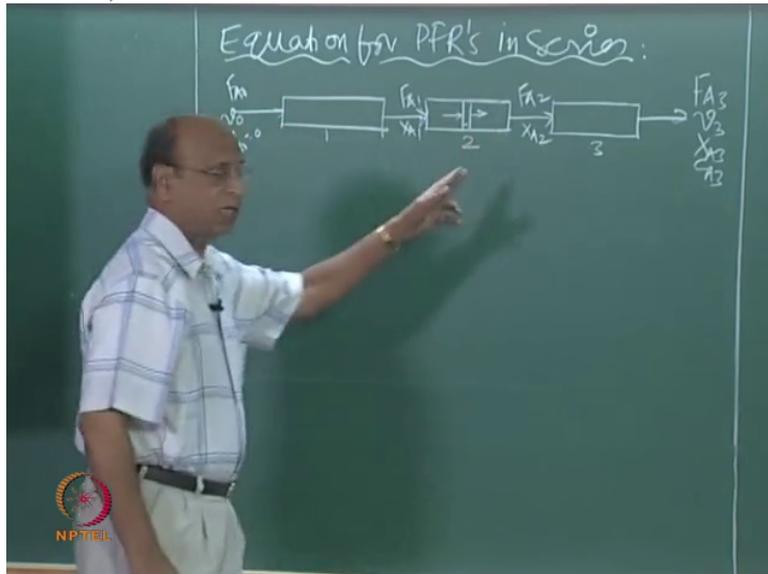


Yeah.

So how do I derive the equation for this? I will take a small, you know normal way of deriving an equation for PFR is, because it is a distributed parameter system, you take a small slice. Write what is entering, what is leaving and all that. So that you have done many times.

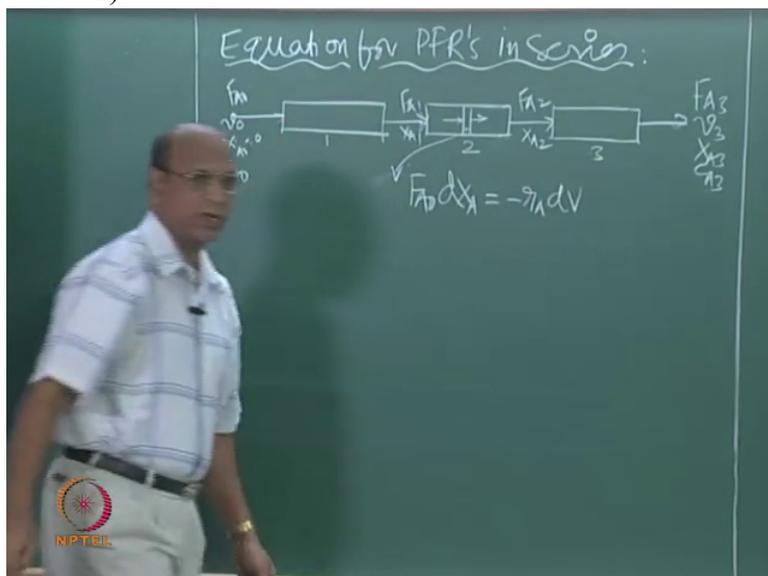
So that is why I will take a small strip here, entering, leaving and all

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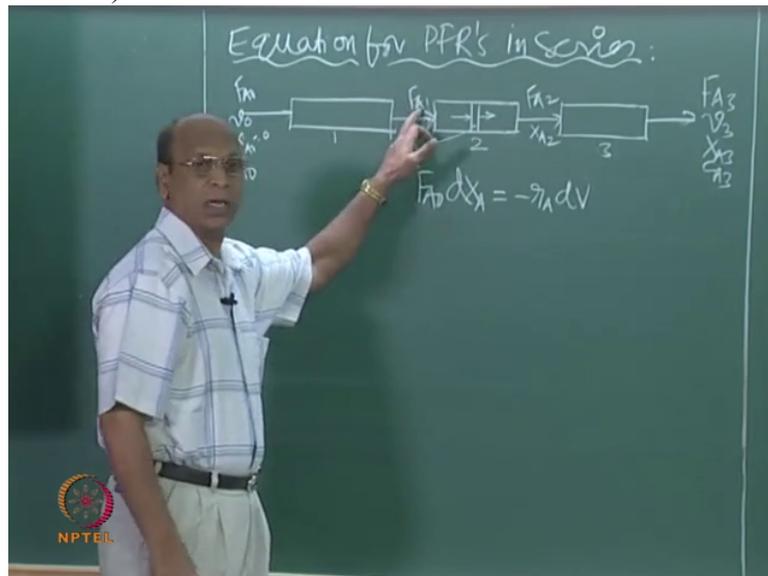
that. So this equation already we know, that gives me  $F A \text{ naught } d X A \text{ equal to minus } r A d V$ . This is the equation.

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Where you get confused is why this  $F A \text{ naught}$  but I have here  $F A 1$ .

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Ok, you do not get? You do not get that doubt? Yeah, why?

How do you define conversion?

(Professor – student conversation starts)

Student: Conversion is with respect to input only,

Professor: Yeah with respect to input, which input you are now talking? Is it this input or this input?

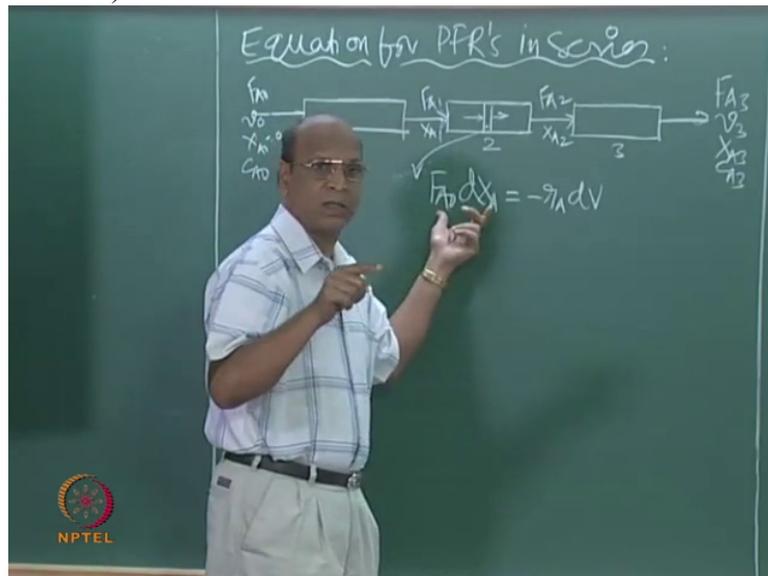
Student: 0:05:06.9

Professor: F A naught.

(Professor – student conversation ends)

Because you are writing the conversion based on F A naught that is why this F A 1, X A 1 will not come into picture unless otherwise it appears in terms of only boundary conditions. Ok. So, using this if I extend that logic that means this F A naught d X A you got

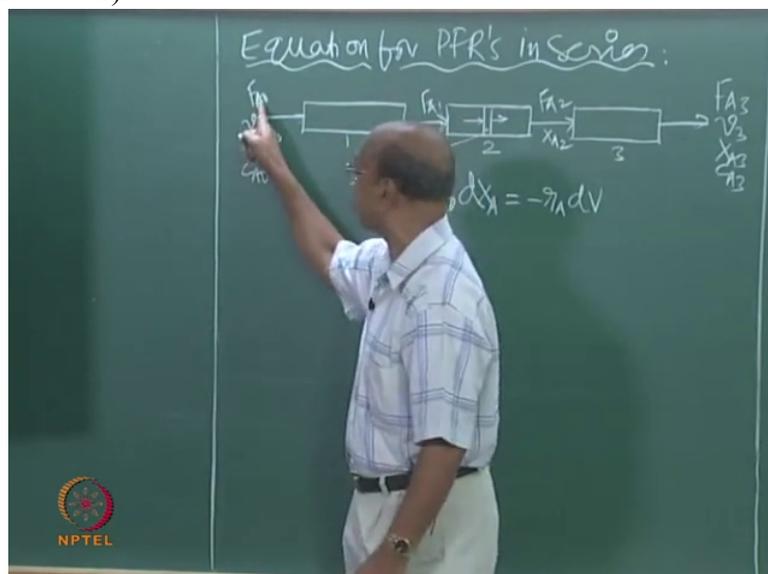
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by defining  $F_A$  equal to  $F_{A0} (1 - X_A)$ . At least these things you have to remember.

$F_A$  equal to  $F_{A0} (1 - X_A)$ . That is general one. So that this definition is based on

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$F_{A0}$  naught, not somewhere else inside, Ok. So if I have that, then I will have total volume  $V$  by  $F_{A0}$  naught is  $\sum V_i F_{A0}$  which is also equal to  $V_1 F_{A0}$  plus  $V_2 F_{A0}$  plus  $V_3 F_{A0}$ ,

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$$F_A dX_A = -g_A dV$$

$$\frac{V}{F_{A0}} = \sum \frac{V_i}{F_{A0}} = \frac{V_1}{F_{A0}} + \frac{V_2}{F_{A0}} + \frac{V_3}{F_{A0}}$$

Ok, good.

So now if I write this, this will be  $V$  by  $F_{A0}$  equal to integral zero to  $X_A$ , I am writing for this,  $X_{A1}$ ,  $dX_A$  by minus  $r_A$ , plus, yeah exactly,  $X_{A1}$  to  $X_{A2}$   $dX_A$  by minus  $R_A$  plus  $X_{A2}$  to  $X_{A3}$   $dX_A$  by minus  $r_A$ . This is equation, if I take that is starting equation, yeah.

Now in fact your condition is automatically

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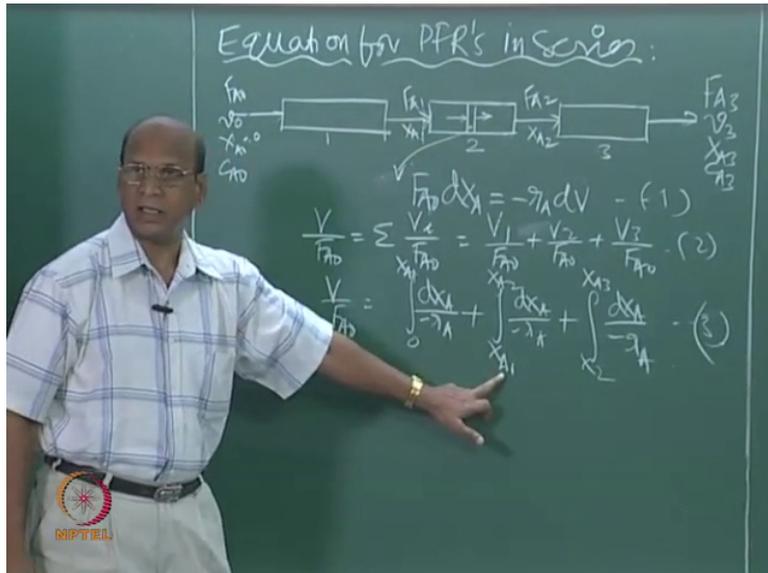
$$F_A dX_A = -g_A dV \quad (1)$$

$$\frac{V}{F_{A0}} = \sum \frac{V_i}{F_{A0}} = \frac{V_1}{F_{A0}} + \frac{V_2}{F_{A0}} + \frac{V_3}{F_{A0}} \quad (2)$$

$$\frac{V}{F_{A0}} = \int_0^{X_{A1}} \frac{dX_A}{-r_A} + \int_{X_{A1}}^{X_{A2}} \frac{dX_A}{-R_A} + \int_{X_{A2}}^{X_{A3}} \frac{dX_A}{-r_A} \quad (3)$$

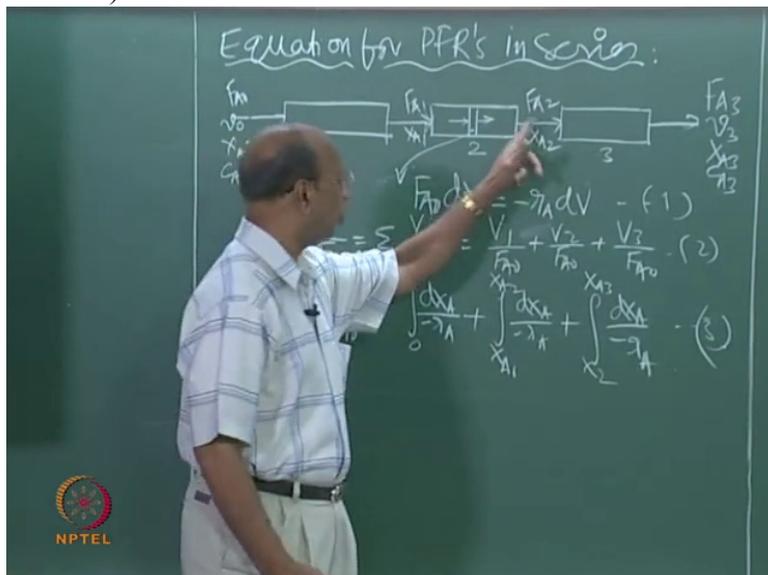
met and those people who still bother that no, no, no, no I think he is wrong, this  $F_{A1}$  must come.  $F_{A1}$  automatically comes there because you have lower limit as

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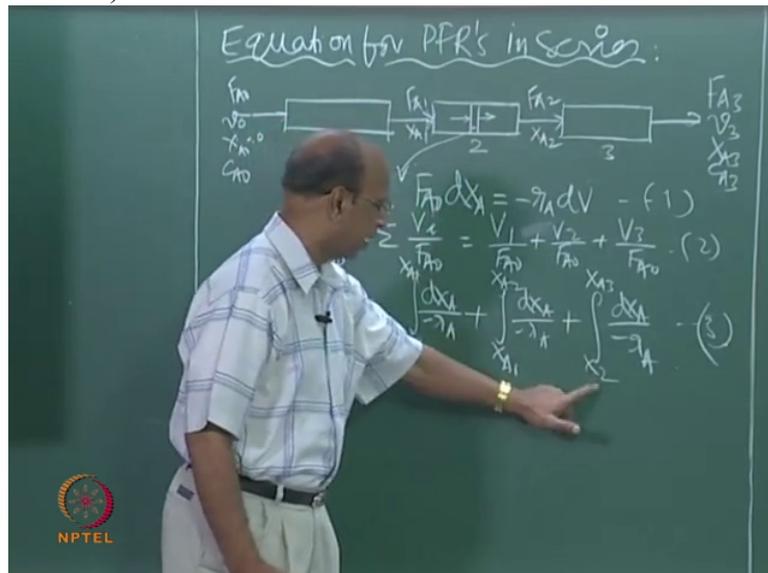
X A 1. Similarly here

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X A 2, F A 2 will automatically

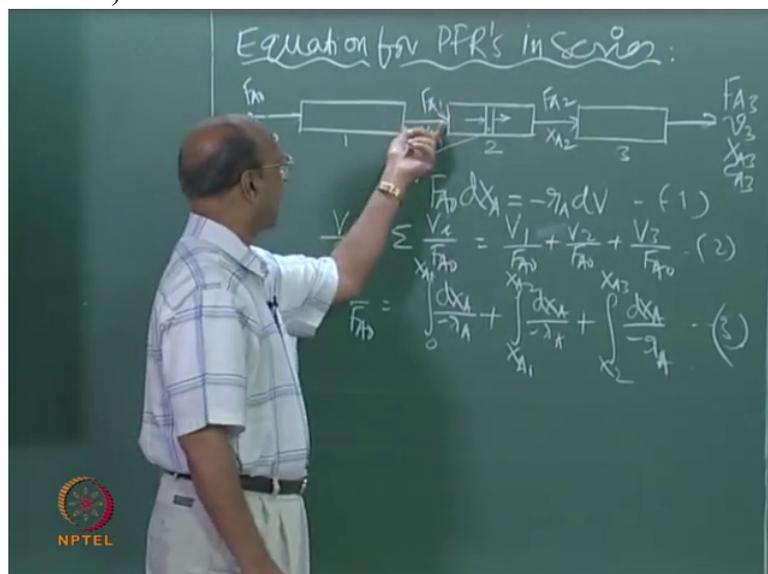
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come because you have  $X_{A2}$  at entrance. This is one of the problems you have when you are actually trying to derive in the sixth chapter beginning, Ok. Many students asked me.

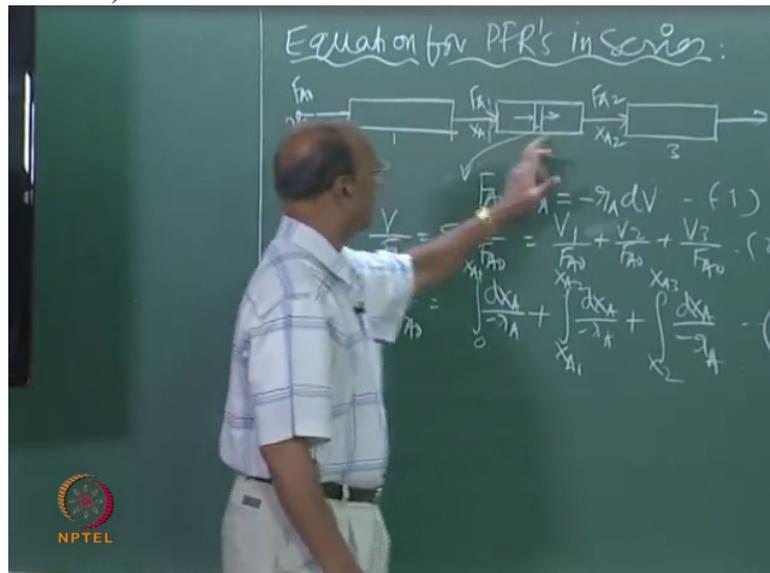
So that is the reason again I am now trying to tell where you can have this kind of confusion. And mathematically speaking this

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this gap in between is imaginary gap for us. So when I have 3 reactors that will also

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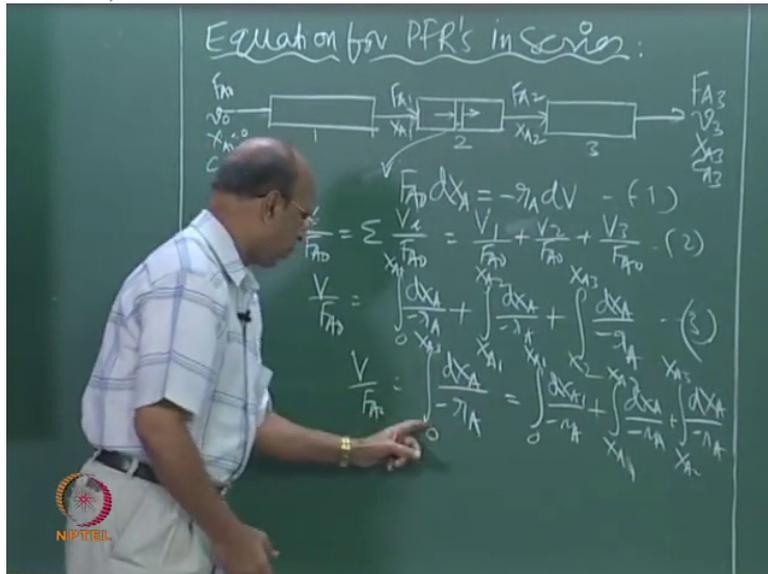
be equivalent to as 1 reactor where that is entering with  $F_{A0}$  and coming out as  $F_{A3}$ . How do I write that equation? Assuming that I have only one, you know this plus this plus this, all three is just only one tube and even if you have the connections, connections is directly given through flanges, you do not have. But to show as diagram, we are writing this.

This does not mean that there is an arrow between these 2 pipes, Ok and through arrow it is going, right? Through arrow nothing can go. Only it can show that the direction where it is going. So when I have all these three together what is the equation you have?

Total if I take,  $V$  by  $F_{A0}$  is equal to zero to, what is the upper limit?  $X_{A3}$  d  $X_A$  by minus  $r_A$  and if you are able to imagine this one as just simple calculus, you know integral expression, cannot I split this as  $X_{A1}$  d  $X_A$  by minus  $r_A$  plus  $X_{A1}$  to  $X_{A2}$  d  $X_A$  by minus  $r_A$  plus  $X_{A2}$ ,  $X_{A3}$  d  $X_A$  by minus  $r_A$ . This is pure mathematical thing, right?

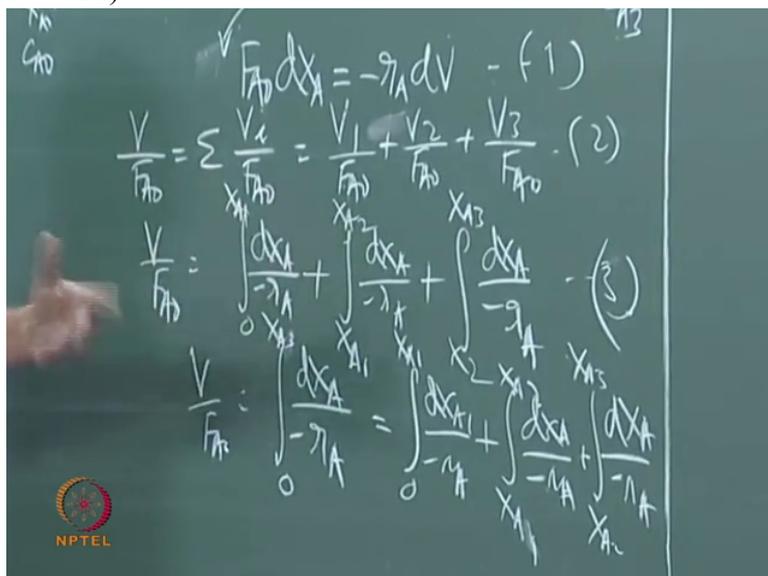
Because this integral, this integral I can split into,

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yeah 3 parts. I mean even 10 parts also, even 100 parts, right? Ok because this one, till X 1, then X A 2, X A 3,

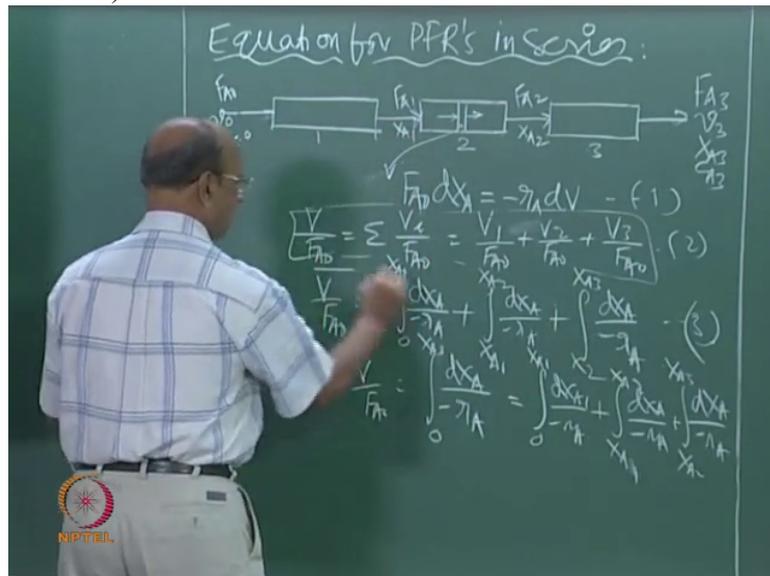
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like this. That is why this equation is also exactly this equation. That is why you should not worry.

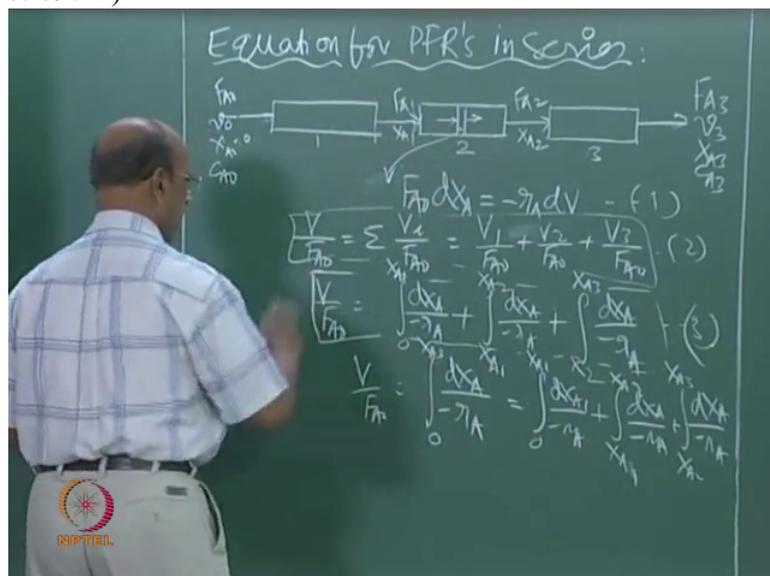
That is why general thumb rule is that even if I have any number of mixed flow reactors in series, so that is equivalent to as a single plug flow reaction expression. That is what is this. That is what is this and automatically that also comes as,

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yeah,

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that is the one. So I think there is no problem there. Good.

So yesterday after the class some students were coming and asking me, how do you get that for MFR,  $1 - \tau$  we plotted versus, you know that slope, on the curve, on the graph where I have  $-r_A$  versus  $C_A$ , Ok. So this may be bothering some more people.

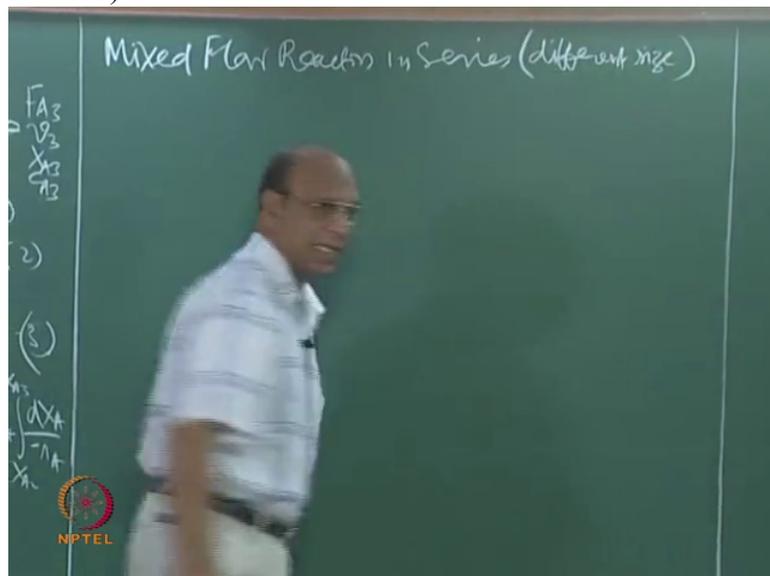
This is simple calculus again. Because we are excellent in forgetting whatever we taught earlier, that is why you are not able to recall that. But I think I do not blame that student.

There are many, I told you no, all Indians are sisters, brothers and sisters. So some more people will have that doubt but he is bold enough to come and ask.

Other people would have forgotten thinking that, Ok does not matter, whether is 1 by minus tau or 2 by minus tau, why should I care? Ok, so with that attitude you again, you know your cobwebs will not be removed. So that is why I thought I will also tell that one in a more systematic way, so mixed flow reactors in series.

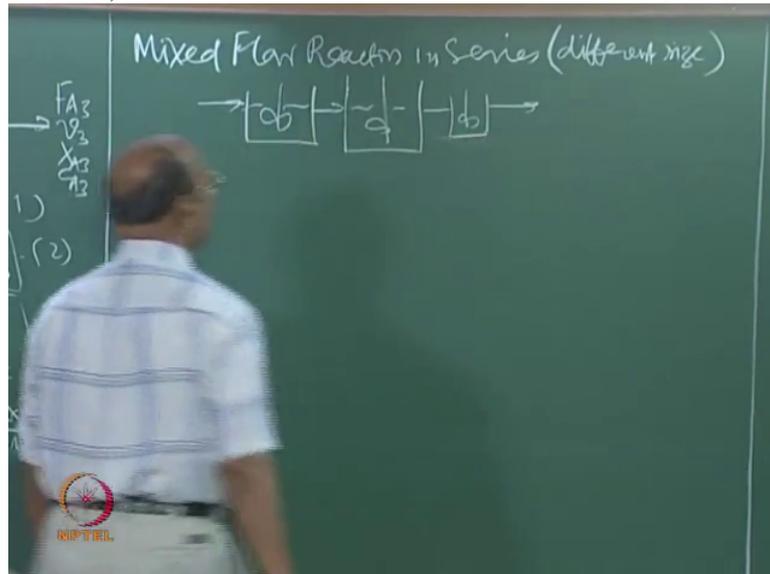
Ok, they can be of different size or they can be of the same size, does not matter. Ok, for us does not matter. May be different size also we will take, you know. Different size means you know,

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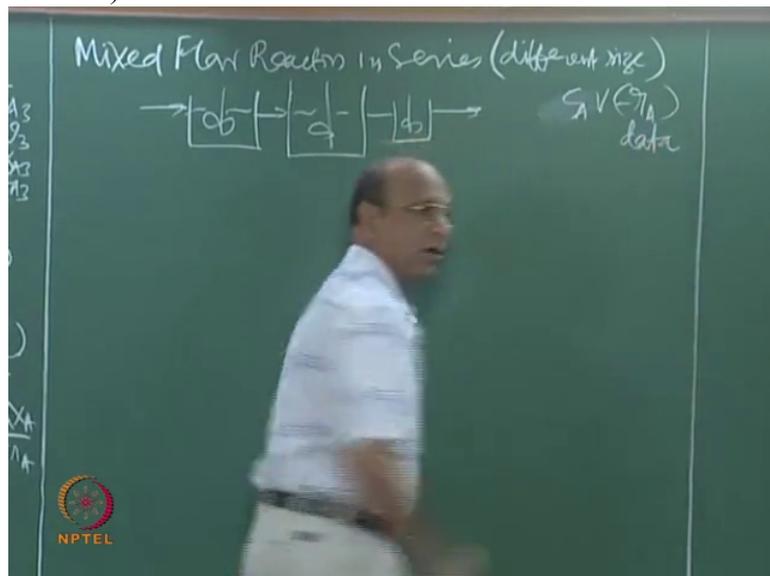
the slopes will change, Ok good. So now when I have this, let me say this is 1, then I have another big one here, not looking big one, yeah another small one here, yeah.

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And here the condition is that I have only  $C_A$  versus  $\tau$  data.

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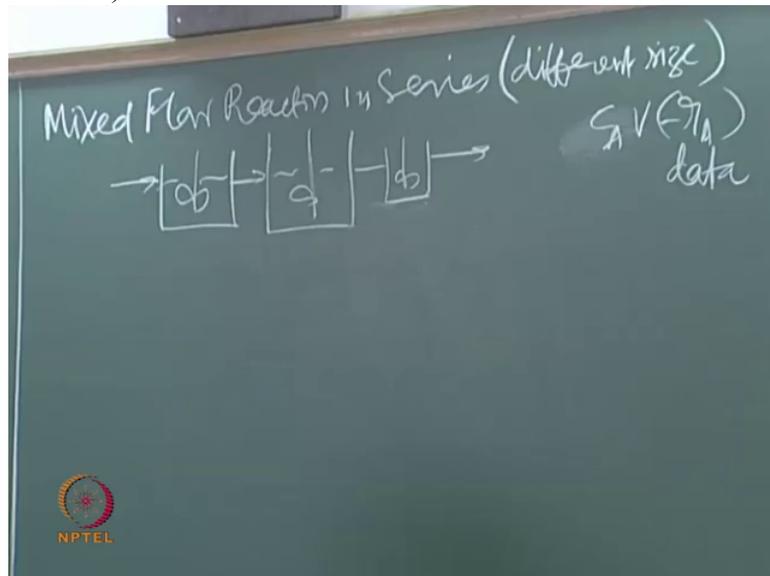


I do not have a rate expression, Ok. Normally what we do, the moment I have  $\tau$  versus  $C_A$  data, then I will try to find out what is the order of reaction and then substitute that in the equation and then do it.

So in industry you may not have that kind of time where you search for a kind of rate expression and then use that rate expression and all that. So that is why most of the time in industry particularly they say that exploratory research, where they try different things, for them the easiest one is once they have this  $\tau$  versus  $C_A$  or  $X_A$  versus  $\tau$ , simply plot that and try to find out which reactor is better.

You can also get very good idea which reactor is better even if I plot  $1/r_A$  versus

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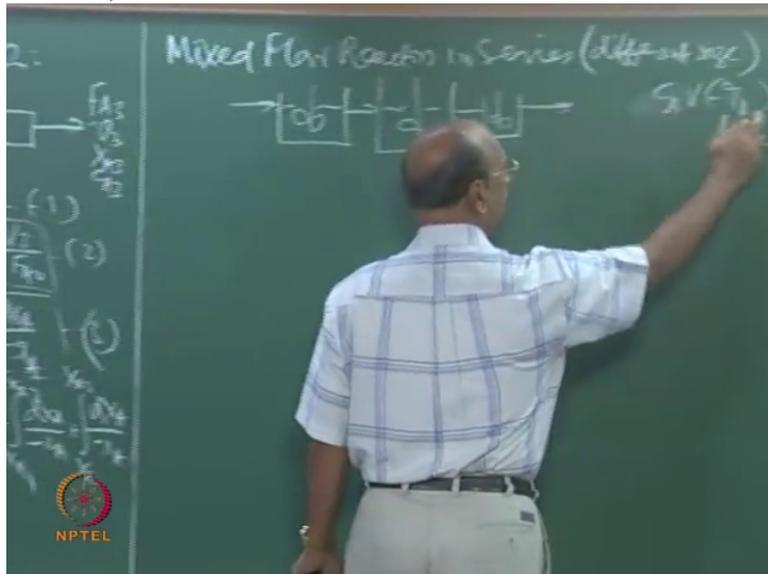
$X_A$  or  $1/r_A$  versus  $C_A$ ,  $C_A$  also one can write. Only thing is coordinates reversed, Ok. So but I think our, usual thing is I think for any general case,  $1/r_A$  versus  $X_A$ . Then depending on the shape of the curve, you can choose which reactor is the best reactor.

Ok, so that means when you are using mixed flow, you know what is the area you have to take. When you are using plug flow, you know what is the area you have to take. Wherever the area is coming, larger and larger, that is bad for industry. That means volume is larger. So that is why you also have some increasing, decreasing like autocatalytic reactions, Ok.

So that is why clearly the moment you plot  $1/r_A$  versus  $X_A$  or  $C_A$ , then you will very clearly know what reactor is the best reactor. Or what combinations of reactors are the best reactor. Ok, so that is why that remember, always. You know these are the general things I am repeating so many times, right? Ok good.

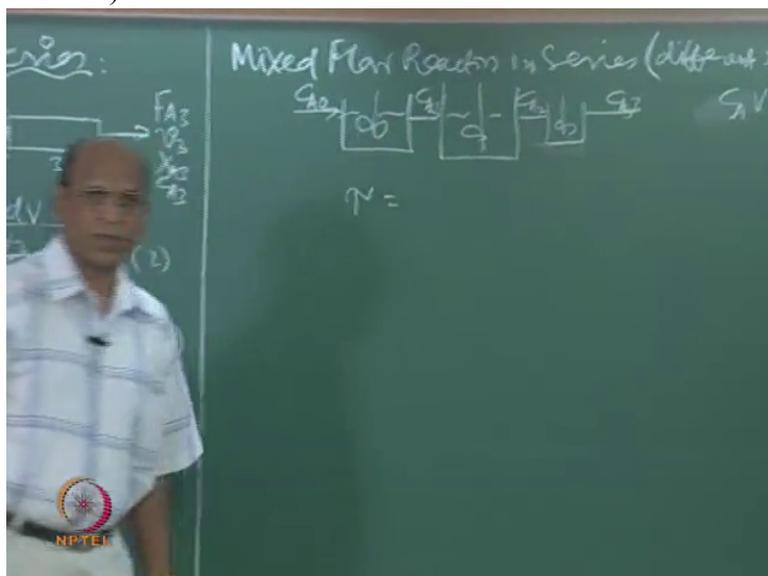
So in this case I know that I have only  $1/r_A$ ,

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minus  $r_A$  versus  $C_A$  data. But here I do not have to plot as  $1 - R_A$  versus  $C_A$ , Ok, why? I have a general expression for, Ok, this is  $C_{A0}$ ,  $C_{A1}$ ,  $C_{A2}$ ,  $C_{A3}$ ,

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Ok, different reactors so then if I write for this one, this is  $C_{A0} - C_{A1}$  by minus  $R_A$ . This much you know definitely.

We have written in terms of  $V$  by  $F_{A0}$  but that can be easily converted.

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$$N = \frac{C_{A0} - C_{A1}}{-r_A}$$

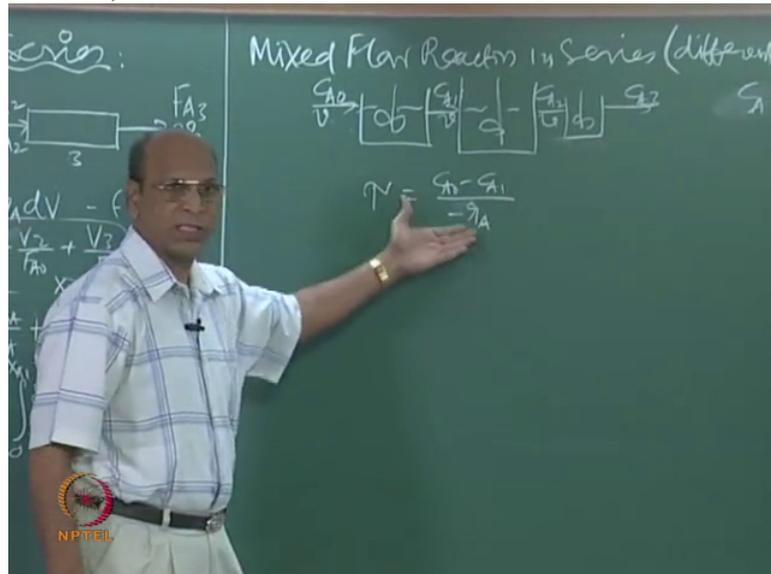
I also told you how easily one can write the mass balance equation in terms of concentration and volumetric flow rate.  $V C A$  so  $V$  into  $C A$  naught entering, and all this is  $V$  because there is no

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volume change, Ok. Yeah.

Volume change also can also be easily dealt with. There is not a problem here. Because volume change is already there in minus  $r_A$ . So that is why even if you have volume change, the equation is

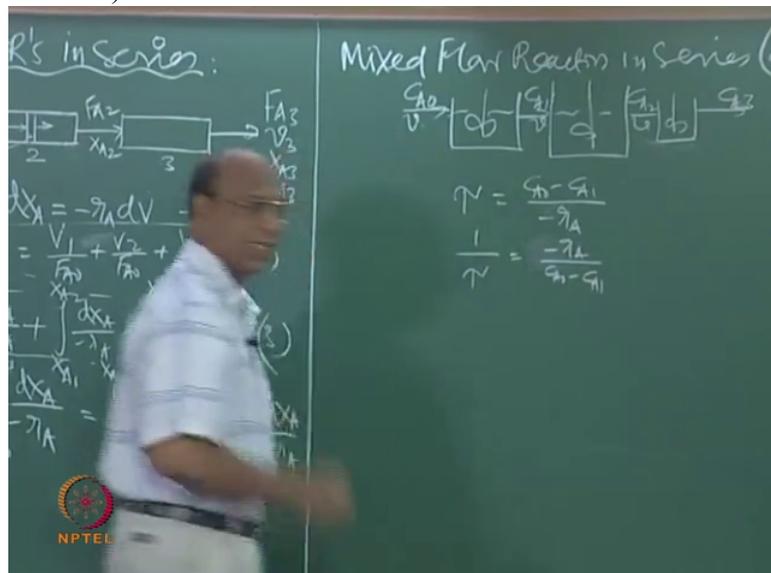
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same. This will not change. Ok, this is the one.

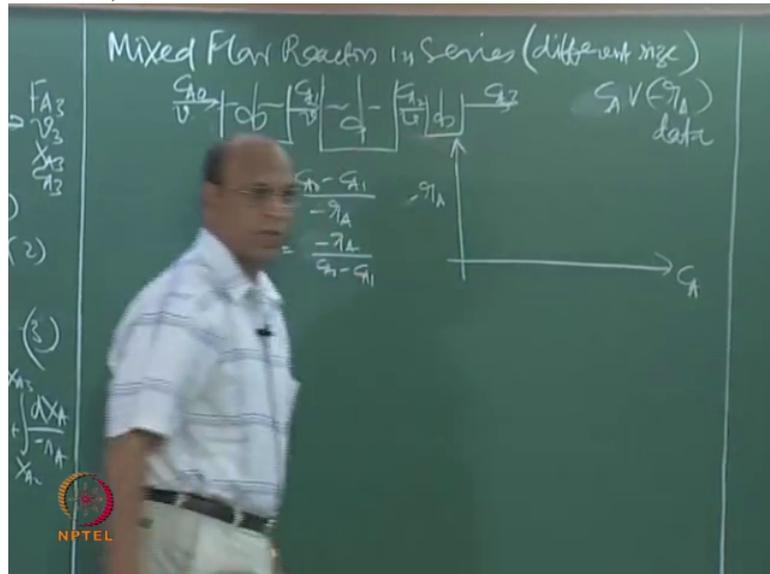
So why we have to plot in that way is now I will write, try to write that equation as  $1$  by  $\tau$  equal to minus  $r_A$  by  $C_A$  naught minus  $C_A 1$ ,

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Ok. And that is why minus  $r_A$  versus  $C_A$  if I plot, this is minus  $r_A$  versus  $C_A$  when I plot,

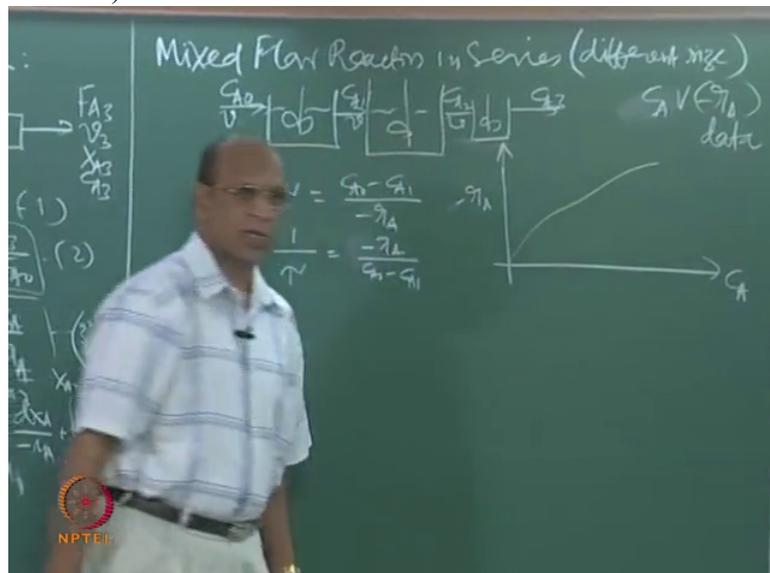
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so I may get normally, as concentration increases, for many reactions what will happen to rate, rate should increase.

So that is why you may get something like this.

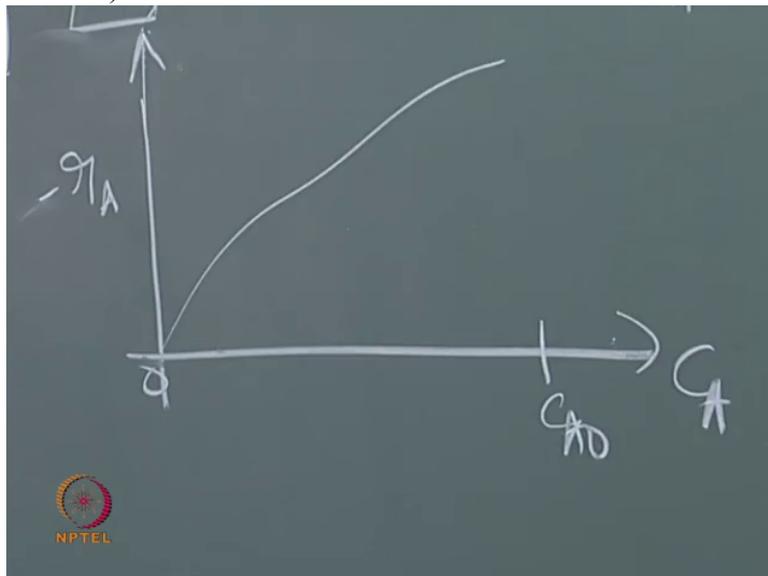
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It need not be beautiful curve where you have only increasing, you know linearly or with you know quadratic equations, so that kind of things need not be there, Ok. So simple this one.

So my  $C_A$  naught is here. Because I am moving in this direction. This is zero.

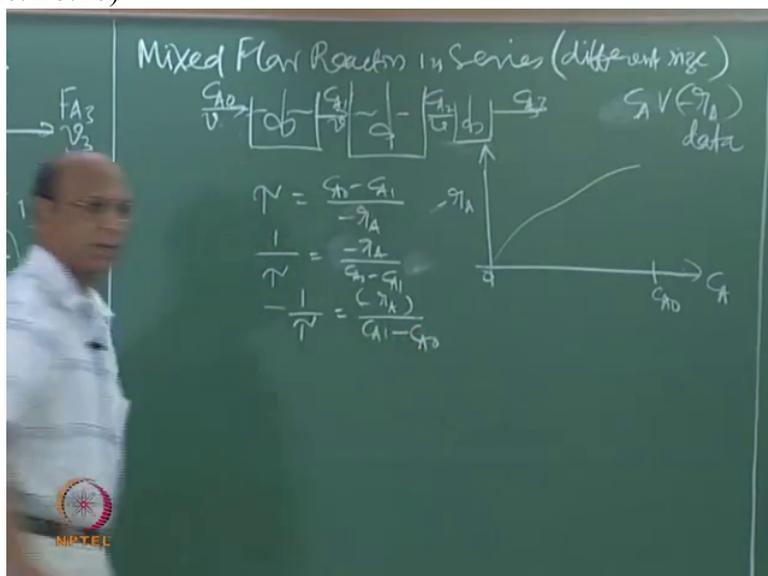
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$C_A$  naught is high, right? So now this will tell me that, if I draw a line here with  $1/\tau$ , I think Ramkrishna was also pointing out, Sir it must be  $1/\tau$ , yeah minus and that minus comes because I told him, I think that negative itself shows minus, but anyway still equation also we can change.

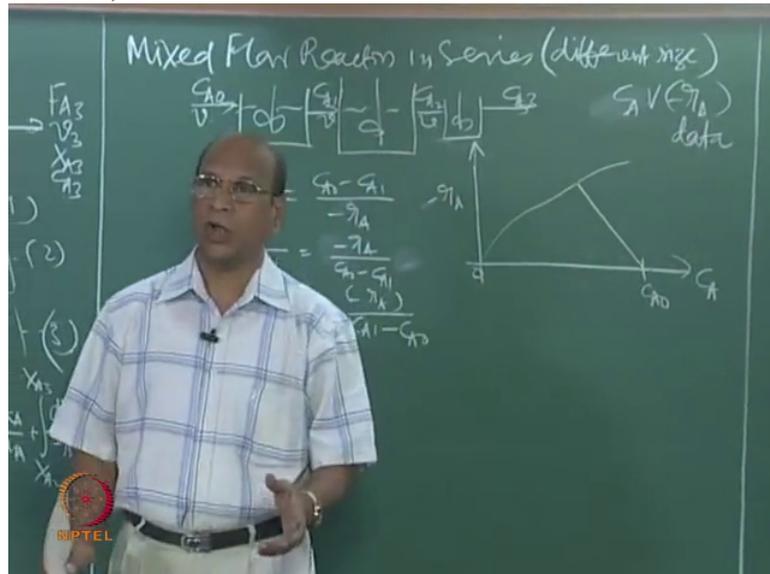
So that equation, now instead of writing this, this also, yeah so minus  $1/\tau$  as minus  $r_A$  into  $C_A$ , sorry divided by  $C_A$ ,  $C_A$  naught.

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Ok, why we should do that? I will first draw this one because  $C_A$  versus minus  $r_A$  I know.

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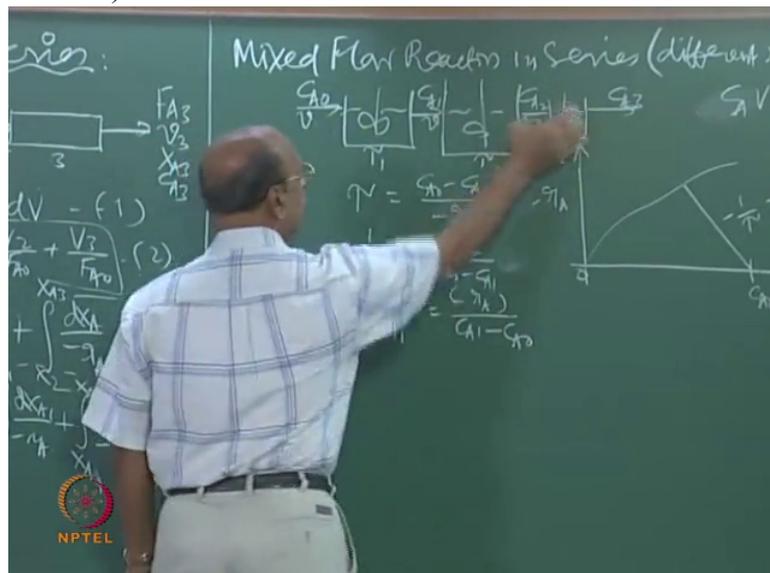


I have to also know the size of the reactor, Ok.

Otherwise how do I know tau? Tau is volume by volumetric flow rate. That I should know. Otherwise I cannot use this method. But I can also use that method but it will be a trial and error method. Ok I will tell you later.

So this one is, this is minus 1 by tau which is nothing but, or may be tau 1, because I have 3 different tanks, Ok this is tau 1, tau 2, tau 3, because volumes are changing,

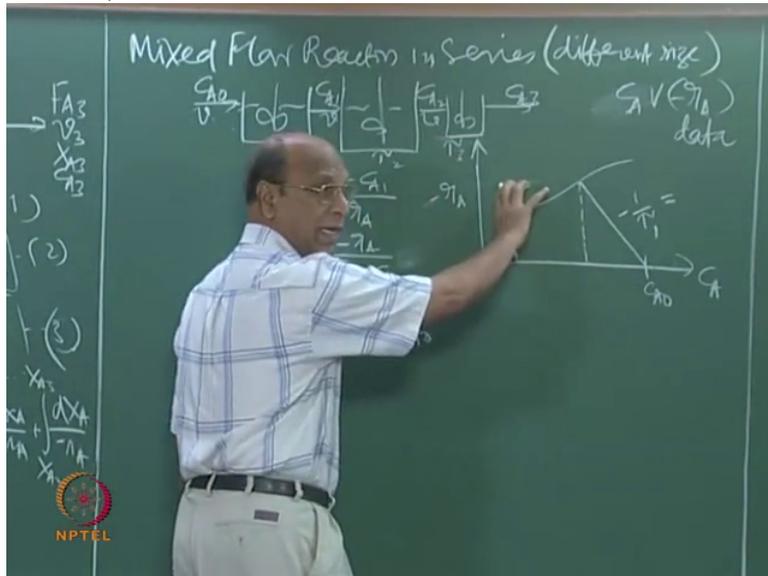
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volumetric flow rate is same so tau will change. Ok, that is why tau 1, Ok, so this is tau 1, and I told that we have to draw a line here. Ok.

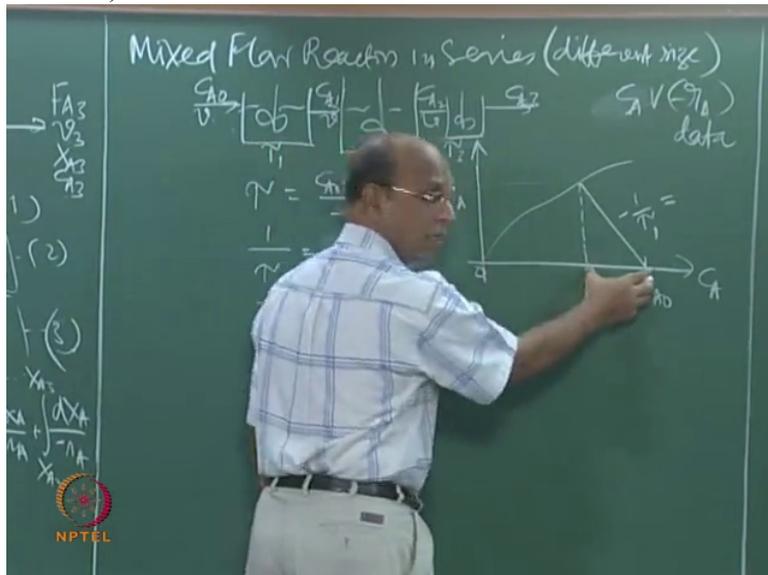
And this equation tells me this is minus r A. What is this, d y

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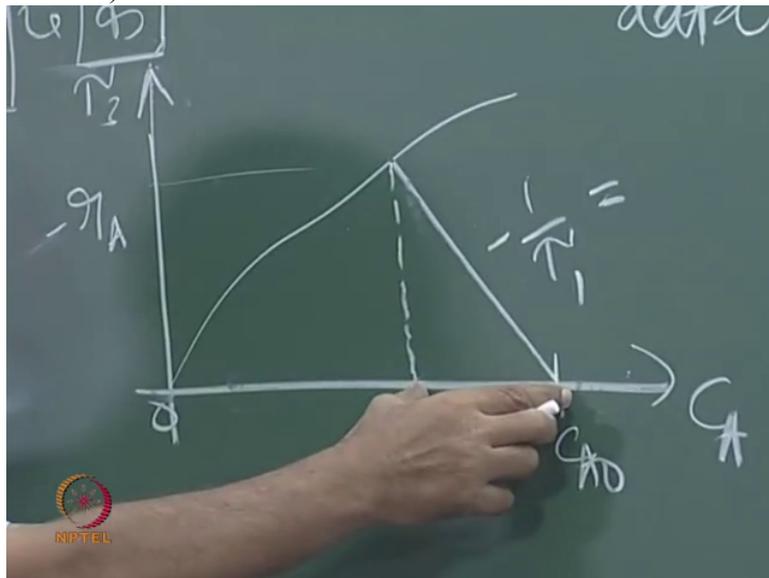


and this one is

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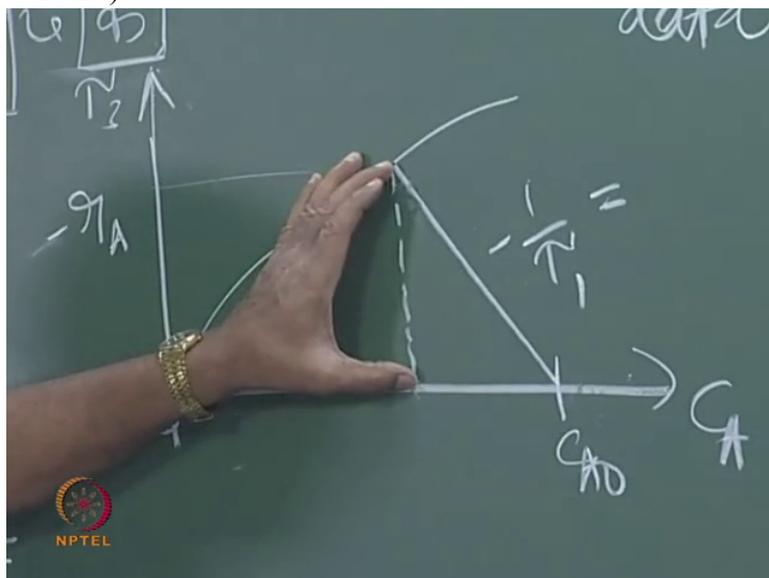


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$d x$ . This slope is nothing but  $d y$  by  $d x$ . Ok.  $d y$ , here I am starting from zero, that is why this entire thing is  $d y$ . What is the

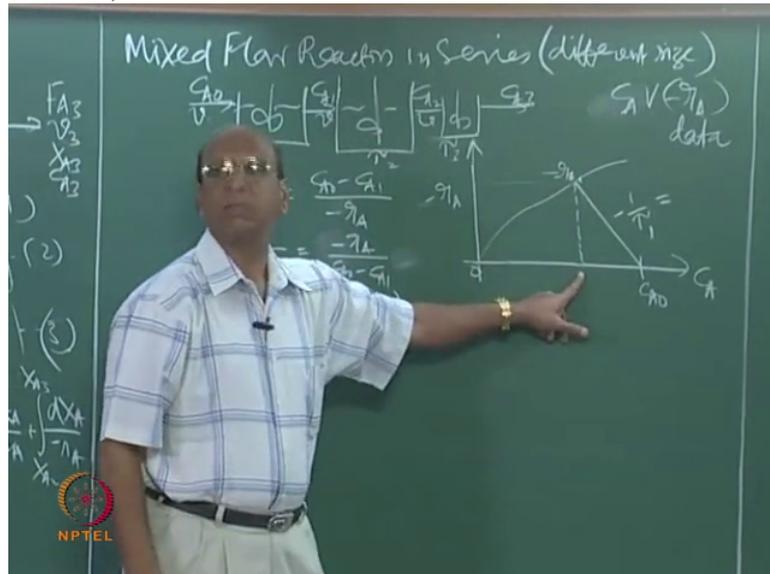
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value of this? minus  $r_A$ , Ok, this is some  $r_A$  here, right and correspondingly what is

(Professor – student conversation starts)

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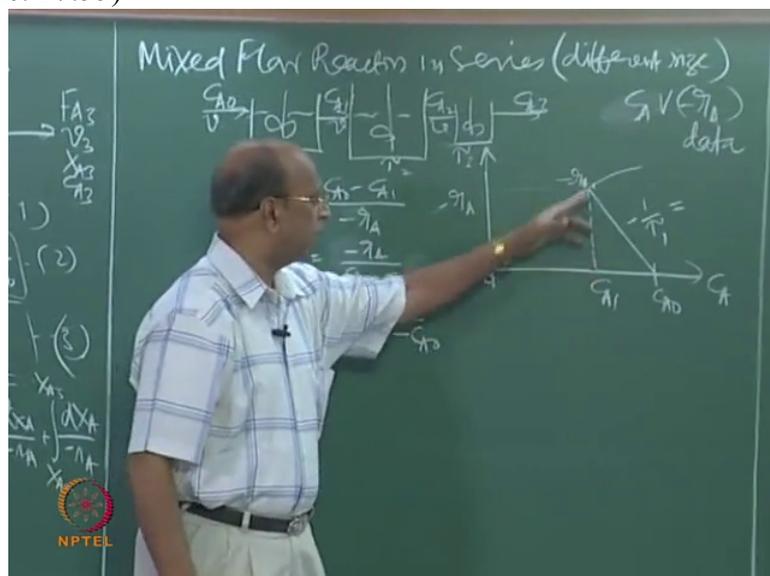


Student: C A 1

Professor: It must be, because yeah, C A 1 is the one which is leaving and throughout also I have C A 1. This is where some people were asking doubts, Sir how, why we have to drop it down? We have to drop it down because that is only possibility.

(Professor – student conversation ends)

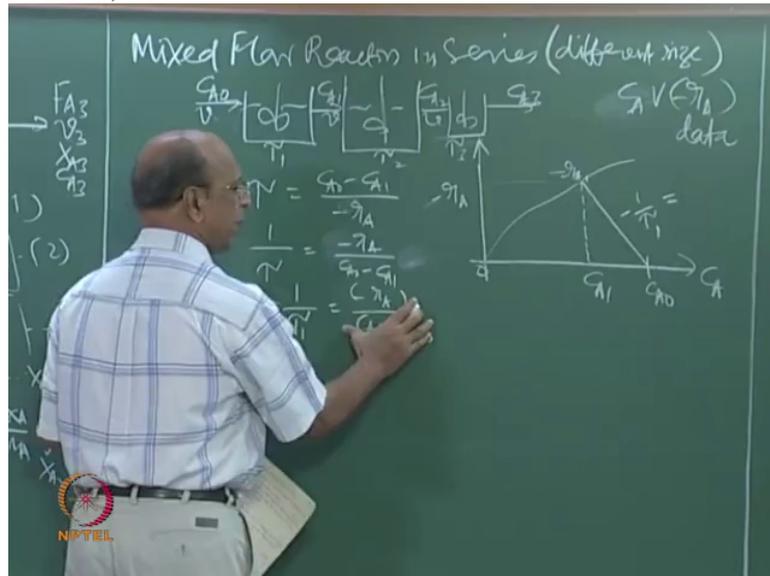
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Because you know from here to here, that change inside the reactor will occur so this is C A

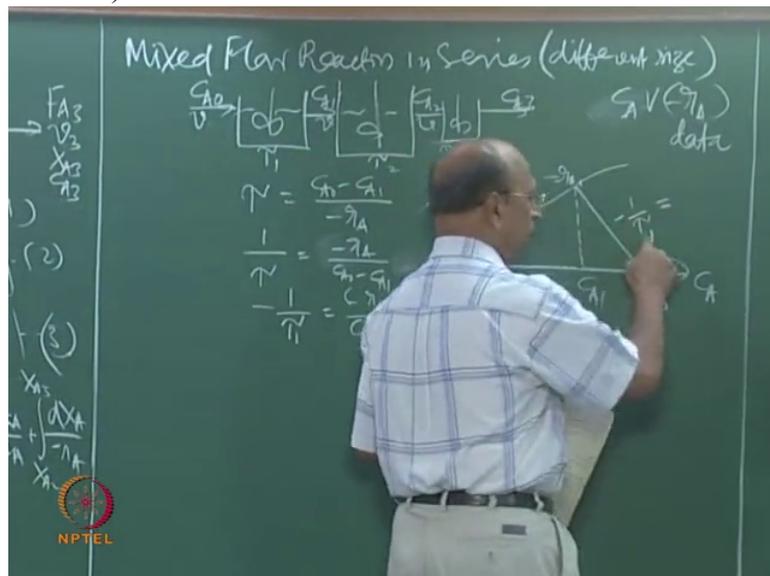
1. Now you see this equation is just nothing

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but  $\frac{dy}{dx}$ , because our mind is always in terms of  $\frac{dy}{dx}$  whatsoever. And that is why you have this one as minus 1 by tau

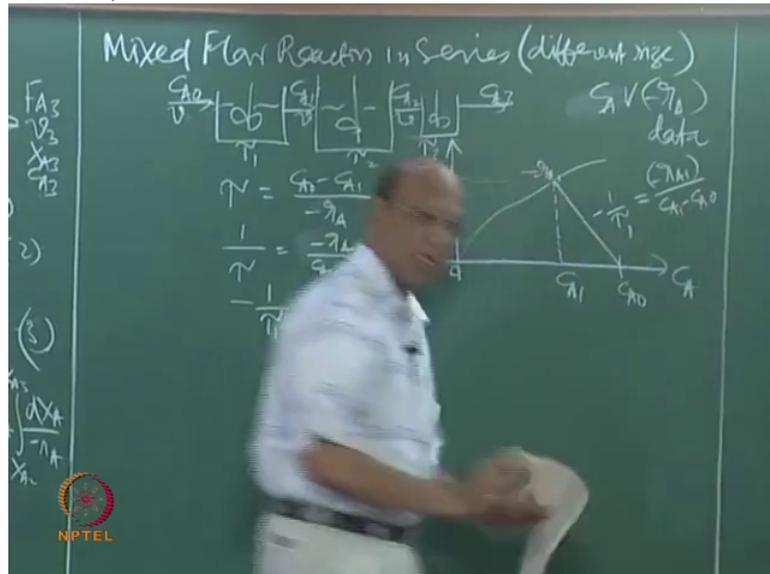
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because negative things will show me, I mean negative slope.

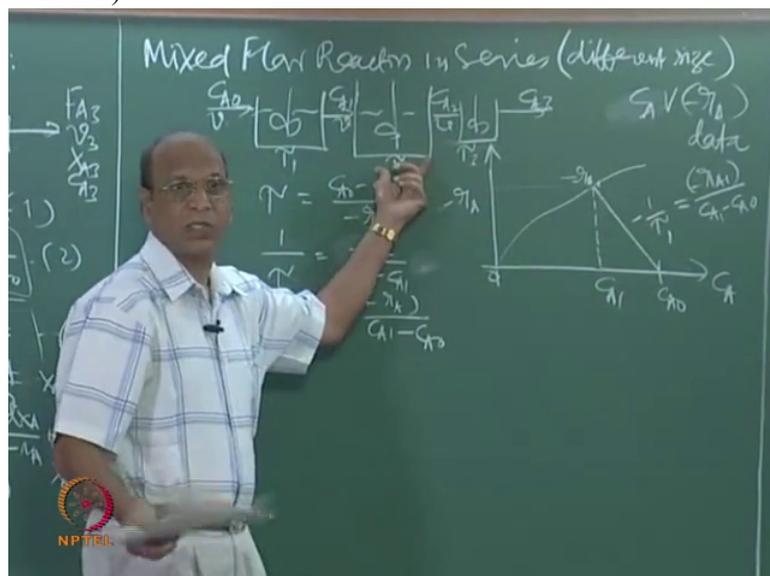
This is nothing but minus  $r_{A1}$  divided by  $C_{A1}$  minus

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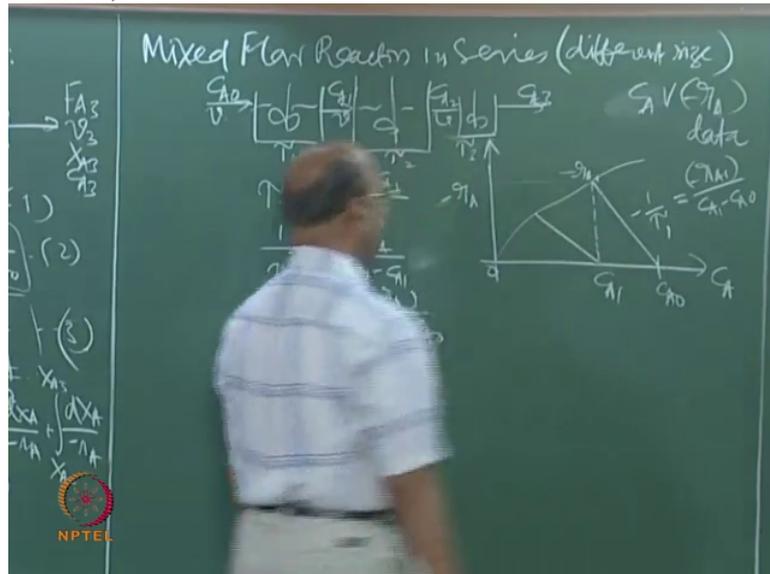
C A naught. That is right. So similarly you can also go to, yeah. Now as I told you here, I have tau is different.

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Here tau is large. So I may have, 1 by tau is small, right. So I may have not same thing, small means yeah like this. It may go

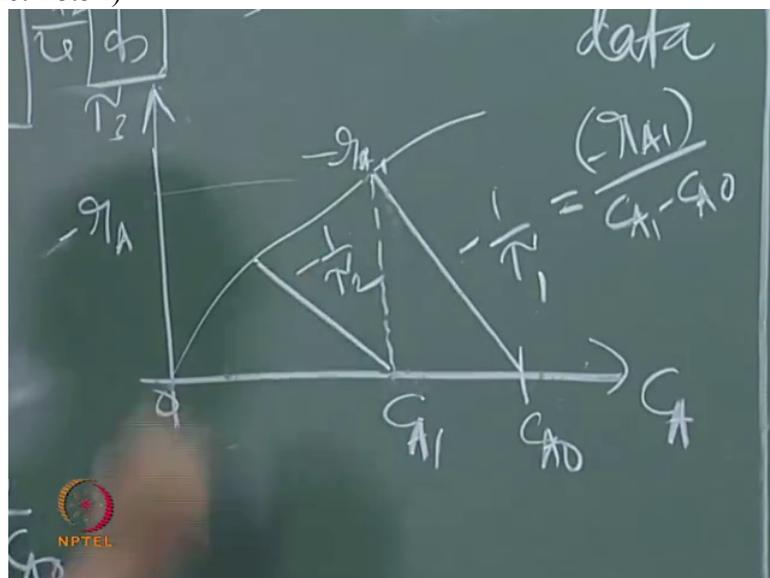
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like that. So this slope, they look parallel?

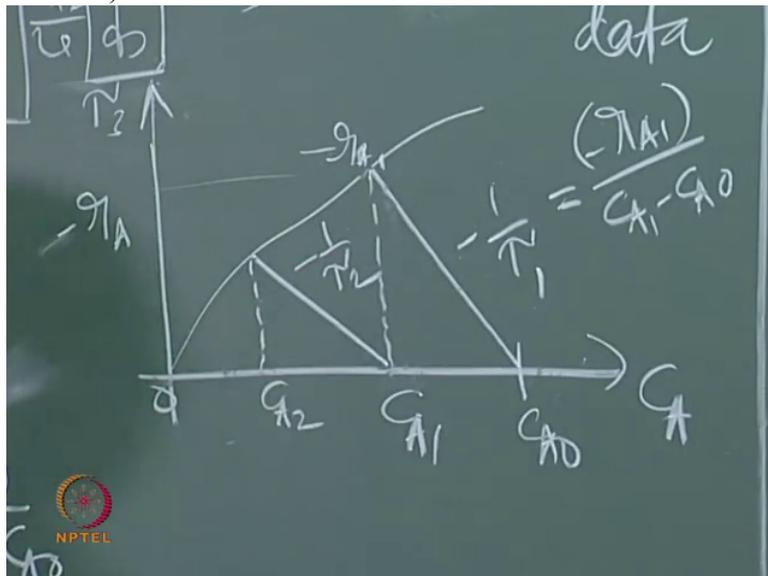
I think they are not parallel anyway, yeah, so this is nothing but for me this is minus 1 by tau 2.

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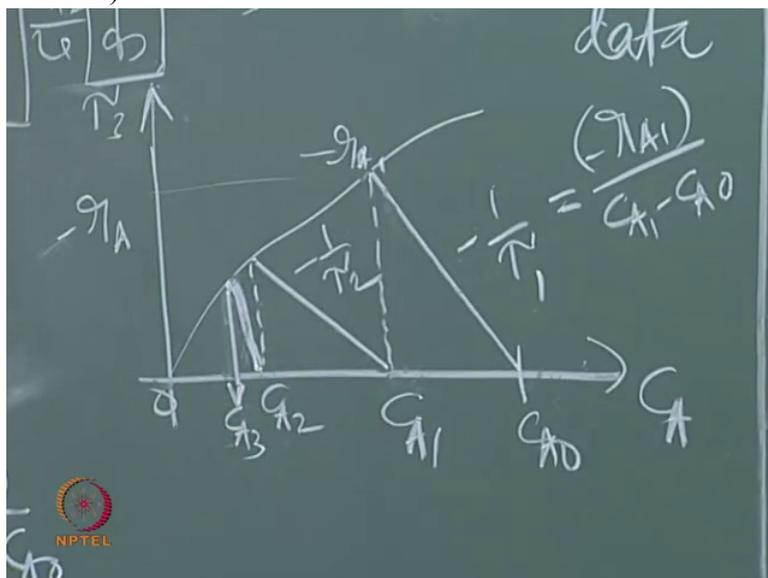
Now I have to draw parallel. This

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is  $C_{A2}$ . Now next one is smaller one. So it may go like this, right? Yeah. So now you have, yeah. Ok. This is the final one,

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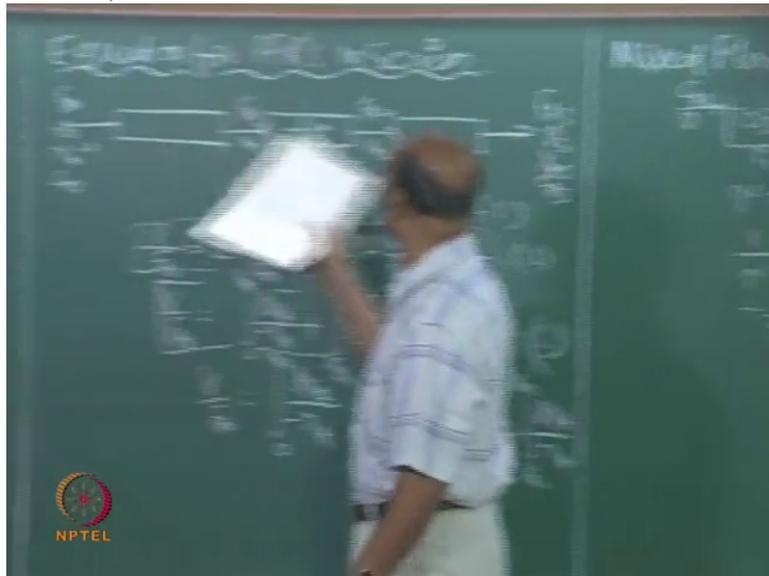


$C_{A3}$ .

So for a known, this is for known  $\tau$ . Ok this is for known  $\tau$ ,



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That is same everywhere

Student: By using the same pump

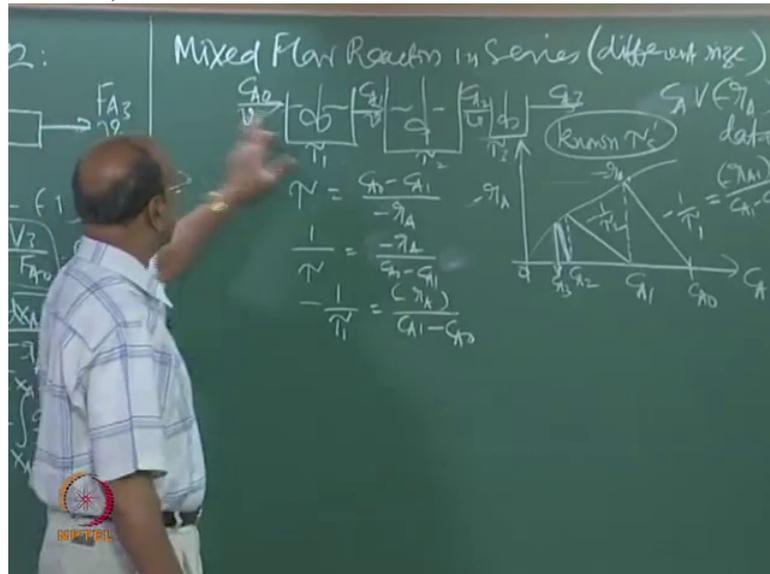
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we can...

Professor: Same pump can be there, I think

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that is only, you know

Student: Flow rate will be same if we use two equal volume tanks, tau will also be same for both

Professor: So what? It is not a problem for me.

Student: Connecting the plug flow reactor

Professor: Conversion depends on, you know, you already calculated conversion based on, you have large tank one and small tank one.

(Professor – student conversation ends)

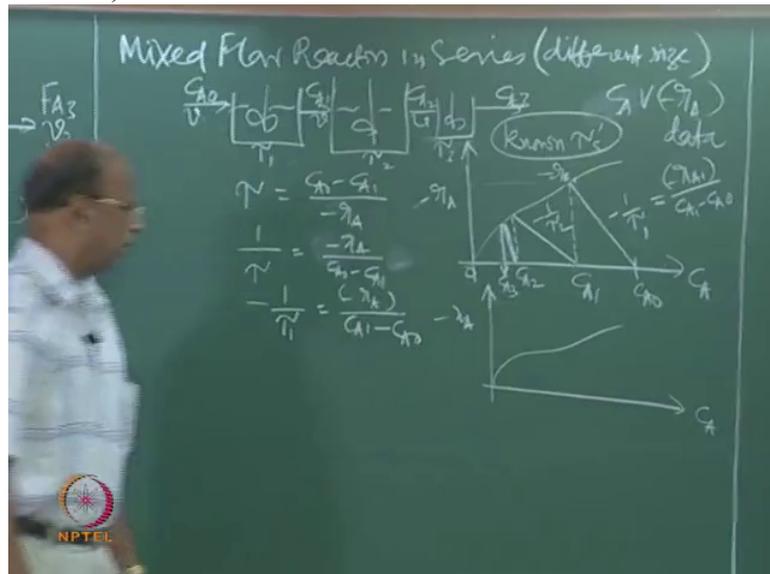
That is not a thing but he does not want to use. He will hesitate to use. No, no, no, I think 2 different tanks; I do not want to use that. Can he say that? Only two equal tanks I would like to use. Yeah, excellent. That is only reason. Because as far as fabrication is concerned, I can only design for one system and then say that Ok, this is the height, this is the diameter, and these are the fittings, all that is fixed.

But the moment it is changing, one meter cube, 1 point 5 meter cube, 1 point 7 meter cube, many things will change including my cutting the plate, to bend it and then again weld it to have the tank, you know cylindrical tank, most of the time. That is why most of the time they do not want to use different reactors.

And fortunately for us, when they, there are some conditions when we have to use larger tank and smaller tank, Ok. There are conditions. I will just tell you that. Ok, so now this is the one. This is the method.

Now if I do not know kinetics, I mean to, I know only conversion. So that means I have another curve. So this is minus  $r_A$  versus  $C_A$ . So again I may have the rate, you know

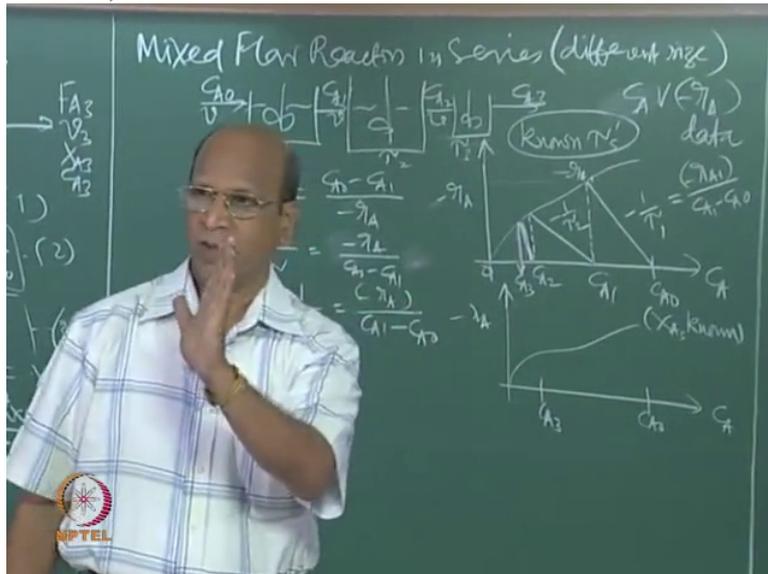
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minus  $r_A$  versus  $C_A$  thing there. I know  $C_A$  naught. Here I do not know, only  $X_A$  is known. So that means I know only  $C_A$  final or  $X_A$  3 if I am taking 3. Ok, yeah, now how do I find out all these?

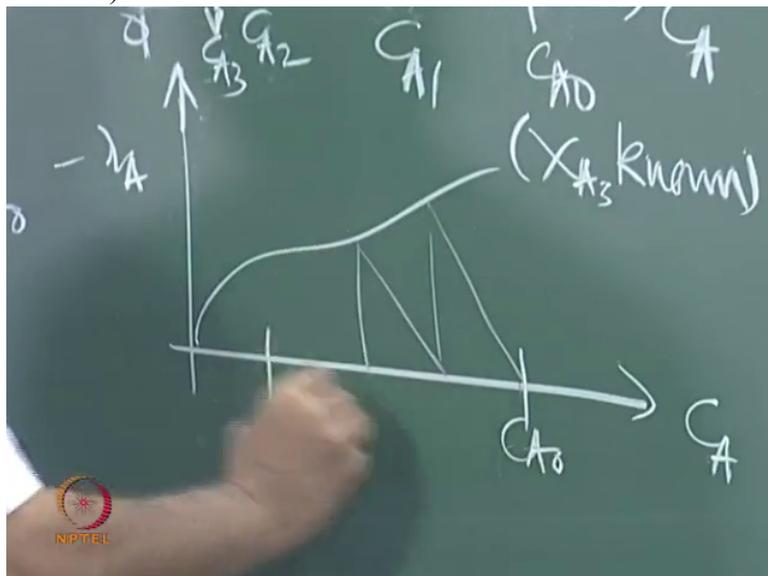
Best thing is that to assume that we have equal, so that slopes are equal.

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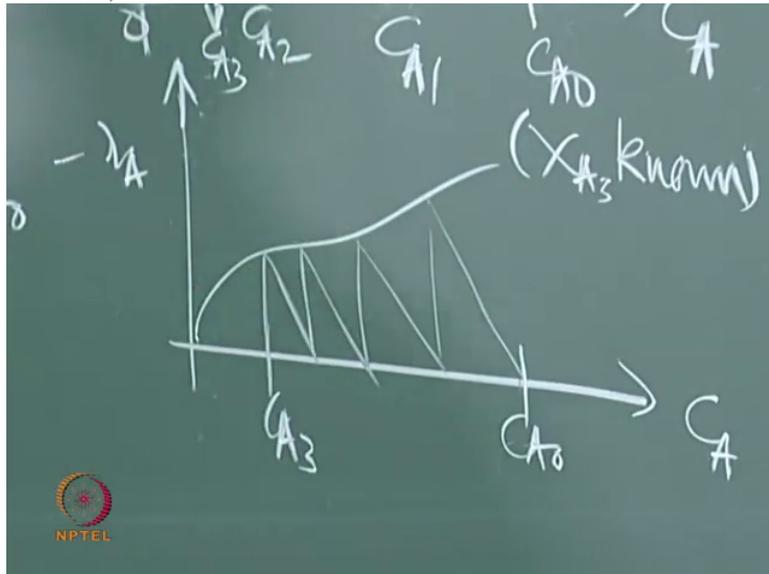
Now try to draw those slopes. Ok, here 1, here Ok, like this, here one like this,

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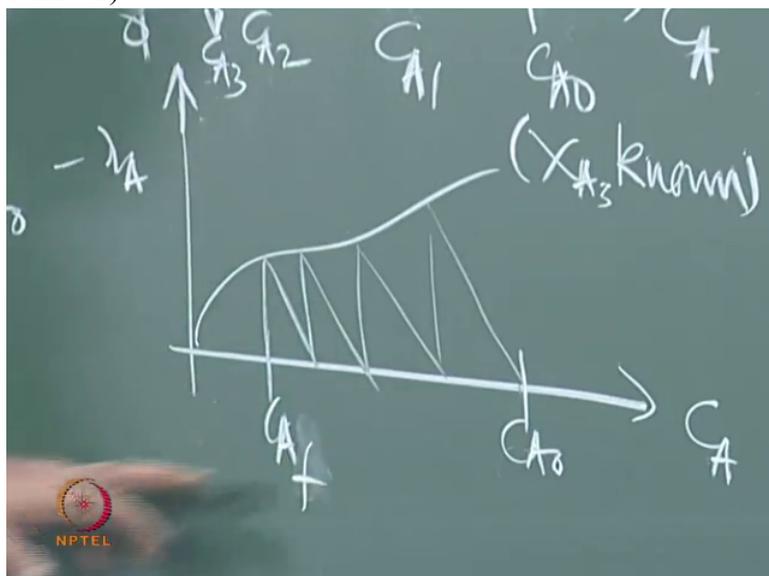
here one like this, here one like this, here one like this.

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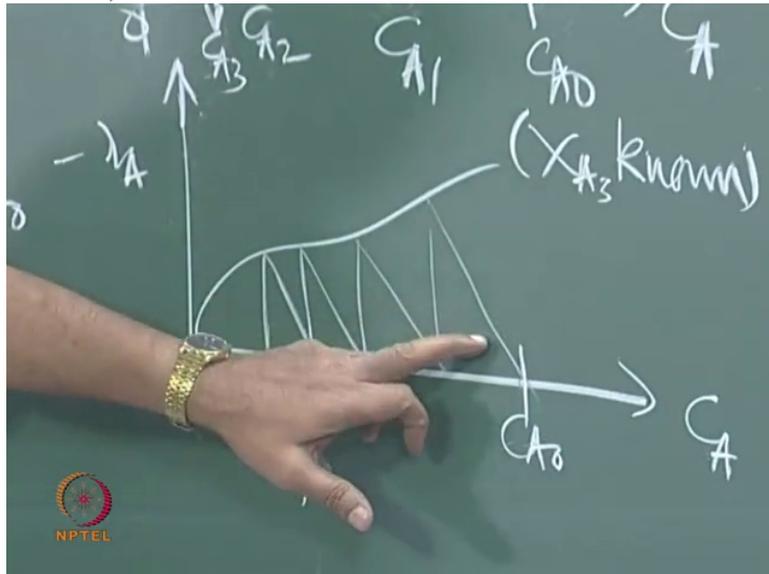
If you are able to match this fortunately then you will have, I think I do not have to put here, X A f I will put here.

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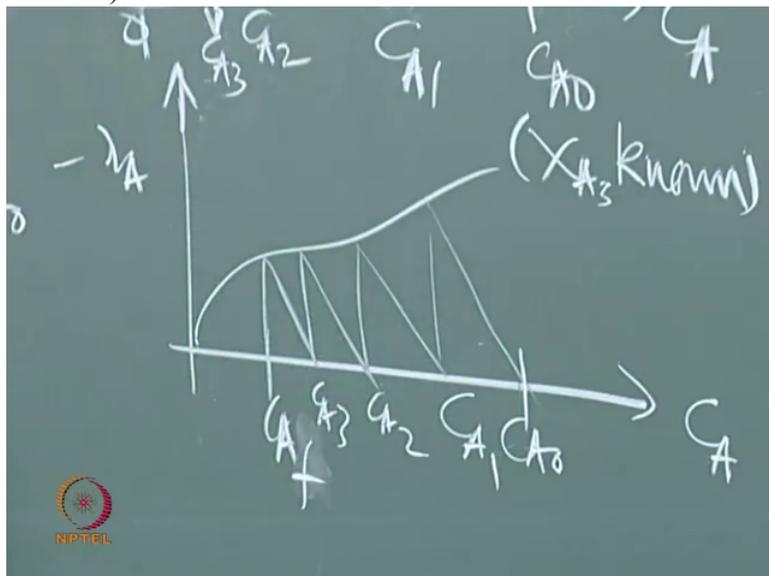
Then it is the same slope, same slope, same slope, same slope then 4 tanks I have. So I am very lucky. Then the first slope which we have

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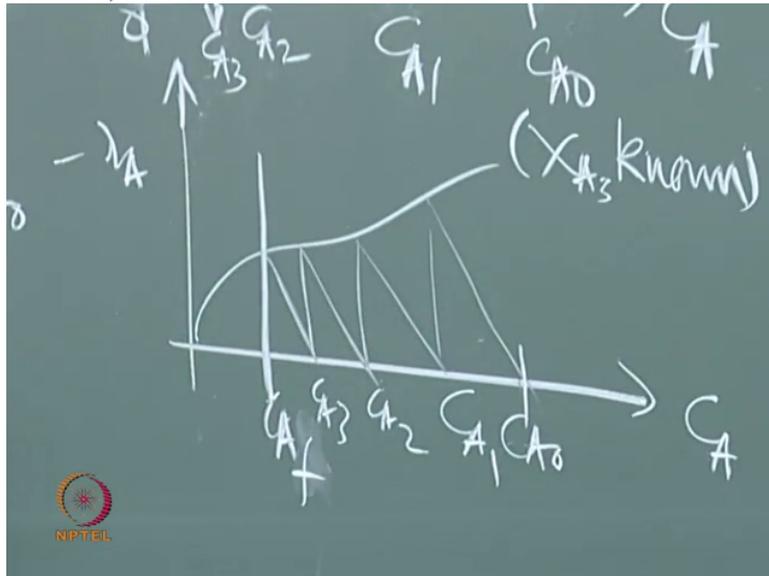
taken may be giving me, this is  $C_{A1}$ ,  $C_{A2}$ ,  $C_{A3}$  etc.

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So finally when it match, because this line is fixed. Because that is your final conversion.

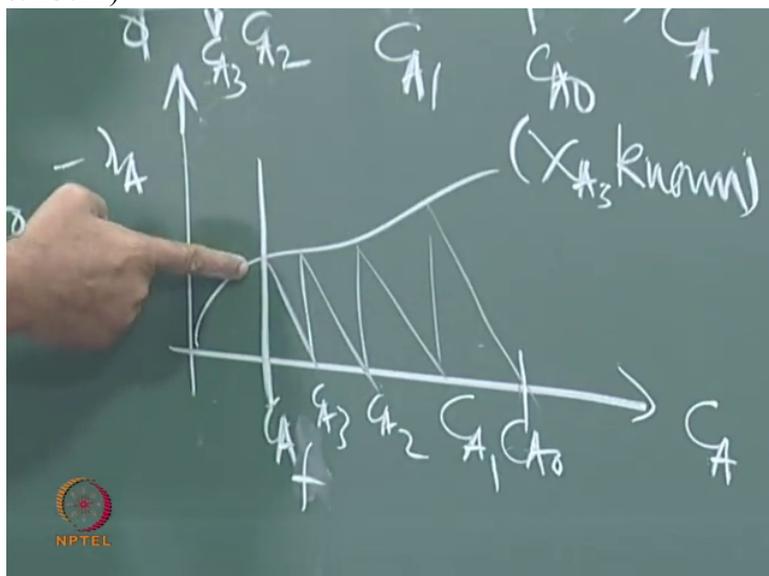
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How do I draw?

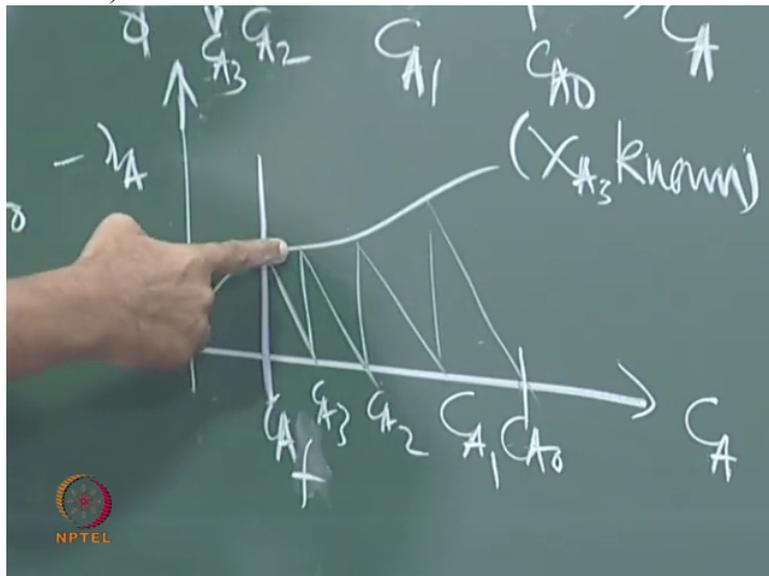
And if it is not coinciding, let us say it is going here.

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Or let us say it stopped here.

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Then again I will change my slope. So once I can find out that kind of parallel slopes, that is nothing but  $1$  by  $\tau$ . So then you can find out, yeah, volumetric flow rate you know, so volume we can find out and then how many number of tanks also you can find. This is by trial and error method. Good, Ok excellent.

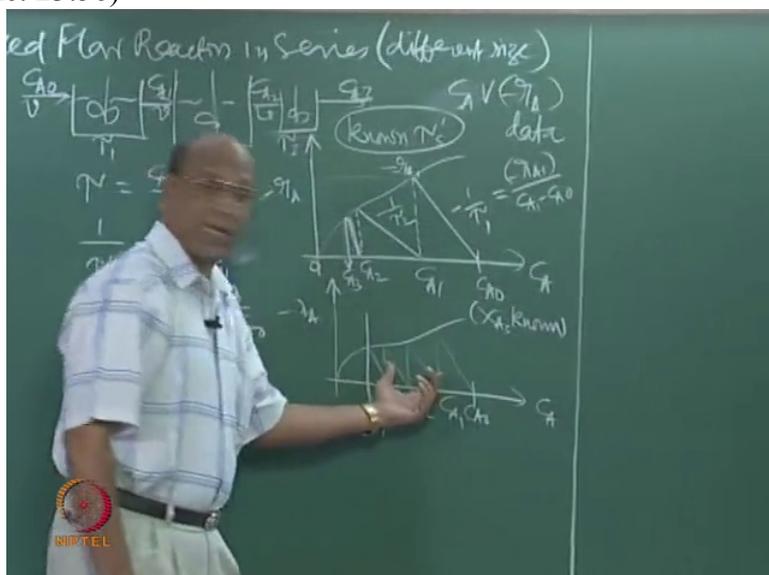
So I think these things are now very clear. So this is over now. So the next one is

(Professor – student conversation starts)

Student: 0:23:53.3 or not?

Professor: You do not know. Even you may not know that one earlier.

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But if you know number of tanks, then you have to draw the slopes again such that, yeah, only that 3 will come.

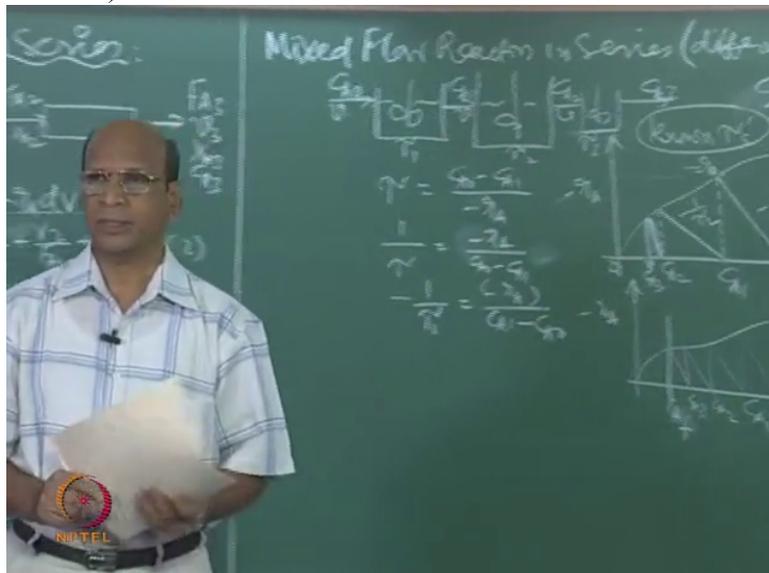
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That means again you have to adjust the, these things such that you will get that.

Student: Sir, number of tanks

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are not given. How can you find,

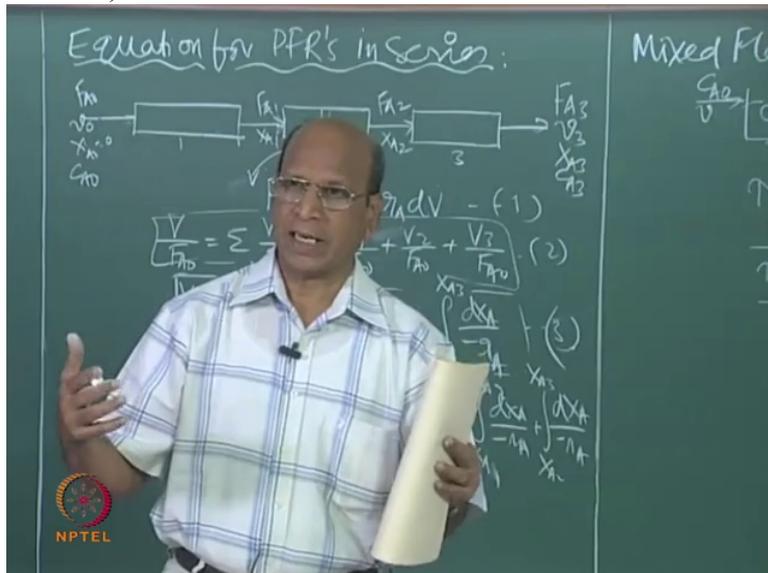
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so many tanks can be there. So many parallel and series will be there

Professor: Yeah, I think that can be drawn. I think, you know number of tanks, that is what, is the problem there. Because it is

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by trial and error when you are doing it, there are many possibilities, Ok.

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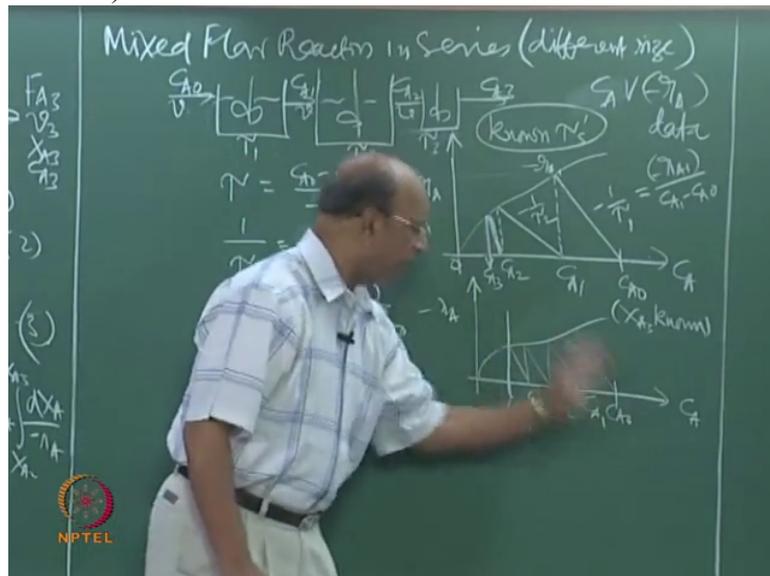
(Professor – student conversation ends)

So that is why if you are able to fix that that is easy for you. If you are not able to fix that, then as he said any number can be drawn. But only thing is that final one should match

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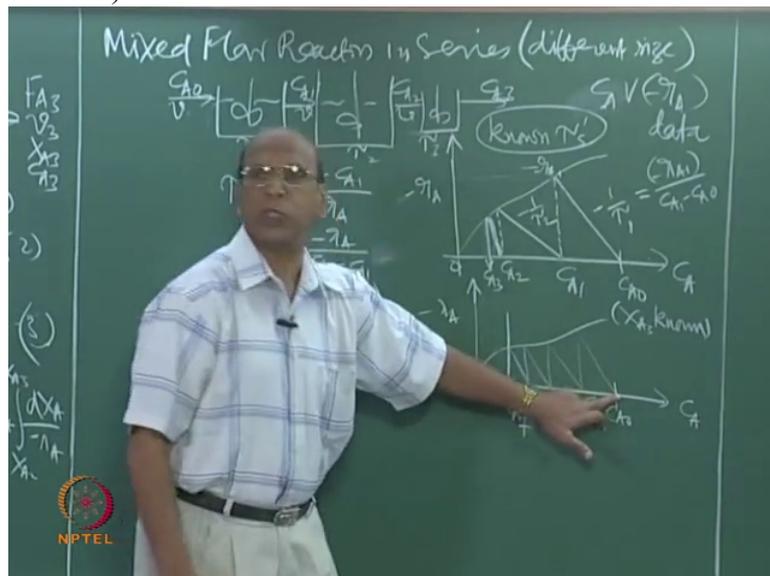
here, Ok. Or otherwise you can also start from here, draw the line, and move parallel

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lines; again it will go through this point,

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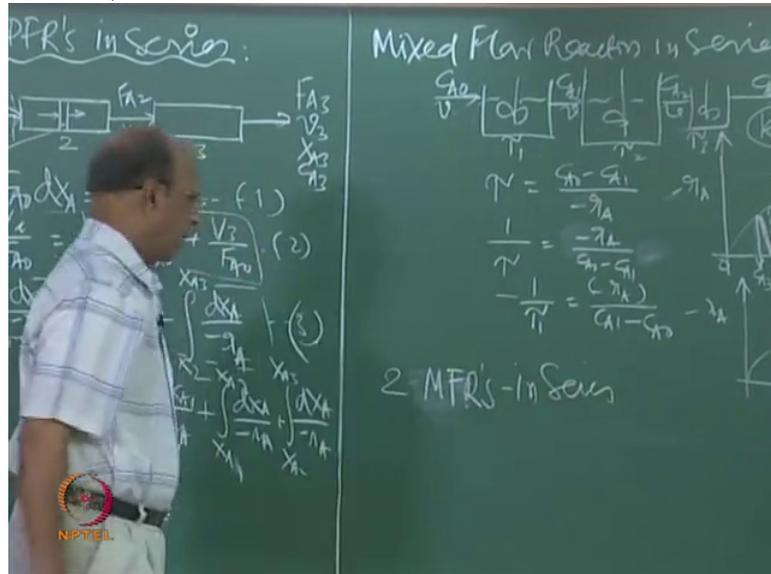


right. That is by trial and error, good. Ok. So that is one thing.

Now we have another problem. This is also nice problem. I have two tanks, Ok, right and I gave you this one as an exercise. I have two tanks in series. I want to find out the optimal size of these tanks, right? Optimal size means given conversion what will be the minimum volumes of these two? Right and what will be the, the conversion is fixed there, right but similarly if I have the volume fixed, what will be the maximum conversion you get, maximum conversion you get.

These are the two problems, optimization problems what we have. But the first one, we will take 2 tanks, two MFRs in series.

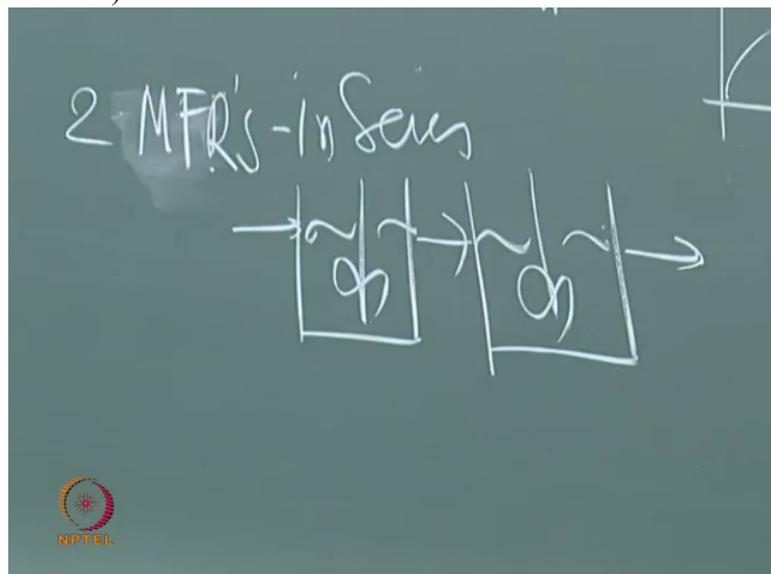
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And I have, in fact I do not know if they are equal in volume also. I do not know whether they are equal in volume also, right?

So there is a nice, again graphical procedure here for this kind of work, for going to optimization. I will tell you another thing.

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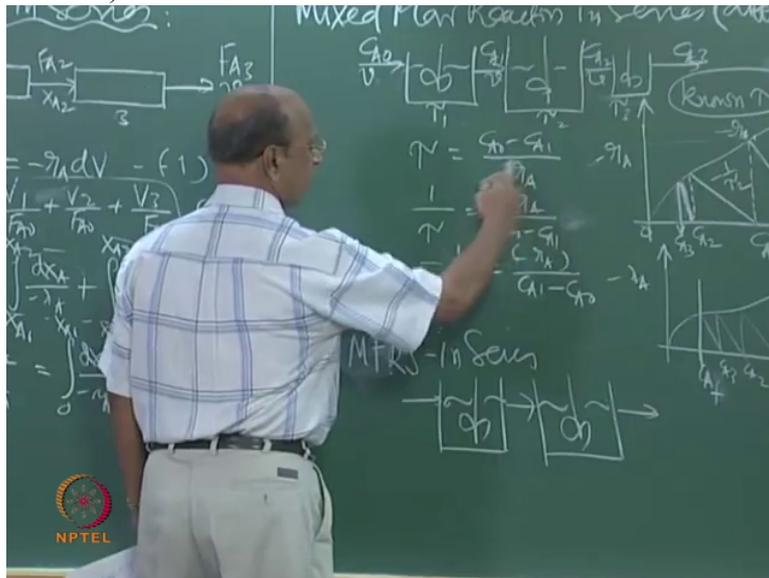


It is equivalent method to McCabe Thiele method Ok, where the stages and 45 degrees line and all that, that is one of the oldest method many people are not using.

But there what you know is, you do not know the, kinetics are known to me. For example order of reaction is known to me in that case. Ok, and now I would like to find out what is the conversion if I know the size.

So I mean kinetics are known to me and also the size is also known to me means tau is also known to me. But that is very good for particularly complicated rate equations like second order. If you put second order in this equation  $V$  by  $F A$  naught, yeah in this equation,

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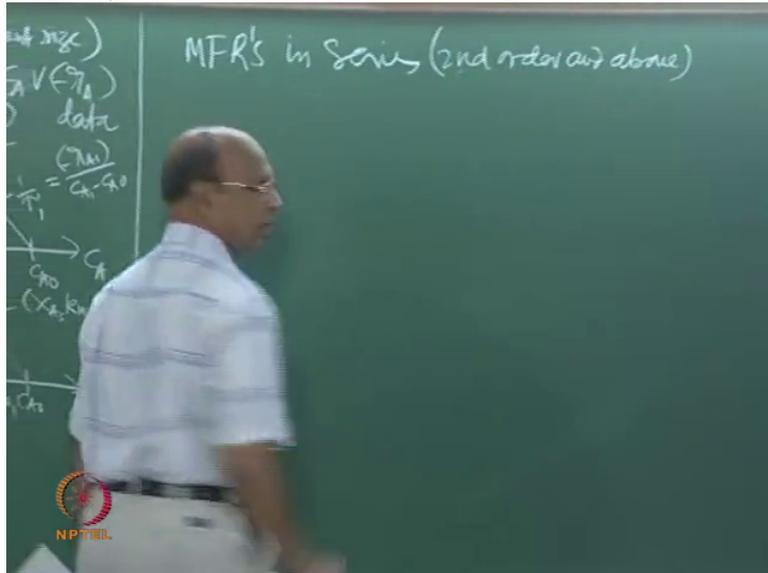
right, you have here  $k C A$  square or  $k C A$  1 square, right then you will get a quadratic equation.

If you see Levenspiel book for tanks in series second order he will put many roots, many square roots. So under those conditions, instead of using you know how many square roots you know you have to find, you know the graphical procedure is one of the easiest and beautiful procedures. Very nice procedures, Ok. So that procedure we will just discuss.

For generally order 1 also one can put but it will be very useful for orders greater than 1. Ok. Or of course less than 1 where you get algebraic equations are complicated equations. So that I think, yeah this we will postpone and then we will take this one first.

This is 2 tanks in fact, yeah. This, yeah, so this one let us say for second order and above. MFRs in series second order and above,

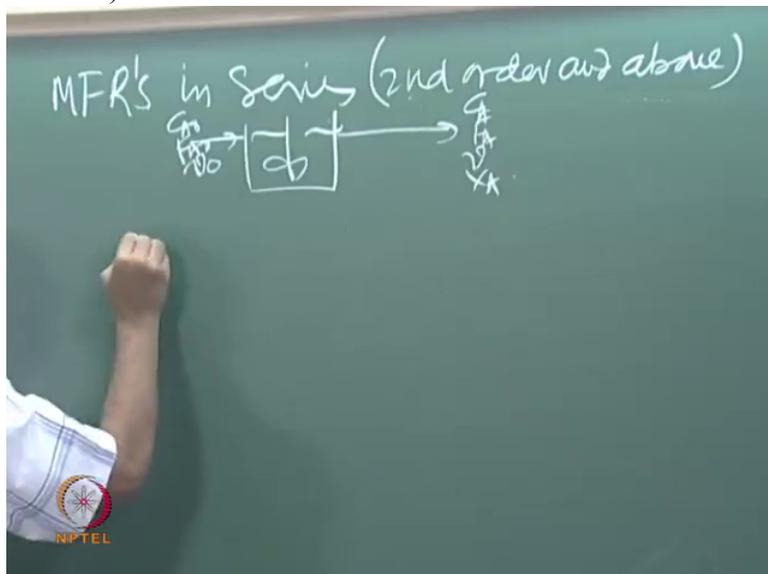
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Ok. So first I would like a reaction for a particular reactor. So then we will have, yeah. So this is  $C_A$  naught, let me write for 1 first Ok, and  $F A$  naught,  $v$  naught and this will be  $C A$ ,  $F A$ ,  $v$  and  $X A$ , good.

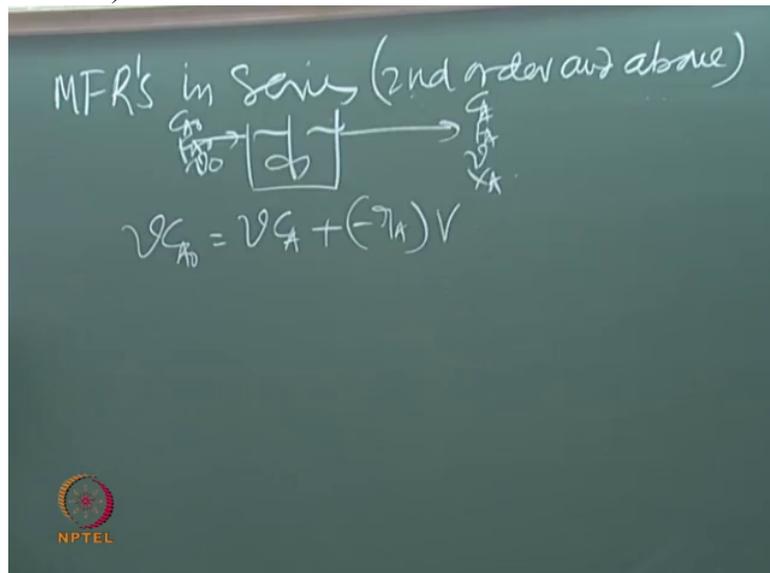
So what do you

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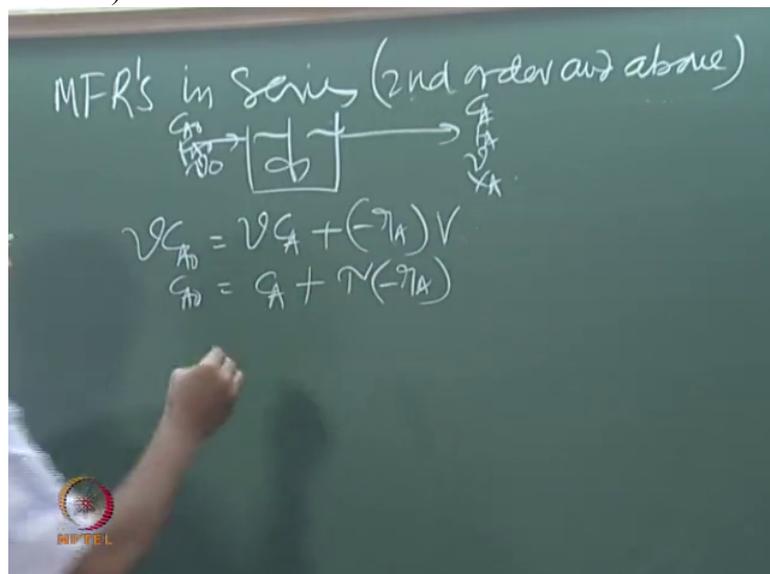
write here?  $v C A$  naught equal to  $v C A$  plus minus  $r A$  into  $V$ ,

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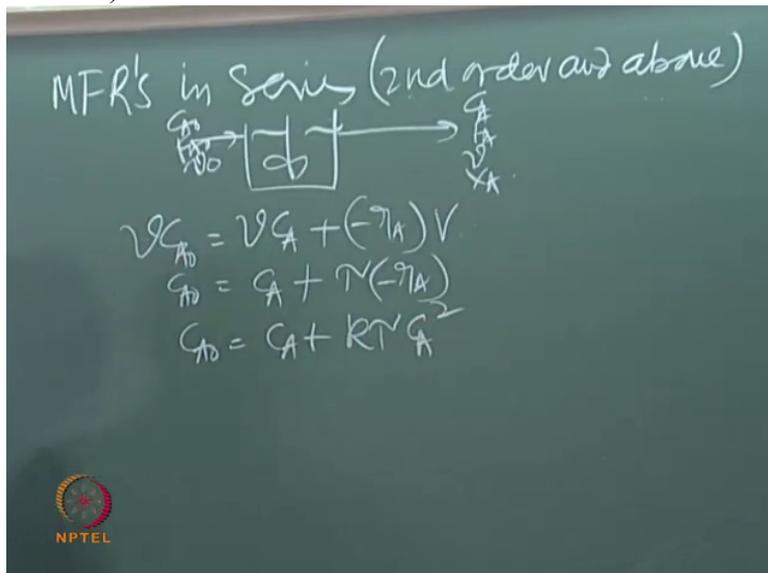
correct, minus  $r_A$  into  $V$ . Now this I know that second order and above. Let me take second order. So that second order will be, Ok, before second order I can also write this equation as  $C_A$  naught equal to  $C_A$  plus  $\tau$  into minus  $r_A$

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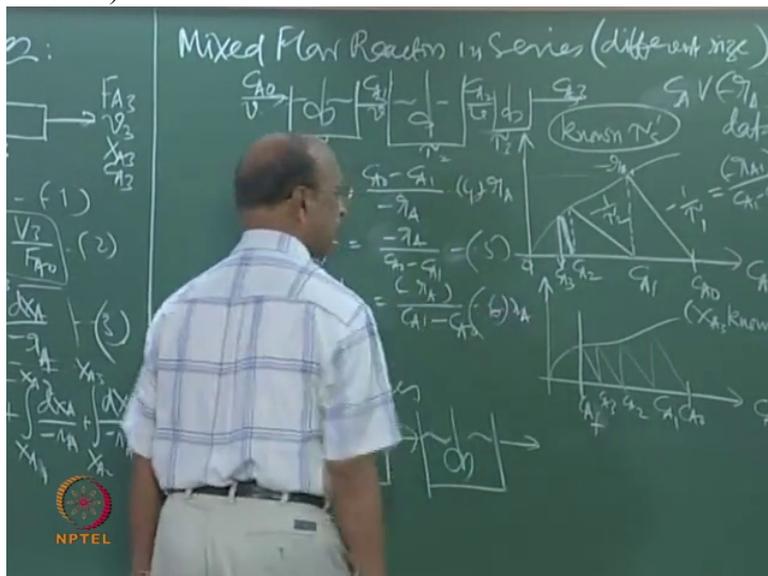
where this one is greater than  $C_A$  naught,  $C_A$ , it is second order so  $k \tau C_A$  square,

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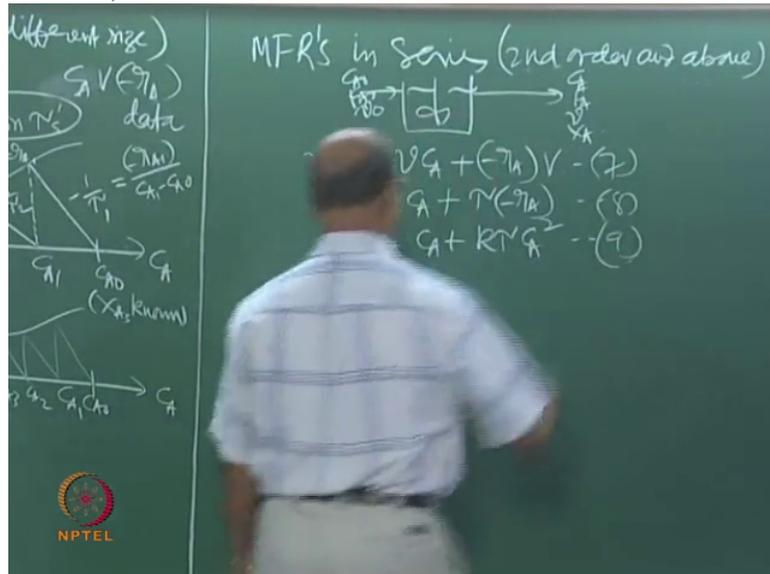
This is also 3, this is 4, this is 5, this is 6,

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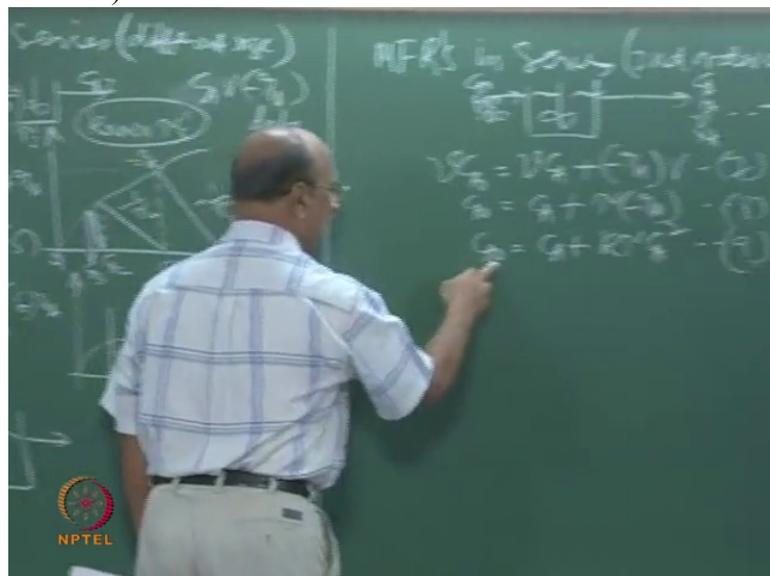
Ok all this, Ok, this is 7, 8, 9.

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Ok. So this is for one reactor I have written. But when I have n number of tanks like this and then I have, Ok, this is nth tank, this is ith tank. When I have this kind of situation, I can simply write this one for

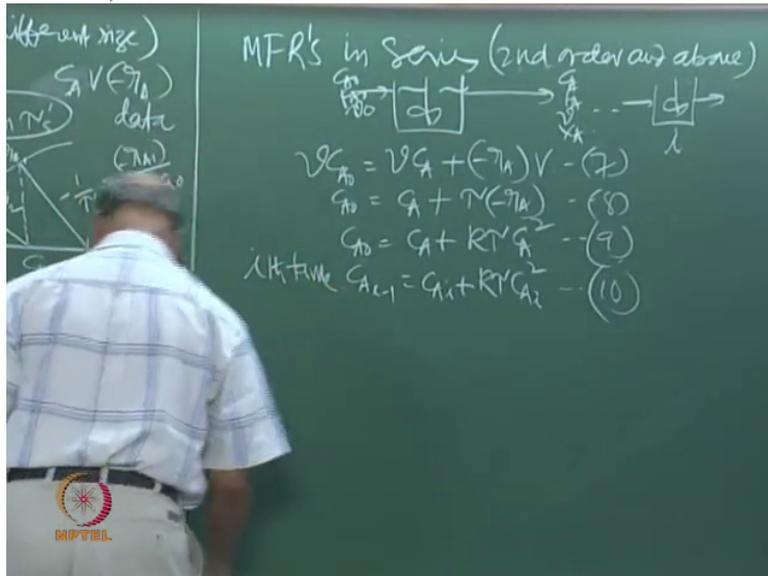
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ith tank. Ok, what is that?

$C_{A,i-1} - k\tau C_{A,i}^2$  for ith tank, Ok for ith tank.

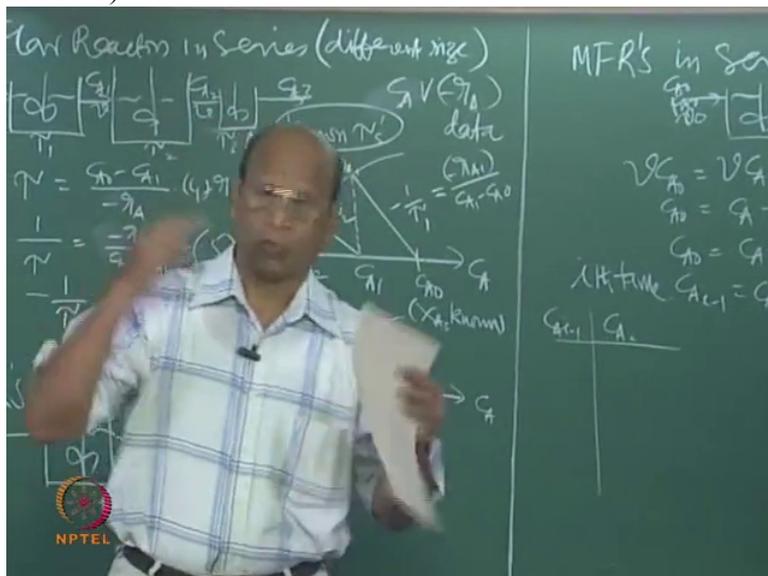
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So now this is fine. But now I do not want to solve this quadratic equation and then sometimes we also do not know which is positive, you may get one positive root and another negative root and then you know, so it is really difficult. So that is why one of the easiest engineering way to solve it is again using the graph.

And this is very easy also to plot because for me, I will simply try to find out what is  $C_{A,i} - 1$  versus  $C_{A,i}$  but for given

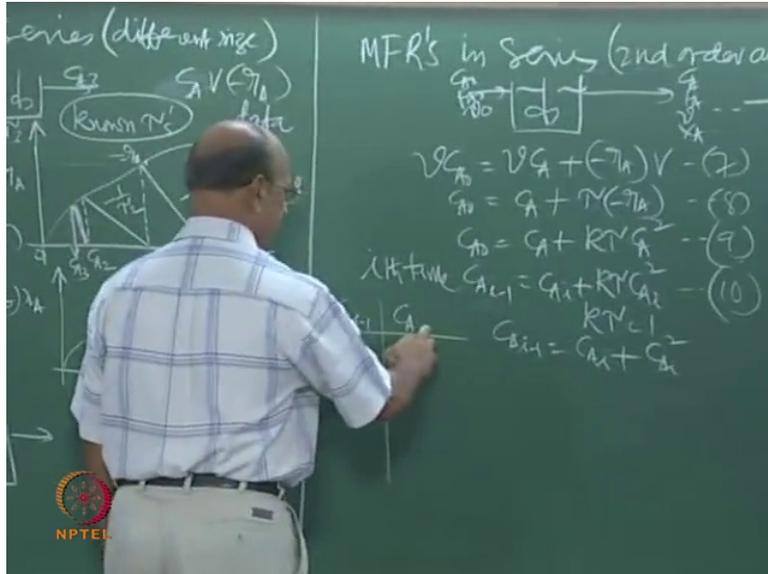
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tau. It is second order I told. That means kinetics I know. I want to only find out what is the conversion. Right. That means k I know, tau I know because I know the volume of the reactor so that I can find out what is the conversion. Ok, yeah.

That is what is the normal, may not be exactly normal all the time so here  $k\tau$  is known. So if  $k\tau$  is known, let me say that  $k\tau$  equal to 1, just for simplicity. So then what is the equation I have,  $C_{A_i}$  minus 1,  $C_{A_i}$ ,  $C_{A_i}$  square? So what I do is, I assume  $C_{A_i}$

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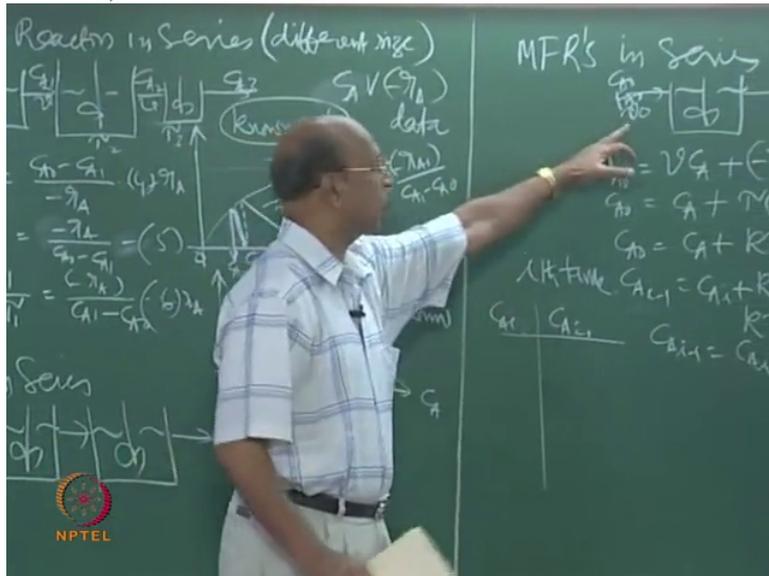
because that is easy.

Otherwise every time if I assume  $C_{A_0}$  and then try to solve  $C_{A_1}$ , again I am not solving my problem. Correct no? Again I have to do quadratic equation. So I do not have to draw graph again. So the simplest one is I think maybe I will write reverse. Because it is very easy.

I have also done using computer. It is very easy but only thing you have to be careful is that when I have 3 reactors, 4 reactors, initial concentration you know but final concentration how much it will come so that is why your range of these values must be very, very large where this entire, all the tanks, that must be within this range. I am talking about numerical values.

If we have  $C_{A_0}$  equal to 100, that is what

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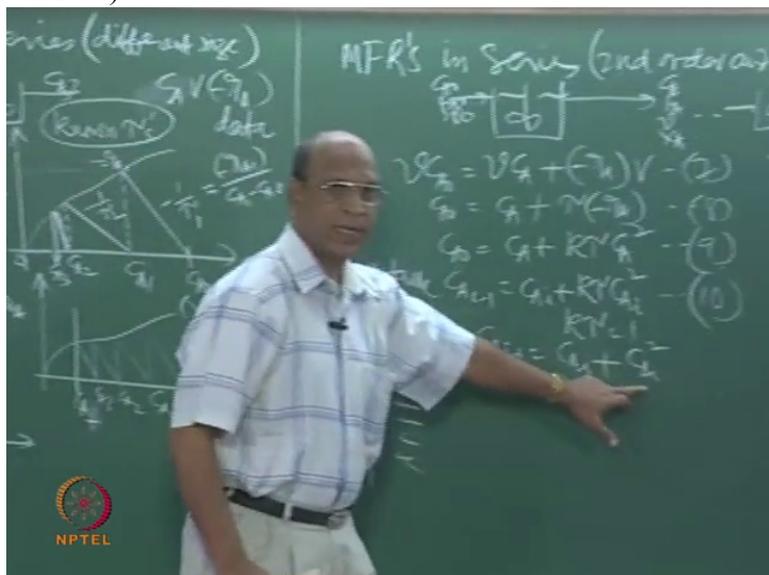


is entering here and you do not know what is outlet, you have to find out some conversion, right so under those conditions you should know what is the range because you have to draw the line, yeah some line, it is not straight line. Ok.

So like that we will have, if I start with yeah, some values, I think I cannot write but numerical values. Because the moment I write C A 1, C A 2 again you will confuse with, Ok those things. I do not want to write. These are the numerical values based on these.

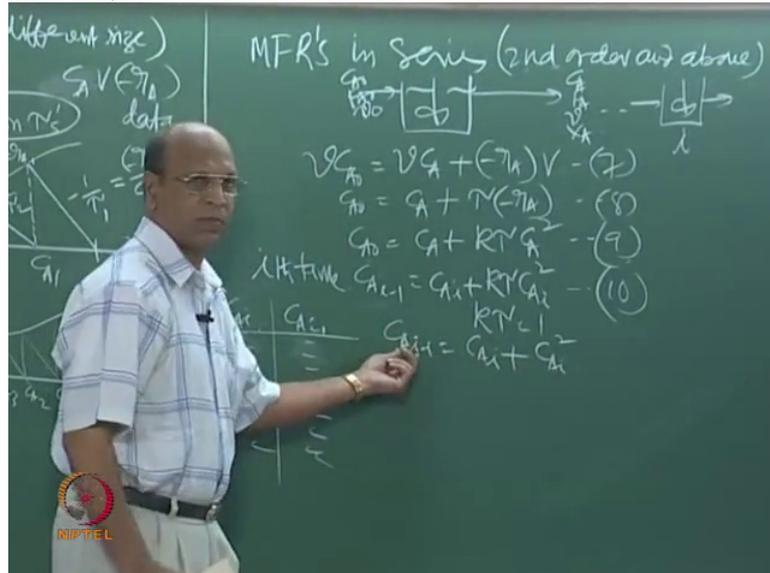
That means I will

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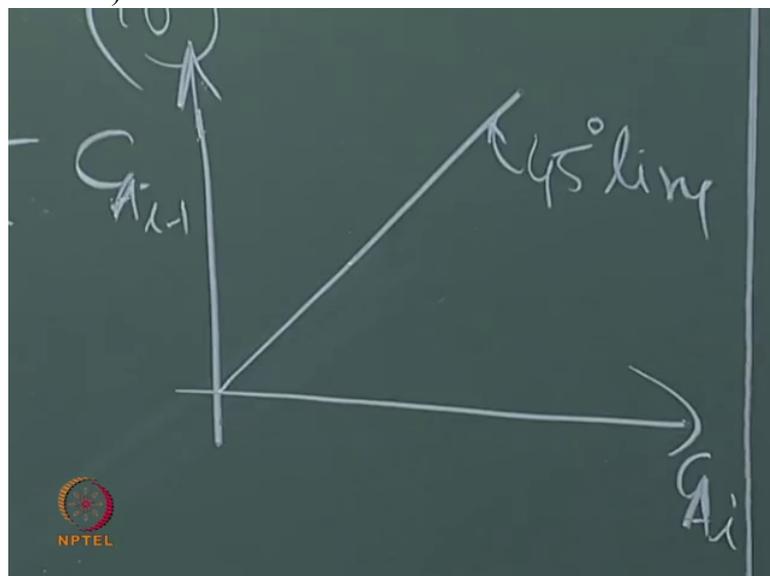
start with C A i equal to 1, calculate this.

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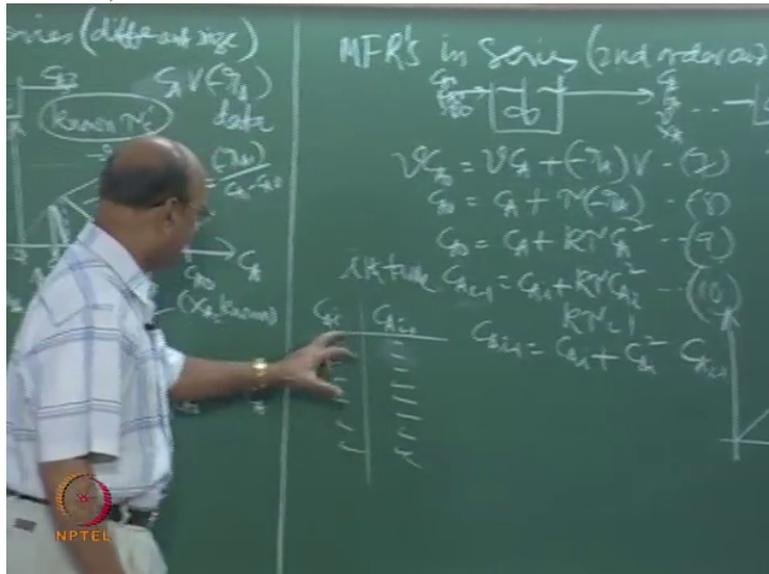
$C_{A,i}$  equal to 2, 4, 6, 8, 12. Like that you know, simply dragging no. You go to Excel and simply drag; you will get those values, good. So now what we have to plot is  $C_{A,i-1}$  versus  $C_{A,i}$ ,  $C_{A,i-1}$  versus  $C_{A,i}$ , right and first I will have 45 degrees line. This is 45 degrees line, good.

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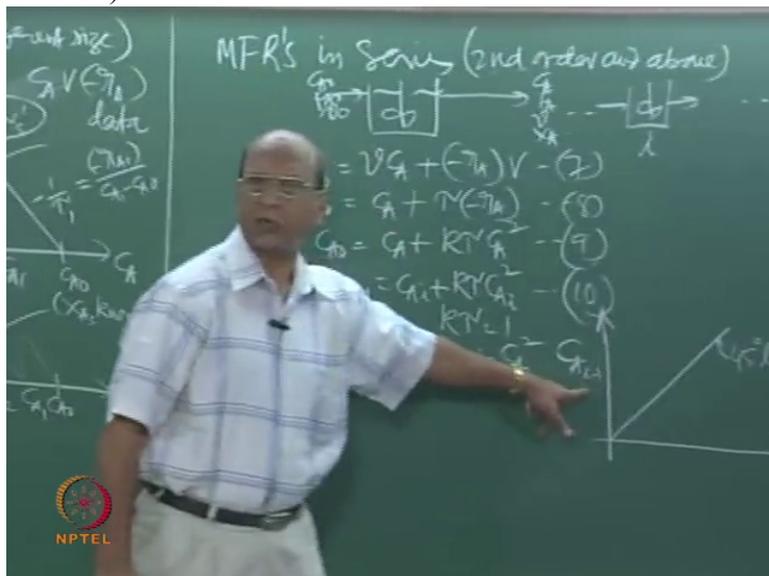
Ok. 45 degrees line is easy for me to draw. That is no problem, good. So the next one is I have to draw is this data. This data I have again,

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$C_{A i}$  and  $C_{A i-1}$  values,

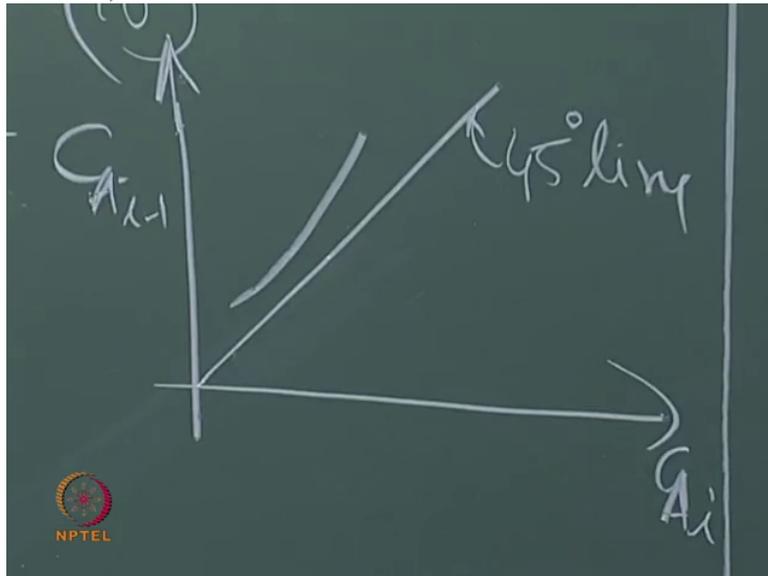
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Ok. So depending on what is the  $k\tau$  you have given, that means you know the reactor slope, you can find out  $k\tau$ . May be 2, may be 5, may be 10, may be 1 point 2, may be 3 point 5 like that.

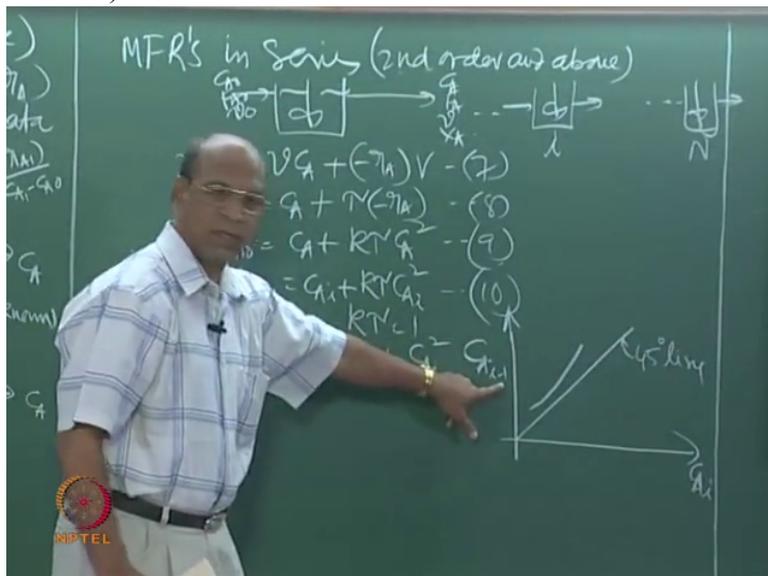
Then you will get a line, something like this.

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But only thing is this line should be beyond the input and output values of this entire system.  
 Ok. So now what is the starting point for  $i$  equal to 1?

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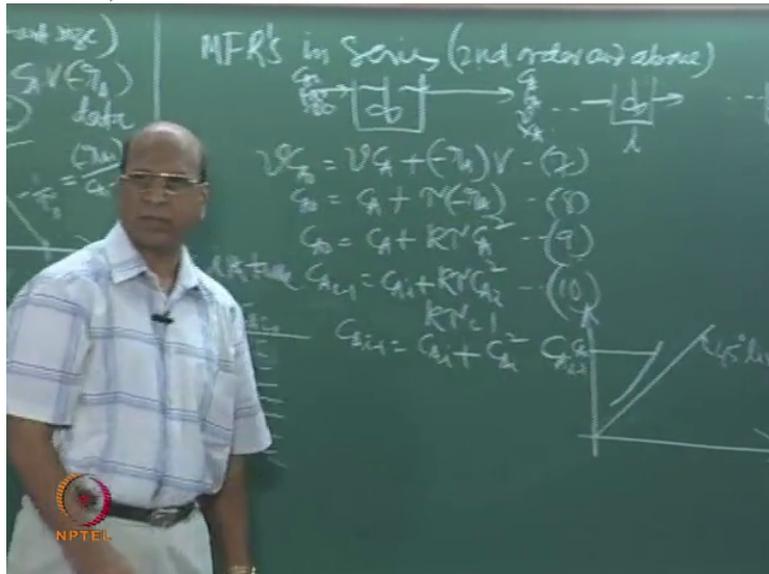
$i$  equal to 1 means  $C_A$  naught

(Professor – student conversation starts)

Student:  $C_A$  naught

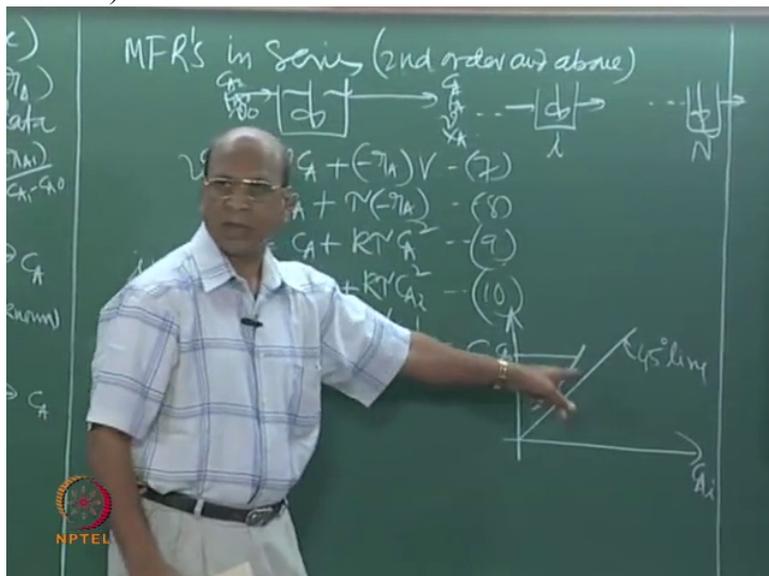
Professor: So  $C_A$  naught I know that is what is entering. So then I will draw, this is  $C_A$  naught, then I draw a straight line.

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That is C A. Now that will touch this particular line, right,

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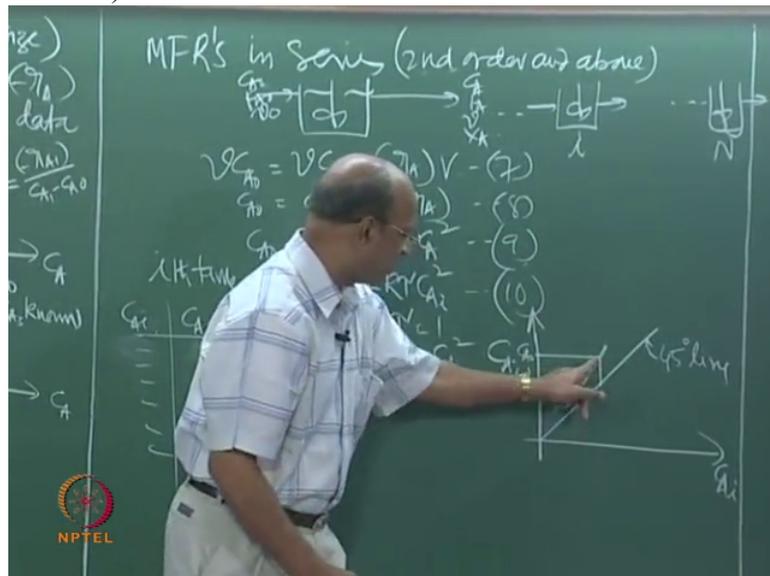


so I cannot go further. That means there is no change.

(Professor – student conversation ends)

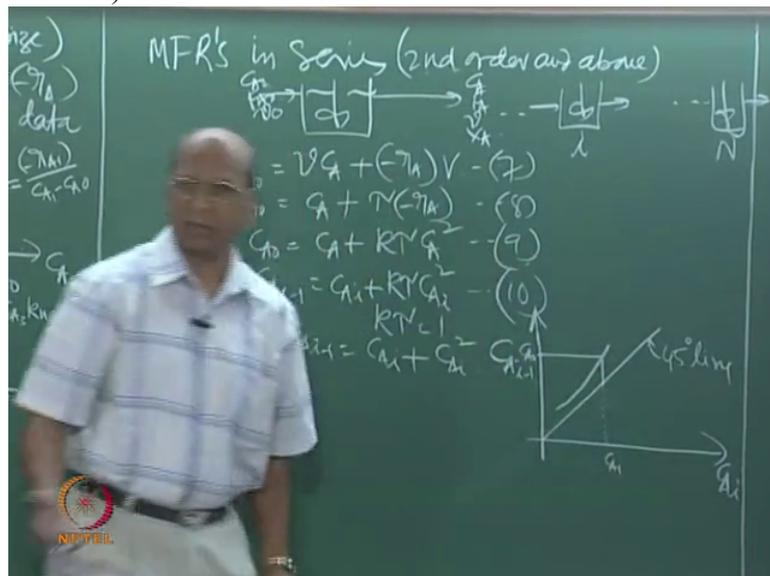
If I straight go here till 45 there is no change at all. But there must be change because reaction is occurring. So that is why only possible way for me is this. And this distance, that means

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this distance is nothing but you have here  $C_{A1}$ ,

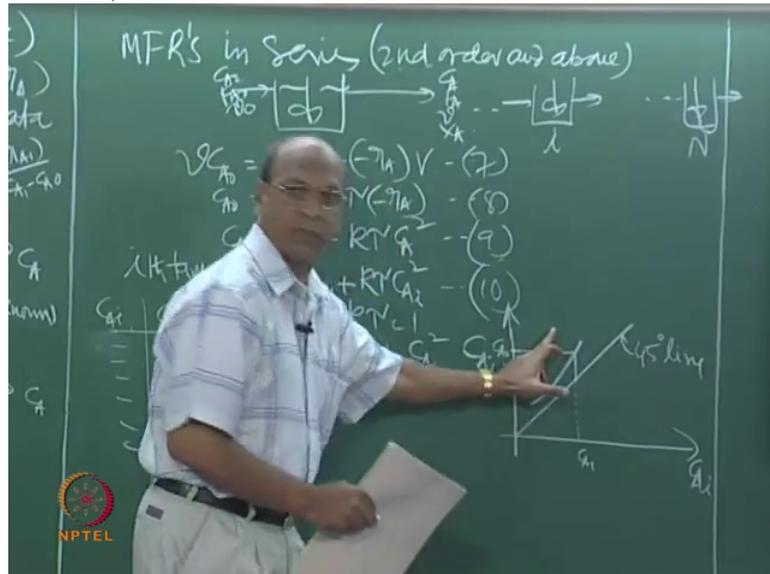
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that is what is coming out of reactor.

But if this curve is you know  $k\tau$  equal to large for example, then it may go like this.

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So then automatically that value will change. The distance between these two, that is what no, in McCabe Thiele diagram also, the distance between equilibrium line and operating line, if it is large then you will have minimum number or maximum number?

(Professor – student conversation starts)

Student:

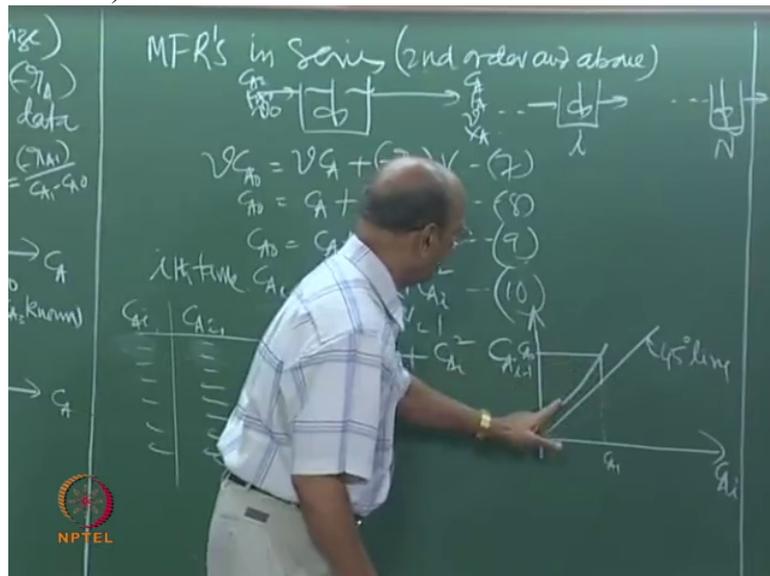
Professor: Minimum, same thing here also.

(Professor – student conversation ends)

If  $k\tau$  is large means what, large size reactor. When you have large size reactor what will be the conversion? Definitely more, more than compared to the small size reactor, you know  $k\tau$  large, volume large so  $X$  will be more. So for  $X$  will be more means  $C_A$  will be small.

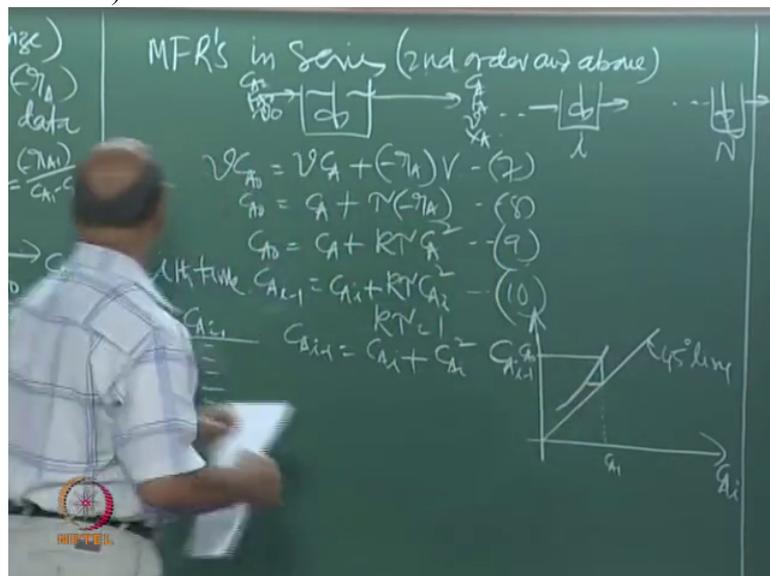
So naturally then you will have, if it goes like this then

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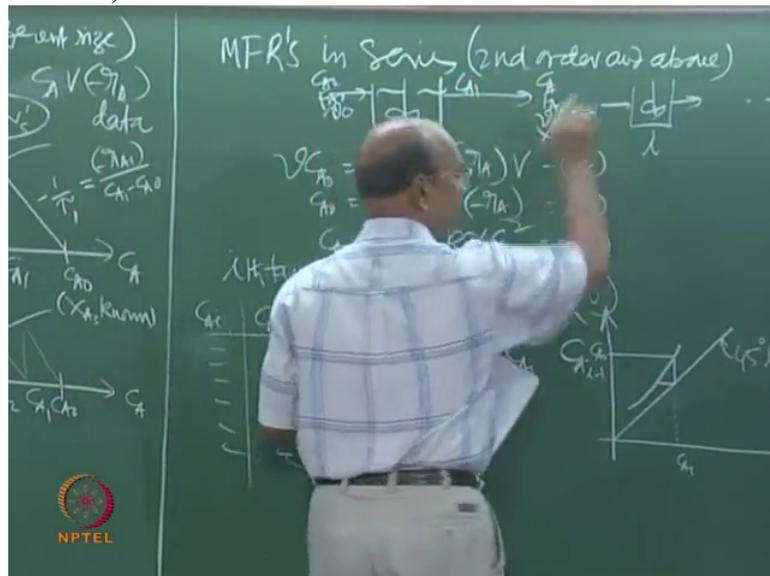
it goes like this, here only cutting so  $C_{A1}$  will be less so conversion will be more. Ok, good. So now what you do is, this is 45 degrees line. This same line is going to the next reactor, Why?

(Refer Slide Time: 35:53)



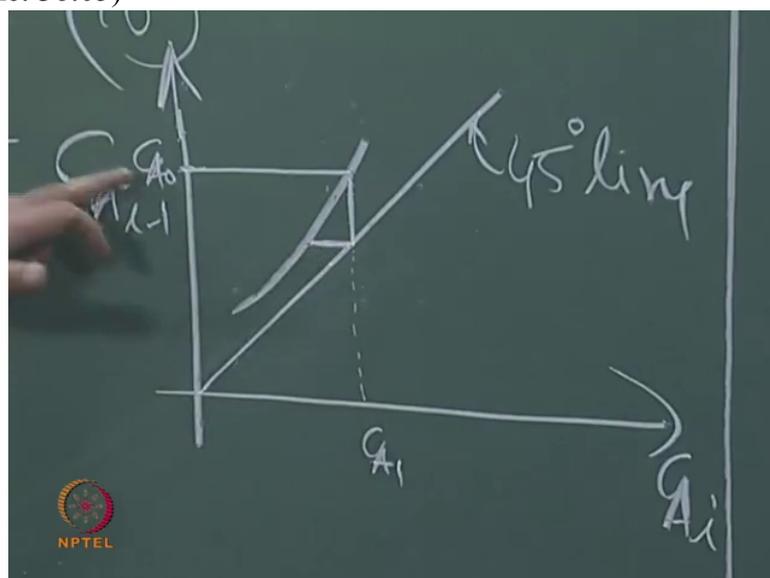
This  $C_{A1}$  is also entering

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$C_{A2}$ , that is not changing. So that is why this line gives me, this is again horizontal line. There is no concentration change. Because this is the input.

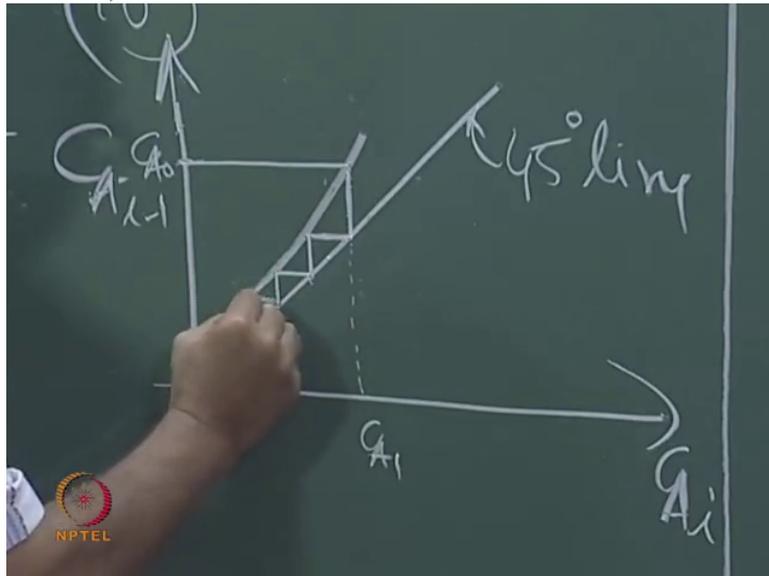
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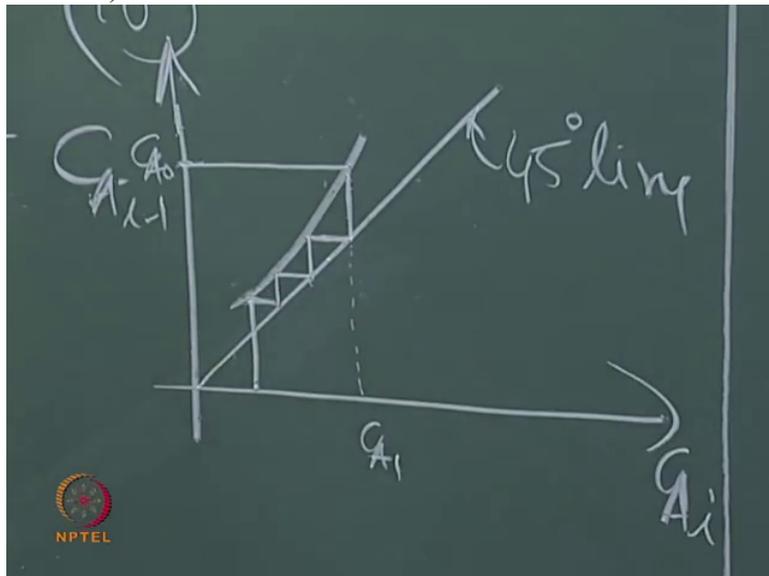
We started with  $C_{A0}$ . Next one input is  $C_{A1}$ . Next one input is  $C_{A2}$ . So that is how.

Like that you go and then if you,

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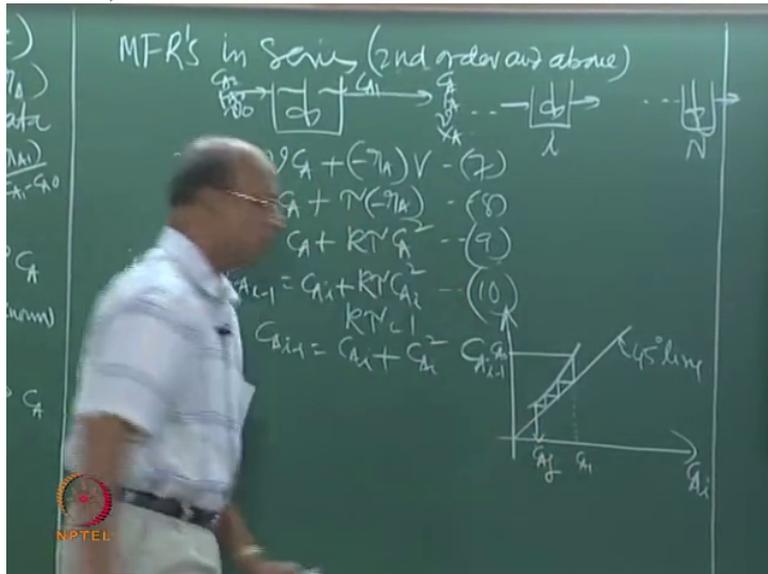


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if you get the, you know conversion as let us say 90 percent, if you are able to reach 90 percent conversion equal to that corresponding concentrations, then that is the

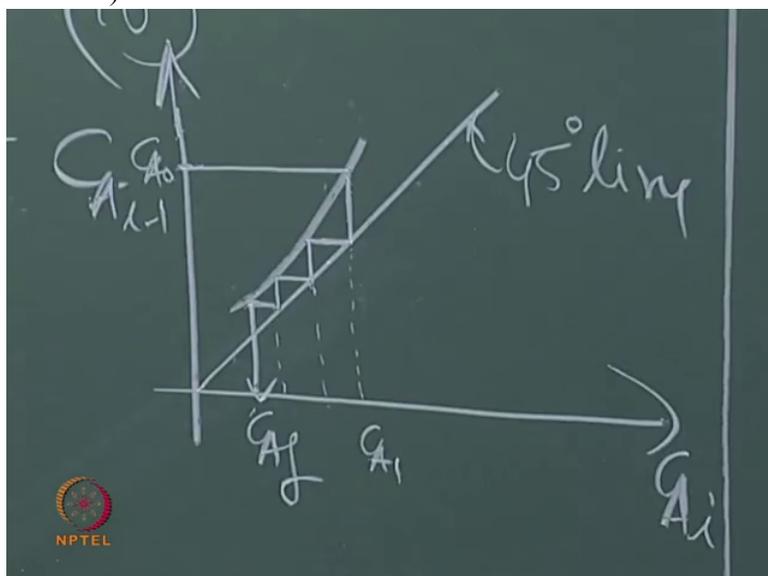
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final. And you can also have so many tanks. What is the assumption here?

Because tau we have not changed. Tau is same. So that means they are equal number of tanks, I mean equal sized tanks. So then you can easily find out. This method, has anyone seen this method?

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You have seen that, which book you have seen? Ah, he is a 1960s man, 70s man.

(Professor – student conversation starts)

Student: Same

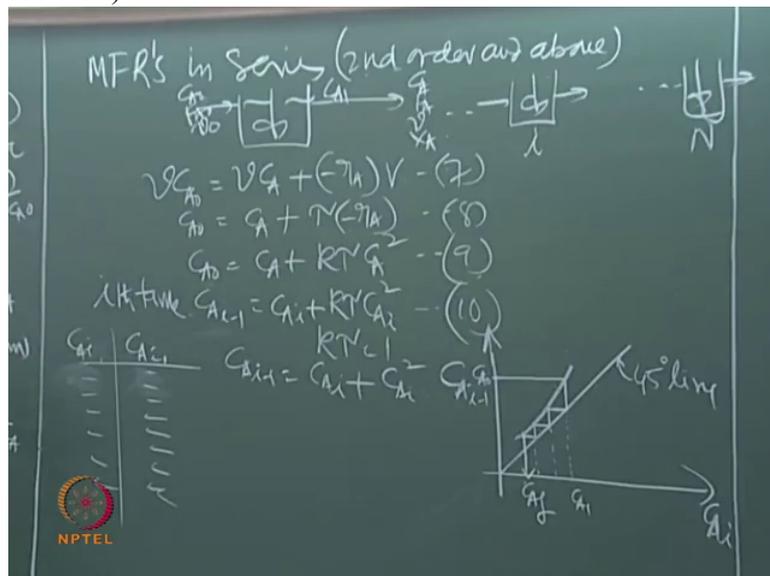
Professor: Yeah, I think old books only; afterwards they stopped it, Ok.

(Professor – student conversation ends)

But I thought you know that is really thrilling for me because it is the generalization of your, yeah McCabe and all that you know, what are the equilibrium stage processes what we have used. So that is the reason why I thought I had to just definitely mention this, which is very easy.

And for second order I do not want to remember all that square roots, square root, square root, square root. Ok, in Levenspiel book, and not only Levenspiel book, any book. So and we are capable of making mistakes there where as here simple graph, calculate and then plot and then you can beautifully solve,

(Refer Slide Time: 37:30)



Ok. I am a dangerous fellow. So that is why in the examination I can give this kind of questions. Because more, I like graphs. I like pictures. Yeah anything?

(Professor – student conversation starts)

Student: If the volume

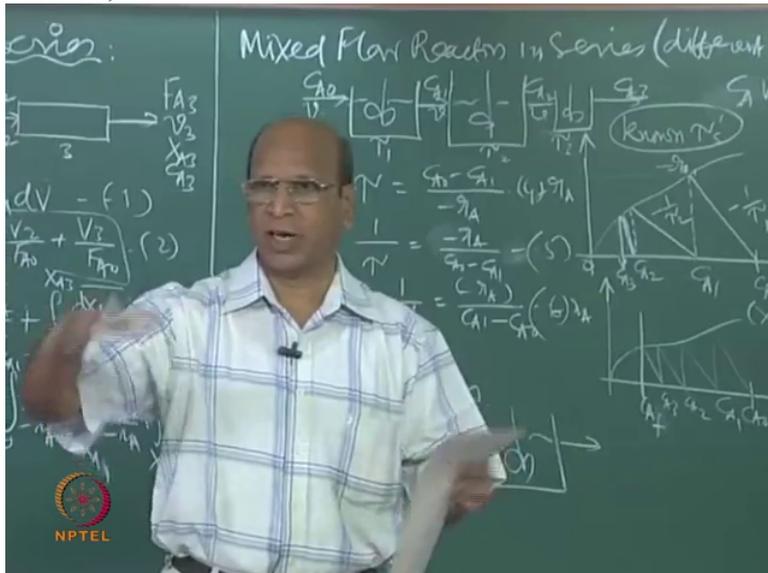
Professor: Ok, if the tau is changing you cannot use that, (laugh). Because

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you know each plate is equally same for us in,

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in McCabe Kaelin method,

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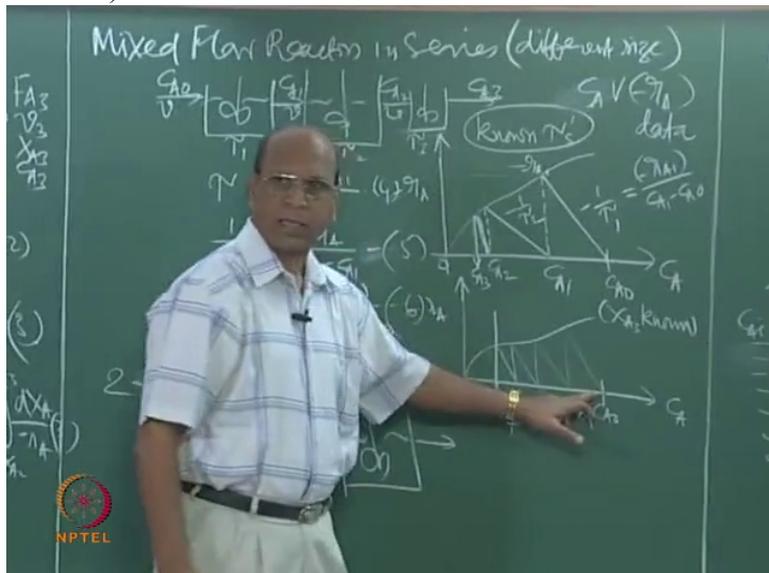


no

Student: Classical method, between that plot between  $r_A$  and  $C_A$ , from there, first method.

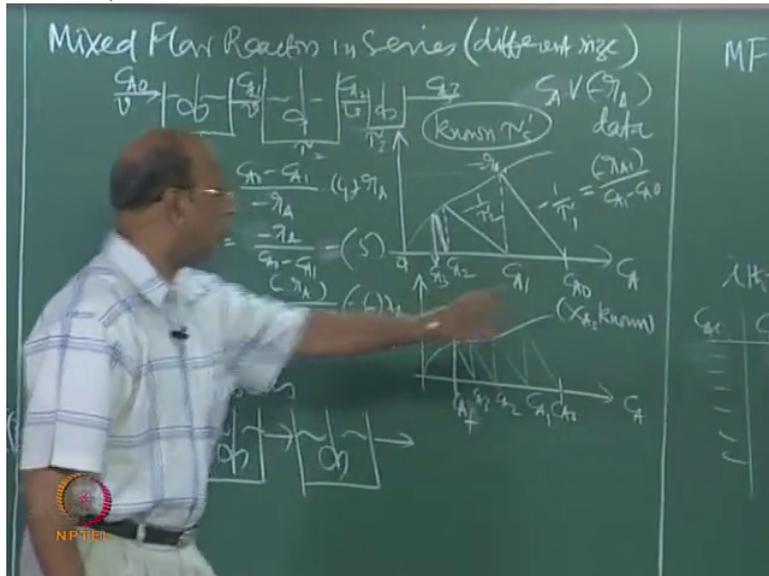
Professor: Which method? Yes,

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but  $\tau$  is changing means you have to use this method. Because slope will change automatically. But there you cannot. Ok, yeah. With everything there will be some drawbacks or some advantages so what you have to take, very good. So this is the one which I want to tell you.

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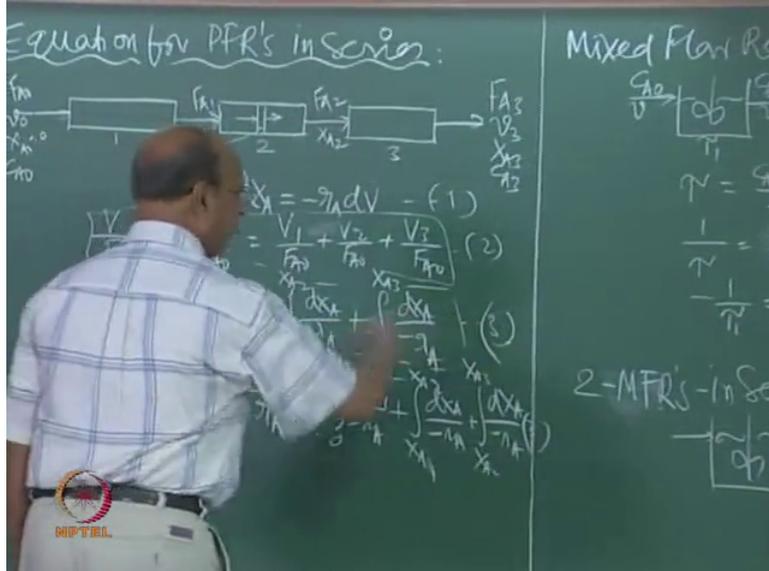


This is also very good.

(Professor – student conversation ends)

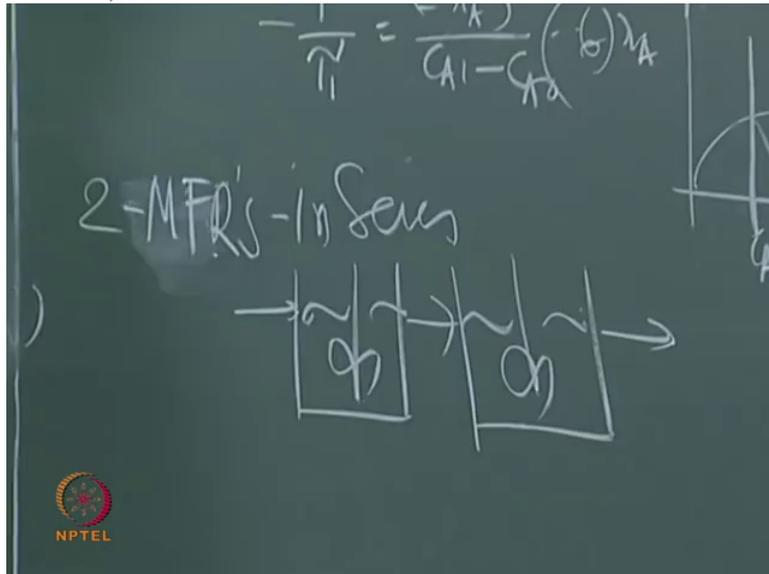
So the next one is this M F R in series.

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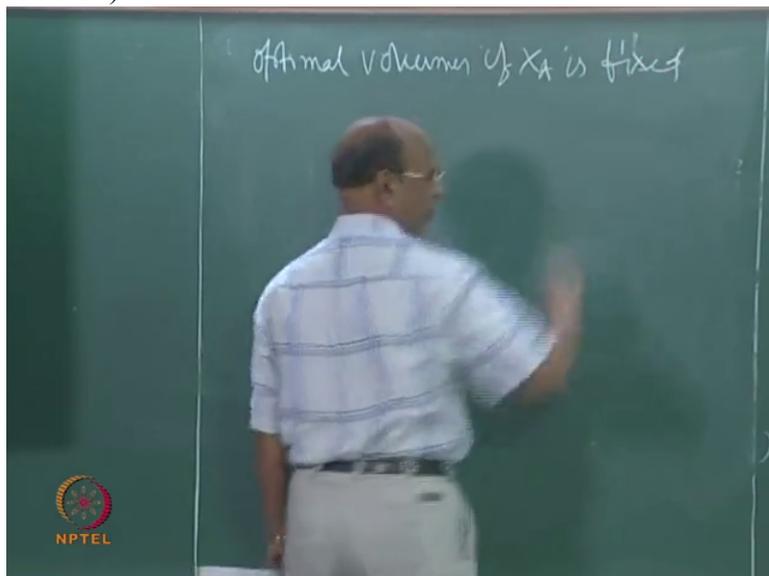
Ok. So M F R in series, there are two tanks. So here I have

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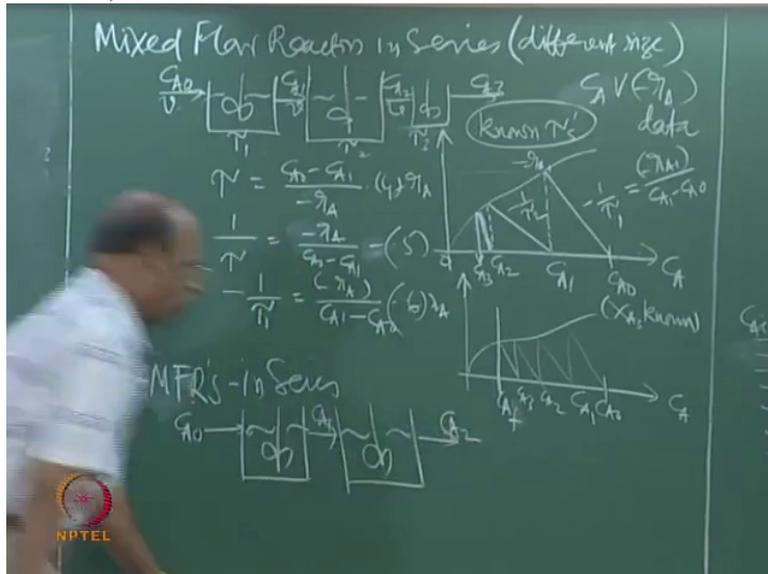
two tanks, my idea is to find out optimal volumes if  $X_A$  is fixed,

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Ok, good. That means I have here,  $C_{A2}$ , this is  $C_{A1}$ , this is

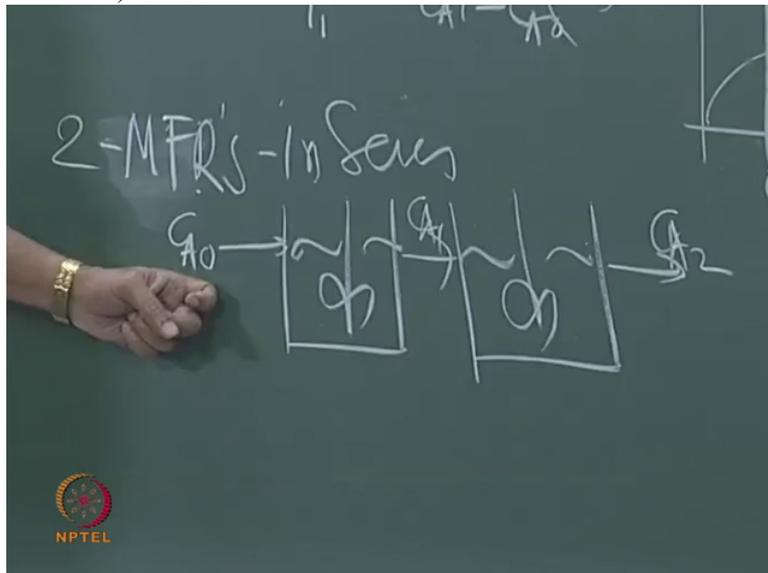
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C A 2, my X A 2 is fixed. Ok, good and analytical way also one can solve that.

But analytical, you know differentiation, for first order it is very simple, that is what I have given to you. Ok. Ok let me tell analytical. So I have to write, the volume should be minimum. So that means my function is V as a function of, V as a function of, which one is the variable here? Because C A naught is fixed.

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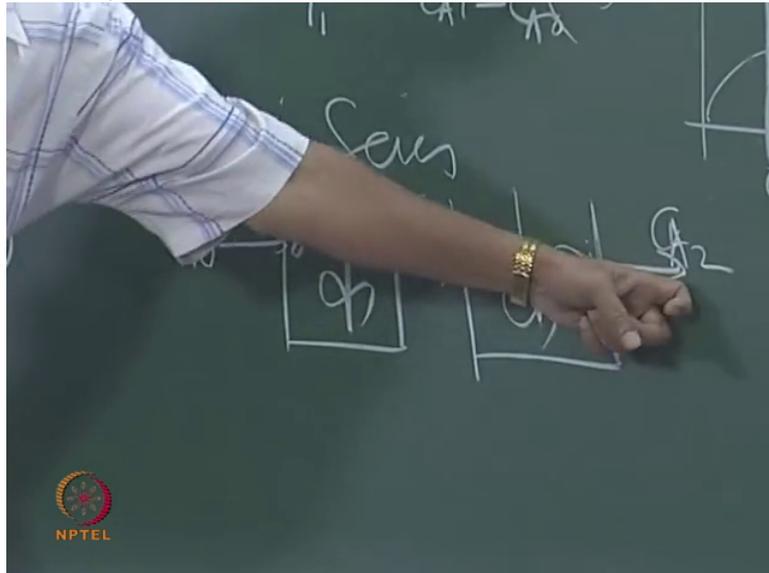


(Professor – student conversation starts)

Student: C A 1

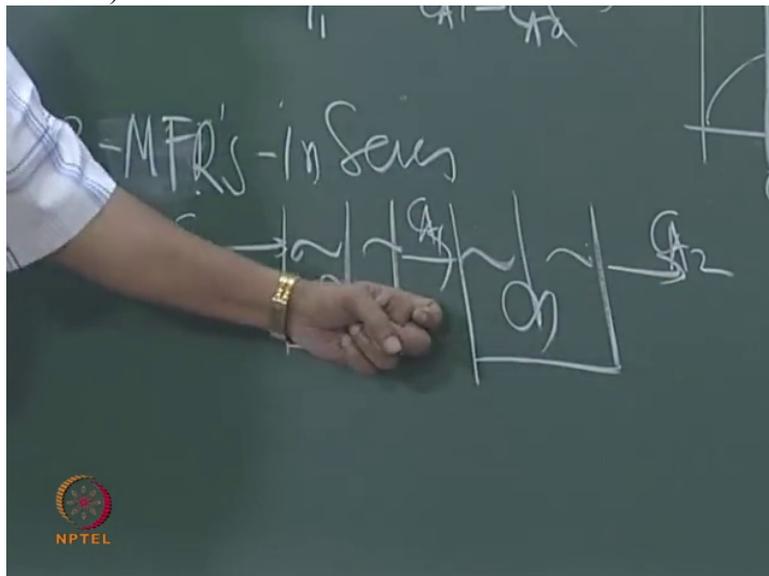
Professor: C A 2 is

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fixed, so only  $CA_1$ ,

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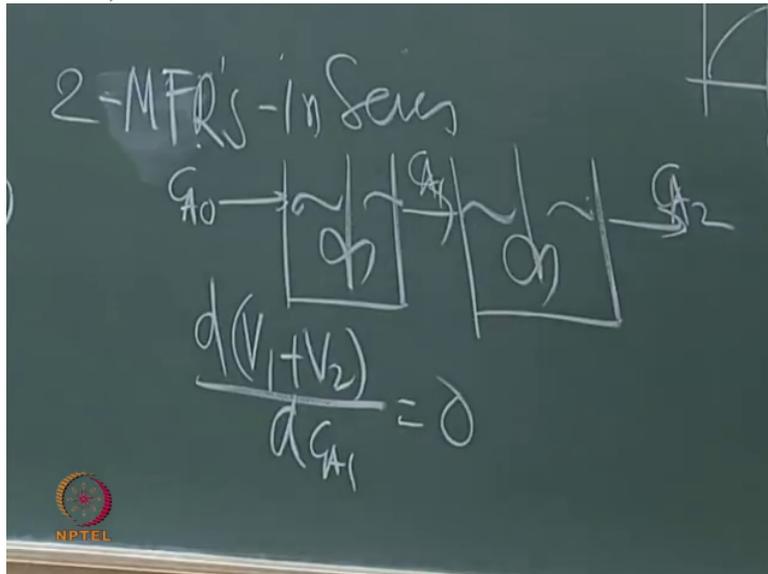


right and also the  $\tau$  values, yeah the volumetric flow rate also is fixed, so only thing I can manipulate is  $CA_1$  so that my volume will be minimum.

(Professor – student conversation ends)

So that is why I have here,  $V_1$  plus  $V_2$ ,  $d$  by  $d$   $CA_1$  should be equal to zero. This is simple again calculus, you know,

(Refer Slide Time: 39:40)



maxima minima problem. You know, optimization problem, right? Ok so this is what is mathematical. Analytically we can solve that. Abhijeet, getting confused?

Ok simply I mean first derivative you take and second derivative you take again to find whether you have maxima or minima, that is all, simple calculus, you know that is what you have learnt may be in third year, second year or first year of your degree but I think maybe you do not know how to use that here, Ok. That is one method.

But using graphical method also, Levenspiel does, you know that wonderful work. As far as that is concerned it is very nice problem. You know that means can I now minimize the rectangles. Not rectangles actually, the area, Ok.

So what he does is he takes this kind of, you know two reactors where I have let us say, this is  $C_A 1$ , this is  $\tau 1$ , this is  $\tau 2$ ,

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is fixed

Mixed Flow Reactors in Series (different  $\tau$ )

$$V = \frac{G_0 - G_1}{-r_A} = (4) \tau_1$$

$$\frac{1}{\tau_1} = \frac{-r_A}{G_0 - G_1} = (-5)$$

$$-\frac{1}{\tau_1} = \frac{(-r_A)}{G_1 - G_0} = (-6) r_A$$

2-MFR's in Series

$$\frac{d(V_1 + V_2)}{dG_1} = 0$$

NPTEL

yeah so then I have, here we can use our normal method,  $X_A$  1, sorry versus

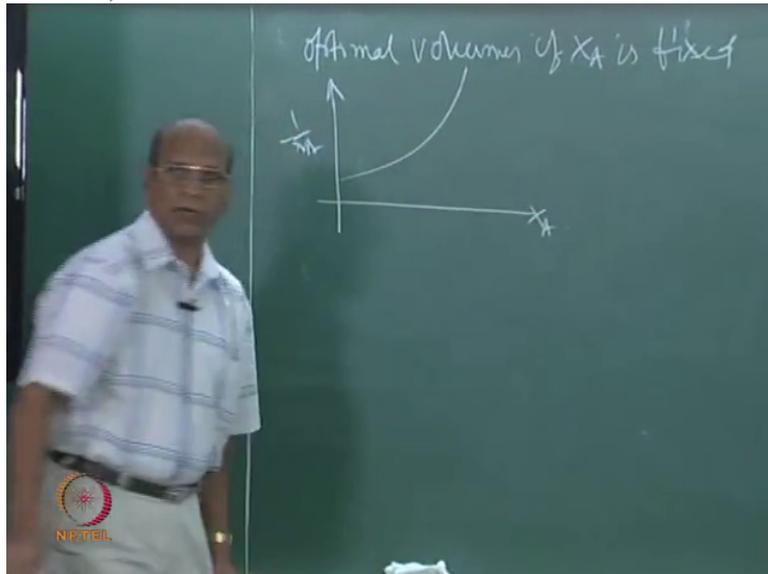
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Optimal volumes of  $X_A$  is

NPTEL

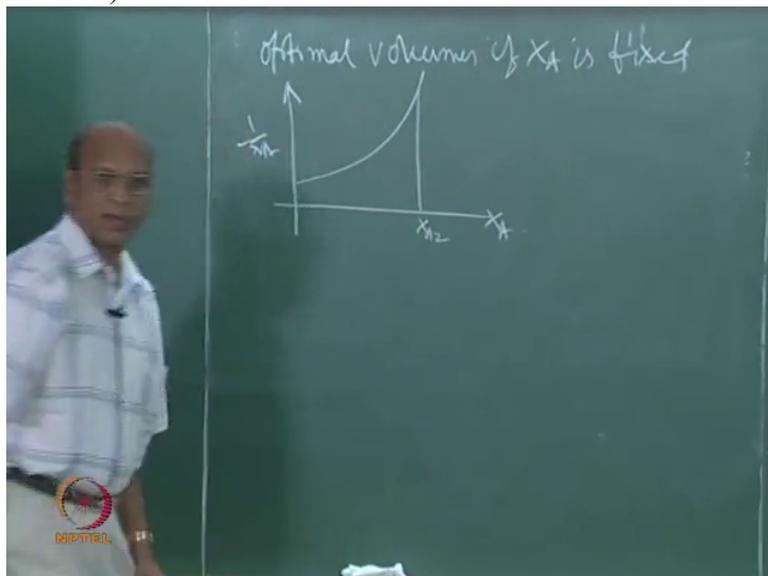
$X_A$ , then you will have this kind of

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curve. And  $X_2$  is fixed for me. So this is the one,

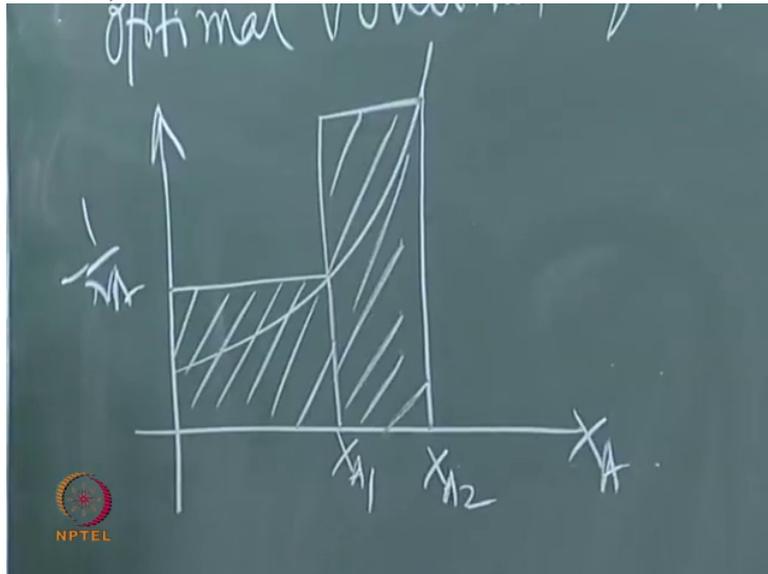
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right?

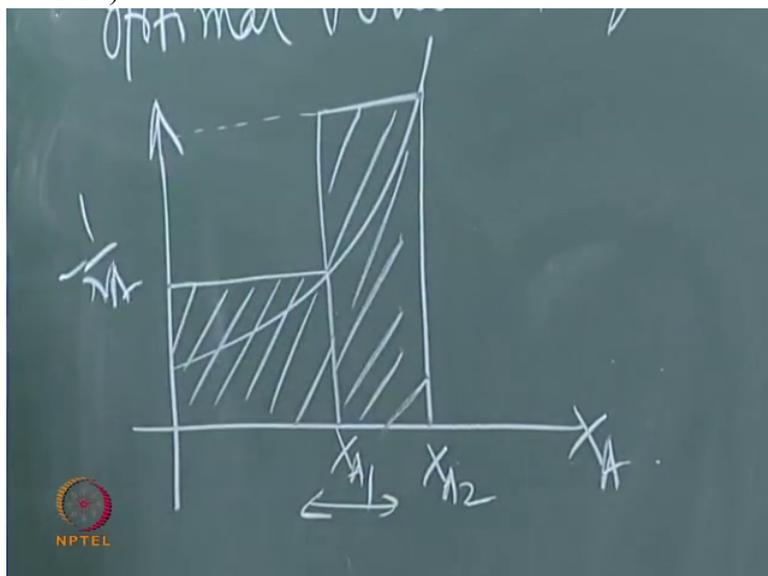
So if I do not know  $X_1$ , so that means I mean I have to say simply somewhere here I have  $X_1$ , what are the areas I have to take? I have to take this area, plus I have to also take this area.

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But this one can move this side, that side so that, you know if this is the total area, if this is the total area,

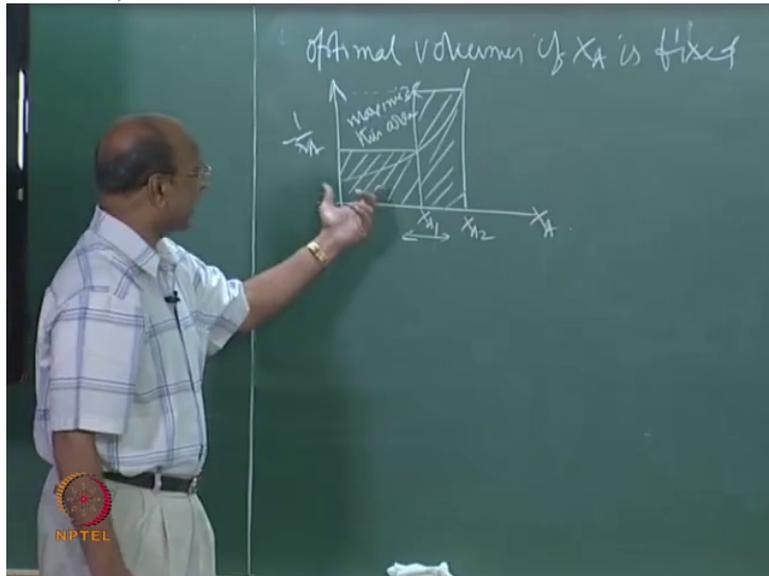
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how do you maximize this? Maximize this area. Why?

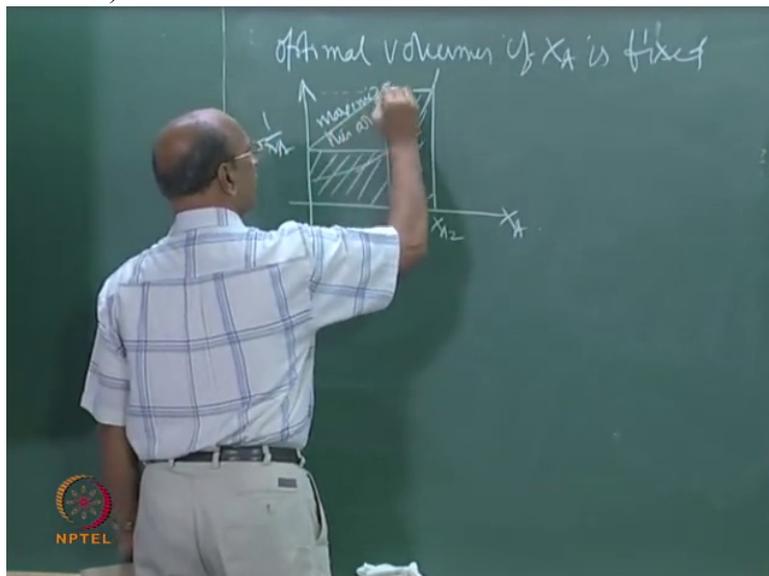
If I maximize this area then these things will become minimum, Ok, so that is the one.

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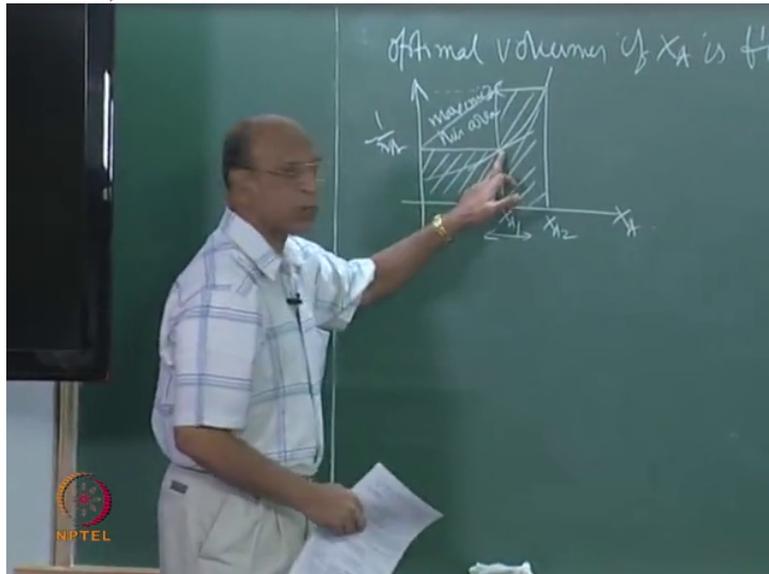
He has nicely given calculus method where I think, that I do not want to give, Ok. You know the condition comes as equating slopes. That means if this slope at this point and if

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this diagonal, if they are parallel, slope at this

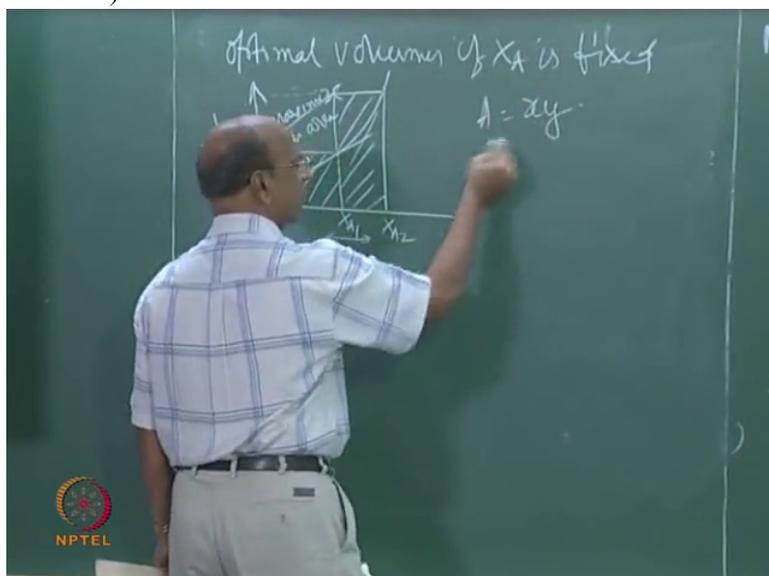
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point because  $X_A$  is moving this way, that way.

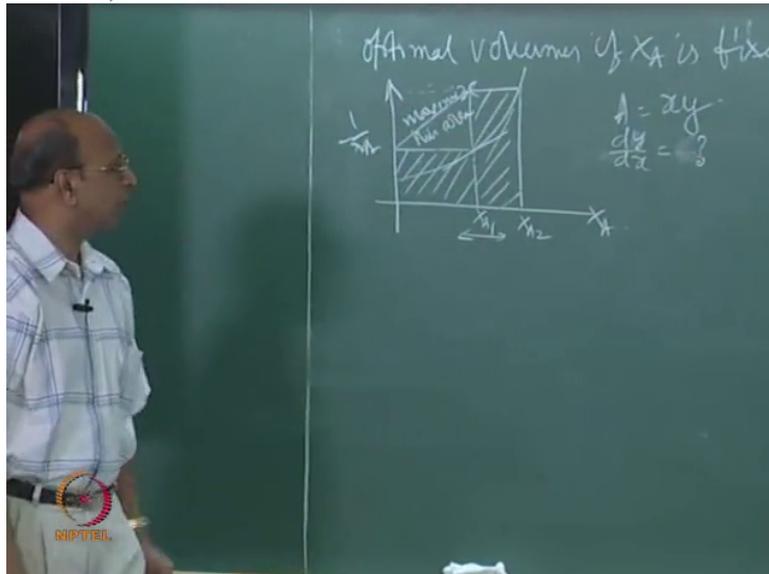
So if draw a slope there and if I also draw the diagonal in this, this is rectangle, if they are parallel, that means this will be the maximum and he will prove that with a simple you know  $x, y$ . Area equal to  $x$  into  $y$ . You differentiate this

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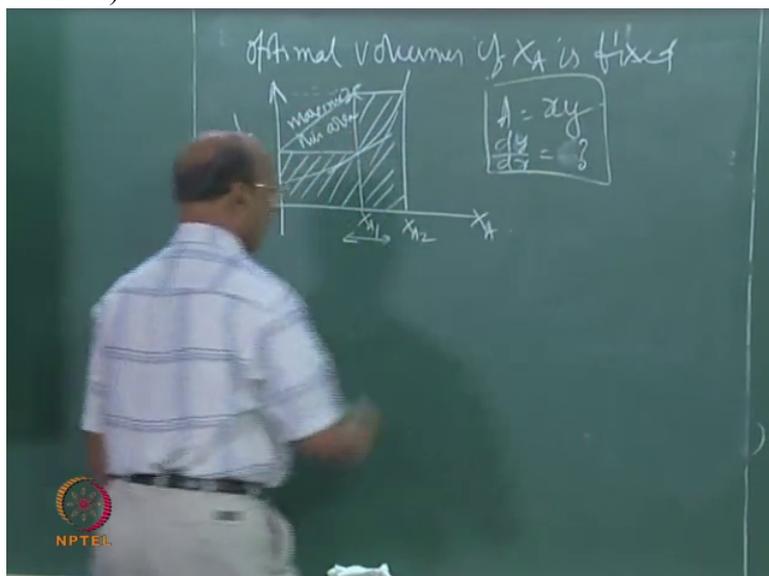
and then equate it to  $\frac{dy}{dx}$ . From this  $\frac{dy}{dx}$  equal to what?

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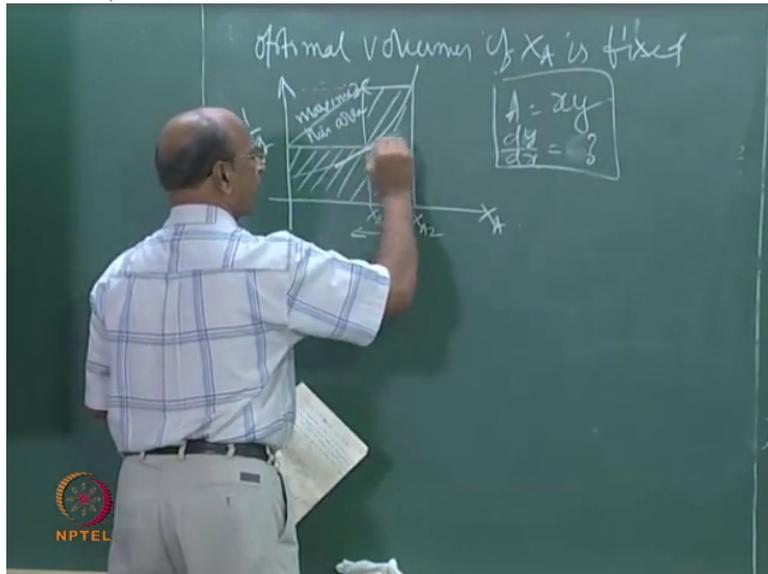
Ok, area should be maximum; I think that is the condition what you get there. Yeah to maximize d A, Ok anyway this is the one, I think just go through that, this part. Ok,

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now based on this, that means what you have to remember is the slope at that point whatever you are able to take,

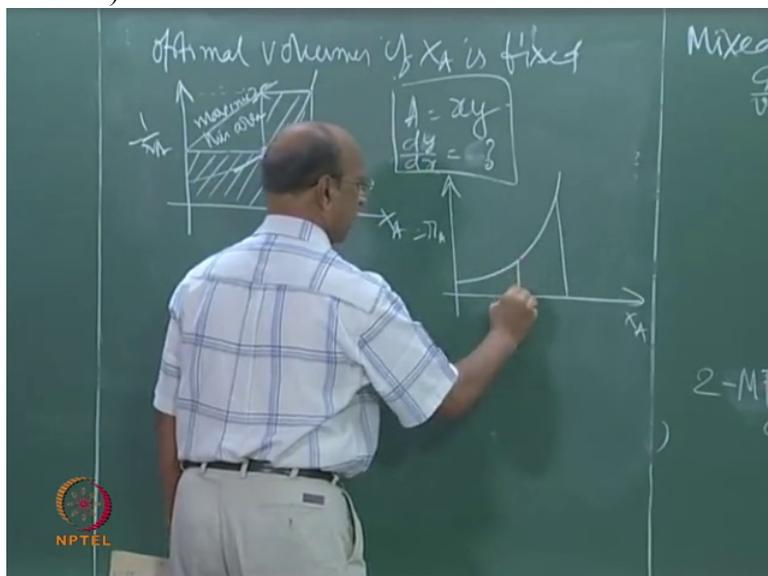
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that point, the tangent. Ok. That is the tangent. And this diagonal must be parallel.

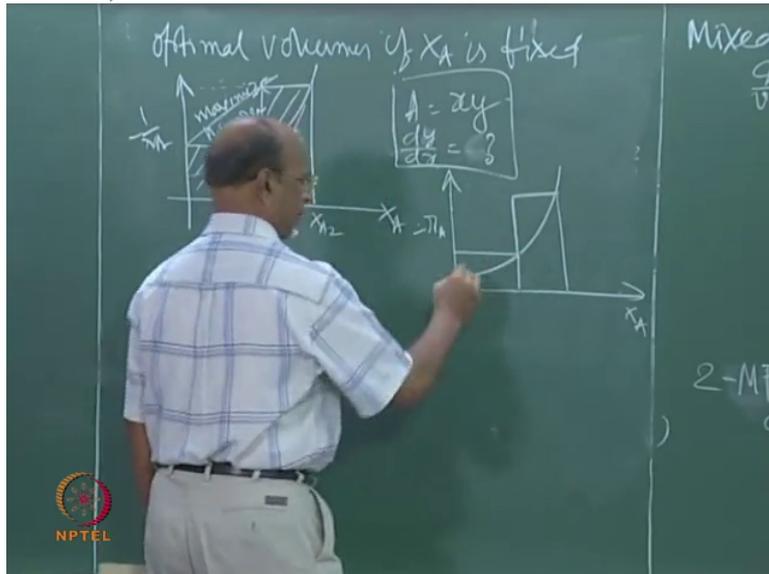
If you have that, then you have the minimum volume for both the reactors. So if I have, so like this, if I have, maybe I will try this.

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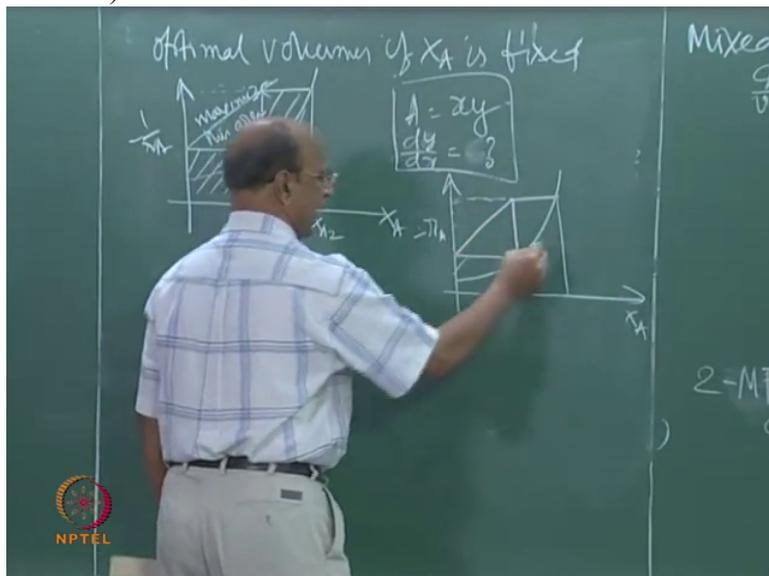
So this will be, yeah. So this is another area.

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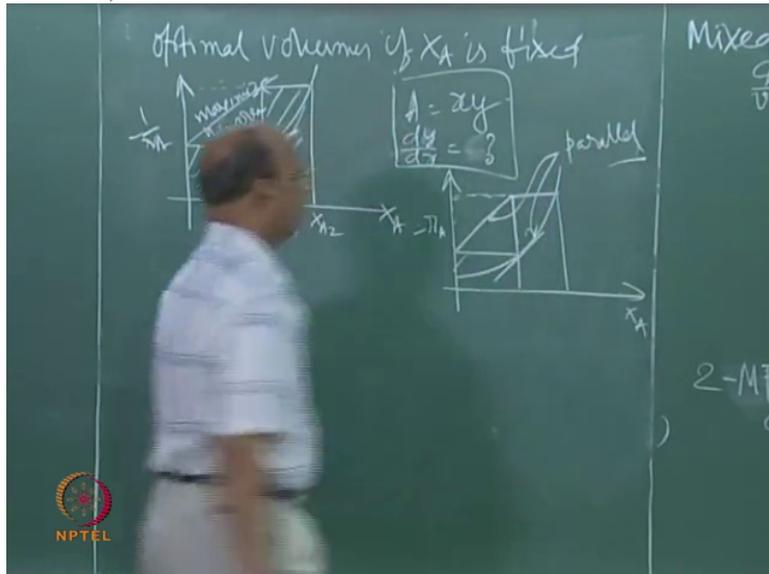
So I have now this triangle, sorry, this rectangle. So now this diagonal

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and, I will try to manage, if these two are parallel. Ok. So how do I write?

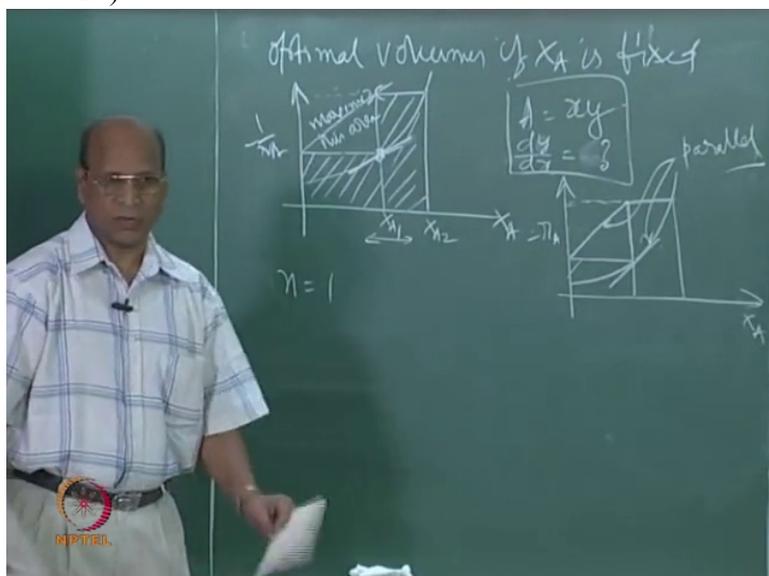
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Ok. Sum of the volumes will be minimum. The total volume, that is why  $V_1$  plus  $V_2$  together is the minimum, Ok good.

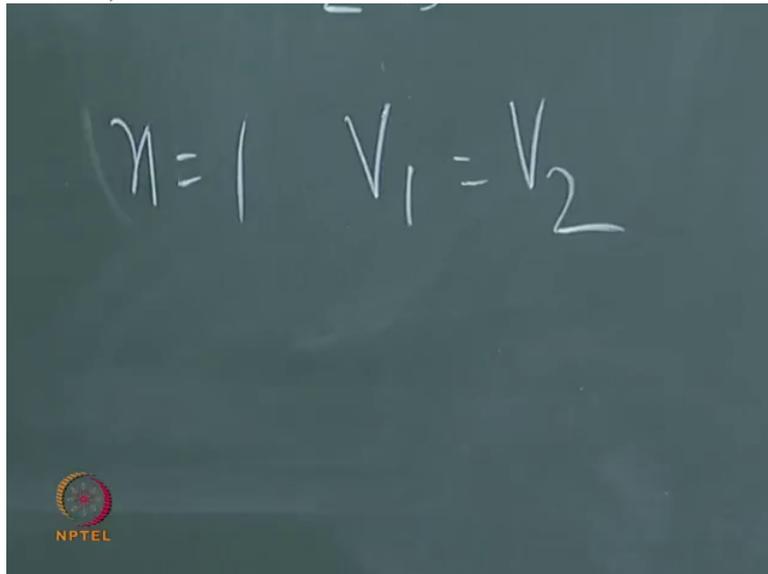
So what I want to tell here is, and again I do not want to repeat that in the next class, that is why I am just, not given this, quickly going, is that if I have  $n$  equal to 1,

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first order reaction then  $V_1$  equal to  $V_2$ . That means

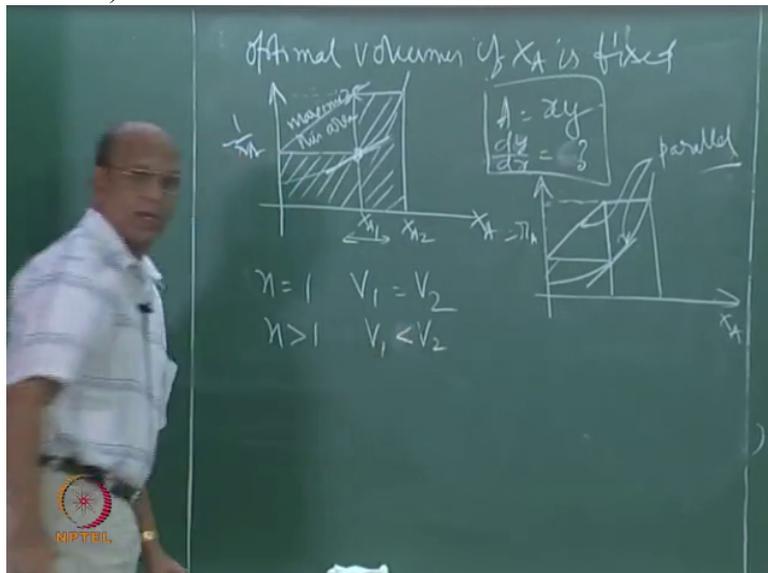
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both the reactors will have same.

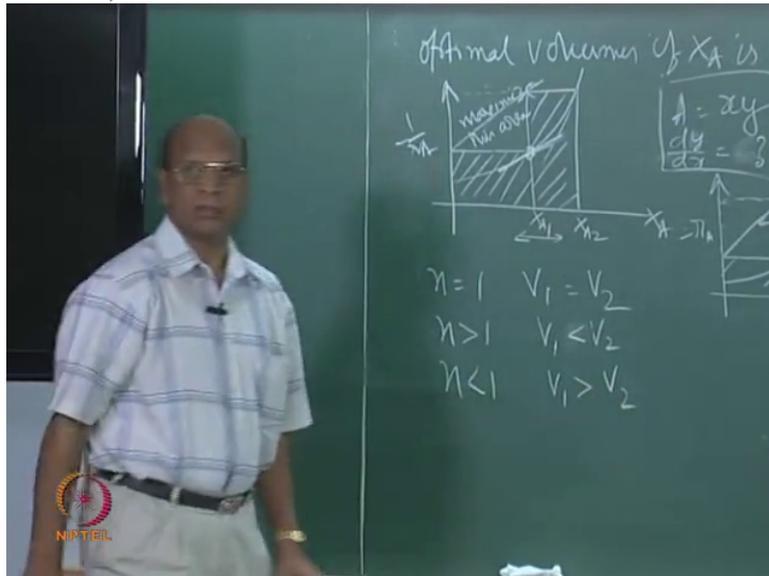
If  $n$  is greater than 1,  $V_1$  will be greater than  $V_2$ , reverse.

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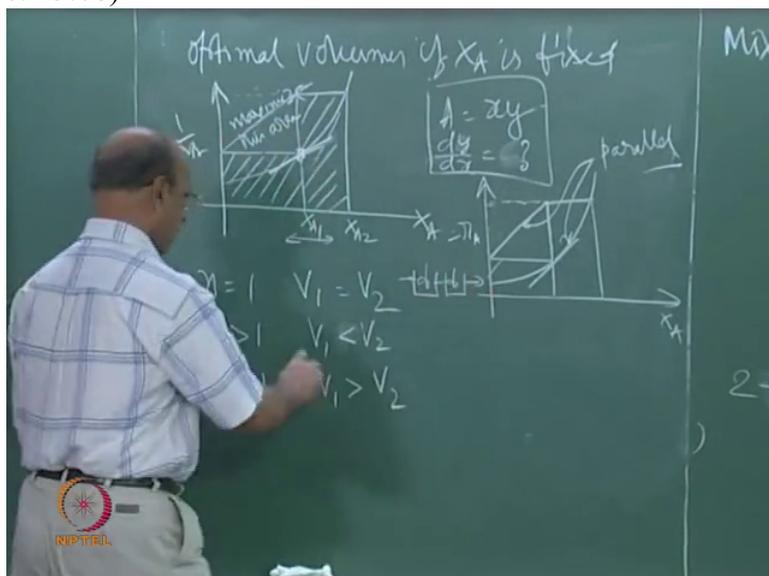
$V_1$  will be smaller and  $V_2$  will be larger. If  $n$  is less than 1,  $V_1$  is greater than  $V_2$ .

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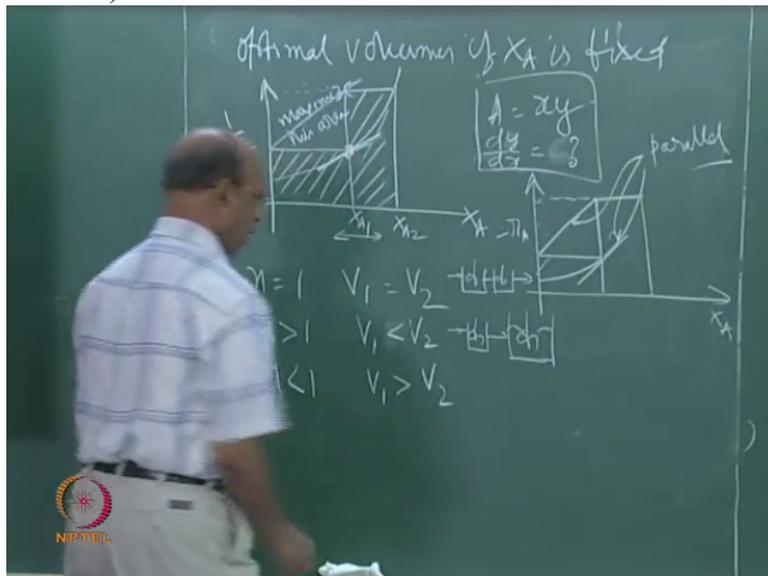
I think to give the simple idea, this one is  $V$ , exactly same  $V$  like this and in this

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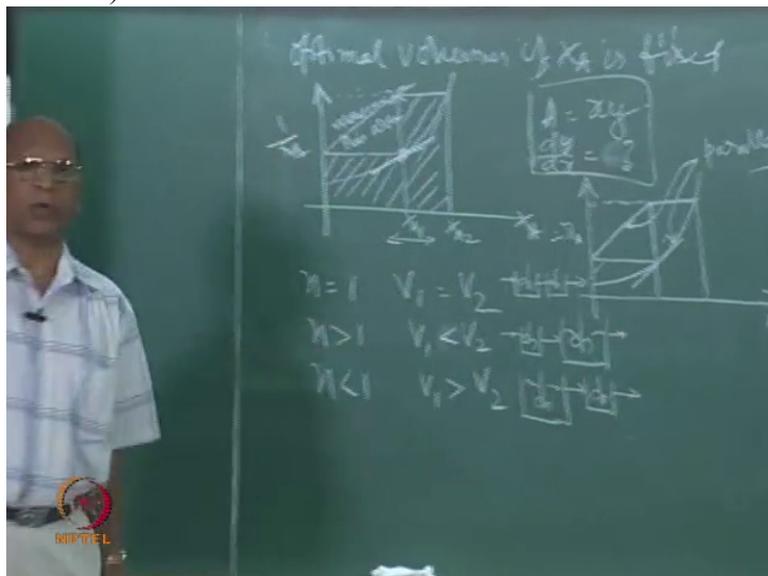
case we have like this, you have like this

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and in this case you have like this, yeah. That is the setup what

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we have.  $y$  is very important there.

(Professor – student conversation starts)

Student: In the first,  $n$  is greater than 1, so in that minus  $r$   $A$  equal to  $k$   $C$   $A$ , means we have to use

Student:  $k$   $C$   $A$  square

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Professor:  $C A$  square

Student: Greater than 1

Professor: Greater than 1

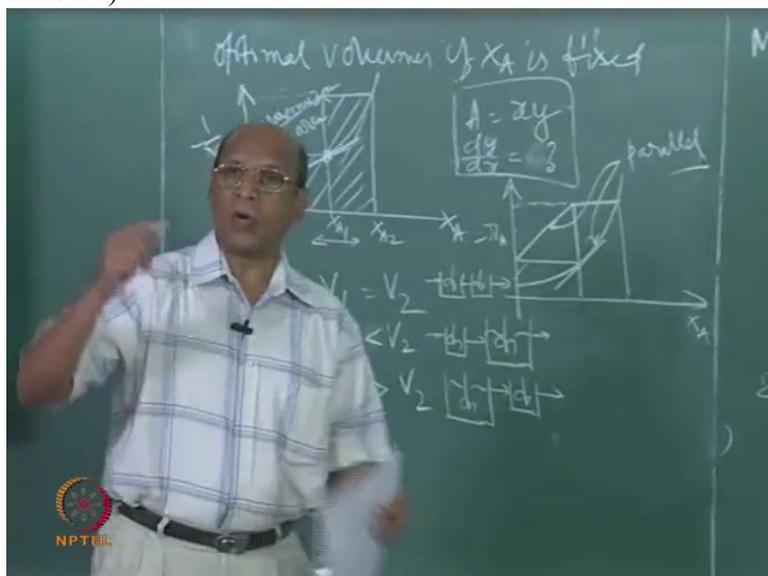
Student: Minus  $r A$  equal to  $k C A$  squared. So ultimate aim is that to increase the rate of reaction. Yeah? So in greater than 1, if  $C A$  is greater then  $r A$  will be greater. So means in  $C S$   $T R$  conversion, as you add the....

Professor: No, if  $r A$  is greater volume will be less for a given conversion that is why.

Student: 0:46:06.8 say that.

Professor: Ok yeah, that is why whenever we have

(Refer Slide Time: 46:11)



higher orders, Ok so then try to put smaller volumes when you have 2 C S T Rs so that you will get, you know that means when you have a larger volume, what is happening to the concentration, decreases.

(Professor – student conversation ends)

So when it is decreasing it will not be very high, Ok because  $1 - rA$ . So that is why you have to maintain as high as concentration as possible for second order reaction and orders greater than 1 because when you have  $C A^2$  and if you are maintaining larger concentration, what will be  $r$

(Professor – student conversation starts)

Student:  $r$  will be high

Professor:  $r$  will be high. So  $1 - r$  will be

Student: Less

Professor: So volume will be

Student: Less

Professor: Less?

Student: Yes

Professor: Because all  $r$ s will be only in the denominator.

Student: Yeah

Professor: Whether is batch reactor or plug flow reactor or mixed flow reactor, all these things are same only, Ok. All are in the denominator, so that is the reason. But the reverse, that means maintain high concentration, Ok now I will tell you, if I have a plug flow reactor and then mixed flow reactor, I have second order reaction. How do I connect these?

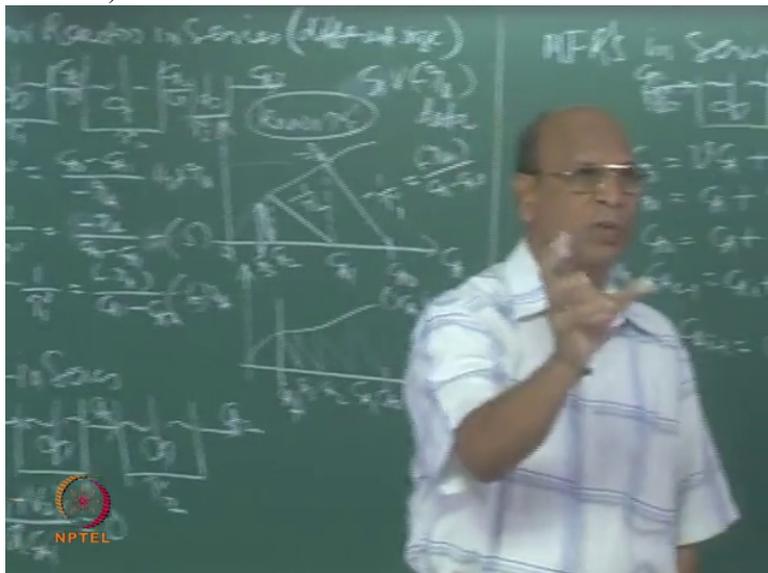
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Student: 0:47:22.2

Professor: Gone? The first one you have understood? Why I have to keep a smaller

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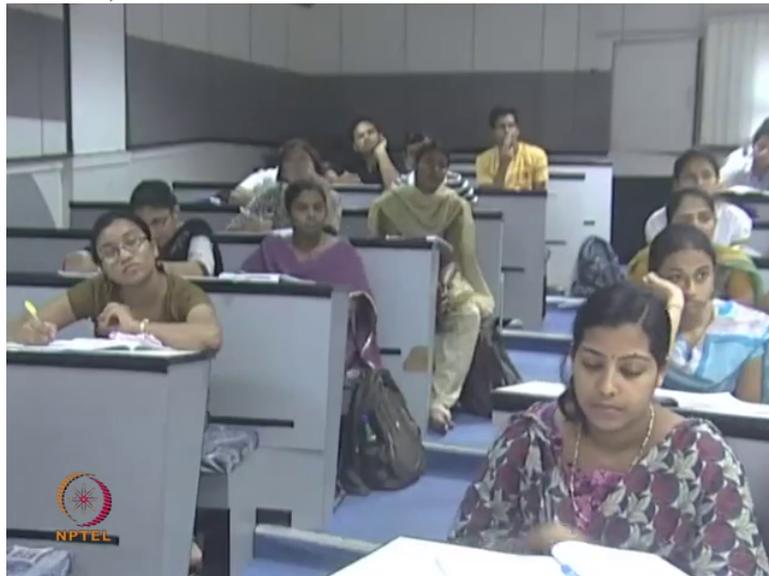


volume for  $n$  greater than 2, Sir, Ok  $n$  equal to 2, for example. First I have to put a small C S T R and then later a larger, you know C S T R. Why?

(Professor – student conversation ends)

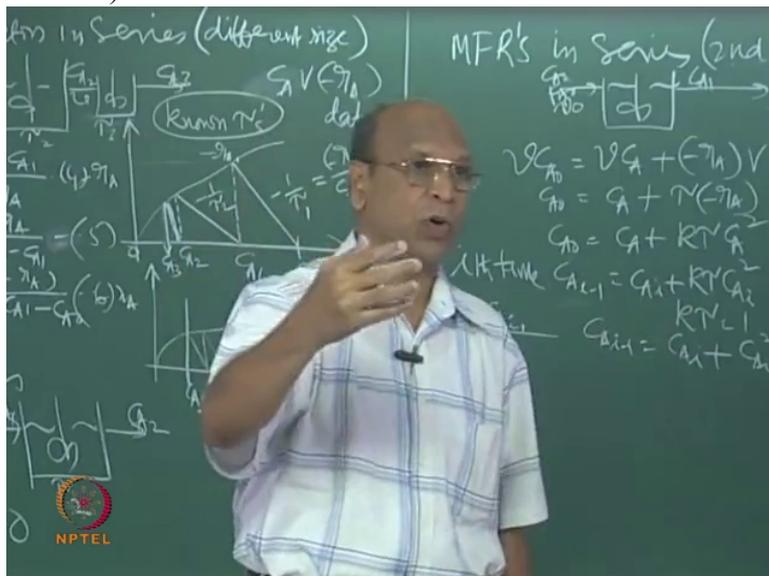
Because here I want to use my concentrations as

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optimally as possible. That means to maintain concentration as high

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as possible. When I have a smaller  $CSTR$ , then definitely concentration will be high. When concentration is high, I have  $-r_A$  equal to  $k C_A^2$ , Ok.

Squared concentration, large concentration into whole squared will give me more rate and I by rate will be small so that I can give for given conversion, smaller volume. Or for given volume higher conversion, mean just opposite problem. Ok, now you understood that? No. Still not understood. Ok. Now if I have a plug flow and mixed flow, in which reactors you can keep concentrations high?

(Professor – student conversation starts)

Student: P F

Professor: Because maximum possible, that is all. I think there is no other reactor which can beat that. So but my order is more than 1, that means second order. So I have to keep the concentrations as high as

Student: Possible

Professor: possible. That is why keep first

Student: P F R

Professor: P F R and then later?

Student: C S T R

Professor: C S T R but the same thing is reversed if I have n less than 1.

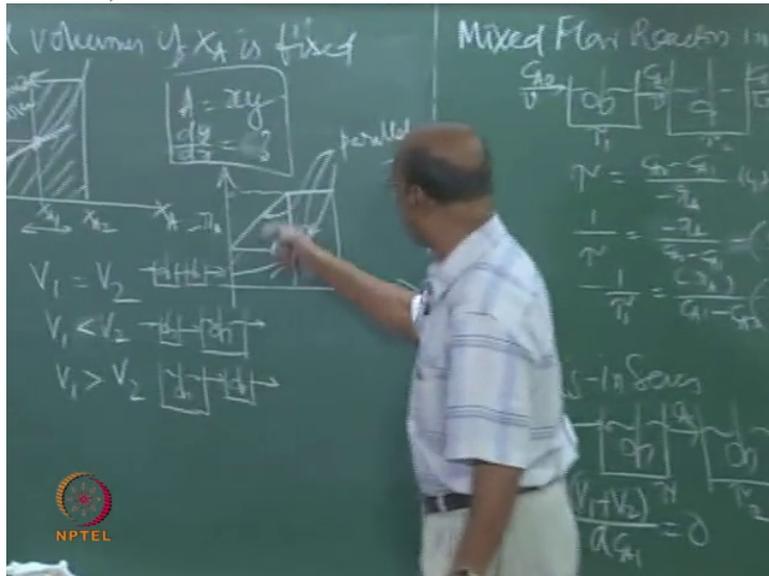
(Professor – student conversation ends)

But I am not able to find out any physical explanation for that unless you calculate numerically you will see that. This is easy to explain to me, because keep concentrations as high as possible and all that. Last 30 years I am not able to get good explanation but there the explanation is through mathematics.

You solve for half order, first order and second order, you will beautifully find what is the difference. Whatever we told here exactly will happen there. But then why for first order it is same? Even if I have 2 tanks, smaller tank and larger tank, Ok first two tanks, two tanks, first two C S T Rs, for first order whatever way you keep, does not matter. And this exercise will give me that.

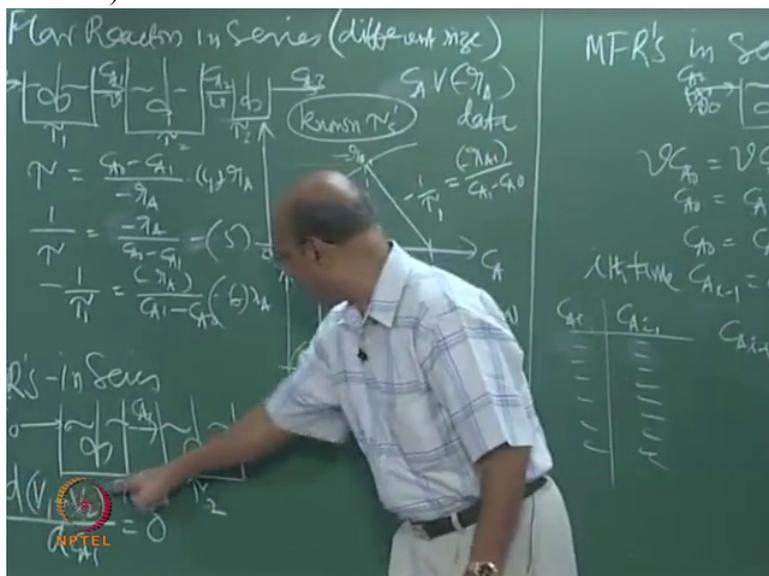
When I do

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this exercise it will give me. If I take second order reaction, analytically you can do. If I take second order reaction

(Refer Slide Time: 49:35)



here and then differentiate with respect to  $C_{A1}$  you will see that the first one will be smaller, second one will be larger.

But if you do the same thing for  $n$  equal 1, first one and second one will have the exactly same volume. And you again repeat the same exercise for  $n$  equal to point 5. Then the first reactor will be larger and second reactor will be smaller. Ok, so that is why whenever you have second order reaction, keep the concentrations as high as possible.

And only for first order reaction, I ask you to think. Why? You get same conversion, whether you get larger reactor or smaller reactor. That means concentration really does not matter there, right. And not only that. I gave you one problem. I do not know how many of you have done it. I think maybe you are waiting for that day, I think 28 is last submission day means twenty seventh night only you will do it, Ok? Yeah P F R

(Professor – student conversation starts)

Student: Same expressions

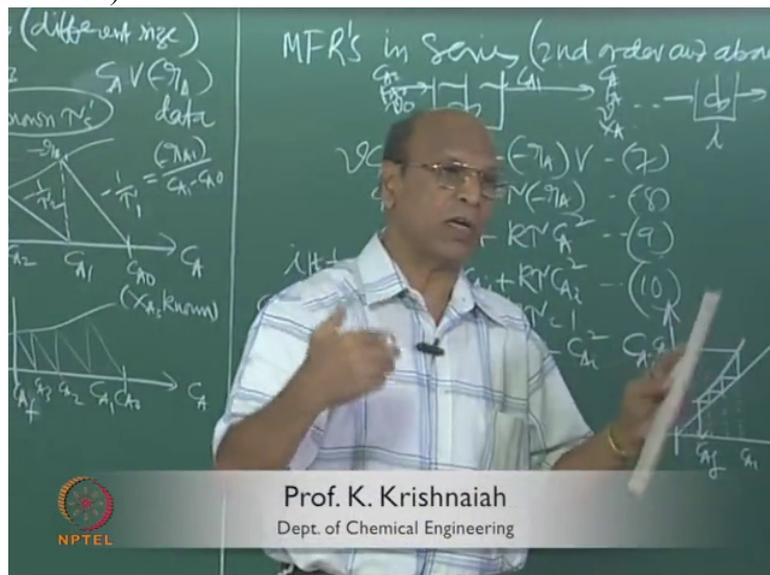
Professor: Yeah, I think, because you have done, you are able to tell that. I think you know they have not done, so they do not know that. Ok.

(Professor – student conversation ends)

So I have given the exercise as plug flow reactor followed by C S T R, Ok. Does not matter, volumes may be different. Ok, or same volumes also we can take, I think, it does not matter. The same thing is reversed. First you put plug flow, mixed flow. Then same thing, instead of reversing you pump from this side. Now you will have first is mixed flow, second is plug flow.

Which one will give you more

(Refer Slide Time: 51:11)



conversion with respect to first order reaction?

(Professor – student conversation starts)

Student: Same

Professor: Same. Now think about the reason why it is same. So I will stop here.

(Professor – student conversation ends)