

Particle Characterization
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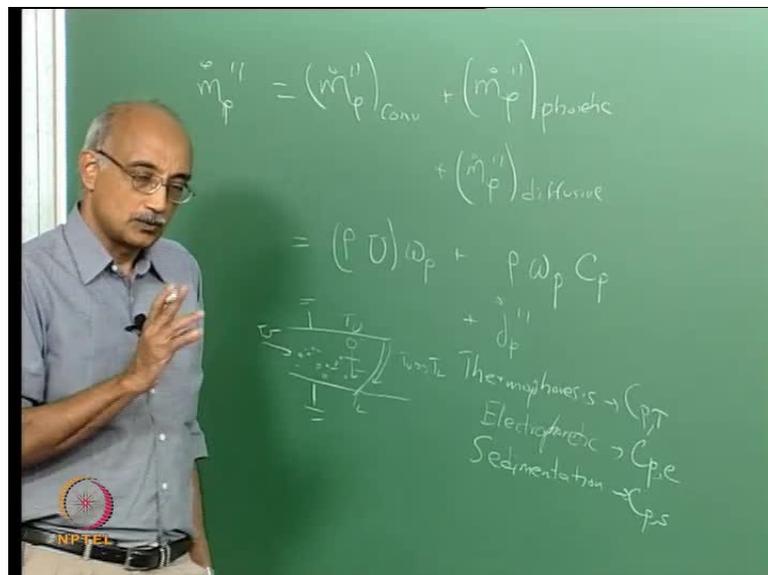
Module No.#9

Lecture No. #24

Transport Properties: Deposition fluxes and Rates

Welcome to the 24 lecture in our particle characterization course. In the last lecture, we continued our discussion of particle transport phenomena and specifically focused on drag forces as well as inertial forces that dictate the movement of particles in fluids. And you wrote down expressions for relative mass fluxes that can be used as reference values in order to non-dimensionalize the actual prevailing particle transport rates or particle transport fluxes, so let us start by revisiting that.

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Again, going back to our expression for total mass flux of particles \dot{m}_p'' , this has three components conductor plus phoretic plus diffuser. All these transport mechanisms come into play all the time, but the relative magnitudes will depend predominantly on the size distribution of the particles. So, if you have extremely fine particles then, diffusive effects **are** important if you have particles that are somewhat

intermediate in size range, then convective effects as well as some of the phoretic effects become important.

If we have large particles then, particularly phoretic forces that are associated with the body force field become important and those are what we referred to as inertial forces. So, the convective flux can be written as the convective flow of the fluid, which is density of the fluid multiplied by the velocity of the fluid multiplied by the mass fraction of particles in the fluid that gives you the convective mass flux of particles in suspension. The phoretic phenomena can be expressed again as $\rho \omega_p$ times a velocity that is considered a phoretic velocity - C_p . So, the difference between this term and this term is that, here the velocity term refers to the velocity of the carrier fluid, whereas the C_p is a term that is specific to a field that is being applied on top of the prevailing flow field.

For example, if you have flow that is occurring in a pipe line, then you will experience convective flow because this velocity may be some u . So, there is a convective flux if you have particles that are suspended in this fluid then, the flux of fluid associated with the flux of the carrier fluid is what is referenced in this first term. But now, let us say that on top of this force field you apply a temperature field, so that let us say that here you have a t_u and a t_L where t_u is, let us say, much greater than t_L , so you are essentially setting up a temperature field across the flow field.

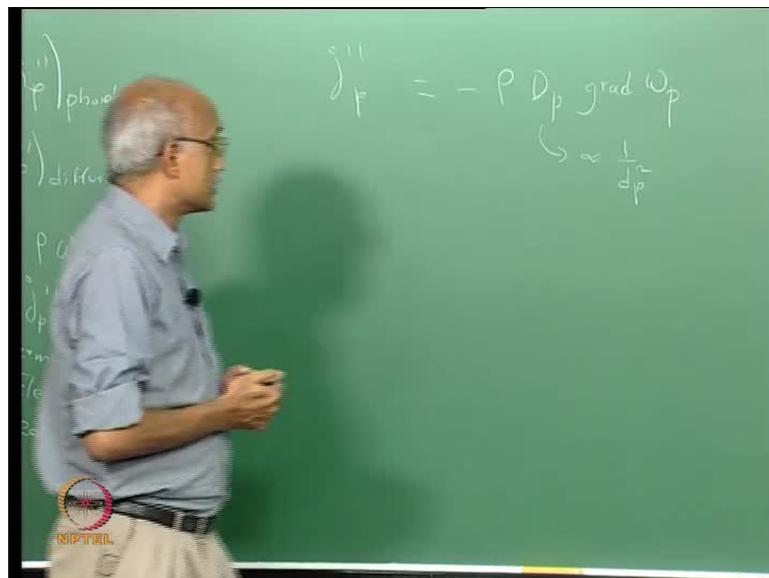
Now, this is going to have its own influence on the drift of the particles, because essentially the particles that are near the hot wall are going to have higher energy levels. The particles that are near the cold wall are going to have a lower energy level and the system wants to equilibrate, so there will be a drift of particles from the hot wall towards the cold wall and that is called thermophoresis.

And the corresponding velocity is written as $C_{p,t}$, so that is the thermophoretic velocity of the particle which is induced by the presence of the temperature gradient. Or let us say that, you apply an electric field of some kind and setup a potential difference across the flow. Now again, we are going to induce a velocity of the particle because of the associated electrical field and that is what we called an electrophoretic field which is typically referred to as $C_{p,e}$. Or in the simplest case, you can even think about if the particles are large enough then, in addition to flowing in this direction because of the

convective motion, they are also gonna experience gravity and they are going to start settling in the flow. So, that is gonna essentially induce an another velocity component to the particles that are being transported.

So, the gravitational or sedimentational field will then impose a velocity which will be the settling velocity. So, it will be the balance between the body force that is trying to drag the particles down and the drag force that is associated with the flow. So, in the second term you essentially substitute the appropriate field related or phoretic velocity.

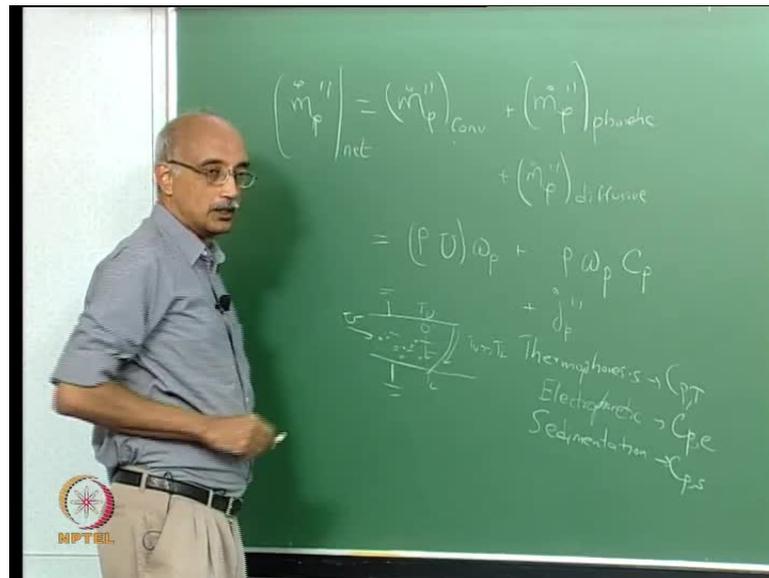
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Now, the third term is the diffusive term and that is conventionally written as $j_p^{(3)}$ dot double prime, particularly to reference diffusion, so m stands for total mass flux and j stands for diffusional mass flux. And if you look at this term in a little more detail $j_p^{(3)}$ dot double prime and again **i am** the notation is dot stands for per unit time and double prime stands for per unit area, so this represents a flux of particles by a diffusive mechanism. As we have seen you can write this as some minus ρ times D_p times a gradient in ω_p , so this is a brownian diffusive flux of particles which is driven by a concentration gradient of particles.

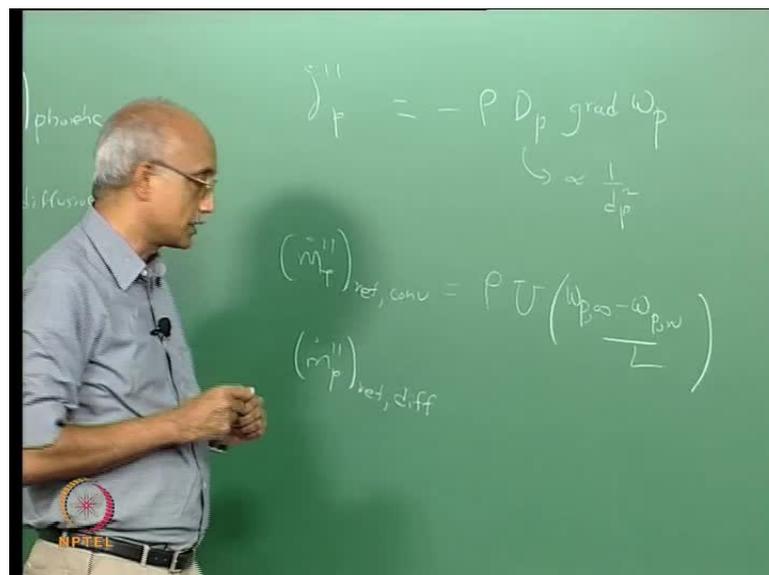
And this term is particularly important for fine particles because this D_p goes as one over particle diameter for fine particles, and in fact, for super fine particles, it goes as one over D_p squared because of the stokes cunningham correction factor affect that we discussed in one of the previous lectures.

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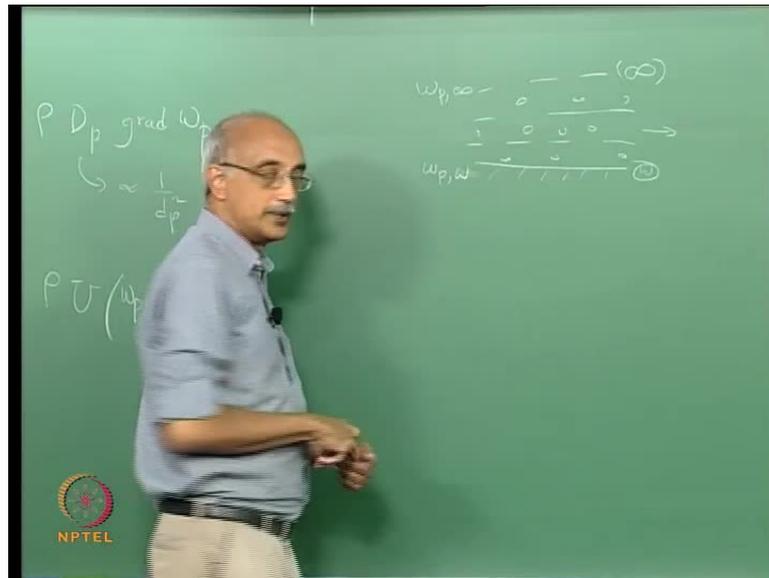
So, as the particle size decreases, this term gains in importance, so let us say that for a given particle size distribution in a flow you have calculated the convective contribution, you have calculated the phoretic contribution, you have calculated the diffusive contribution, you have added it all up and this is your net total flux particles that are flowing past a surface, this is a dimensional value **right** and as chemical engineers we always try to non-dimensionalize.

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So, we have to come up with appropriate reference values that we can divide by, so reference fluxes have to be defined, so \dot{m}_p double prime reference has to be defined essentially in two limits in the convection dominated regime and in the diffusion dominated regime.

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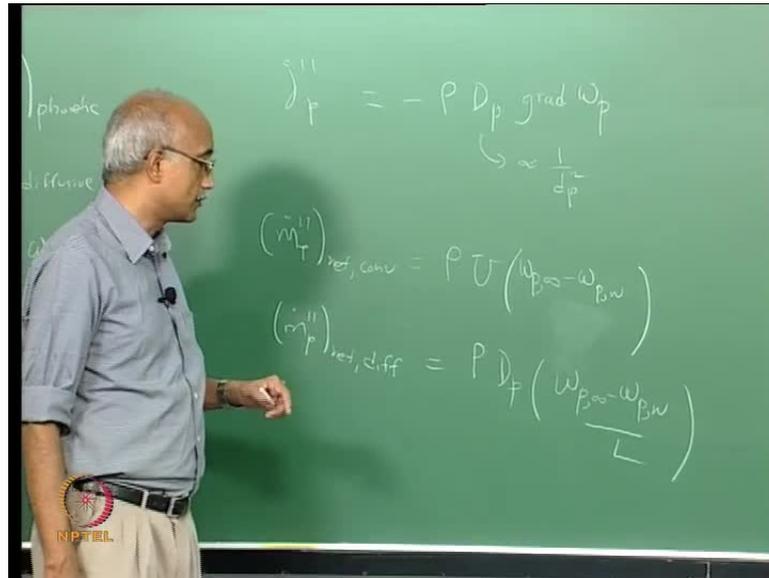


It turns out that the reference flux you use is different in the two different cases, in the case where convection is dominant, this is simply given by $\rho u \omega_p$ infinity minus $\omega_p w$ over L , where referencing back to this figure if your surface is here and you call it w , infinity refers to a location that is far away from the surface.

So, if you have fluid flow occurring in this direction with particles suspended in the fluid then, the particle mass fraction far away from the surface is represented as ω_p infinity, the particle mass fraction in the vicinity of the surface is represented as $\omega_p w$ and L is a characteristic dimension of the system. So, it depends on what the collection surface is for example, if the collection surface is a heat exchanger tube in a power plant then, the characteristic dimension would be the length of the tube.

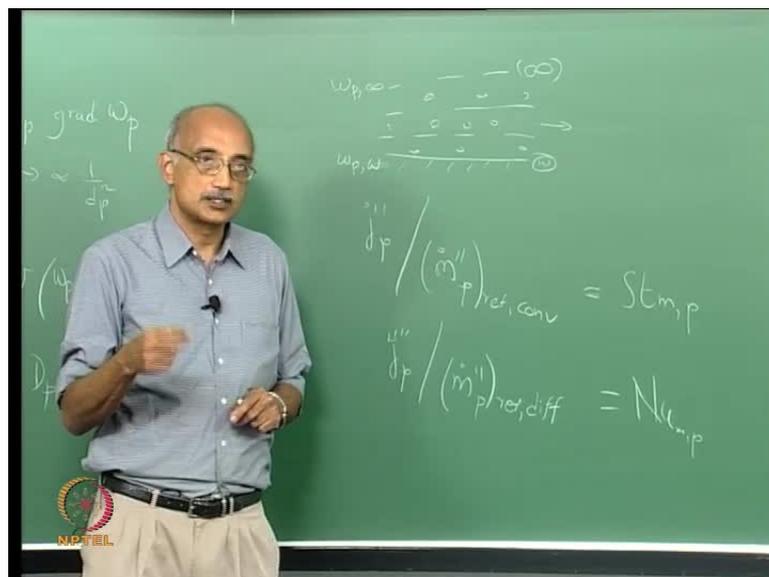
Or if you have a collection plate on which you are trying to gather particles for analysis then, again the reference length may be the one that is longitudinal to the flow. So, L is your reference length and this is your gradient in mass fraction in the case where you are talking about.

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Diffusional reference flux, this can become rho times Dp times, sorry, there is no division by L here, this itself has units of flux, the division by L comes when you are trying to define the reference flux for diffusion which is omega p infinity minus omega p w divided by L. So, if you look at this, this is basically a simplified form of this, where you are assuming that properties are constant across the diffusion length or diffusion region. So, these are the two reference fluxes corresponding to convection dominated flow and diffusion dominated flow.

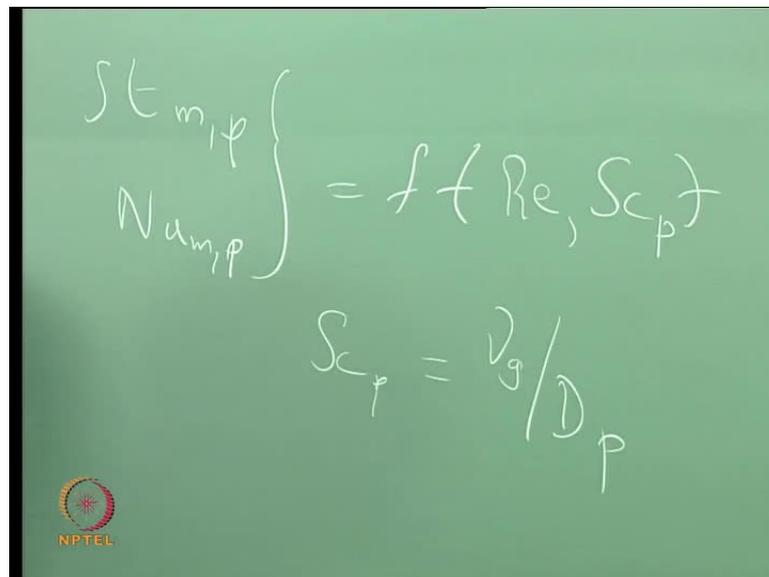
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Again, D_p is the brownian diffusivity of the particle, now if you take your diffusive flux j_p dot double prime and divided by your m dot p reference convective that is the reference mass flux corresponding to convection dominated flow, this is referred to as Stanton number. The Stanton number for mass transfer for particles, if you take the same diffusive flux and divided by m dot double prime reference diffusive, this is known as nusselt number.

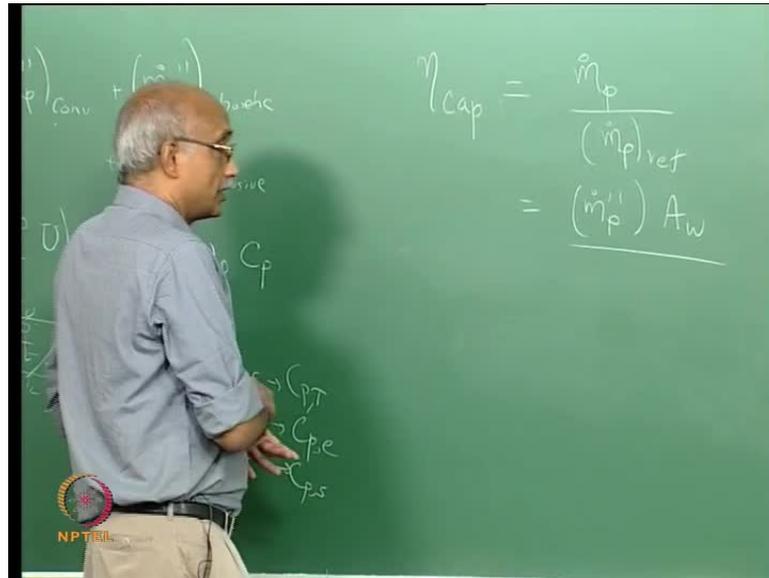
So, both the Stanton number and in nusselt number are frequently used in literature to represent non-dimensional values of particle mass fluxes. So, you have to be aware of the difference between the two and where it is appropriate to use what.

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$$\left. \begin{array}{l} St_{m,p} \\ Nu_{m,p} \end{array} \right\} = f(Re, Sc_p)$$
$$Sc_p = \nu_g / D_p$$

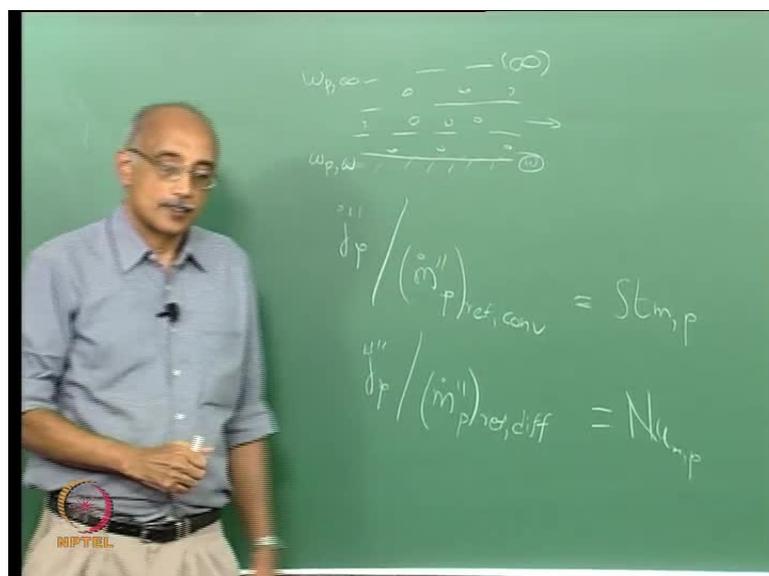
Now, both of these values stanton numbers as well as nusselt number are functions of two other non-dimensional numbers, the Reynolds number and the Schmidt number. Reynolds number of course, is $d v \rho$ by μ or $d v$ by ν , Schmidt number of course, in this case for a particle equals ν for the gas divided by the brownian diffusivity of the particle ν by d .

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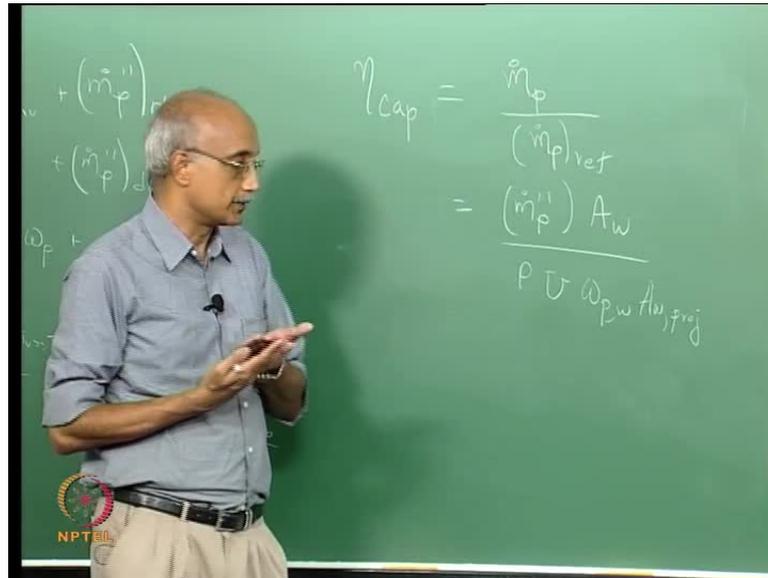


Now, there was one other non-dimensional parameter that we referred to in the last lecture, and that was the capture efficiency. The capture efficiency referred to as eta cap, is a ratio of actual particles captured on a surface to the total inventory of particles flowing across the surface. So, the capture efficiency can be written simply as \dot{m}_p dot that is a rate at which particles are being collected on a surface divided by an \dot{m}_p dot reference, because this, now is \dot{m}_p dot double prime times an area divided by what would be a reference value for inertial capture of particles. Well, the situation that we are looking at here is, you have a surface and particles are flowing across it.

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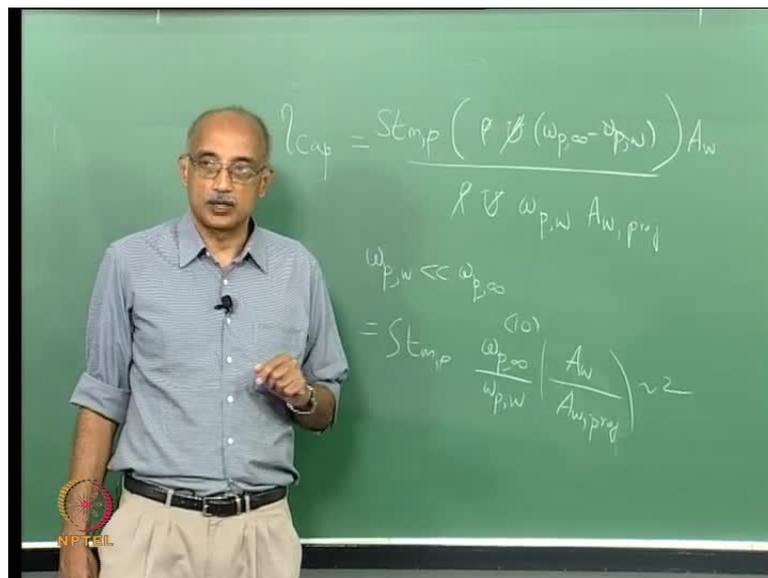


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If you look at the total inventory of particles that are flowing across the surface, that is, basically $\rho u \omega_p w$ times. So, this is the total flux of particles flowing across the surface multiplied by the projected area of the surface a_w projected. So, this is the expression for capture efficiency, it is simply the total mass that actually collects on the surface to the potential collectable mass flow across the surface.

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So, the higher the capture efficiency greater the probability, that more of these flowing particles will be entrained and deposited on the surface. So, if **you if** I write this

numerator now in terms of, for example, the Stanton number, you can write this capture efficiency η_{cap} as Stanton number for mass transfer times the reference value **right** that we had written earlier, which was $\rho u \omega_p \infty - \omega_p w$ and this is now being divided by the maximum collection rate which is multiplied by a_w divided by $\rho u \omega_p w$ times a_w projected. So, if you look at this expressions, the ρu gets cancelled out, let us assume that $\omega_p w$ is much smaller than $\omega_p \infty$.

So, in this expression essentially the $\omega_p w$ goes away in comparison to $\omega_p \infty$, this is the case where essentially particles that come to a surface have a much greater tendency to attach the surface than to stay in the flow and then you have a w and a w projected. So, the relationship between capture efficiency and the Stanton number can be simply written as $\omega_p \infty$ over $\omega_p w$ times a_w over projected area of the surface. Again the difference between these two values is a_w , is total area of the object, a_w projected is its projected element which is actually sticking out in the flow and over which the particles are flowing.

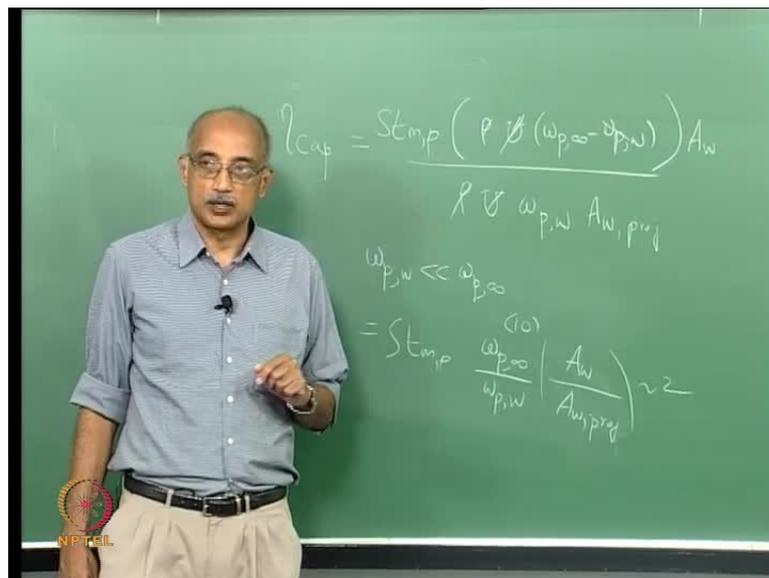
Now, typically this ratio is of the order of - I mean - it is never equal to 1, because it is very difficult to imagine situations by the entire object is exposed to particulate flow tends to be roughly half - I mean - if you look at the projected area to total area that tends to be about 0.5, so the inverse of that, so this quantity within the parenthesis roughly of the order of 2. And the $\omega_p \infty$ to $\omega_p w$ ratio, let us say that in the case where this is much smaller than this, this could be as much as 10. So, what that leads to is, the result that says that the capture efficiency compared to the Stanton number can be as much as an order of magnitude higher, just as a ball forecasting. Now, we saw in the last class that this capture efficiency then, is relatable to the stocks number.

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If you plot stocks number verses capture efficiency that has an s shaped curve, where it varies from 0 to 1 as you increase a stocks number. Again, remember that this was the curve for inertial capture, whereas if you were taking total or net capture of particles, that will have a slightly different behavior because, diffusional transfer can come into play at very, very, small particle sizes.

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So, the relationship between stocks number and capture efficiency is captured here, and from that we can relate the stocks number two parameters such as the Stanton number

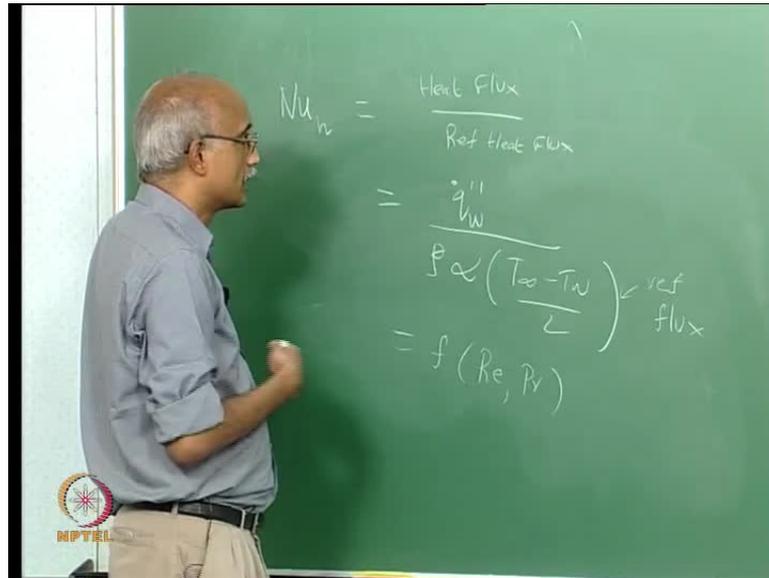
and the Nusselt number. So, the key thing to remember is, in terms of characterizing particle transport, there are 5 important dimensionless parameters. The Reynolds number and the Schmidt number are important to characterize because, the three dimensionless parameters that actually reference a mass flux which is the capture efficiency Stanton number and Nusselt number or all related to the Reynolds number and the Schmidt number.

So, these are the 5 critical dimensionless parameters that are used to characterize particle transport and particle deposition on to surfaces. Now, how do we estimate these Nusselt numbers and Stanton numbers - I mean - this approach is really only useful if it is possible to estimate the corresponding dimensionless parameters because for example, if we can characterize the prevailing Stanton number and Nusselt number, then we can simply multiply them by the reference values to get the actual fluxes of particles, but it is always difficult to measure mass fluxes - I mean - if you imagine in order to measure a mass flux you have to have a detection device that is capable of measuring transport of mass in micro gram levels, and it is actually not easy to design such an instrument.

So, even though these are convenient representations, Stanton number for mass transfer and the Nusselt number for mass transfer are very difficult to measure, so how do you measure them or how do you estimate them, that is done essentially by using what are known as Analogy conditions. The basic assumption here is that there is a perfect analogy between mass transfer and heat transfer, if that is true then, you can do heat transfer measurements and from that you can actually estimate the corresponding mass transfer parameters.

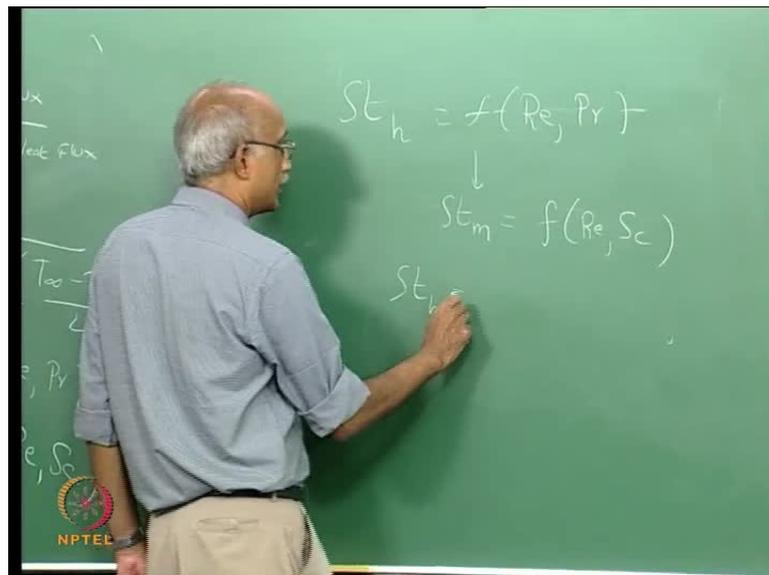
So, that is the trick that is usually resorted to, because temperature measurements are easy to do, all you need is a thermometer or an RTD or a thermocouple and you are set. So, unlike the challenge of measuring mass fractions and mass fluxes, measurement of heat fluxes and temperatures is trivial, technologies well established and inexpensive. So, you can take a lot of data and experiments on heat transfer are always easier to do and because, you can make a lot of measurements your confidence in the data also tends to be higher. So, the strategy that is generally used is to do heat transfer measurements and from that extract mass transfer properties.

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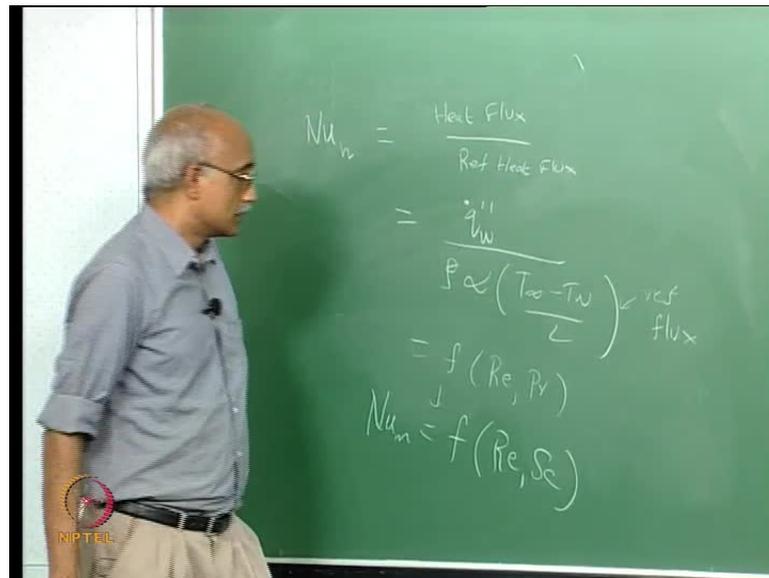
So, how do we do that? So, what we can do is, for the same system you can actually evaluate, for example, a nusselt number for heat transfer, which would be equal to reference a prevailing heat flux to a reference heat flux. For example, you can write this as some q_w dot double prime over row times, what is the corresponding parameter for mass diffusivity in the case of energy? It is, so that is alpha **right** times the temperature differential t infinity minus t w over L.

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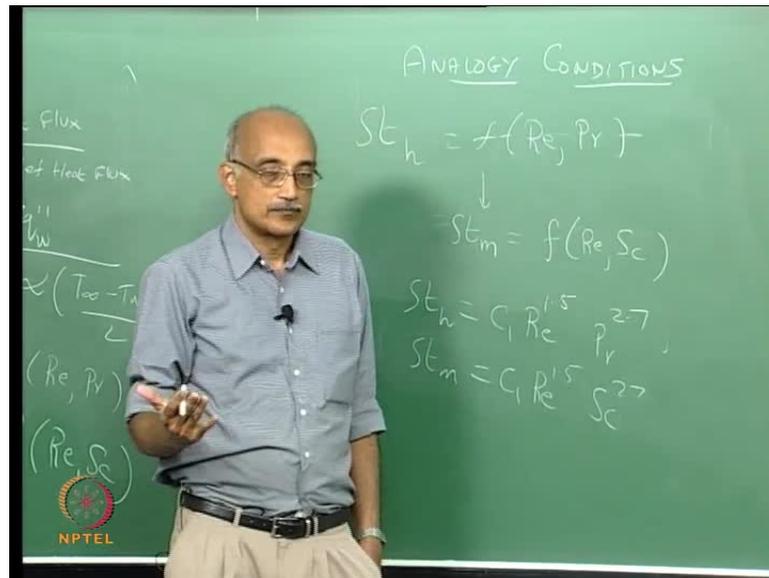
Now here, basically what we have represented is a reference flux to the prevailing flux, and this turns out to be a function of Reynolds number and Prandtl number which is easy to measure, like I said for a given temperature gradient, you can measure the heat flux and use the data to estimate the prevailing nusselt numbers. Similarly, you can evaluate the Stanton number for heat transfer has a function of Reynolds number and prandtl number.

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From this, you can then evaluate s t m as the same function of Reynolds number and Schmitt number, similarly, here from this you can estimate the nusslet number for mass transfer has the same function of Reynolds number and Schmitt number. So, that is how the analogy works, now what is the basic assumption here? You are saying that the functional dependence of the heat transfer nusselt number on Reynolds number and prandtl number is the same as the functional dependence of the mass transfer Nusselt number on Reynolds number and Schmitt number and similarly, for the Stanton number.

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So, what this says is, let us say that, St_h equals some constant C_1 times Reynolds number to the power 1.5 times Prandtl number to the power 2.7. What this analogy tells us is, then St_m will be equal to the same constant times Reynolds number to the power 0.5 times Schmidt number to the power 0.7, so there is perfect analogy between heat transfer and mass transfer.

And similarly, for the Nusselt number, but what is again the underline principle here, we are saying that the mechanism of heat transfer is the same as the mechanism for mass transfer. If that is true and they are both following the same mechanism for transport then, you can apply these so called Analogy Conditions.

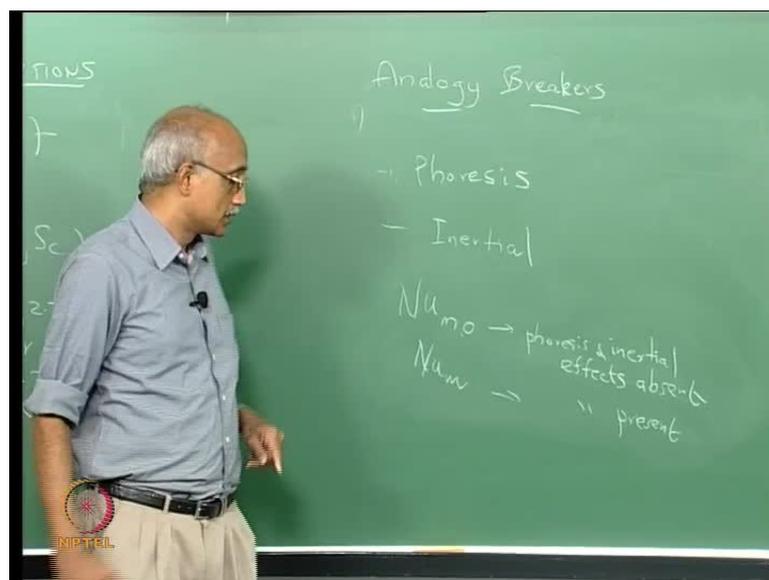
By the way, the analogy between heat transfer and mass transfer is much easier to established compare to momentum transfer; it is more difficult to apply analogies between heat transfer and momentum transfer, and mass transfer and momentum transfer because there are more factors that can lead to breakage of this analogies when you start taking momentum transfer into account.

Something as simple as pressure gradients can violate this analogy condition, but the analogy between heat transfer and mass transfer is widely applicable in a surprisingly long range of conditions. And so, this is very gainfully done to simplify experimentation and yet obtain critical information particularly as it pertains mass transfer.

Now, the basic assumption like I said is that, mechanistically heat transfer and mass transfer are occurring in the same way, but it turns out that, that is not exactly true because, as we have seen there are at least two phenomena that cause this analogy to be violated. The first one is phoresis, that is, mass transfer can occur under phoretic conditions due to an applied field in a very different manner compared to heat transfer.

So, as soon as an external force field comes into play, if the system has a steep temperature gradient or the system has a electrical gradient or the system has a gravitational gradient, the analogy breaks down particularly for particles that are sub microns in size in the first two cases. Phoretic phenomena and electro phoretic phenomena; thermo phoretic and electro phoretic phenomena become very important when particle are below 0.1 microns.

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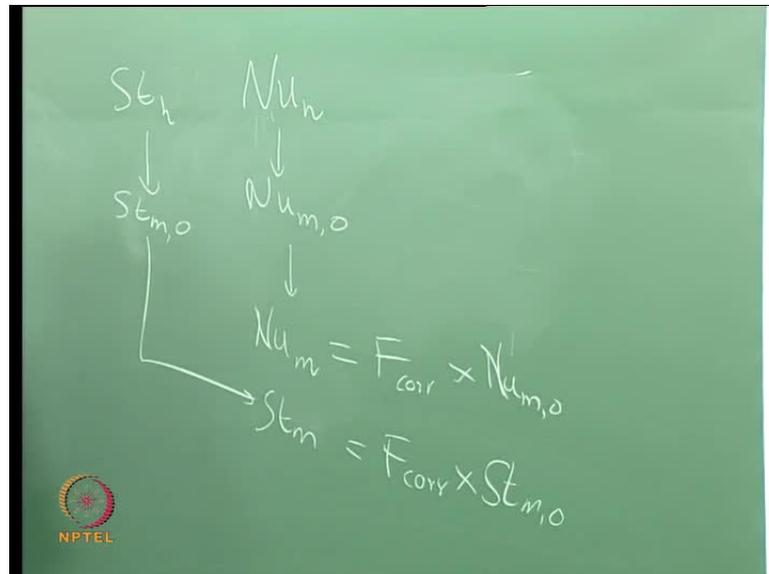


The other extreme, the analogy breaks down when inertial effects become very important, because clearly the inertial effect of inertia on mass transfer is completely unreplicated in heat transfer. So, the two Analogy breakers that we need to be aware of; the first is, phoretic phenomena and the second is inertial phenomena.

Both of them will result in significantly different estimates for the mass transfer coefficients compared to the estimate that was done neglecting these effects, in fact when these phenomena are important you have to talk about a nusselt number 0 and an actual nusselt number, where the nusselt number with a subscript 0 refers to conditions under

which the phoresis and inertial effects are absent, whereas nu_m stands for when these effects are present.

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So, to estimate the actual prevailing nusselt number and similarly, the Stanton number, first you have to calculate its value in the absence of the analogy breaking conditions, and then, you apply a correction factor. So, the procedure that you should follow is first you do your heat transfer measurements, and you obtain nu_h values, by analogy then you can estimate $nu_m 0$ values, that is, the mass transfer coefficients in the absence of analogy breaking conditions, from that you estimate that the actual nusselt numbers which will be a correction factor multiplied by $nu_m 0$.

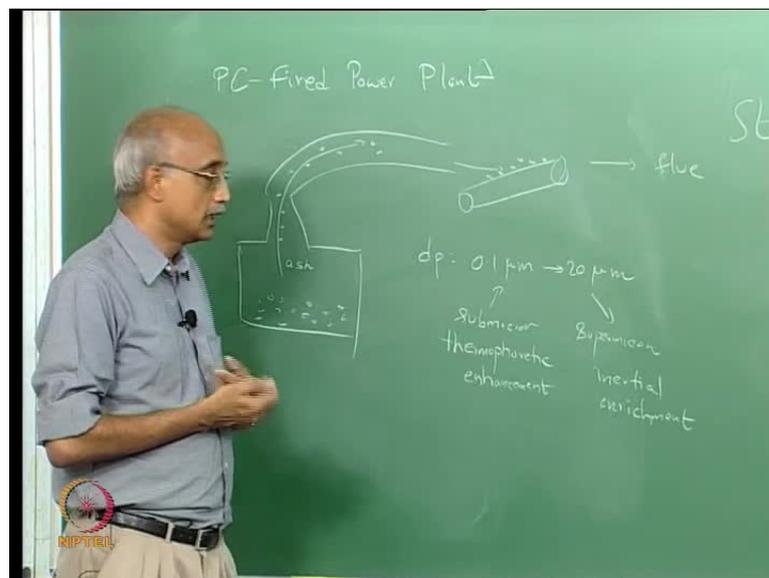
So, you obtain your base line nusselt number and stanton number and you multiply it by a correction factor to arrive at the actual prevailing mass transfer nusselt number. And similarly, to reach st_m , you will **use the** take the same root first you will take Stanton number for heat transfer, from that you will obtain a Stanton number for mass transfer under conditions where the analogy is valid and from that you will take st_m equals again a correction factor times $st_m 0$.

So, that the challenge really then becomes, how do you estimate these correction factors, what are they dependent on. what can be the magnitude - I mean - are we talking about a 10 percent difference, 20 percent difference or can this correction factor result in changes to the nusselt numbers that are orders of magnitude? Well, the answer turns out to be

that, these correction factors can be extremely large. In the case of thermo phoresis for example, if you have a system with a steep temperature gradient, this correction factor can be as much as 250 to 300.

Inertia is an even bigger effect under conditions where particles have significant inertial effects, this correction factor can be as much as tend to the power 4. And the net result of that is that, particularly if you are concerned about particle transport and deposition in your system, you may be surprised at the actual quantities that are collecting on a surface compared to, if you had simply done your estimate based on the 0 values, where you do not take this enhancements into effect. So, what will do is, will go through an illustrative exercise that highlights the role that these phoretic and inertial phenomena play.

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So, let us take the case of a pulverized coal fired power plant, so you are burning coal in this furnace and the combustion gases are coming out like this of course, in a power plant what you are trying to do is capture the or extract heat from the combustion gases **right**. So, you have this heat exchanger tubes that are mounted transfers to the flow of the hot gases and the heat exchanges are suppose to capture heat energy from the combustion gases and then, the rest of the material is vented out as flue gas.

So, ideally that is how it should happen, the tubes should only extract heat from the gases and nothing else, but what happens is, when you are burning coal ash particles survive the combustion process and they get transported along with these combustion gases, so

this ash particles start to collect on the surface. So, in addition to extracting energy from the combustion gases, the surfaces now start extracting mass also from the combustion gases. These ash particles can range in size from submicron 0.1 microns all the way up to 20 microns and larger.

So, you have a sub-micron fraction as well as a super micron fraction of ash particles in your combustion gases. And as we have seen from the earlier discussions, the rate of transport of these ash particles to the heat exchanger surfaces will have a significant enhancement due to thermo phoresis in this size range.

Thermo phoretic enhancement and in the large particles size range you will have an enhancement due to inertial enrichment and because of these reasons what happens frequently in a power plant is, the heat exchanger tubes get foul very quickly, the fouling layer start to build up on these tubes and as it does, it interferes with a heat extraction efficiency of the tubes and it also interferes with a the momentum transverse that is happening.

It actually if the deposit grows to be thick enough it can start interfering with a flow of the combustion gas itself. So, periodically in power plants, they have to shut the plant down and do maintenance, so they actually go in there with hammers, knock of these layers of ash particles that have built up on the tubes or sometimes they have to replace the tubes completely. So, it is both an issue in terms of cost as well as just down time for the plant **right**. So, it is very important to characterize the rate at which these ash particles can be transported to and deposit on heat transfer surfaces, so that we can come up with strategies to minimize them.

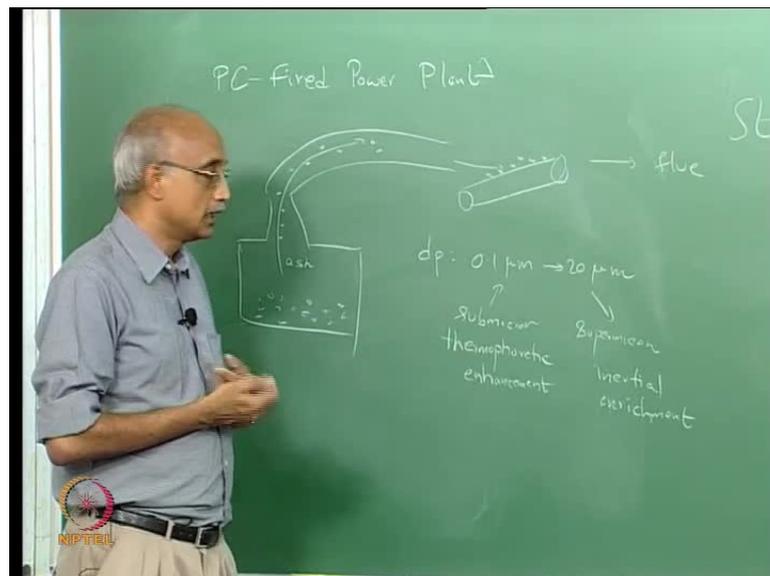
An easy thing that you can think of is if before you burn the coal, you know, first use low ash coal, if you do that then obviously you do not have to deal with the ash deposition problem to the same extent, all the difficulty with that is most Indian coals have high ash.

So, if you want low ash coal you have to import it, so that cost issue, supply issue and all that. So, in India at least power plants have learnt to leave with high ash coals, but the other issue is even if you could import low ash coal table have other impurities in fact most low ash coals that are available in the world have high sulphur.

So, it kind of trade in one problem for the other, because when you have high sulphur coal that you are trying to burn when you have two issues, one is corrosion of the same tubes. Instead of building up of a fouling layer the tubes are start to get corroded, so they will actually failed even faster. The other problem is emissions, if you have high sulphur, you have to deal with SO_2 , SO_3 type of emission.

So, it is not so easy to just take low ash coal and use it, a better option is to actually take a high ash low sulphur coal like indian coals and try to do ash removal, essentially clean the coal before you use it. So, you will see that in most power plants in India before they actually burn the coal, there will be a washing stage, where they will scrub the coal particles to try and see if they can lose on the ash material that is in the coal metrics and try to remove the ash particles which will essentially sink to the bottom, the **coal** the cleaned coal will flow to the top and then, they will try to take that coal and use it.

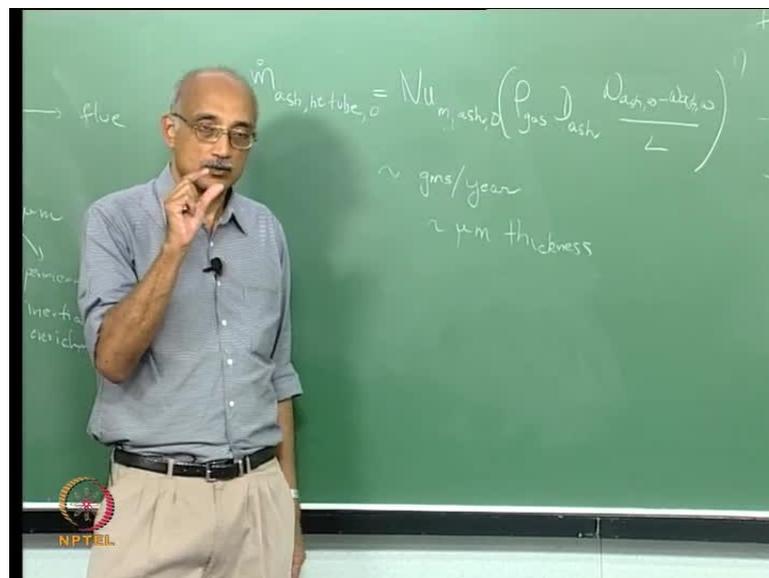
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So, this coal cleaning prior to combustion is a strategy to minimize ash, but that also has a downside, when you wash coal you are going to retain moisture in the coal and any moisture that is retain in the coal reduces its calorific value. So, washing of coal is something that you do not really want to do with water, but then if you use a solvent that brings with its own issues, **how do you deal**, how do you deal with solvent emissions, how do you design a solvent cleaner that will minimize emissions to the atmosphere.

So, all these problems are not that easy they are all heavily interconnected problems and by trying to mitigate one issue you might actually exacerbate another issue, so it something that really have to think through very carefully. Anyway, for the purposes of our discussion what we would like to characterize is, the Stanton number as well as the nusselt number corresponding to the flow of these products, in this case L can be length of the heat transfer cylinder and this is, let us say, d w diameter of the tube, so that the surface on which you have deposition going on essentially becomes phi times d w times L approximately.

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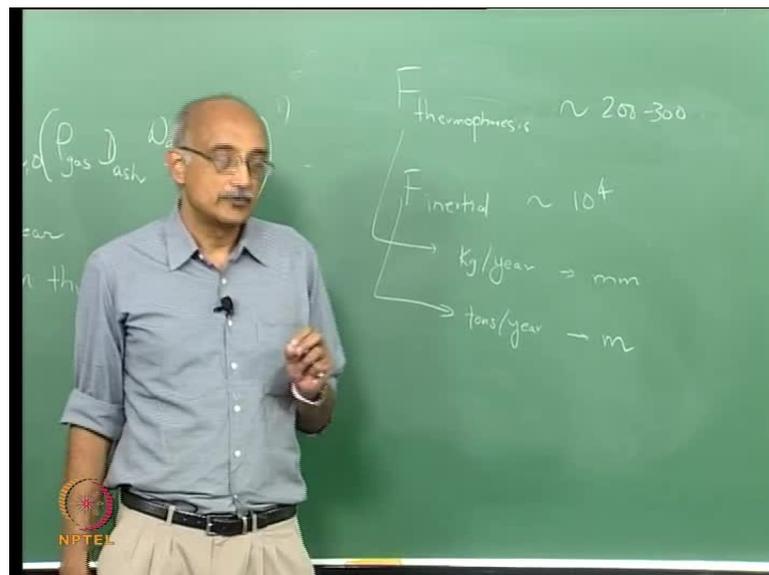
And so, you have to deal with the total deposition of ash to the surface. So, let us say that, some \dot{m} of ash to the heat exchanger tube, again you can estimate this as nusselt number for mass transfer for the ash times the reference value which is, the gas density times the diffusivity of the ash particle times omega of ash infinity minus omega ash w divided by the a characteristic length L. Now, in this expression the nusselt number that we are using is actually what? It is a 0 nusselt number **right**. This is assuming that the process of deposition of ash is analogous to the process of heat transfer from the combustion gases to the heat exchanger tubes, but in reality, so this only represents your 0 condition.

So, if you actually do this calculation for a typical power plant burning coal, the numbers that you come up with or of the order of grams per year, that is, all that will deposit on

the tube which is a very small number - I mean - it is not something you even worry about because, when you talk about grams per year spread over a fairly long pipe, it is not going to amount to much by the way of thickness. I mean, once you know the deposition rate by the way, you can calculate the associative thickness by assuming that we know the density of the ash layer, and by assuming that its uniformly distributed on the surface, you can also get a thickness distribution. And grams per year essentially translates to micrometers of thickness of the fouling layer, but in reality, people that are operating power plants will tell you that within a month they get millimeters of this fouling material to deposit on the surface.

So, clearly something is happening here, that is, more than simple base line deposition without the enhancement conditions that we talked about. So, clearly the findings based on empirical evidence, operational experience is that, the heat transfer to mass transfer analogy is not valid, that is a lot more mass being deposited on these heat exchanger surfaces compare to what you would have predicted, if you were to go by the analogy conditions. So, then it is a matter of calculating the connection factor due to thermophoresis and due to inertia and applying these connection factors in order to obtain the actual prevailing deposition rates and then, see if that matches our experience in terms of plant operation.

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So, when you actually go through that exercise, which we will do tomorrow - I mean - in the next lecture, what you find is, the connection factor due to thermophoresis is approximately of the order of 200 to 300. The correction factor for inertial deposition is of the order of 10 to the power 4, so that if you had done your modeling, taking into account thermophoresis only, you would have predicted kilograms per year of deposited material which roughly translates to millimeters of the fouling layer thickness, whereas if you took in to account inertial effects also you actually predict tons per year of ash material deposition, which will translate to meters of fouling layers.

So, if you just let a power plant operate without doing any cleaning will shut down within a matter of a few months, because that the deposit will get to be too thick. And so, how do you go from having grams to kilograms to tons, because of the correction factors that I mention earlier to account for particularly thermo phoretic effects and inertial effects.

So, what will do is, will stop at these stage, and in the next lecture, we will deal with these correction factors in more detail and look at how to estimate these factors for a power plant environment, any questions? Ok, see you at the next lecture.