

Particle Characterization
Prof. Dr. R. Nagarajan
Department of Chemical Engineering
Indian Institute of Technology, Madras

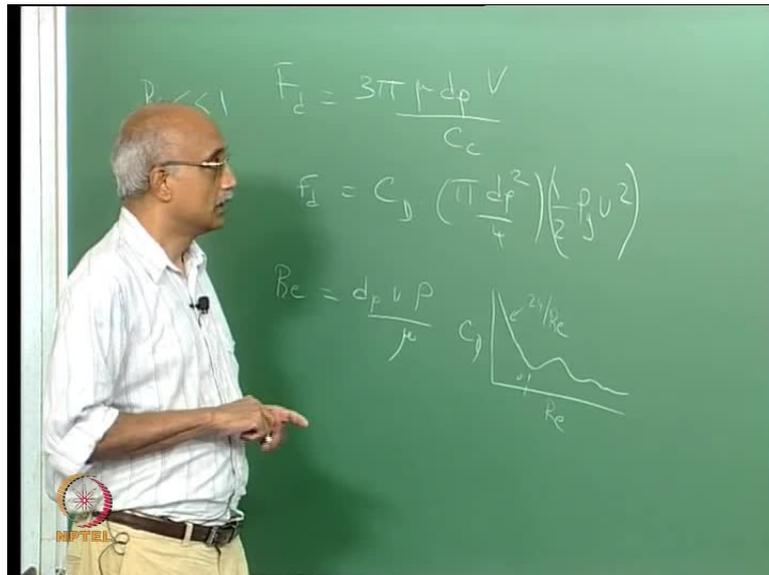
Module No. # 9

Lecture No. # 23

Transport Properties: Drag and Inertia

In the last lecture, we started discussing transport characteristics of particles, and we looked at particle dynamics, particularly the diffusional characteristics of particles, as well as their transport under an electrical field. And later in the lecture, we began our discussion of drag force that is experienced by particles that are suspended in a fluid.

(Refer Slide Time: 00:55)



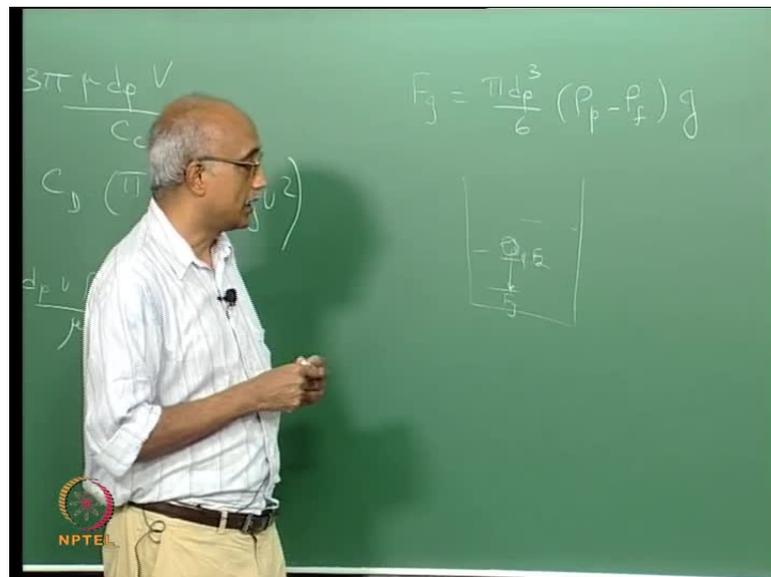
And the gravitational or body forces that act counted to the drag force, particularly under settling conditions for particles. Now, just to recall the drag force of a particle in a flow f depends on the Reynolds number, when Reynolds number is much smaller than one, that is under creep flow conditions.

We said that F drag equals $3\pi\mu d_p v$ over C_c , where μ is the dynamic viscosity of the fluid, d_p is a particle diameter, v is the relative velocity between the particle, and **the**

in the fluid and C_c is the Stokes-Cunningham correction factor, which is approximately proportional to the inverse of particle diameter for very small particle diameters.

When Re exceeds 1, then the drag force is expressed in terms of drag coefficient times πd_p^2 by 4 times half $\rho_{gas} v^2$. Of course, the definition of Reynolds number is $d_p v \rho$ over μ , the Reynolds number versus the drag coefficient relationship again can be expressed as C_d versus Re , where initially there is a steep drop with respect to the Reynolds number with a slope that is expressed as 24 over Re . And then for Reynolds numbers exceeding 0.1, essentially the drag coefficient reaches an isentropic value.

(Refer Slide Time: 02:34)



Now, we also saw that the body force can be represented as $\pi d_p^3 (\rho_p - \rho_f) g$. Here, of course, the $\rho_p - \rho_f$ refers to the differential in densities between the particle phase and the fluid phase.

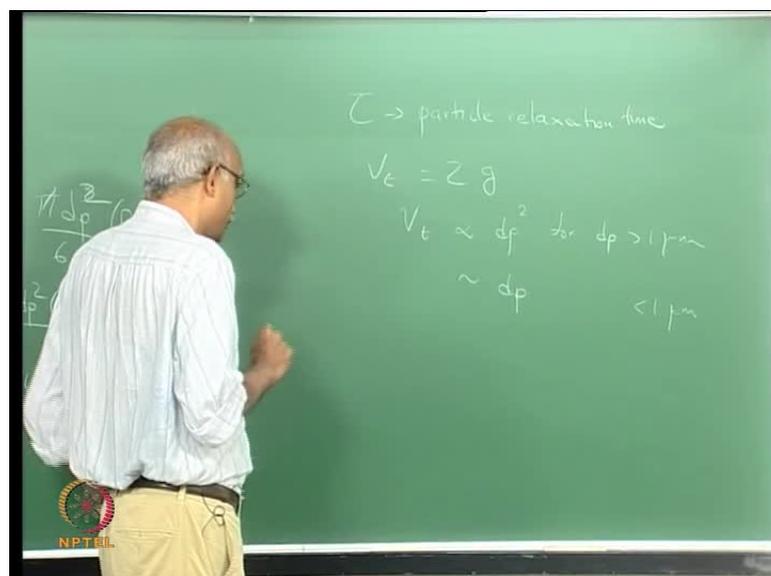
Now, in the case, where you have sedimentation occurring, where you have particles that are suspended in a fluid, the gravitational force f_g is acting in this direction, and the drag force is essentially opposing it, and because of that the particle reaches what is known as a settling velocity or a terminal velocity.

(Refer Slide Time: 03:37)

$$F_d = F_g$$
$$\frac{3\pi\mu d_p V_t}{C_c} = \frac{\pi d_p^3 (\rho_p - \rho_f) g}{6}$$
$$V_t = \frac{d_p^2 (\rho_p - \rho_f) C_c g}{18\mu}$$

And the expression for that can be obtained essentially by equating F_d to F_g . So, when you do that, now let us take the case of very small Reynolds numbers, you know under settling conditions, mostly the Reynolds numbers are very small. So, you can assume that the drag force will correspond to that, so it is $3\pi\mu d_p$ times in this case of course, the velocity will now be the terminal velocity V_t over C_c . And this is equated to πd_p^3 by 6 times ρ_p minus ρ_f times g . So, π cancels out, d_p cancels out, so this V_t equals in the denominator, you will have 18μ , and in the numerator, you have d_p^2 times ρ_p minus ρ_f times C_c times g .

(Refer Slide Time: 04:51)

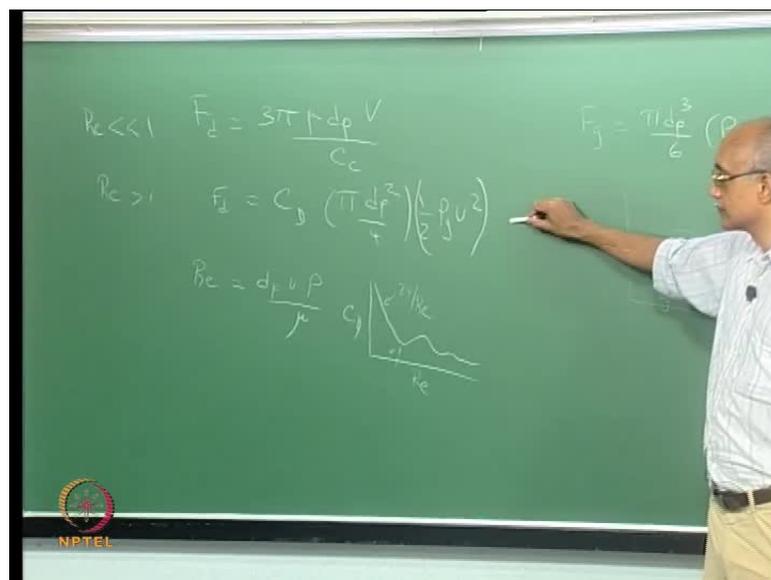


Now, this term is referred to as tau, tau is the so called particle relaxation time. So, you can express the terminal velocity as the acceleration associated with the body force times the characteristic particle relaxation time. So, in other words, V_t this can be simply return as tau time's g.

Now, if you look at this expression, the dependence of the terminal velocity on particle diameter is it goes as d_p squared for large particles, however for finer particles, as we have seen earlier, C_c itself goes as one over d_p . So, for finer particles, that are in the micron and less size range, the actual dependence of the terminal velocity on particle size is linear, it is not quadratic. So, V_t is proportional to d_p squared for d_p values typically in excess of a micron, and it goes roughly as d_p for particles that are less than one micron; now, that is an important consideration.

And in fact, for Nano particles, as you start approaching the Nano regime, the settling velocity dependence on particle size is what do you think, I mean is it going to be linear, quadratic, or less than linear. Actually if you just extrapolate this as particles, becomes finer and finer, the dependence on particle size will keep decreasing. So, for Nano sized particles, in principle, the settling velocities are independent of particle size, it is just like molecules, right.

(Refer Slide Time: 00:55)

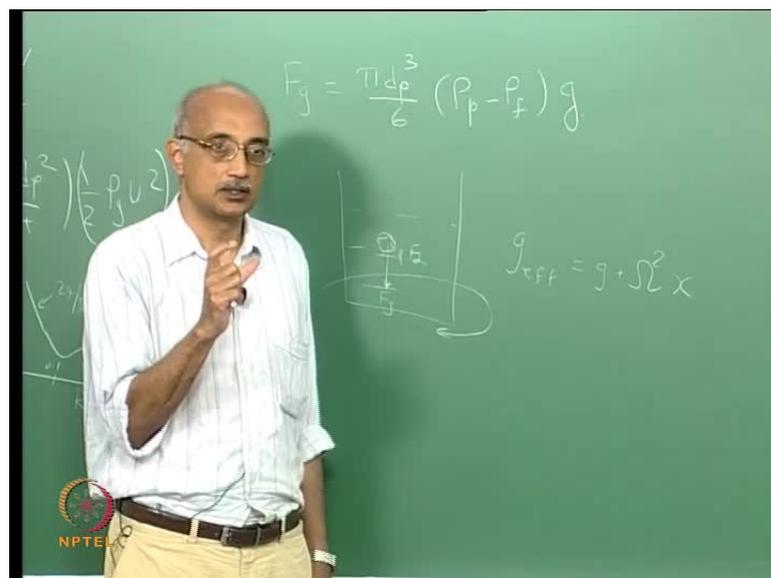


Do molecules settle? Does this **depend** settling time for molecules depend on their size? No, because there just can a randomly moving around, so there is no size dependence per

say, right, and that is exactly what happens. Another thing you should do as an exercise is assuming that the particles are actually moving at this, with this drag force. In other words, the Reynolds number is high enough that this expression applies then what happens? If you equate that to the gravitational force, what kind of expression do you get for terminal velocity?

So, it is suggest you work it out, and you will see that the dependence on particle size is reduced further when you take that expression into account. Again, indicating that **you know** very fine particles have higher velocities, and because of that their motion can be considered to be similar to high Reynolds number motion, and that is another reason why if we actually calculate the particle size dependence of the terminal velocity for very fine particles, you will see that there is virtually no size dependence, so work out that exercise.

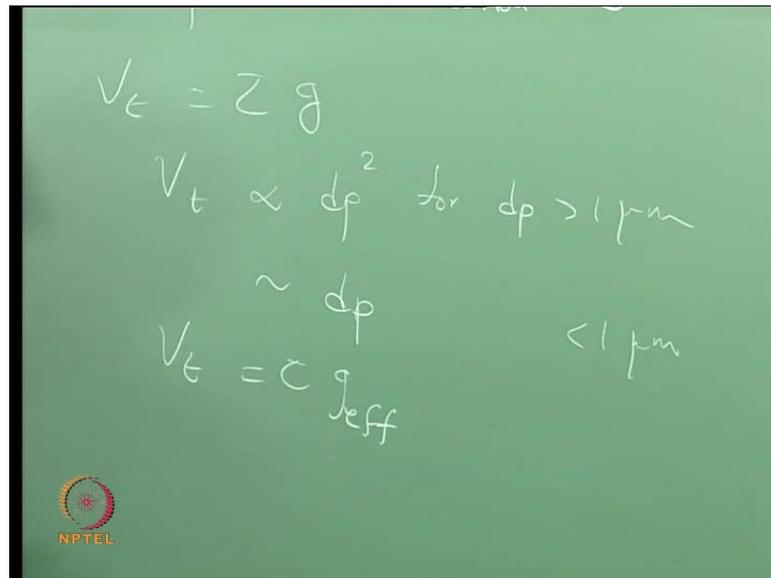
(Refer Slide Time: 08:08)



You should also look at the case where this g does not have to be just gravitational, supposing this whole thing is in a gravitational or in a centrifugal field, so that you are applying a separate body force through your centrifugal motion, then this g effective will be equal to g plus the rotational term, which is **you know** sum sigma squared times x , where sigma is the rotational speed, and x is the distance of separation between the axis of rotation and the centroid of the particle.

So, again, in this case, if you want to find out the settling velocity under centrifugal conditions, where the centrifugal field is acting **in** concert with a gravitational field, you have to substitute for the effective acceleration term, a term that includes gravitational pull plus the pull due to the centrifugal field, that you have superimposed on top of the gravitational field.

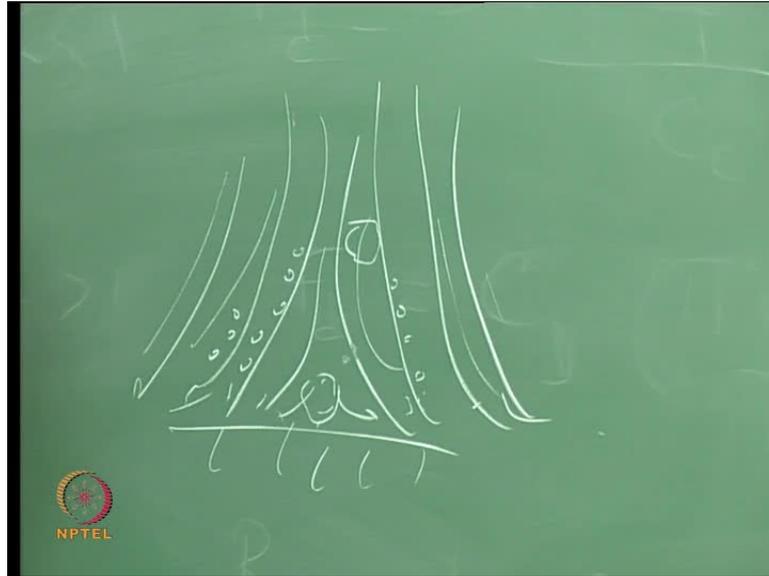
(Refer Slide Time: 09:12)


$$V_t = \tau g$$
$$V_t \propto d_p^2 \text{ for } d_p > 1 \mu m$$
$$\sim d_p \text{ for } d_p < 1 \mu m$$
$$V_t = \tau g_{eff}$$

So, the net effect of that is going to be **that** V_t in general equals τ times g effective, where g effective includes the gravitational field plus any other body force field that you have superimposed on the on that field.

Now, let us look at how this leads us into definition of a dimensionless parameter that governs inertial transport of particles. Now, as I mentioned in the last lecture, as particle size changes, so does the predominant force field. So, if particles are very **very** fine diffusion and electrostatic effect dominate, as particle size increases to an intermediate region, drag begins to make its presence felt, as particle size increases beyond that to super micron values, then inertial effect begins to dominate.

(Refer Slide Time: 10:26)



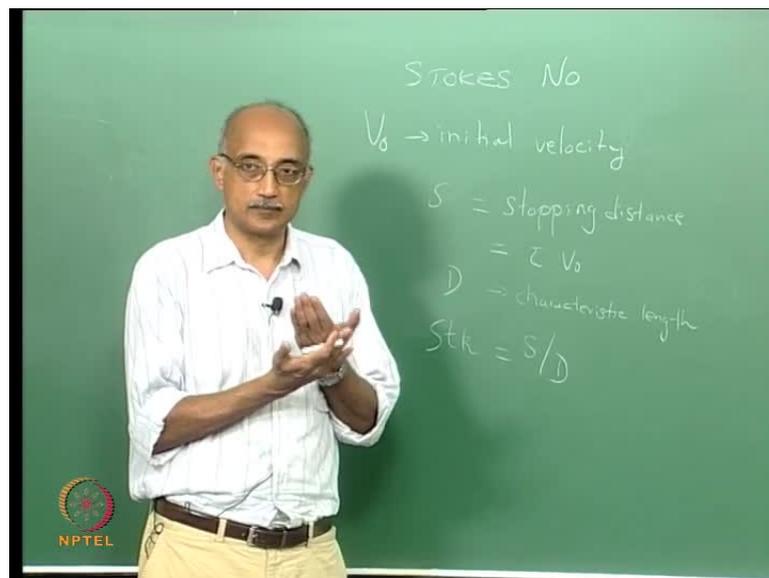
So, what do we mean by inertia? Inertia is what we call as the behavior of the particle, where it starts to deviate significantly from its carrier fluid. So, for example, if you have a solid surface, and the fluid is approaching this solid surface, let us say in a normal direction, then you would expect the fluid streamlines to look something like this, right, with possibly recirculation or stagnation in between.

So, if you have very **very** fine particles, let us start with molecules, what would molecules do? They would basically follow the streamlines, right, they will just keep going along with the fluid streamlines. Let us say the particle size gets bigger, gets to the Nano sizes, what are they going to do? From the most part they are going to do the same thing, we are going to keep following fluid streamlines, but as you keep increasing particle size, **you know** fine particle may follow the streamlines, but what if you have a particle that is this size, is that also going to want to follow the streamline? No, it **its gonna** want to separate itself from the fluid streamline, and follow a different trajectory, and that is because of the inertia that it has.

So, the measure of inertia is really the tendency of the particle to behave in a fashion that is completely different from the carrier fluid in terms of its transport characteristics. And inertial effects will obviously depend on the size of the particle, as the particle gets larger, it will have a greater tendency to separate itself from fluid streamlines.

Now, when that happens, you have to essentially start considering the particle as a separate phase, you really cannot assume that is behaving like a heavy molecule. So, there is a limit of size, once it crosses, which the effects are definitely inertially dominated. How do you find out what that size is? What is that critical size at which inertia becomes extremely significant? These dimensionless parameter that dictates the relevants of inertia is called **the** stokes number.

(Refer Slide Time: 12:56)



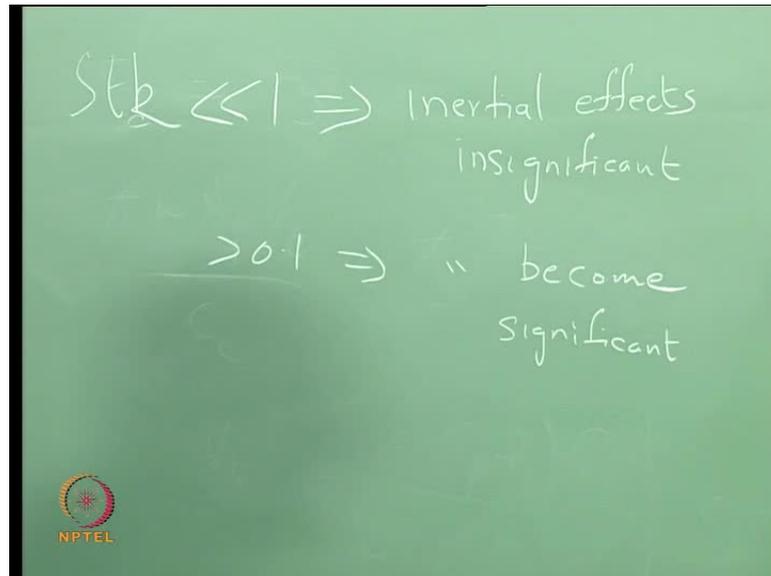
So, what is the definition of stokes number? Well, you start with the definition of what is a characteristics relaxation time for the particle. Now, let us assume that this particle here starts with an initial velocity of V_0 . What is its stopping distance? In other words, how far will it go before it comes to a stop? Now, that is something you can basically obtain by taking the velocity equation and integrating it.

When you do that, you find out that the stopping distance S is simply relatable to the particle relaxation time τ multiplied by V_0 the initial velocity of the particle. So, **the** greater the particle relaxation time, **and** the greater the initial velocity of the particle, the greater will be its stopping distance.

Now, what do you relate this to? Your surface will have some characteristics dimension let us called that some D , depending on the physical nature of the problem the relevant dimension may be longitudinal or transverse to the flow, but there will always be a characteristic length, which is associated with the target substrate.

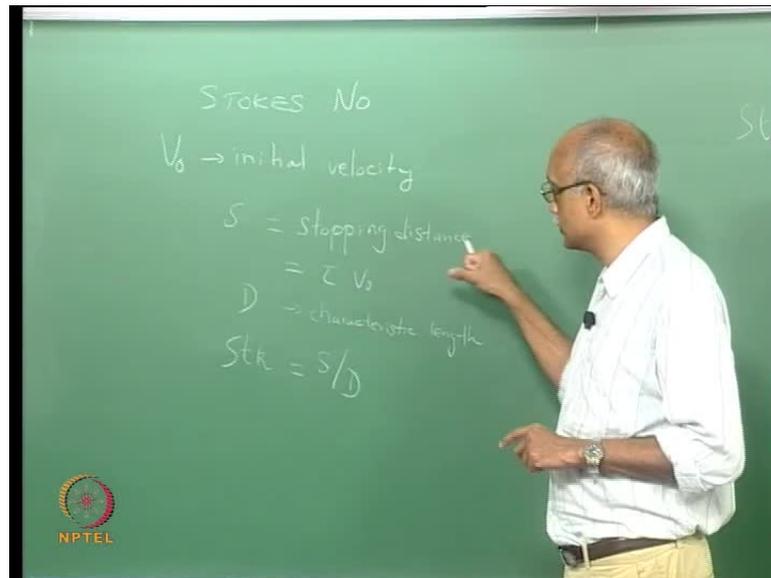
The definition of stokes number is simply S over D ; it is the ratio of the stopping distance of the particle to a characteristic dimension of the surface, which is causing the particle to come to a stop. Now, in **in** terms of the stokes number, **then** what is the physical meaning of the stokes number?

(Refer Slide Time: 14:56)



When **the** stokes number is much smaller than 1, it implies that inertial effects are virtually absent. When stokes number exceeds point 0.1, inertial effects become significant. So, for any physical problem, you calculate the stokes number associated with particle motion, and find out its magnitude, and that will pretty much give you an idea of how important inertial effects are.

(Refer Slide Time: 12:56)

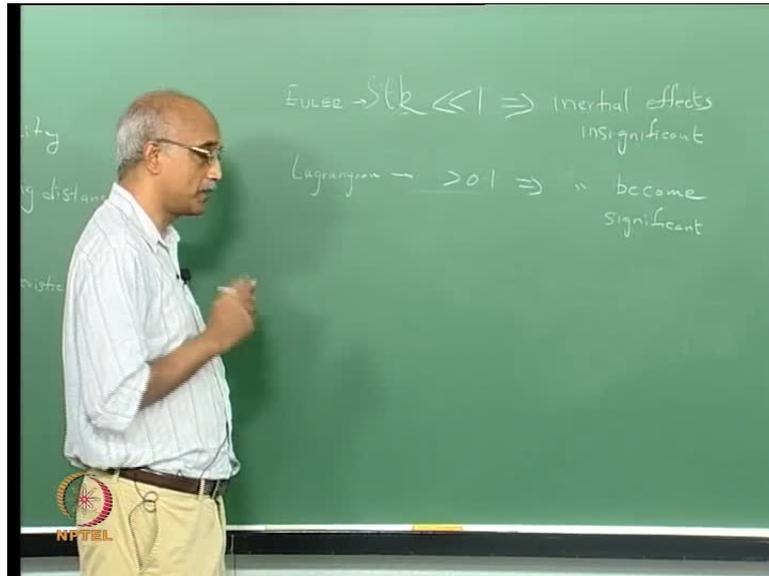


Now, the one of the key things to remember here is, the stokes number is not only related to the stopping distance of the particle, it also refers to the dimension of the surface. In other words, you may have a particle that has a relatively small stopping distance, however if the characteristic surface dimension is also **much small** much smaller, then your stokes number can still be high.

So, it does not depend on the absolute magnitude of your terminal velocity or your stopping distance, it is a relative measure of **well** how much distance it will take to bring the particle to a stop up to a physical dimension of the surface, where it is encountering.

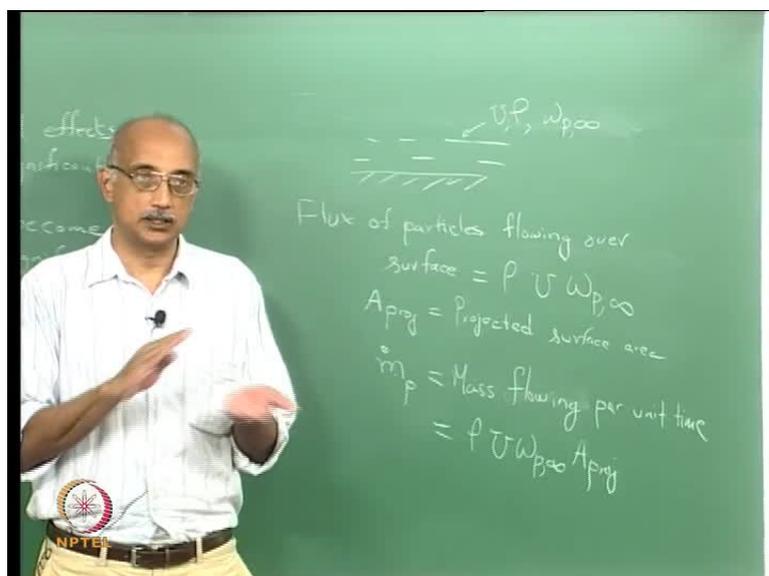
Now, why is stokes number important from a particle transport view point? Well, the reason is of course, when we say inertial effects become dominant, what we are saying is that you really cannot treat the particle as a component of the flow any more, you have to start following that particles separately.

(Refer Slide Time: 17:03)



So, in terms of our analysis of the flow problem, I am sure you have heard about eulerian and lagrangian treatments of flow, so these stokes number much smaller than one lends itself to the eulerian approach, where you do not consider the particle phase separately. However, once stokes number exceeds 0.1, you have to use the lagrangian analysis, where individual particle trajectories have to be tracked **in term** in order to describe the dynamics of particle transport in the problem. So, from a flow analysis view point, from a fluid dynamic modeling view point, from a c f t view point, the stokes number makes a huge difference.

(Refer Slide Time: 17:53)



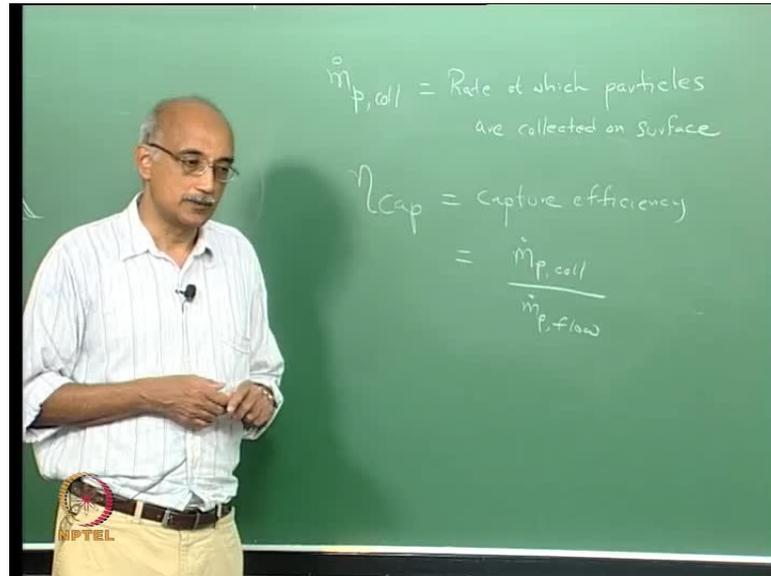
And in terms of particles collecting on a surface, again stokes number has a huge influence. There is a parameter called collection efficiency, so if you have a surface, and you have a flow on top of it, which can be either parallel to the surface or perpendicular to the surface, let us say that this flow is characterized by some velocity u far away from the surface, let say the fluid has some density ρ .

And let us say that the particle mass fraction far away from the surface is some ω_p infinity, where p stands for the particle phase, and infinity stands for far away from the surface, then you can actually estimate a Flux of material across the surface. So, Flux of material or Flux of particles flowing over the surface will be simply equal to ρ times U times ω_p infinity, right. ρU stands for the total Flux of the fluid over the surface, and when you multiplied by the mass fraction of the particles that gives you the Flux of particles across the surface.

Now, let us say that some A projected is the projected area of the surface, that is encountering the flow, then the rate of collection of particles on the surface, which we can write as \dot{m}_p , where m stands for mass, p stands for particles and dot stands for unit time. So, this is mass of particles collected on the surface per unit time, will be simply equal to ρ times U times ω_p infinity times the projected area.

This is mass flowing across the surface; this is the mass of material flowing across the surface per unit time. The parameter capture efficiency reflects what fraction of that material actually collets on that surface. So, let us say that this is the amount of material that is flowing across the surface per unit time.

(Refer Slide Time: 20:47)



Let us say that there is some collection rate $\dot{m}_{p, coll}$, which is, it is called that $\dot{m}_{p, coll}$ collection, which is the rate at which particles are collected on the surface. Now, why is this different from what we have defined earlier? You come back to look at this, you know, when you have a surface which is encountering a flow containing particles, a lot of particles will be flowing across the surface, but not all of them will actually collect on the surface, many of them will strike the surface and rebound, many of them may never encounter the surface.

So, the actual collected material will be smaller than the material that is available for collection. So, this parameter eta capture efficiency is basically the ratio of the 2, so it is the collected material $\dot{m}_{p, coll}$ over $\dot{m}_{p, flow}$. So, the ratios of the rate at which particles are collected on the surface to the rate at which they are flowing across the surface.

Now, in many process industries, this capture efficiency is a **very** very crucial parameter, in some cases, for example in power plants, the rate of fouling of heat exchanger surfaces very much depends on this collection efficiency or capture efficiency.

Ash particles from combustion gases can deposit on heat exchanger surfaces and cause deterioration of heat transfer. So, in power plants that are burning dirty fuels like coal, particle collection on the heat exchanger surfaces and the associated development of a

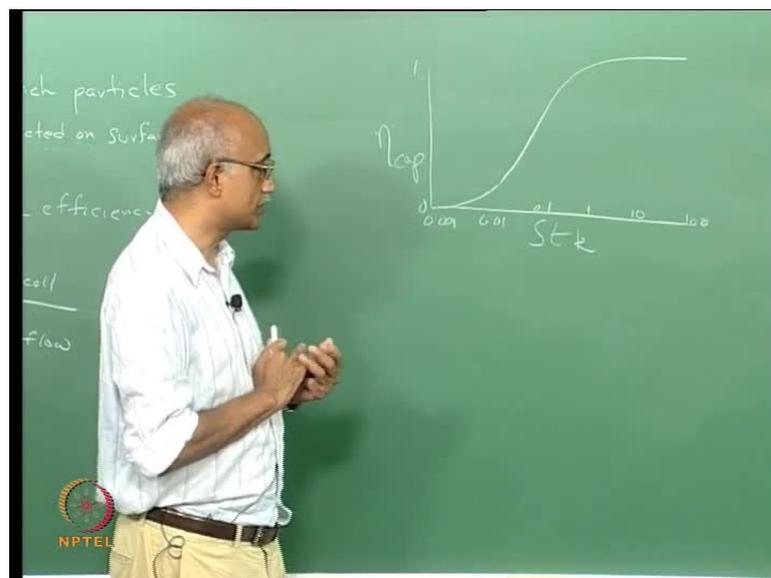
fouling layer are of great concern. So, in those industries, you want to minimize these parameters, you want to keep this capture efficiency as low as possible.

And same thing in micro electronic industries, where you have critical circuits and other components that are potentially being exposed to contamination, and there again what you want to do is minimize your capture efficiency. As long as the particles are not actually collecting on the surface, you do not care, if they are just flowing fast, that is it ok, if the particles that actually collect on your critical surfaces, that can cause reliability and other problems.

So, typically, this capture efficiency is the crucial parameter that must be minimized in many process applications, but if you are designing a filtration system, you want to maximize it, because you want your filter to collect as larger fraction of particles as possible.

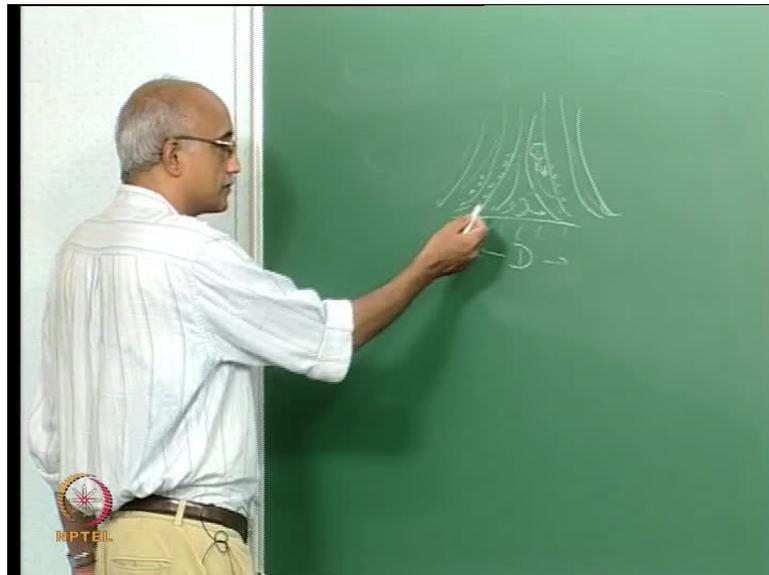
It is not that you always want to minimize it, but you always want to calculate it. The capture efficiency is a very crucial parameter in any system involving particle suspensions coming in contact with solid surfaces. So, how is this related to the stokes number, that is really the key question to answer. Is that a relationship between stokes number and the capture efficiency? If so, what is this qualitative nature and what are the implication of this?

(Refer Slide Time: 24:28)



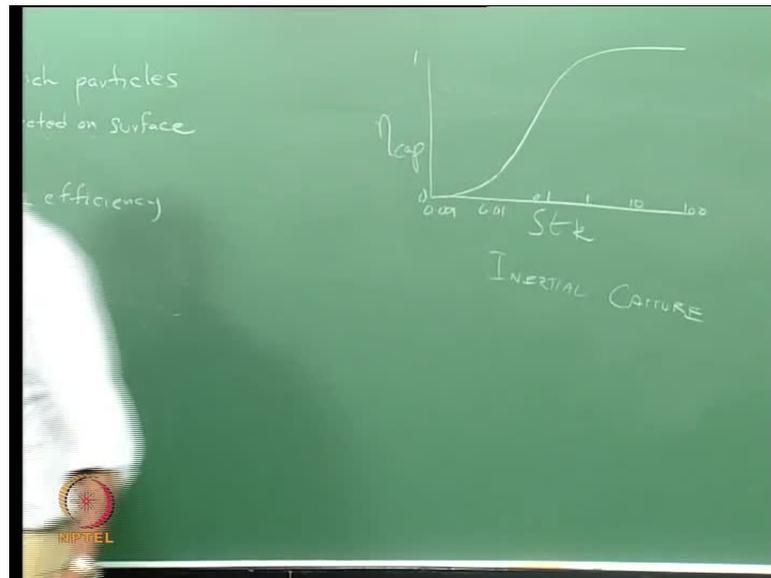
If you plot capture efficiency versus stokes number, what kind of a behavior do you expect? Let us say that it starts from zero and goes up to one, and here you'll start from, let us say 0.001, 0.01, 0.1, 1, what kind of a shape do you expect? It is actually an S shaped curve. So, you'll see a behavior that it looks like that, when you have very low stokes number the capture efficiency is actually very low as well. As the stokes number increases, it undergoes a sharp transition through a high capture efficiency right, around this critical stokes number range, where inertial effects come into play.

(Refer Slide Time: 10:26)



Now, again going back to this figure, it is easy to understand why. When you have small stokes numbers, basically what it means is particles are following fluid flow.

(Refer Slide Time: 25:57)



So, the probability of them getting captured on the surface is low. When the Stokes numbers are high, many of these particles are detaching themselves from the flow streamlines, and directly approaching the surface, and therefore the probability that they will stick to the surface is high, that is why we see this effect of Stokes number on capture efficiency, whereas the Stokes number increases to point 0.1 and beyond, there is a sharp spike in the capture efficiency.

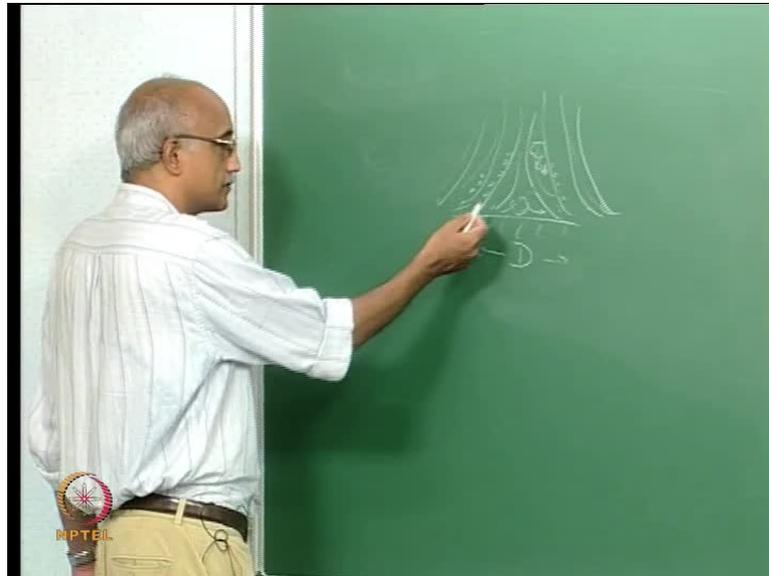
So, here again, if you are trying to minimize this, your approach should be to minimize this, right. If you keep the Stokes number for the problem low, then in principle, your capture efficiency should be reduced as well. How do you reduce Stokes number? Going back to the equation we had written earlier, you either reduce S or you increase d .

Right, but, I mean you said a good strategy, **what do you** what does it mean when you increase d ? If you increase a characteristic dimension of your surface, the capture efficiency may be lower, but the total surface that is available for capturing particles will be much higher. So, that is not a small strategy, how about lowering **you're** the numerator, stopping distance, is that a good strategy? Well, it is, if you are only again concerned about minimizing inertial capture remember that this is characteristic of inertial capture of particles.

If you try to log out your Stokes number by reducing the size of the particles, yes you will be reducing inertial deposition, but what is going to start happening? Diffusional

deposition is going to increase, electrostatic deposition is going to increase and even phoretic phenomenon, which we will talk about later start to increase.

(Refer Slide Time: 10:26)



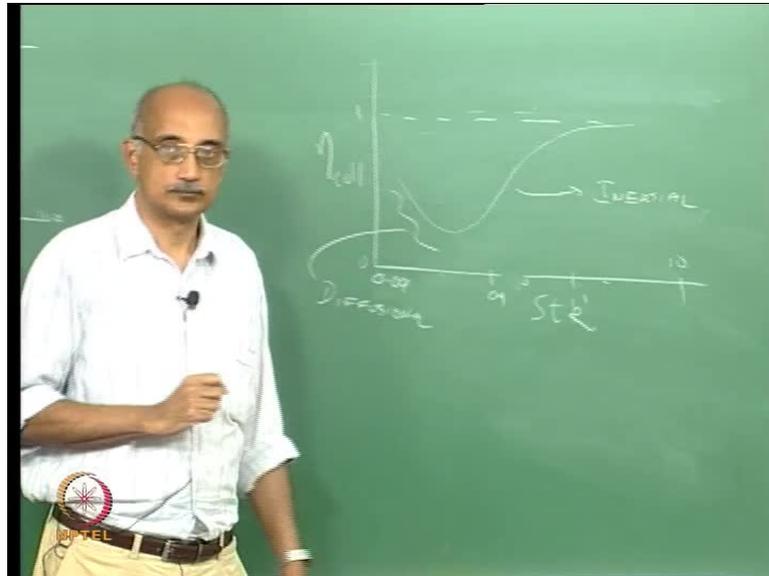
Coming back to this sketch, when particles are very fine, I said that they follow streamlines, but that is not strictly true, right. When do particles deviate from flow streamlines? It happens in both extremes, when the particles are very large, they deviate from flow streamlines because of inertial effects, but when particles are small **and** they also deviate from flow streamlines because of diffusional effects. So, for example, you want to minimize collection of particles on this surface, so you eliminate all super micron size particles and substitute them with Nano sized particles.

Is that good? Well, what that is gonna happen is, all these Nano particles are going to have very high diffusion rates, **and** so they are going to start again depositing on the surface by diffusional means, or if there is a thermal gradient, again **the** finer the particle, the greater will be the effect of what is known as thermal diffusion or thermophoresis, that will also aid in the collection of particles.

So, it is not a **you know** clear cut solution to say that if you do size reduction, **you know** it might seem like a simple strategy, if you can reduce the size of the ash particles for example, that survives the combustion process, that should result in less fouling, but that may be too simplistic view point, because as you reduce size, **yes** you are reducing inertial connection, but you may be increasing diffusional and phoretic collection.

Although, I will say that based on practical experience, it is still pays to do size reduction.

(Refer Slide Time: 29:29)



Because inertial effects are order of magnitude greater in their influence compared to diffusional and phoretic effects. So, you know, **it is** it is an interesting optimization problem again, if you look at overall collection efficiency, not your inertial capture efficiency, but supposing I were to plot η collection versus d_p , where I am including all possible mechanisms of particle collection.

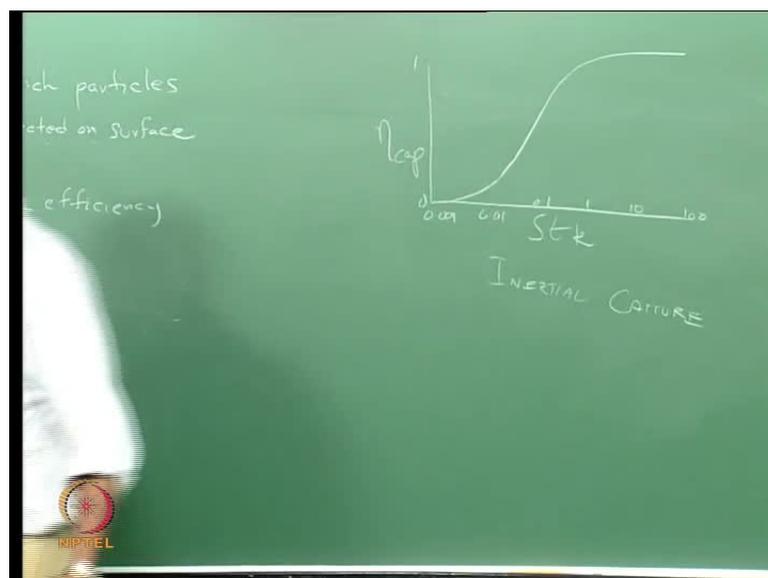
Then, will I see a trend that looks like this? Not really, instead what you will see is a trend that kind of looks like this, where **you know** again going from 0 to 1, when the particles are very **very** fine, let us say that this is one nanometer, the diffusional deposition actually causes an enhancement in collection efficiency, and then as you keep increasing the size, let us say that this is 10 nanometers, a 100 nanometers and so on. You actually get a reduction in overall collection efficiency for a size that is slightly below 0.1 microns, because inertial effects are not set in here, because as we have seen before, inertial effects start to become important, **when** if you write this in terms of the particle diameter, that is probably not good measure, let us go back to our stokes number representation.

So, here again, the stokes number here is 0.001, 0.1, 1, 10 and so on, the minimum occurs below 0.1 microns. And then, as you start crossing 0.1 micron, 0.1 in stokes

number, you start to see an increase in your collection efficiency. So, when you plot collection efficiency as a function of stokes number including all collection mechanisms, then there is actually a fairly high collection efficiency at very **very** fine sizes, which drops rapidly to a, to its minimum value somewhere in the 0.5 stokes number range.

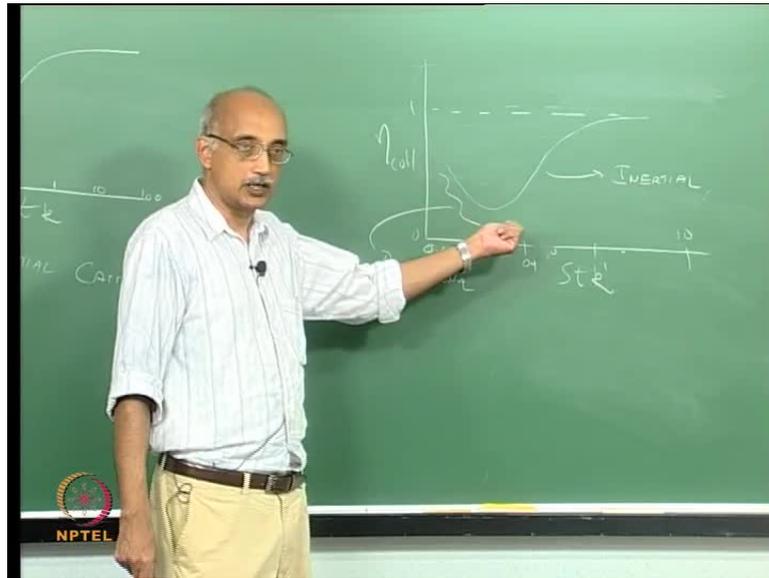
And then as a stokes number starts to approach 0.1 and exceed 0.1, again the numbers pick up. Again, the primary reason is here, you are dominated by diffusional phenomena, and in this part of the curve, you are dominated by inertial phenomena. So, that is why the stokes number is a very critical parameter to characterize the particle transport and particle deposition mechanisms are prevailing in a system. But, at the same time, if you entirely focus inertial effects, and forget about the diffusional effects, you could be making some miscalculation in your analysis.

(Refer Slide Time: 25:57)



So, from this you can see that, it is true that, in the inertially dominated regime, your collection efficiency will be significantly higher compare to diffusional dominated regime, but this non-Miltoic behavior is very important to characterize and understand. So, you should be able to essentially sketch two behaviors, if you want to look at the effect of stokes number on capture efficiency **in the inertial due to** strictly due to inertial mechanisms, then this is the curves that applies.

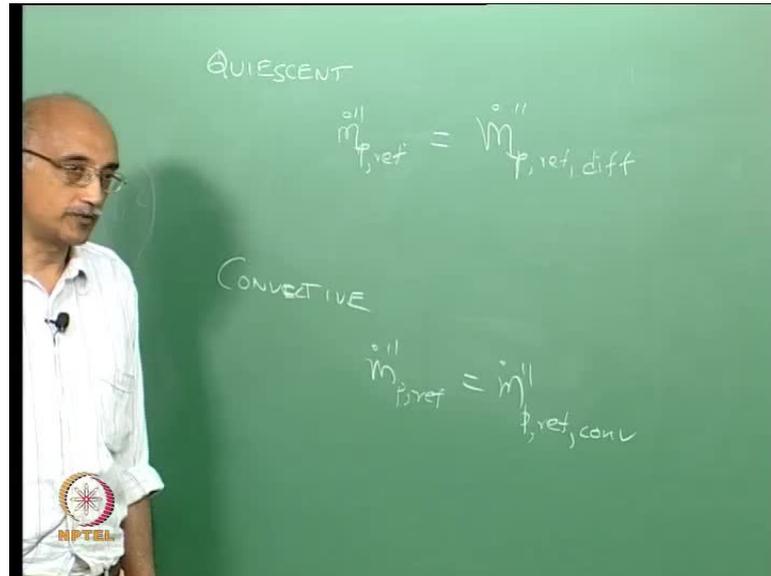
(Refer Slide Time: 29:29)



If you want to look at the effect of stokes number on the overall collection efficiency including all transport mechanisms, that is inertial phoretic diffusional, then this is the kind of qualitative relationship that you will find. So, you ought to be able to distinguish between the two. So, what we have looked at so far is the effect of particle transport and the associated collection efficiencies on surfaces.

Now, as chemical engineers, we always like to non-dimensionalize, right. So, when we look at these particle collection rates on surfaces, and particle diffusion fluxes, you would also like to define certain reference values, which we can use to convert these fluxes into the dimensionless values.

(Refer Slide Time: 34:10)

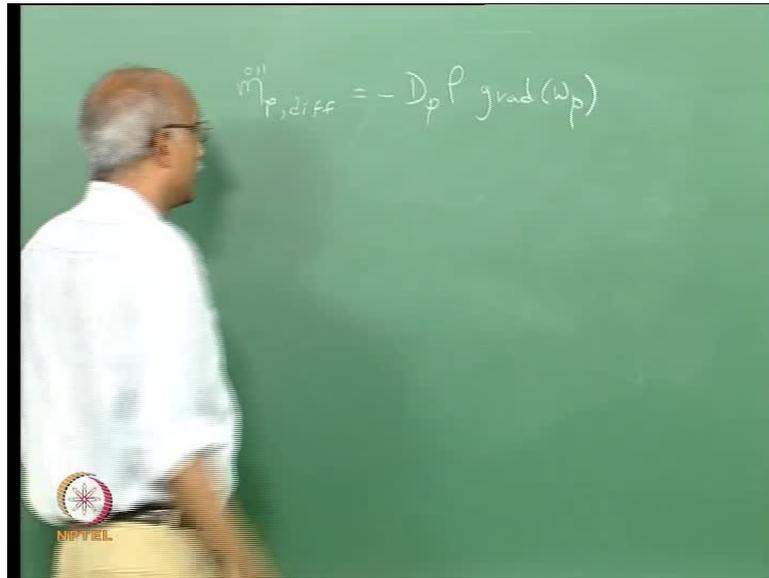


So, what are the relevant non-dimensional parameters in this case? In order to define the reference values, you have to look at two cases, one is the quiescent case, and the other is convective. When we talk about quiescent versus convection, what we are talking about is here, there is essentially an absence of a convective velocity, either forced or natural convection, whereas in this case, there is a significant convective component, which again can be forced convective or natural convective.

Now, in the two cases, how do you define reference fluxes? So, if you take **m p double dot** m p dot double prime reference, where again m stands for mass, p stands for particles, dot stands for per unit time and the double prime stands for per unit area. So, what this is particle mass Flux reference, in the case of quiescent flow, it relates back to the diffusive characteristics of the particle.

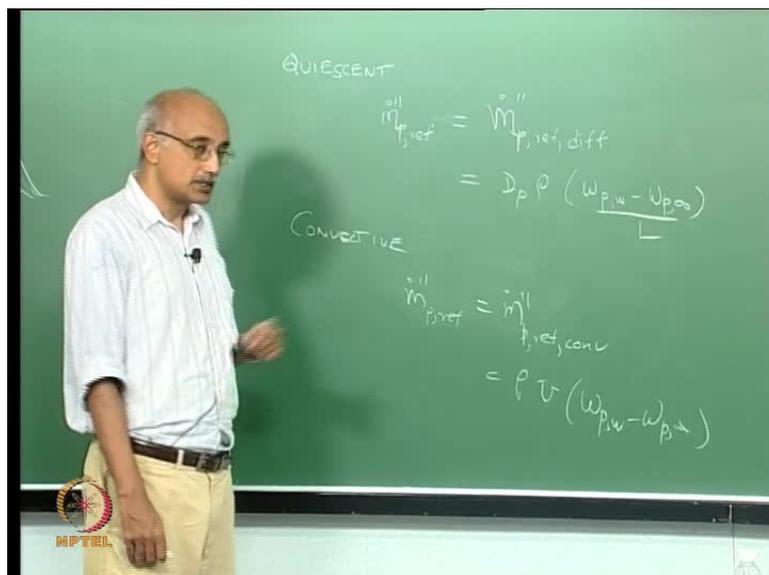
So, this will be an m p dot double prime reference, diffusive, because diffusion becomes the only prevailing transport mechanism, if you have a completely quiescent system, whereas in the case of convective, your m dot double prime p reference is a convective parameter.

(Refer Slide Time: 36:04)



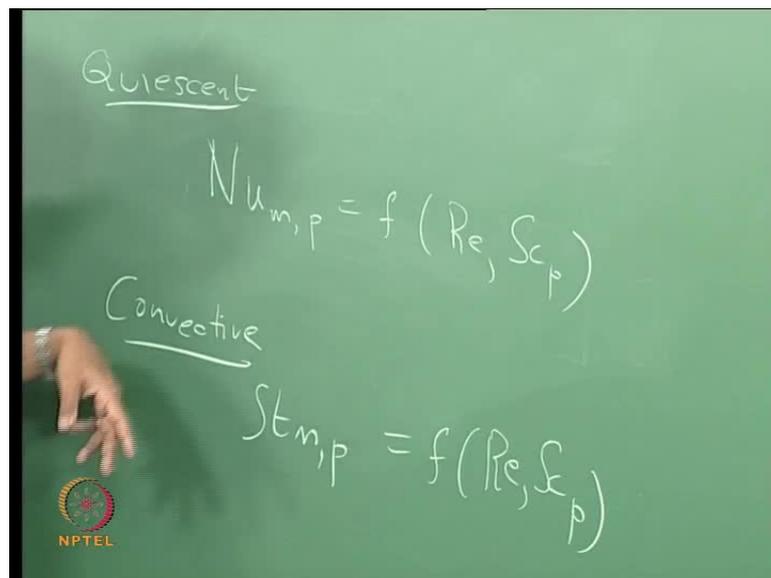
Taking into consideration the fact, that convection does play a major role. So, what is the difference between these two? You have to go back to your expression for diffusive Flux, if you recall the expression for $\dot{m}_{p,diff}''$ equals minus $D_p \rho$ gradient in ω_p , right. The diffusional Flux of particles is proportional to **the** its mass fraction gradient and it is related by the brownian diffusivity D_p and the gas density or fluid density ρ .

(Refer Slide Time: 36:37)



So, from this what would you establish as a reference Flux? This is the reference Flux, therefore is going to be d_p times ρ times ω_p at the surface minus ω_p far away from the surface, the whole thing divided by l . So, the term within the parenthesis is the reference value for the gradient, and for this case, for the convective case, actually the reference value is much easier to think about. It is basically ρ times u , which is the convective Flux of the fluid multiplied by ω_p minus ω_p infinity.

(Refer Slide Time: 37:49)



So, when we actually take the prevailing mass fluxes and we want to non-dimensionalize them, you will divide by this value in the case of quiescent flow and we will divide by this value in the case of convective flow. And the resulting non-dimensional parameters that you obtain are what? In the case of quiescent flow, what is the non-dimensional parameter that are represented mass Flux nusselt number? So, here, you will define nusselt number for mass transfer for a particle, and in the case of convective, you will define a Stanton number for mass transfer of the particle. The nusselt number and Stanton's number are critical non-dimensional parameters that governed transport properties and fluxes of particles in systems, so we will come back to this in the next lecture and discuss in more detail.

It turns out that a nusselt number is a function of primarily the Reynolds number and the schimtt number of the particle. The Stanton number is also a function of the same two parameters, because the relative dependence is going to be very different in the two

cases. In the case of the quiescent flow, the Reynolds number effect, which is a convective effect is going to be significantly lower compare to the diffusive effect, that is represented by the schimtt number, whereas in the case of conductive flow, the Reynolds number effect is going to be significantly greater than the diffusional effect that is represented by the by the schimtt number.

So, we will come back to this and go through the definitions of these numbers and it turns out that you can actually relate these parameters back to your capture efficiency. So, we will do that in the next lecture. Any questions alright, see you at the next lecture then.