Particle Characterization

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Module No. # 9

Lecture No. # 22

Transport Properties: Diffusion and Electrostatic Field Effects.

Welcome to the 22 lecture in our particle characterization course. In the last few lectures, we have dealt with aspects of particle interaction with its surroundings, interfacial properties such as particle adhesion, removal from surfaces, particle cohesion and so on.

Starting with this lecture, we are going to deal with the fourth aspect of particle characterization, if you recall in the earlier lectures, I mentioned that particle characterization, at least in this context of this course, can be classified into a few major categories; we started with morphological characterization which was shape and size related; then we talked about structural characterization; and the third aspect that we have dealt with, is actually, the interfacial characterization of how essentially a static particle interacts with its nearest neighbours.

In starting with this lecture, we are going to discuss the fourth aspect of particle characterization which is transport properties. It is important to know how particles behave in a dynamic mode, because particles that are suspended in fluids have a natural tendency to move around; they are not going to stay in place. Unlike a large object, when you look at a something like a table, it stays where it is; but if you keep breaking it up into smaller and smaller fragments, eventually you will reach a size which is not static anymore. It has a tendency to move, in response to the motion of the fluid around it, or when the particles get to be sufficiently small, they will move on their own without the need of an external force field being applied.

And the dynamic characteristics of particles, again play a huge role in industry. You have to learn how to predict, how to control and how to manipulate the movement of particles, in order to derive the maximum benefit from your particular technology, whatever it is.

So, when we talk about transport characteristics of particles, as we have briefly touched upon in the past, the predominance of the transport mode depends very much on the size of the particle.

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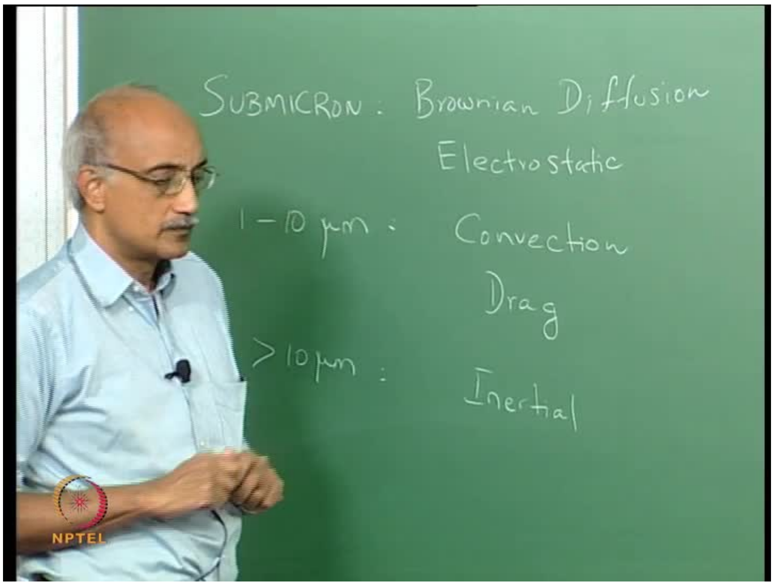

So, when we talk about extremely small particles that are in the sub-micron range, the dominant mode of transport is diffusion, which in the case of particles goes by the name of Brownian Diffusion.

In the very fine particle size range, Electro static effects are also important. Now, as particles become a little larger and you start approaching the micron to a few microns, 1 to I would say around 10 microns. Then essentially, the convective motion of the particle becomes predominant effect in dictating how well the particles move around in a system. So, in this size range, convection, drag, become important forces; and here, we are talking about force convection, as well as, natural convection. And of course, drag can happen in the very low Reynolds number range or in a higher Reynolds number range; but all of these are particularly applicable to particles that are in an intermediate size range.

Now, as particle size is start exceeding the 10 micron range, inertial effects begin to dominate. So, very simply, put the transition in terms of the dominant transport mode, evolves in this fashion from diffusion, and electro static to convection, and drag to inertial effects.

So, in order to understand how particles move, we really have to take each of these and look at them in a little more detail.

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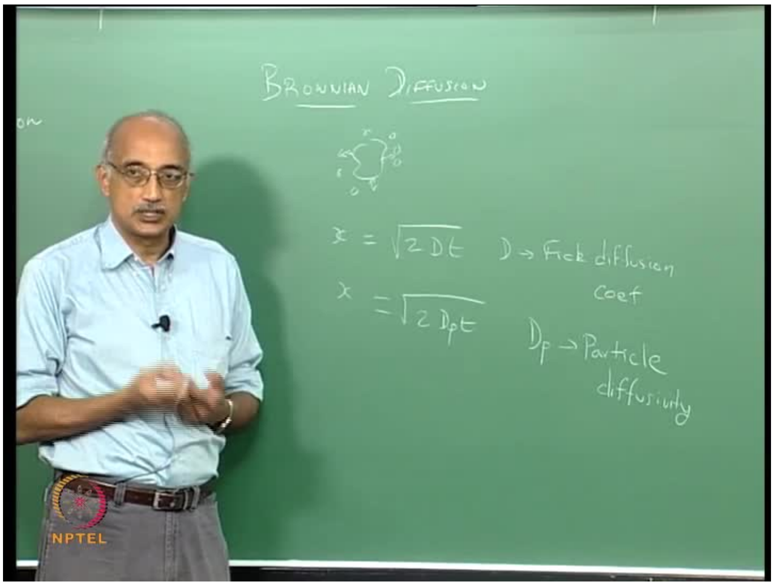

So, we will start with diffusion. I am sure you are all familiar with Fick diffusion of vapour molecules, and it happens by, what is known as a random walk process; when two adjacent molecules hit each other, they exchange energy, and they rebound and move in different directions. So, essentially as a molecule encounters an adjacent molecule, there is a correction in its trajectory and it is goes towards a new place.

Now when we talk about particle diffusion, what is the difference? A particle, in the case of diffusion, can be considered just like a heavy molecule; the interactions are very similar. But here, one of the key things we need to focus on is, when you have a particle, by definition, it is a little larger than the vapor molecules or fluid molecules that are surrounding it; and so, you have all these molecules that are surrounding the particle, and the particle is being repeatedly impacted by the surrounding molecules; as that happens, there is an imbalance in the momentum that develops. So, if there is a preferential direction, in which more collisions happen, and so that, transfer of momentum happens to a larger extent in one direction compare to the others, the particle will have a tendency to move in that direction; it is still, a random walk phenomenon because the collisions that are happening are essentially stochastic in nature.

However, there is a net movement of the particle, due to these collisions of the molecules surrounding it, just like in the case of Fick diffusion, there is a net movement of the molecule because of the collisions that it undergoes. And that distance, in the case of Fick diffusion, is given by 2 times dt, where d is the Fick diffusivity of the molecules, the Fick diffusion coefficient.

In the case of particle diffusion or Brownian diffusion, the same formula holds; except that, now you substitute the diffusivity parameter with the one corresponding to a particle. So, these equations basically say that the particles will experience a net movement due to this repeated and random collisions, and it is goes as the square root of both the diffusivity, as well as, the time over which these the diffusion process is occurring.

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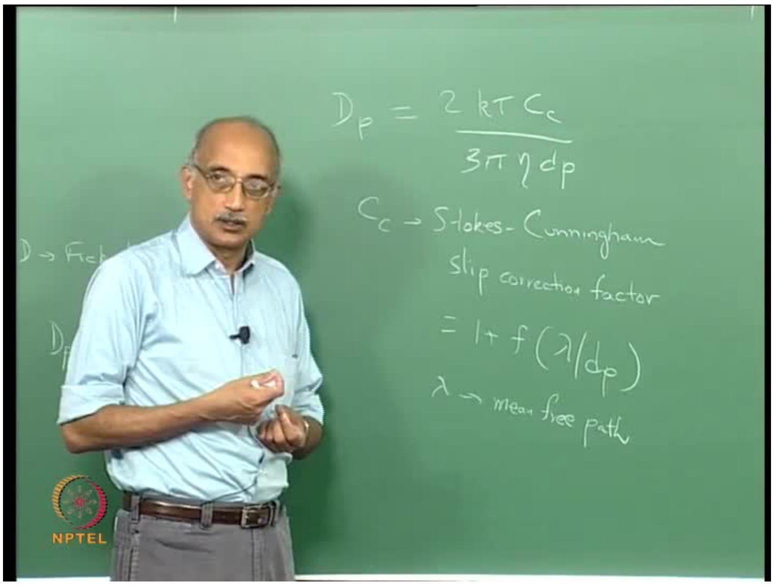

The particle diffusivity itself is defined by two k t C c over three pi eta d p, where k is the Boltzmann's constant, t is the temperature, again the C c factor is the stokes-cunningham slip correction factor. Now, this correction factor, essentially accounts for the fact, that we always assume continuum.

Now, the continuum assumption tends to break down to some extent, at the interface between a particle and a molecule, or at the interface between two molecules. So even though, the fluid system as a whole, may be in continuum flow, again the definition of continuum is a very small Nixon number, where the Nixon number is the ratio the mean free path to the particle diameter.

Now, in general, most flows that we encounter, do satisfy the continuum condition. However, there is a slip between the particle and surrounding molecules, or between one molecule and the molecule around it, which violates the continuum assumption. And in fact, at the interface, what you have is, what is known as free molecule rather than continuum flow. So, this C c is a factor that has been developed essentially to correct for the fact, that the continuum assumption is violated at the interface between the particle and the molecule; and it is equal to 1 plus a function of lambda over D p, where lambda is the prevailing mean free path and D p is particle diameter. So, this correction factor increases rapidly as particle diameter drops.

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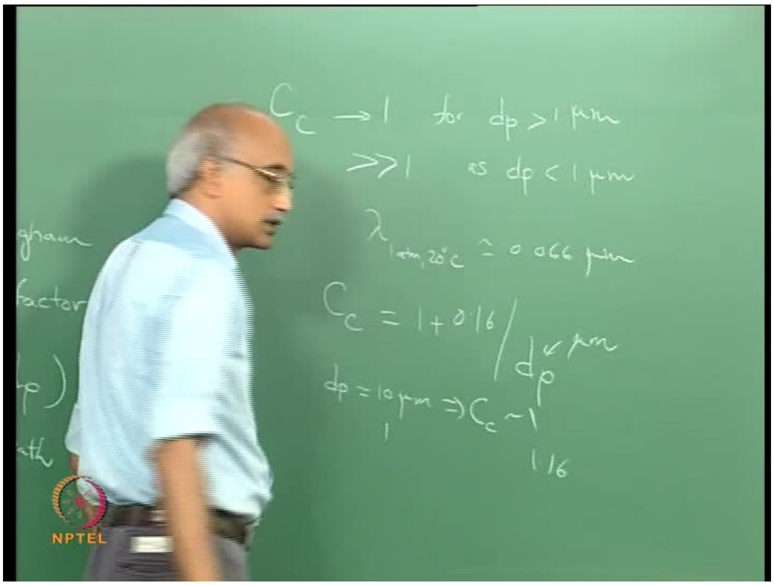

In fact, this factor C c will tend towards 1, for d p values that are in excess of 1 micron; but this C c values can be much larger than 1, as d p drops below a micron. For example, in the case of particle diffusion in air, let us say, you are atmospheric pressure 20 to 25 degree centigrade, the mean free path for air under those conditions, at 1 atmosphere, 20 degree centigrade is about 0.066 micro meter. And under these conditions, it turns out that, you can write C c as equal to 1 plus 0.16 over d p, where the particle diameter is expressed in microns. So, what is the implication of this? When d p is say 10 microns, C c is approximately 1, because this correction factor is 0.01, less than 2 percent. When d p is equal to 1 micron, C c is about 1.16. So, there is a 16 percent correction in, or there is a the C c value here, instead of being 1, is now going to be 1.16.

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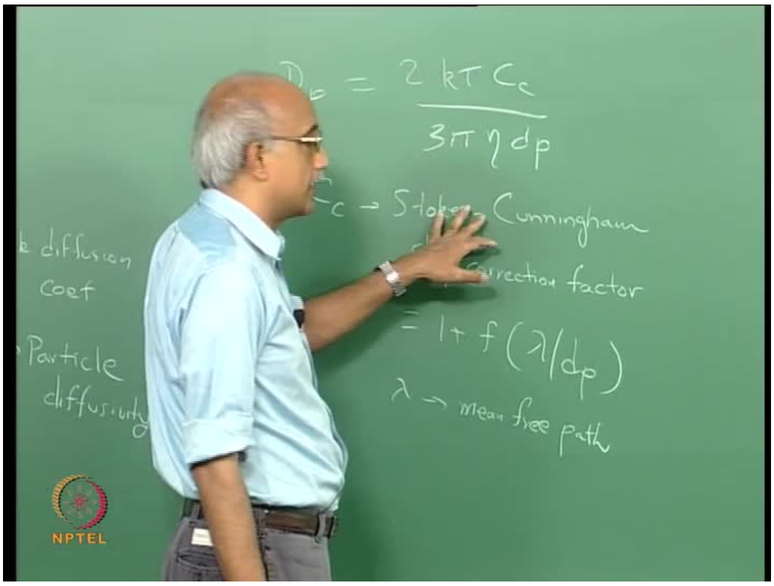

So, in terms of the diffusion coefficient, what does it mean? It increases the diffusion coefficient by 16 percent compared to, if you had neglected this slip correction factor.

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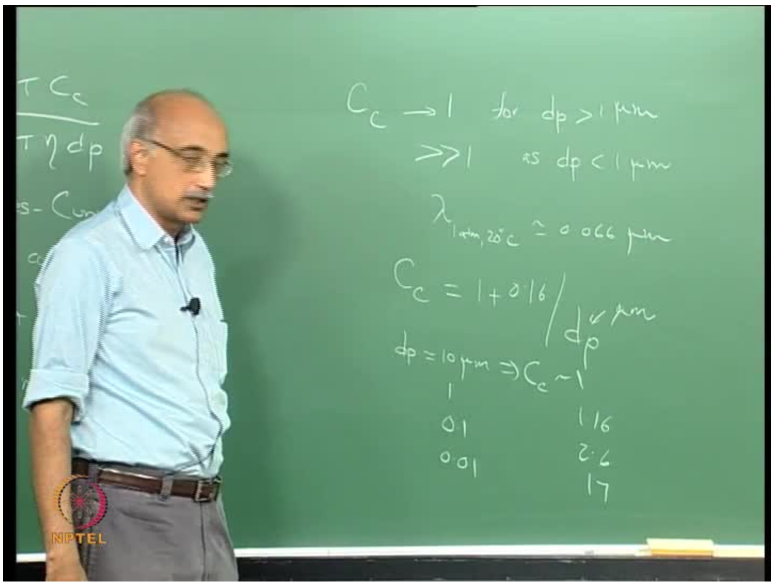

And of course, if this drops further, if d p drops to 0.1, then the C c value, now becomes 2.6; And if it drops to 0.01 or 10 nanometers, this correction factor is as large as 16; so, this become 17. So, you can imagine that, as the particle size drops, the slip correction factor begins to gain in significance, and the particle implication of that is, again if you reexamine the diffusivity value, it is proportional to temperature, as you would expect; as temperature increases, diffusion is faster; everything moves around more quickly; viscosity is in the denominator, diffusion happens faster in a less viscous fluid.

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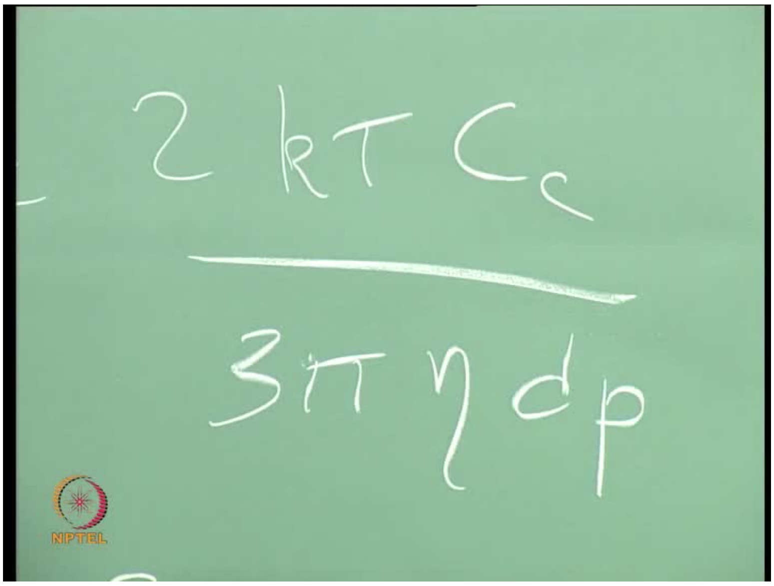

The particle diameter is also in the denominator; however, when C c values are small or when particle diameter are large, there is only a 1 over d p relationship between the Brownian diffusivity and particle diameter. But as a particle size becomes smaller and smaller, this C c value essentially becomes proportional to 1 over d p; so, what that does is, d p value then becomes proportional to 1 over d p square for particles in the sub-micron range.

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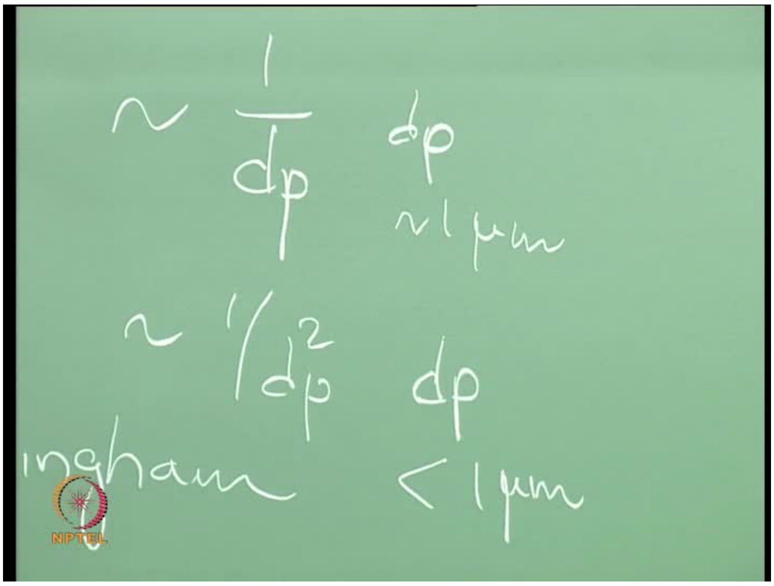

Now that is an important consideration that d p will go as 1 over d p, when d p values are of the order of 1 micron, but it will go as 1 over d p squared when d p values begin to drop well below 1 micron.

So the inverse dependence of diffusivity on size, becomes exacerbated as particle size is becomes smaller and smaller; and also the for a given particle diameter, the value of the Brownian diffusivity is much greater than you would have predicted based on the continuum assumption alone, as the diameters again drop below 1 towards the nanometer sizes.

So, very important to keep in mind, this correction factor for the slip between the particle and the surrounding molecules. Now, let us get back to our discussion of Brownian diffusion. So, we have said that essentially that distance that the particle moves, once you have calculated the Brownian diffusivity, d p will be equal to square root of 2 times d p times t. What is the Fick first law of diffusion, anybody knows? It relates flux to gradient.

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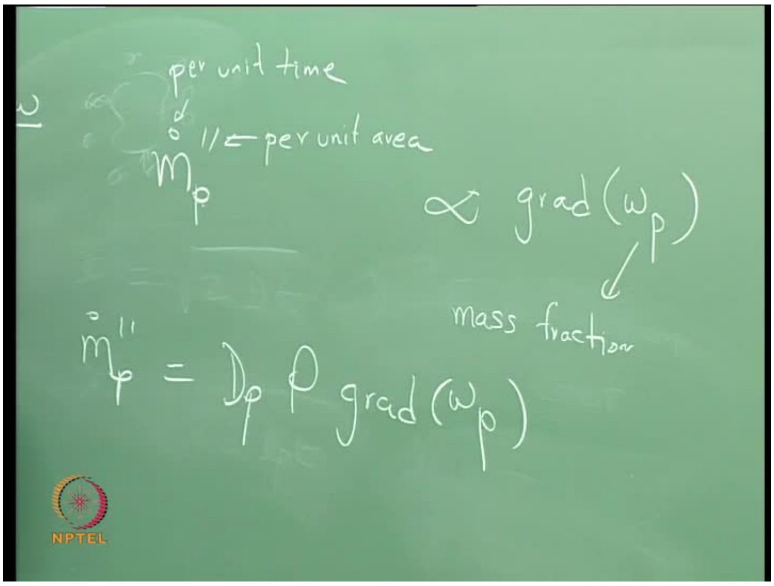

So, similarly for particles also, there is a first law of diffusion which basically would say, that, if you look at the particle flux, transport flux due to diffusion, let us call that some m p dot double prime, where p represents the particle, the dot represents per unit time and the double prime represents per unit area. So, m p dot double prime represents mass of particles transported per unit time per unit area, which is the definition of mass flux.

So, the Ficks law has extended to particles, would basically say that this is proportional to gradient in omega p, where omega p is the mass fraction of the particle.

According to the Ficks diffusion law, flux always happens down a concentration gradient, and it is proportional to the concentration gradient. In its full form, you can write this as m p dot double prime equals d p times rho times gradient in omega p, where d p is again the Brownian diffusivity, rho is the density of the surrounding fluid, and gradient in omega p is the mass fraction gradient that drives the diffusion process; by the way, there is usually a negative sign associated with it, because diffusion is conventionally regarded as occurring towards the control volume; we will discuss those aspects in more detail later on, but for the time being, just look at how particle flux is related to the diffusivity parameter, the density of the fluid and the gradient of the mass fraction.

Now, this law is applicable under what conditions? What is a basic assumption here? Among many, one of the assumptions is essentially it is a steady state process, you have a assume you are assuming here that you are essentially have achieved a steady state gradient in the mass fraction

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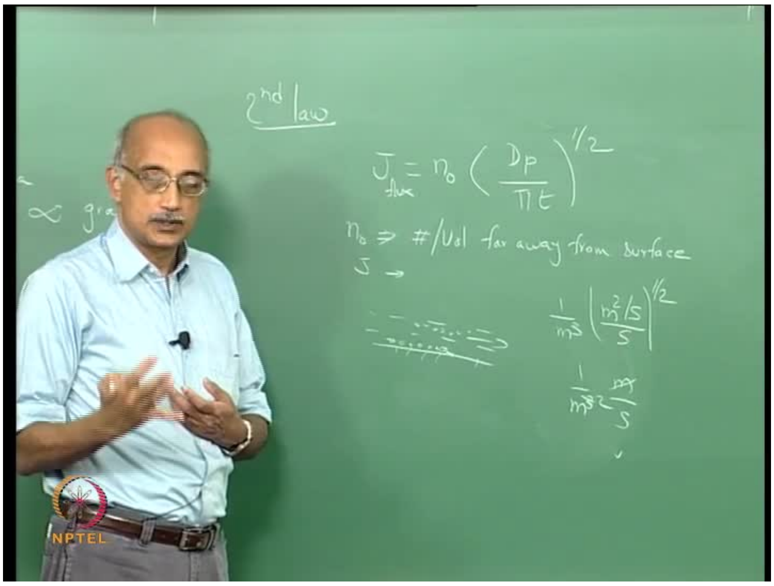

and there is a flux associated with it, which is also occurring in a steady or quasi steady manner. The second law of diffusion accounts for the time dependence, are you familiar with the second law of Fick diffusion? a Fick second law of diffusion dictates, how or the time dependence of molecular diffusion; and again, there is a corresponding second law of diffusion for particle diffusion which says J equals N 0 times d p over pi t to the power half, where N 0 is the number of particles per unit volume far away from a surface.

By the way, you have one other things, you have to consider is, why does flux even happen, I mean why does mass diffuse in a certain direction? It is because you establish a gradient in the direction. But how does the gradient get established? If you have a free flowing fluid, and you are assuming that particle movement is completely random, then in theory they should not be a gradient. Essentially, the concentration should be roughly uniform everywhere, if concentration distribution is accruing only through diffusion processes; so, what leads to a gradient? Well actually, if you, if you are just looking at the flow of a fluid in a free volume, diffusion phenomena are virtually absent due to concentration gradients alone. Diffusion still happens because of other factors such as thermal gradients or other energy gradients, but not to a significant extent, based on the mass gradient. However, when you look at a surface over which a fluid is flowing, under these conditions, mass flux due to a concentration gradient becomes highly likely, what is the reason for that?

The reason is that the fluid molecules that are adjacent to the surface have a tendency to strike the surface and get attached to them. And similarly, particles that are flowing adjacent to the surface have a tendency to touch the surface, develop an adhesive force and there by become attach to the surface. So, what does that create? It creates a concentration gradient, because you have large number of particles in the main stream or far away from the surface; but you have fewer particles in the gas phase or in the fluid phase, adjacent to the surface, and that is what setting up your concentration gradient and initiating your diffusion process.

So, N 0 then represents the number concentration of particles number per unit volume, far from the surface that is in the main stream of fluid flow; and d p again is a Brownian diffusivity; and t is time, what is J?

Look at the units, N 0 is per meter cube, d p is meter square per second, t is second to the power half. So, this is basically 1 over meter cube times meters per second which is again a flux term.

So, this J is the number flux, it is not the mass flux, but the flux in terms of number of particles that are transported towards a surface per unit area of the surface per unit time. So, if we take this J value and you integrate it over time you will get a rate of deposition.

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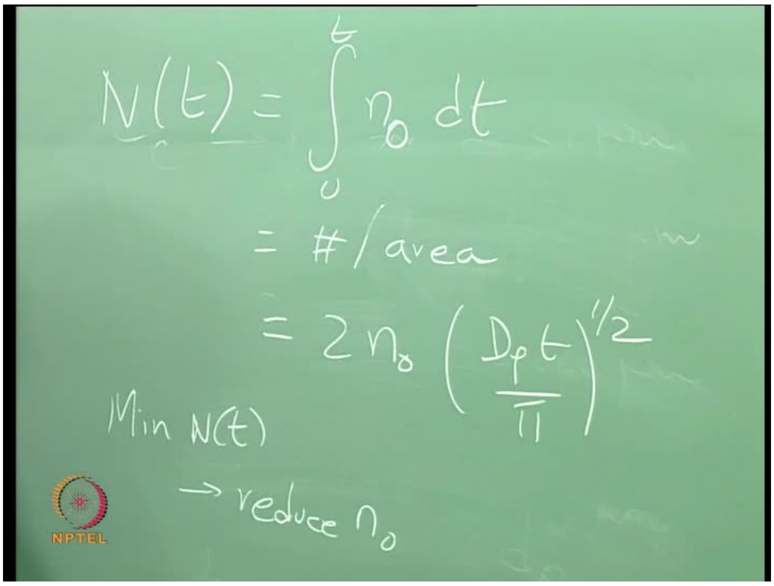

So, let us take n of t is being equal to integral 0 t N 0 D t. so, this is the number per time. Sorry, number per area that has accumulated over a certain period of time from 0 to t. so, what would this be if you go back and do this integration? This is two times N 0 times d p t over pi to the power half. And so, the number concentration on the surface per unit area, essentially scales as these parameters.

Now, what is the implication of this formula? Suppose you have an industrial problem where you have a surface that is exposed to the flow of a fluid and you are trying to minimize deposition of the particles on to the surface, it happens very frequently; for example, this may be the product gases from coal combustion in a power plant and these may be your heat exchanger tubes; you do not want ash particles, for example to deposit on the heat exchanger tubes, and thereby, reduce the efficiency of heat transfer.

So, you want to minimize ash particle deposition by diffusion or this could be something happening in a clean room, where you are trying to take a micro electronic product and you have contaminant particles floating around the clean room air, and again, you want to minimize the deposition of these contaminants on to this surface. How do you do that? This equation actually gives you a lot of clues. If you want to minimize n of t, you can reduce N 0; in other words reduce the inventory of particles to begin with, if you have a clean room, design the clean room; so that you minimize a number of particles that can enter the clean room, and you design your equipment, and processes, and work station in such a way, that generation of particles is also minimized. So, by a combination of these strategies, you can reduce N 0. And that has a direct relationship, because n of t is proportional to N 0. So, any reduction that you make in the number concentration of particles far away from the surface will have a direct influence on reducing the number concentration particles, depositing on the surface.

The second strategy is to reduce t. Now, what do you mean by reducing time? Deposition of particles on a surface, only occurs during the time period, where the surface is open to the environment. So, if I am making an integrated circuit and you want to keep it as clean as possible, then you would only expose it to the clean room environment for as shorter time as possible. And then, as soon as, the necessary processing is done, I will cover it with something, you know putting it in a wafer carrier that has a, you know, good sealing led and so on; you can actually protect the surface from being exposed to the surrounding environment.

So you try to control t, minimize t. Another way to think about this is, if I am running a manufacturing process which is contamination sensitive, I do not want to build up wip, have you heard of wip? Wip stands for work in progress.

You know in manufacturing, there is something called C F M - continuous flow manufacturing - where you essentially use a pull strategy, as it is known. So, basically you do not build up inventory at every stage of your process; the end of the line tells you I am now ready to make a product; so, it pulls components. So, once you receive that signal, then you start sending material through your process; and you do it in a fashion, where it is a continuous flow, which means there is minimum time of waiting for components to be assembled in in the next stage of the process. And what you are minimizing then, it is what is called work in progress or wip. So, you do not have components just sitting around in a room for a long period time, waiting for somebody to use them to build a product; it is also known as just in time manufacturing, the components are delivered to the process, just in time, just when you are ready to use them.

So, this combination of C F M, just in time, wip minimization, all these strategies are good from a manufacturability view point, you know, if you talk to the manufacturing engineer, the first things that will they will preach to you is these aspects of efficient manufacturing, but it turns out that, they also have very positive implications for minimizing the accumulation of contaminants on the surface.

Because they have a direct influence on time t. Of course, the other factor where you have some control is d p. So, how do you minimize flux using D p as a controllable parameter, while you reduce D p, you reduce diffusional flux.

Actually, you basically, will want to, what you want to do is minimize the diffusivity of the particle. How do you do that? By having larger particles in your system, but is that a smart strategy as particles becomes larger? Yes, their diffusion rates will slow down, but all these other transport mechanisms will start kicking in.

So, you may be reducing some of your diffusional fluxes, but the cost of increasing your convective fluxes, your inertial fluxes, and so on. So, you have to think about it in a more systemic fashion. So, actually turns out, there is an optimum size range of particle which will actually minimize the overall deposition; but optimum size range though, varies from application to application, it also depends on product sensitivities, you know, if you have a product that is very very sensitive, let us say to sub-micron particles, then this is a good strategy to minimize diffusion, because even though that might increase the mass flux of larger particles by the other transport mechanisms, they do not affect your product, so you do not care.

So, you really have to understand the sensitivities of the product, you are manufacturing; whether it is a micro electronic product, or it is a pharma product, or whether it is a fertilizer, or whether it is a food product, it does not matter. You have to understand, is your product sensitive to particles and if so, what is the size range of particles that it is most sensitive to; and the way you try to control the transport mechanisms that are present in your system, is by essentially adopting an appropriate strategy; you cannot minimize everything, well you can, but it is going to cost you so much, your product becomes economically enviable.

So, you have to have a practical strategy that says, you do not try to get zero particles in your product, but you are trying to have an optimum control strategy, where you try to identify the most sensitive product particle sizes and you have the maximum containment in those size ranges; and for the less sensitive particles size ranges, you do not worry about it so much.

So, it is a combination of strategies that that you need to evolve by keeping in mind, the a holistic picture of what your product requirements are.

So, we have discussed diffusion. By the way, there are other types of diffusive phenomena, such as thermal diffusion, which we will talk a about later in this course; but the next most predominant mode of transport for fine particles is electrically induced motion.

Now, electro static forces are important because all particles on earth carry a charge. Now, the charges may be large or they may be small, but every particle does carry a charge; and that happens because, essentially in cosmos, you have ions that are present, and essentially when these particles collide with any molecules in air, they pickup charge.

Now, if the charging of the particles is essentially happening by a random mechanism of collision with ions that are present in the atmosphere, then the charge distribution follows a Boltzmann distribution. So, the net charge on an assembly or collection of particles may be 0. However, the individual particles will carry positive and negative charges, depending on what ions or electrons they have collided with.

On the other hand, if you have intentionally charged a set of particles, obviously they will not have a Boltzmann distribution, I mean they would not have an equilibrium distribution that essentially results in a net 0 value; but they will have an overall positive charge or overall negative charge depending on the charging field that you have applied to the particles. Now, under these conditions you setup a force, which we have actually looked at in the context of particle to surface interactions, and that is the columbic force.

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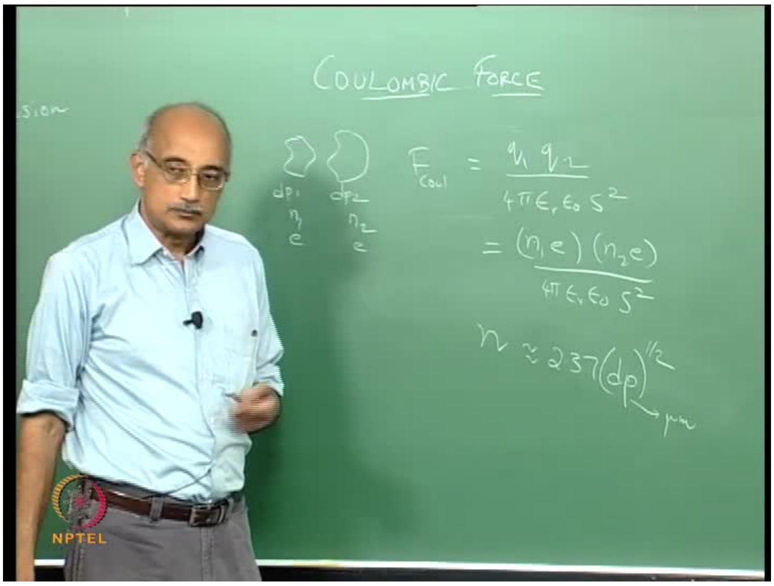

So, if you have two particles, and let us say, they have sizes d p 1, d p 2 and they have, the first particle has n charges, n 1 charges, the second particle has n 2 charges; of course, each charge unit is e, then the columbic force of interaction between them which is equal to q 1 q 2 over 4 pi epsilon r epsilon 0 s squared, can be written as, n 1 e times n 2 e divided by the denominator, s square, of course s is the distance of separation between the particles. Now, if you look at this equation, n 1 and n 2 are the charges carried by each particle. Now, what is the value of an n 1 and n 2 and is that a dependence of charge on size?

It turns out that, there is very sensitive dependence of particle charge on particle size. In fact, n is roughly equal to 2.37 times d p to the power half, where d p is expressed in microns. This is the average number of charges carried by a particle which has been exposed, essentially to a Boltzmann distribution of charges.

And so, when you look at this equation, there is clearly a drop in the charge, as the size decreases. And therefore, if you have two particles n 1, the value of n 1 and n 2 will very much depend on the sizes of the particles. It turns out that, even this expression does not fully capture the dependence of number of charges on particle size.

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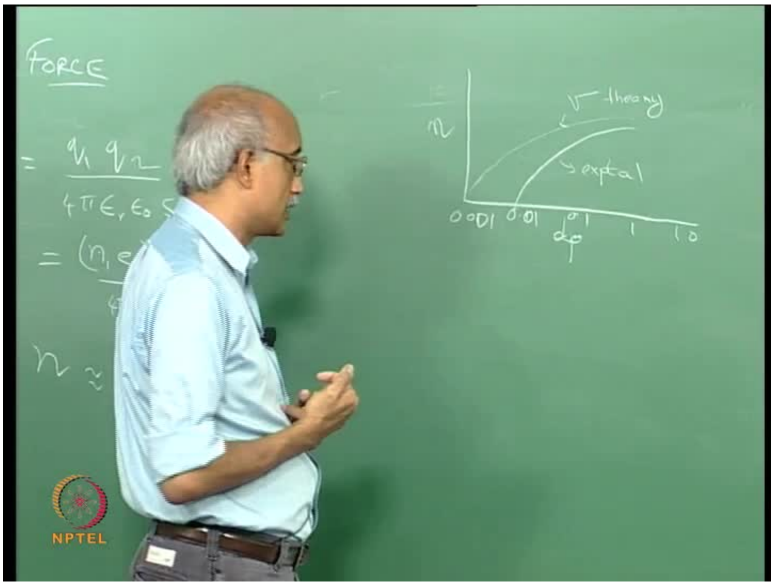

Because essentially, this is, if you plot d p verses n, you know what this is predicting is a square root dependence, which is valid all the way from 0.01 microns to much larger sizes.

Actually this is, according to this curve sensory 0.01 and so on. If you actually measure charges on particles, it turns out that this slope is much steeper, it actually drops to virtually 0 by the time you get to 10 nanometers in size. So, the charging effect in a neutral field is important primarily for particles that are larger than roughly 10 nanometers; if the particles is in the nanometer range 1 to 10 nano meters, you can kind of ignore charging effects, unless there is a charge field that has been induced to to deliberately setup a charge on the particle.

So, this is basically what predicted by the square root theory; and it actually, severely, under estimates charge pickup by, or it severely over estimates charge pickup by particles in the very fine size range. So, the the practical implication of this and what we have discussed about in terms of Fick diffusion or Brownian diffusion is that, as you start approaching nano dimensions, diffusive phenomena become much more important than electro static phenomena.

As you start approaching the 0.01 to 1 micron range, then electro static effects become comparable to diffusive effects. As you exceed sizes of 1 micron, the diffusion drops quite rapidly, electro static effects drop less rapidly. So, electro static phenomena continue to play a role beyond a size where diffusion stops being an important phenomena.

We have looked at essentially, two transport mechanisms that are particularly important for fine particles. the as you transition into larger sizes, as I was mentioning earlier, the macroscopic forces that are transfer to the particle by the surrounding fluid, start to become increasingly important, and drag forces are among the more important forces that a particle wou

d experience in a larger size range.

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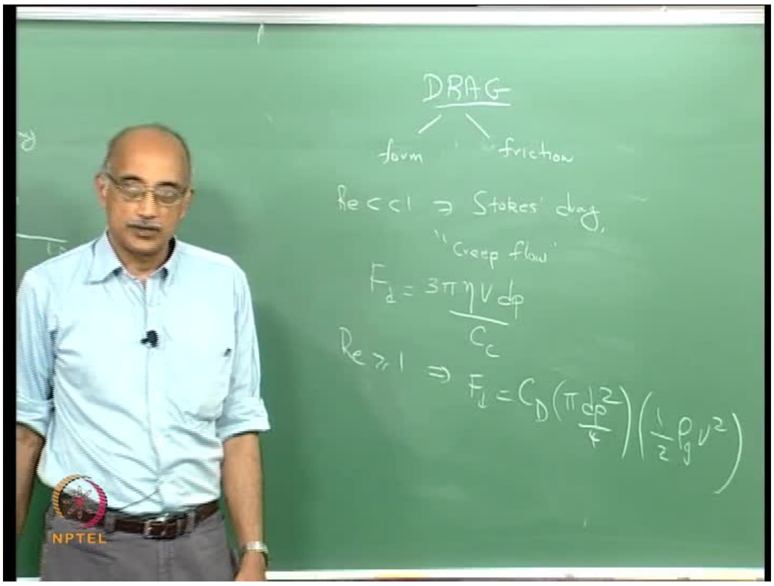

Now, what is drag? Drag is basically a frictional force, that a particle experiences as it is traveling through a fluid; actually, it turns out that, there are two types of drags, that is form drag and friction drag. What is form drag? Form drag happens because of the particle trying to push molecules, out its way.

Friction drag occurs because of the friction between adjacent layers of molecules and the particle as it is travelling. The net drag is actually a combination of both. Now, when you look at a drag force on a particle and let us assume, it is a spherical particle, you know, that there are two formulations depending on the Reynolds number, I am sure you have studied this in fluid mechanics.

So, when the Reynolds number is very small and Reynolds number much smaller than 1, you are in the stokes drag regime.

What type of flow is it when Reynolds number is much smaller than 1 is called? Creep flow, whether it is laminar or turbulent depends on the transition Reynolds number. So, you can have a high Reynolds number and it can still be laminar flow. However, when the Reynolds number is much smaller than 1, this is called creep flow. And the expression for drag in this limit is, F d equals three pi eta v d p by C c, where eta is a viscosity of a fluid, v is a differential velocity between the fluid and the particle, D p is the particle diameter, and C c once again is these stokes cunningham slip correction factor. When r e approaches and exceeds one, the drag force becomes a drag coefficient multiplied by area by pi D p square by 4 times half rho g v square. So, c d is the drag coefficient, pi D p square by 4 is the surface area of the particle, and rho g is the density of the fluid in which the particles are being carried; and this c d parameter has a relationship to the Reynolds number.

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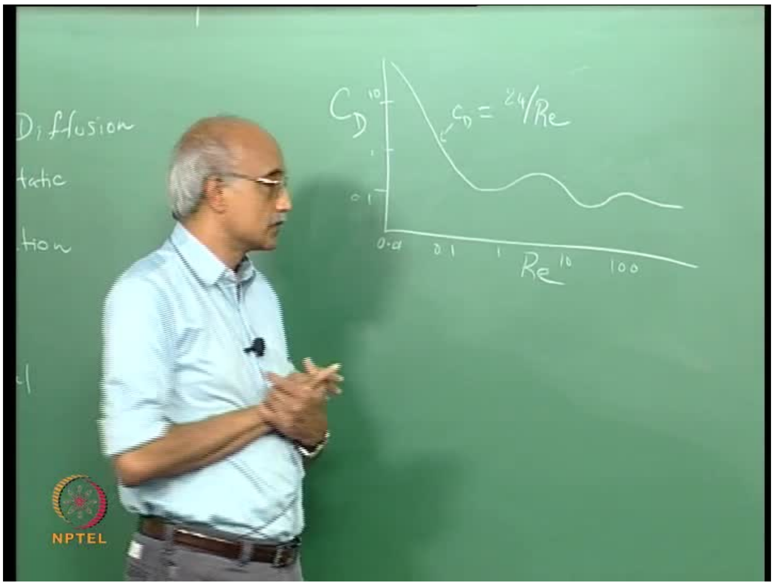

If you plot c d verses r e, what does it going to look like? There is a linear portion. And then more of a cyclical behavior, and what is the relationship here, I mean this is a log log plot. So, this goes from point 0.01, 0.1, 1, 10, 100 and so on. And this value is 0.1, 1, 10, and so on.

So in this limit of Reynolds number being less than 0.1, what is a relationship between drag coefficient and Reynolds number? 24 divided by R e. So, in the low Reynolds number regime, the drag coefficient and the Reynolds number are simply related by c d equals 24 over r e; and for higher Reynolds numbers, the drag coefficient does not drop or increase significantly. What happens in the turbulent regime compared to the laminar regime? Is drag higher or smaller or you cannot say?

Yeah, I mean in in the turbulent regime, all you can say is, fluctuations in the Reynolds number do not have a significant effect on the drag force. Mainly because in the turbulent regime, the flow itself changes its character. You are you are essentially having a situation where parcels of fluid are being flung about in turbulent eddies. So, it very much depend on, it becomes a probabilistic process; if these object is impacted by several eddies in a period of time, it will experience high drag; but there may be periods of time, where again, given the unpredictable nature of fully turbulent flow, the particle may experience low drag; but if you so if you integrate drag force over time, essentially laminar verses turbulent, does not have a significant influence on the drag force that is experienced by the particle.

So, when you look at drag force, the expression for drag that we wrote down, again contains a lot of useful tips. One of the questions that you always face is, do you want to minimize drag? Again it depends on the application. If this particle is actually, let us say an airplane that is flying, then you want to minimize drag.

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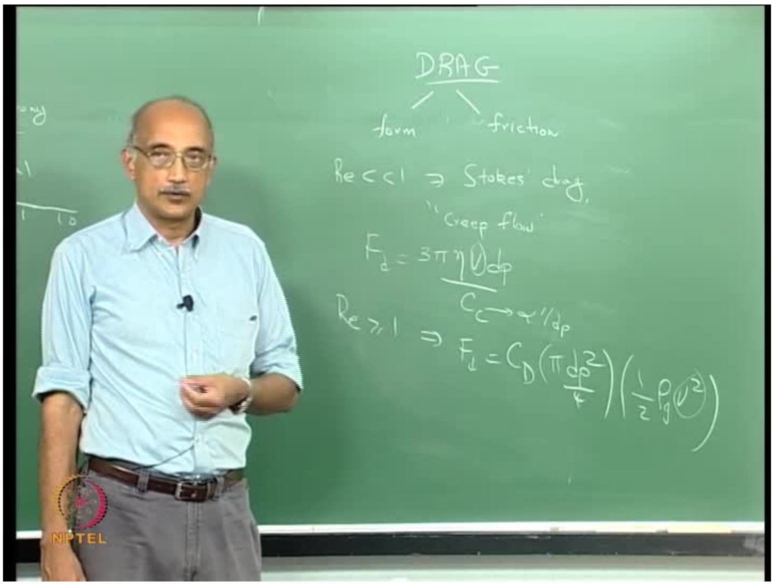

So, what would this tell you? Again if you minimize the viscosity of the fluid, if you minimize a relative velocity, if you minimize the size of the object, whether it is a particle, or a plane that minimizes your drag force; and of course the stokes cunningham correction factor being in the denominator, something had we careful about, you know, it may look apparently like the drag force is proportional to d p; but remember that, as particles becomes smaller, the C c is actually proportional to 1 over d p.

So the actual dependence of drag force on particle size is d p squared for very fine particles, it is not equals to d p. Same as, for the case of a larger Reynolds number drag, where the drag force is proportional to d p squared.

Some of the other dependence that are interesting are, for example, the linear dependence on velocity in the stokes flow regime, or creep flow regime as opposed to a v squared dependence at higher velocities; again this has practical implications for, when you are trying to remove particle from a surface. As we discussed in in the lectures on particle removal from surfaces, one of the methods that is commonly used to remove particles is to blow air or spray liquid on the surface, and thereby induce a drag force.

Now, what these expressions tell you is, for higher raynolds number flows, increase in velocity will have a much more significant effect compared to increasing the velocity of flow in the in the lower Reynolds number regimes.

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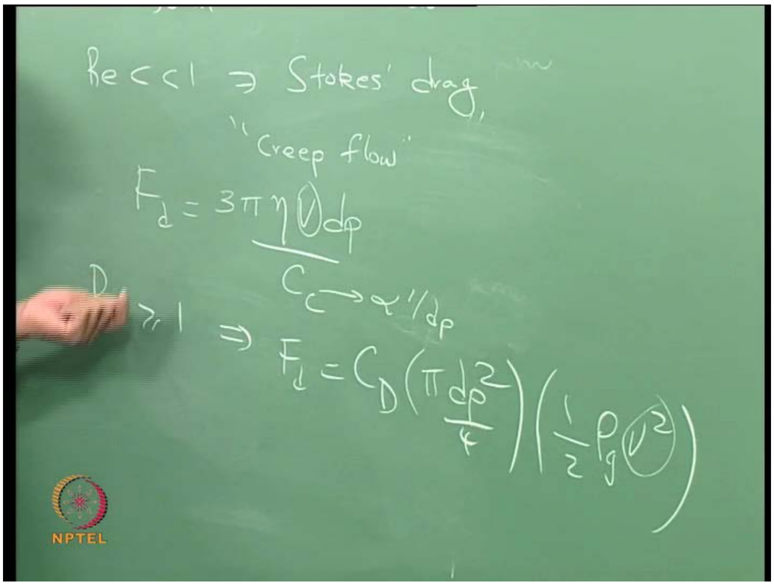

And in general, if you compare drag in this limit to drag in this limit, which do you think is higher? Is the drag force higher in the small Reynolds number limit, or in the higher Reynolds number limit? Yeah, I mean the drag force will increase as you go to higher Reynolds numbers; however, the drag coefficient, as you can see, actually decreases as you as you go to higher Reynolds numbers, until it eventually reaches an asymptotic or a or a Plato value. Now what is this drag force usually balanced by? Some kind of a body force. So, as the fluid is flowing around the object and exerting drag on it, there is, because the particle or the object is the body, there is a body force that is acting on it, which will essentially balance the drag force. So, that is the gravitational force, or the body force in general.

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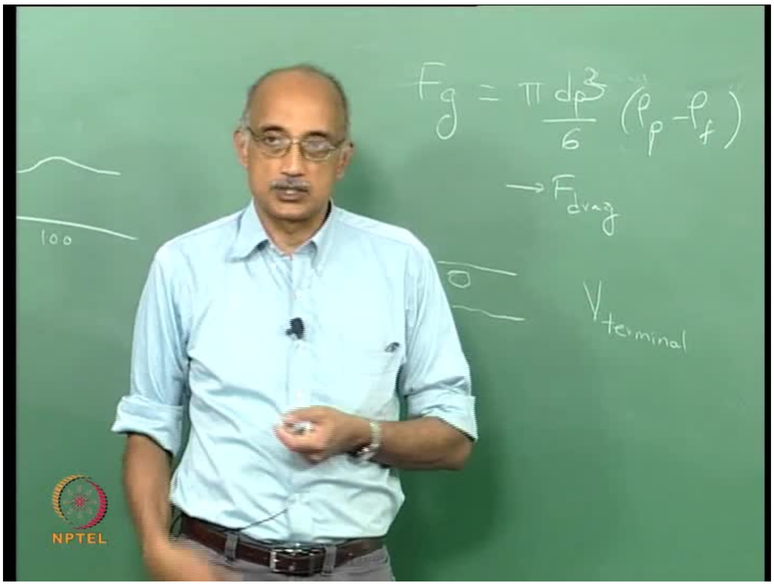

Which is written as pi d p squared or actually pi dp cube by 6 times rho p minus rho f times g. It is a, it is a volumetric force and when this force will, when this force exactly balances drag force for a particle that is entrained in a fluid, what velocity does it reach? Terminal velocity, which is a steady state velocity that represents the balance between body force and the drag force. So, we can actually estimate what this terminal velocity is for any object that is immersed in any fluid, and that is the terminal or settling velocity that the particle will reach, when it is, for example, dropping through a fluid and being subjected to a body force that pulls it down, and a drag force that is trying to resist the motion particle under the body force.

We will stop lecture, at this point. In the next lecture, we will begin by, essentially deriving an expression for the terminal velocity, which will also be used then to establish the inertial force on the on the particle, as the particle size becomes larger. Any questions? I will see you at the next class.