CFD APPLICATIONS IN CHEMICAL PROCESSES

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Week-11

Lecture 51: Turbulence Modeling

Hello everyone welcome back with another lecture on turbulence modeling that we are doing in the course CFD modeling CFD applications in chemical processes which is essential for the CFD modeling of various processes including turbulent systems. So, in the last lecture, we discussed about different strategies, different modeling, why do we require the various models such as Reynolds stress models, two equations models, one equation model, large eddy simulations or the large eddy model and the direct numerical simulation. So, the utility of this, the reason, the background of it we briefly discussed. And then we understood the concept of DNS as well as the alias.

That means the direct numerical simulation and the larger is simulation. So, although these are more favorable methods compared to because this particularly DNS does not involve any approximation. It is the direct solution of the direct modeling of the Nivea-Stokes equations. But, LES also is close to the DNS because 80 percent of the scales are resolved through the direct simulation like the DNS, but it requires a sub grid models that we have understood in the last lecture. But the problem is still the industrial scale problem or the commercial scale problems are hardly solved or can be solved

by the LES modeling because of the enormous computational power that it requires. And also sometimes we may not require that high resolution of the turbulent scales that we need to understand. So, the point is We require some strategies those are in fact widely used in practical applications where the scales are clearly separate. So, here although in LES we filtered out some scales or the in this modeling strategy, but as I mentioned it still requires 80 percent of the scales it actually solves. So the point is this solutions that we will be now talking about that contains or in fact are based on the foundation of the Reynolds decomposition.

we have seen the Reynolds decomposition. In the previous lecture, it was proposed by Reynolds that the instantaneous variables can be split into two parts: one is the mean, and the other is the fluctuating part. This means if I write the velocity at an instant, the instantaneous velocity, so, this proposes that we can write the instantaneous velocity in its mean and fluctuating components.

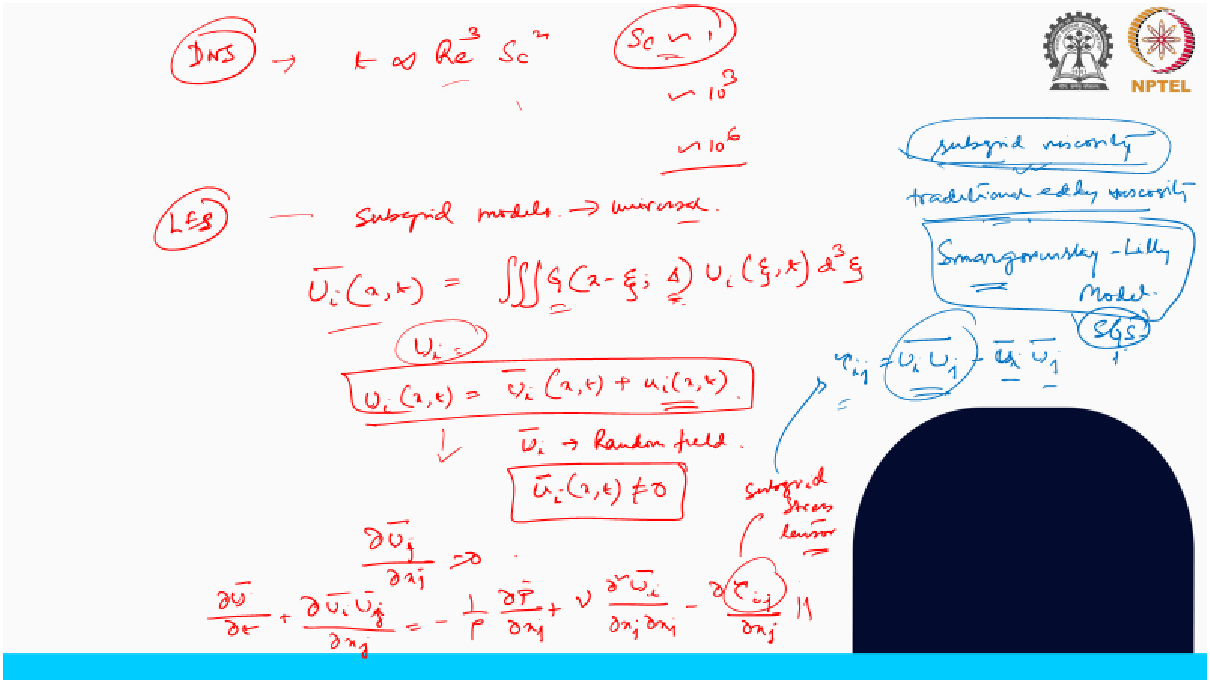
Similarly, the pressure can also be split into the mean and the fluctuating part. Through this method, the turbulence modeling that we will now discuss in this lecture, as well as in the coming lecture, is that the flow—mainly the flow that we describe by the mean flow velocity and turbulent quantities. or the properties of the quantities.

What happens is that we average over a reasonable time period, and the turbulent fluctuations are separated from the non-turbulent quantities. So, we have some average part, which we call the mean of the flow or the parameter or the variable, and the other we separate out as the fluctuating component or the turbulent quantities. The set of equations that we derive from the Navier-Stokes equation is essentially why it is called the RANS model or the Reynolds-averaged Navier-Stokes equations or the Navier-Stokes models.

So, RANS stands for Reynolds Average Navier-Stokes and if it is model or it is a modeling strategies. So, the point is in several places or situations we need to simulate non steady flows. where we require the instantaneous velocity and in those by the RANS model what it does those instantaneous variables are averaged over a period of time. So, this period of time over which it is averaged is large compared to the turbulent time scales or the turbulence time scales. the averaging period, but it is small compared to the mean flow time scale.

So, that means it has actually it contains that means this time derivative that we have from the Reynolds RANS models those of the main flow that actually accounts for the variations of the time scales that are larger than those of the turbulence. So, the turbulent time scales the smallest ones are not captured here it is averaged and the averaging period of time that is done which are greater than the time turbulence time scale. sufficiently smaller than the mean flow time scale that gives us the turbulence influence in the equation, but not at the scales where we have seen earlier at the level of DNS or the LES. So, if we remember in Einstein notation this

continuity equation and we had momentum equation. This is the Navier-Stokes equation and this is the continuity equation. So, these equations of mean variables can be derived by substituting such decomposition in the expression. So, once we do that say we substitute like this we will not go into the detailed derivations though, but I am just showing you the approach.

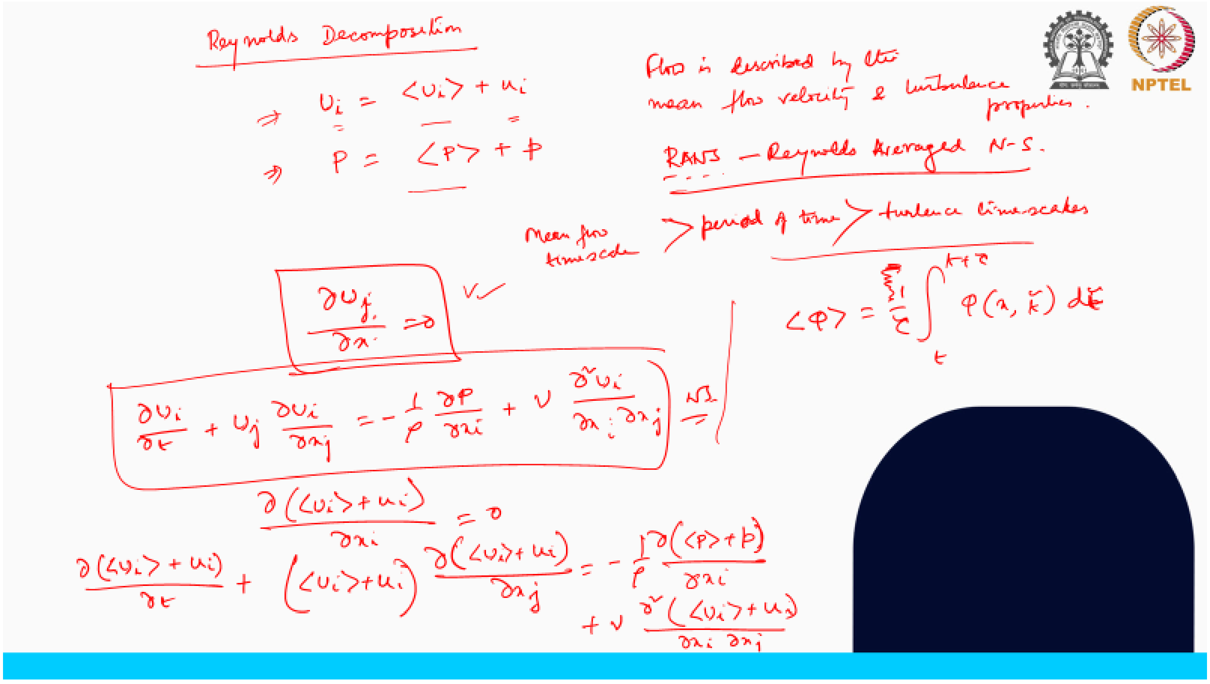


So, this U is replaced here in its decomposed form and this for Navier Stokes it is quite it is just the so this one is x j. So, simply this quantities the decomposed form is written in the Navier-Stokes equation and then what happens we do the averaging the time averaging using say such function that for phi t 2 t plus tau phi x function of x and t bar t hat and this t tilde that we have. So, if this is the time averaging definition that we apply here.

so this is 1 by tau. If we if we apply it this in these equations, then what we get is basically the continuity equation would become this averaged of the whole the continuity equation and the momentum similarly that looks like xj this is which is equals to minus 1 by rho we had this form already we knew this it is just the averaging. Now, the other thing that we have to remember that all terms linear in fluctuating variable will be 0 on averaging which means u i

is equals to Q j these are 0, but when we have this non-linear term. then it is not the case. The averaging of this non-linear term does not result in 0 because what we have here again if we expand this is multiplied by which is essentially equals to u i u j this is the quantity plus u i j plus uj ui plus uij which means if we separate it out this is this results in this first term that comes plus small u small j where these are 0 which essentially means this quantity plus u j. So, this is equals to this value. Okay and accordingly again this continuity and Navier-Stokes are further simplified, but we have to understand that once we simplify this it results into some additional term that is in fact is of quite importance for us. So, when we replace these in this expressions

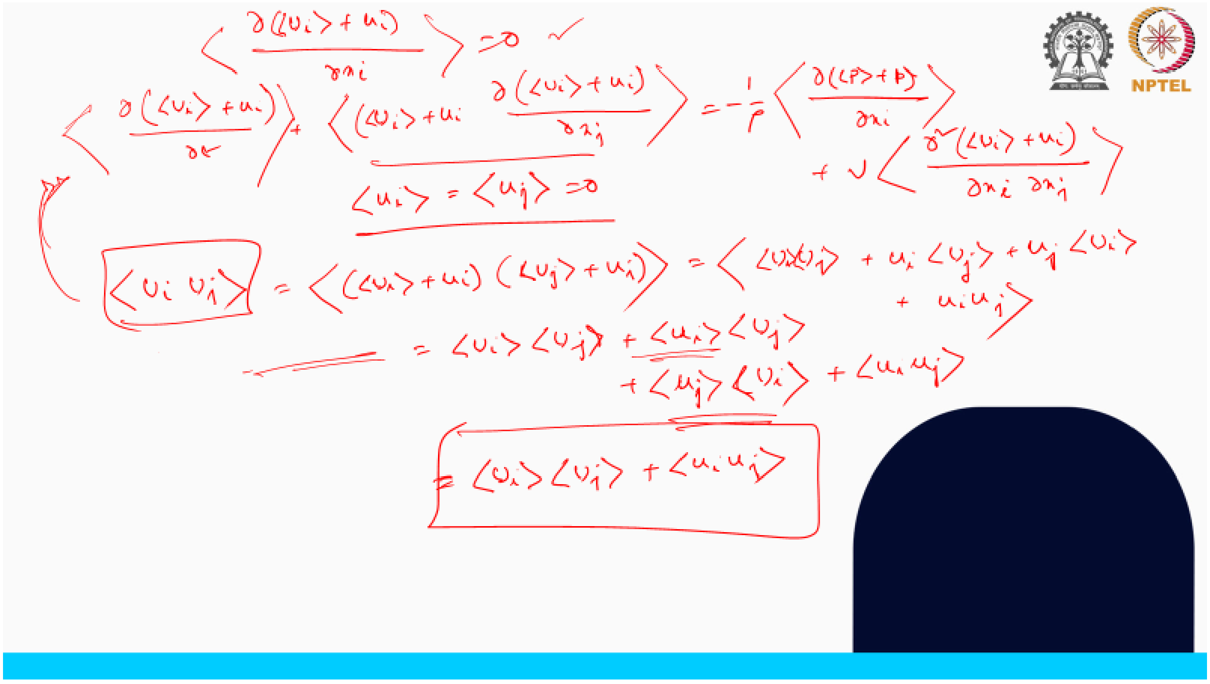
what it happens is that the continuity equation is simply that we have x i is equals to 0, but this Navier Stokes equation after incorporating those parameters and after rearranging it results into this kind of a form. X j is equals to minus 1 by 2 X i nu X j square minus X j. So, after rearranging replacing this and rearranging the Navier Stokes equation what we get is essentially a form like this which looks very close to the Navier Stokes equation. In fact, when we further write this in the form it is further rearrangement we just take this quantity out of the system and then we will write. So, this is new I have taken out rho outside. So, this becomes x j x i minus rho of this quantity u i u j. So, this looks pretty much similar in fact, till this part it is similar. to the conventional Levi Stokes equation that we see that the original Levi Stokes equation, but the additional term is this one. that comes from the replace or the replacement of the decomposed dependent variable in its decomposed form by Reynolds decomposition and integrating it over a period of time.



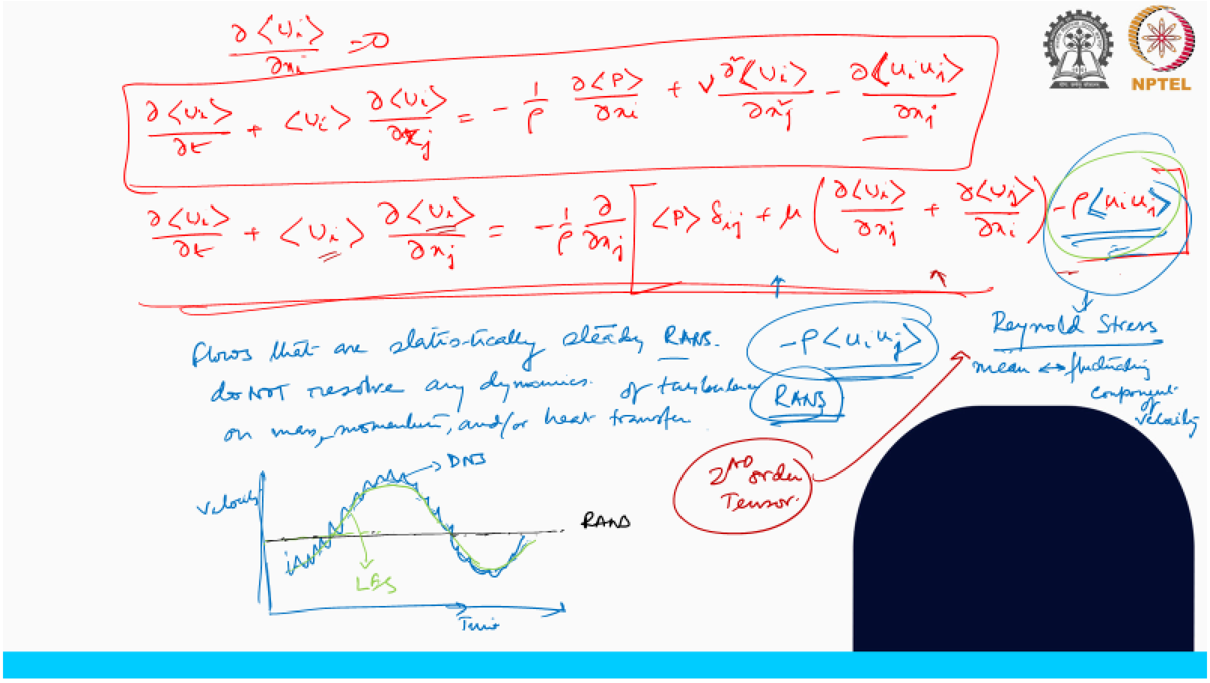
So, we get this term as an additional term to the original Navier-Stokes equation, but definitely with the mean quantities here. but this additional term is actually unknown and this is called Reynolds stress. So, this is called the Reynolds stress terms and this actually acts as a contributing quantity Between the mean and the fluctuating part of the mean and fluctuating part, or the fluctuating component of turbulence or the velocity. So, the mean velocity and the fluctuating component—their interactions

are taken into account by Reynolds stress star. In several places, you would find that essentially, so this is slightly wrong as I have mentioned here. It is eventually this—the averaged part. So, sometimes what you will see is that this quantity is also commonly called Reynolds stress, but the accurate form is this term. This whole term is the Reynolds stress term.

So, the Reynolds stress term contains the product of velocity fluctuations, and this term has to be modeled in order to solve this equation. And that is the whole motto, or the whole crux, of the RANS modeling approach. So, this thing has to be modeled by some means. Somehow, we have to come up with some closure to close this equation so that we can solve it. Now, the point is averaging over all time scales of turbulence—what do we do?



Averaging over the longest time scale of the turbulence. So, if we do the averaging of the whole turbulent time scales, which means we essentially take the averaging of the longest time scales. So, which means that the dynamic behavior, or the dynamics, that will be resolved by the simulation would be of the mean flow. So, by RANS modeling any of the ways that we will see, when we understand the dynamics of the system, we essentially resolve the dynamics of the mean flow. Because this averaging that we do over the turbulent time scales, and if we take all time scales, that means we essentially are averaging by the longest time scale in turbulence.



So, the flows that are statistically steady, ok. These Reynolds-averaged equations do not resolve any dynamics of, say, any effect of turbulence on, say, be that mass, momentum or heat transfer, so and or heat transfer. So, what essentially happens is that, schematically, if you think of that, we have this is the time and this is the velocity.

So if we apply DNS, so DNS essentially captures, say, if this is my profile, this is the DNS simulation that it captures. But LES would eventually filter out these small-scale fluctuations and would capture this dotted line that I am showing. And these small-scale fluctuations that we consider in LES. So, if I, in fact, draw it in a different color. So, LES captures this without the fluctuation part that is captured precisely by the DNS.

This fluctuating part in LES we consider that those are of more universal in nature and the small scales are superimposed or superposed on the large scale. So, these small scales are resolved by the subgrid models, but RANS essentially resolves this kind of mean flow velocity. So, this is the RANS resolution; the fluctuating one is DNS, and this one is LES. So, this is how the resolved time scales in steady turbulent flow would look if we apply RANS, DNS, or LES.

Okay, so this statistical averaging process that we have seen leads to these unknown quantities, and these Reynolds stress terms are essentially second-order tensors. And that represents a second-order moment of velocity components at a single point in space. So, these stresses actually appear as additional or fictitious stress tensors in this equation—the momentum equation we are seeing. This fluctuating part interacts and influences the mean flow by this quantity. So, this appears as additional and fictitious source terms when turbulence is present, which interacts with the mean velocities or the mean flow field.

Just to account for the turbulence, the turbulence intensity, turbulence properties, and so on. So, this means the Reynolds stress essentially looks similar to the viscous stress. ok, but the point is it is not the same, it is not the part of the fluid stress, but it results we have seen that it comes out as from the derivations when we use the decomposed independent decomposed variables in the standard Navier-Stokes equation and this came out as an additional quantity. It represents the average momentum flux due to the velocity fluctuations, but not the fluid stress.

So, we have to remember that Reynolds stresses are not conventional fluid stresses, but they signify the average momentum flux due to the velocity fluctuations. They characterize the transfer of momentum by turbulence. Now, modeling this term. This is the essence of the other processes of the strategies inside RANS. So, in this case, you remember inside RANS we defined several or spoke about different models. So, RANS came out like this, and then in order to close those Reynolds traces.

We have several strategies that we will discuss or briefly have an overview of these systems, particularly the Spalart-Allmaras model that I spoke about, which is frequently used in our next class. So, until then, I would request you to go through this concept once again and see how these stress terms are different and how they appear, although we have roughly gone through the derivations. but it is worth to do it on your own so that you realize that how these stress terms appear for the case of turbulence modeling in the standard Navier-Stokes equations which is modified by the mean velocity field. So, thank you for your attention today.

We will see you in the next class.