CFD APPLICATIONS IN CHEMICAL PROCESSES

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Week-10

Lecture 49: Turbulence Modeling

Hello everyone, welcome back with another lecture on turbulence modeling that we are discussing as a part of safety applications in chemical processes. In the last lecture we introduced Kolmogorov's hypothesis because those are the cornerstone or the fundamental of turbulence modeling. that we should understand because based on that this philosophy several turbulence models have been derived or the genesis of the turbulence models occurs for the CFD studies. Now, the point is I told that in the last class that we have to look at three hypotheses that Kolmogorov proposed and that started with the first one. The first one was that we that has a genesis that Kolmogorov argued that at high sufficiently high Reynolds number there are reasons

to expect that the directional information are essentially lost and in the chaotic scale reduction process. So, the small scale turbulences or the small scale eddies that have small time scales with high frequencies those motions are statistically independent of the large scale turbulence and of the mean flow. So, which means in other words that somewhere in the process whereby turbulence eddies are reduced in size in all directional information are lost. So, the small size turbulences or the eddies that are actually of say the computationally intensive processes because if we have to resolve all the time scales and all the length scales then those small scales with high frequency eddies actually are difficult to resolve. So, it is hypothesized that at sufficiently high Reynolds number the small scales of turbulent flow are statistically isotropic. So, this is the first hypothesis which based on which few models are derived or developed. Kolmogorov also argued that the statistics of this small scale motions are universal that is similar in every high Reynolds number flow. Now this is actually comes from the understanding or comes from the assumption that for the small scales motions the isotropic scales the transfer of energy to successively smaller scales and the energy dissipations are predominant process or the dominant process, which leads to the conclusion that the energy transfer rate and kinematic viscosity these two are the important parameters that determines the statistics of small scale motions. So, the turbulent structures the turbulent structures much smaller than this anisotropic structures which are the large scales are universal and irrespective of the

situational flow that that happens with every high Reynolds number flow it is universal it is. So, the point is the first as a second hypothesis the similarity between the different types of high Reynolds number flows what he proposed is that in every turbulent flow at high Reynolds number the statistics. So, again at high Reynolds number the statistics of small scale motions have a universal form that is independent or can be uniquely we can say

because we have to estimate those or we have to calculate those. So, these small scale motions have a universal form that is uniquely determined by viscosity and dissipation rate which is say epsilon. So this is the first similarity hypothesis or in chronology if you think of this is the second hypothesis of Kolmogorov where it is stated that in every turbulent flow at sufficiently high Reynolds number the statistics of small scales motion have a universal form that is uniquely determined by the viscosity nu and the dissipation rate epsilon.

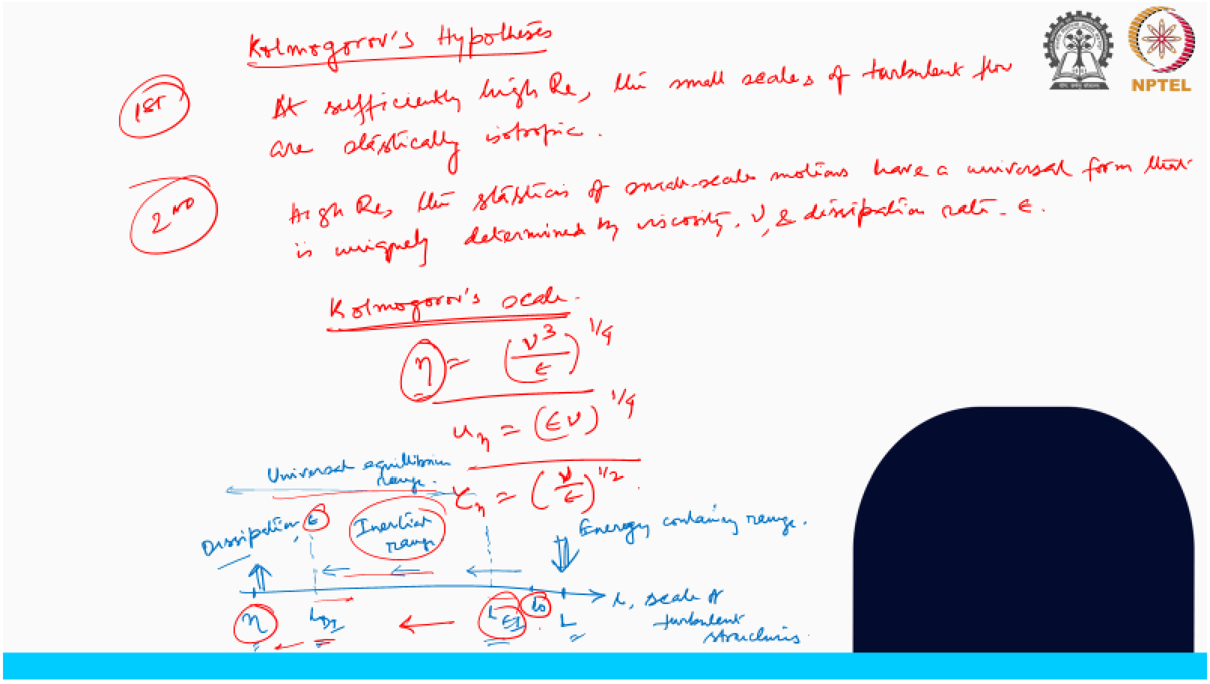
The point is again just to reiterate that the troubles in turbulence modeling are the small-scale eddies. So Kolmogorov tried to find out how to resolve this small scales eddies in a less complicated manner. So, if we have to define the time scale, velocity scale and the length scale in the dissipative region that is where the small scales motions through the viscous dissipation increases the temperature or through the heat.

it increases the fluid temperature in turbulence. So, these scales, if we define them. So, these scales are then called the Kolmogorov scales, which are, say, if I define them. to the power of 1/4. So, the scale for dissipation of turbulent kinetic energy is this.

The velocity scale is defined by this parameter, and the Kolmogorov time scale is mu by epsilon to the power of half. So, the Kolmogorov scale, this one eta, characterizes the size of the smallest scale or the smallest turbulent eddies that can be modeled. So, this is the scale for dissipation of kinetic energy. We also define the velocity scale and the time scale in the Kolmogorov range. So, by definition, the Kolmogorov's length and velocity scales give rise to the Reynolds number equal to 1.

And therefore, the smallest motion in turbulence is indeed laminar, which we can determine simply through the viscous forces. Now, there exist—so, we can understand—there exist different length scales. So, this cascade of energy transfers from the mean flow. The energy that is extracted from the mean flow, which we discussed in the energy cascade mechanism. The energy that is supplied by the mean flow to the large eddies that subsequently is transferred to the smaller eddies and the smallest eddies

then dissipates into heat in the fluid and also near the wall damped by the increase in the surface temperature of the contained wall. Now, so, if we think of this scale, say, the length scale that is the scale then say this is the length scale of the domain. So, we and this is the eta which is the Kolmogorov length scale, the smallest length scale of the turbulent eddies and this is the largest one that can happen because this is the domain size or the characteristic length of the domain where the turbulence is occurring or happening.



Now there exist say several length scales. So, here at this large at the larger eddies what happens? This is the energy containing range. So, this range we can say energy containing range. where the production of the turbulent energy is happening and it is supplied to the larger radius.

Then this energy transfer happens and here the dissipation of energy that is happening. this range this L D I and this L E I this range and. So, from L E I to this whole range of the Kalamogorov is called the universal equilibrium range. this is the dissipation range that is demarcated by LDI and in between this. So, this LDI essentially demarcates this device section which is demarcated by dissipation range of this out of the universal equilibrium range and the other one is the inertial range.

So, universal equilibrium range is divided into two parts which is dissipation range and the inertial range by this length scale d i and l e i to eta this whole range we are calling this as the universal equilibrium range. Now the utility of this range understanding is because that within the range this which is smaller than the L E I the time scales are small compared to the time scale of the larger which means the smaller eddies can quickly adapt to maintain the dynamics equilibrium of the energy transfer rate that is imposed or that is coming from the larger eddies. Now, the statistics of this since the smaller-scale motions are universal in the range from here.

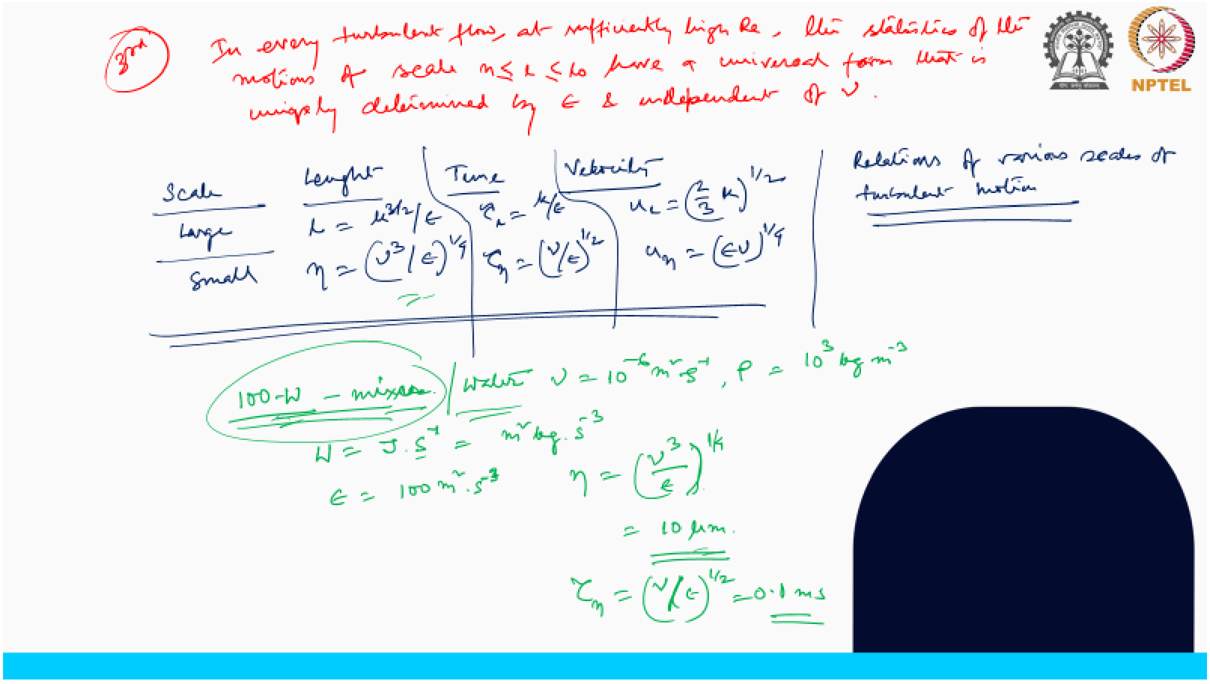
Towards the Kolmogorov length scale, and that is why it is often designated or called the universal equilibrium range. So, Kolmogorov also stated a second similarity hypothesis that takes into account a special range of structures within which the universal range of viscosity plays a negligible role in the motion. So, the viscosity inside this universal equilibrium range within a certain range that plays a negligible role which means that the only the energy dissipation that is the epsilon determines the statistics and the motion of the motion.

Though smaller it is in that range, and that is called the inertial range. So, Kolmogorov's second similarity hypothesis, or I would label that as the third one, is that in every turbulent flow again at sufficiently high Reynolds number, the statistics of the motions of scale that we have seen here within the range of this L0 and eta. Ok. Within the range of L0 and eta, a special range much smaller than this L0 and much larger than the eta.

So, this is the inertial range. So, this is much larger than the eta, but much smaller than L0, a special range that has a universal form that is uniquely determined by epsilon and is independent of nu. So, this sub-range between eta and L0 is called the inertial range, and beyond this range, we call that the dissipative range. So, based on these two hypothesis these two similarity hypothesis the motion of the in this case if you look at it this similarity analysis these two the second and

the third one is says that the motion in the inertial sub range are can be solely determined by the inertial effects wherever the motion in the dissipation range Can be addressed solely by the viscous effects. So, we have understood this energy cascade effect ok, how this energy getting transferred and all and then between these different scales of length and the velocity there exists certain correlations ok. So, if Those are to be mentioned.

So, if I say we have two different scales, one is the large one, the other one is the small. So, large scale and the small scale where I have to look at the length scale, I have the time scale and the velocity scale. So, to summarize, this L length scale is essentially k by k to the power 3/2 divided by the epsilon, and the smallest scale eta is nu cube by epsilon to the power 1/4. For the large scale, the time scale is k by epsilon, but in this case, the Kolmogorov length scale, the smallest scale, is nu by epsilon to the power half, and the velocity scale U L. So, these are all corresponding to the subscript L, that is the large scale, 2/3 k to the power half wherever theta this U eta is nu to the power 1/4.



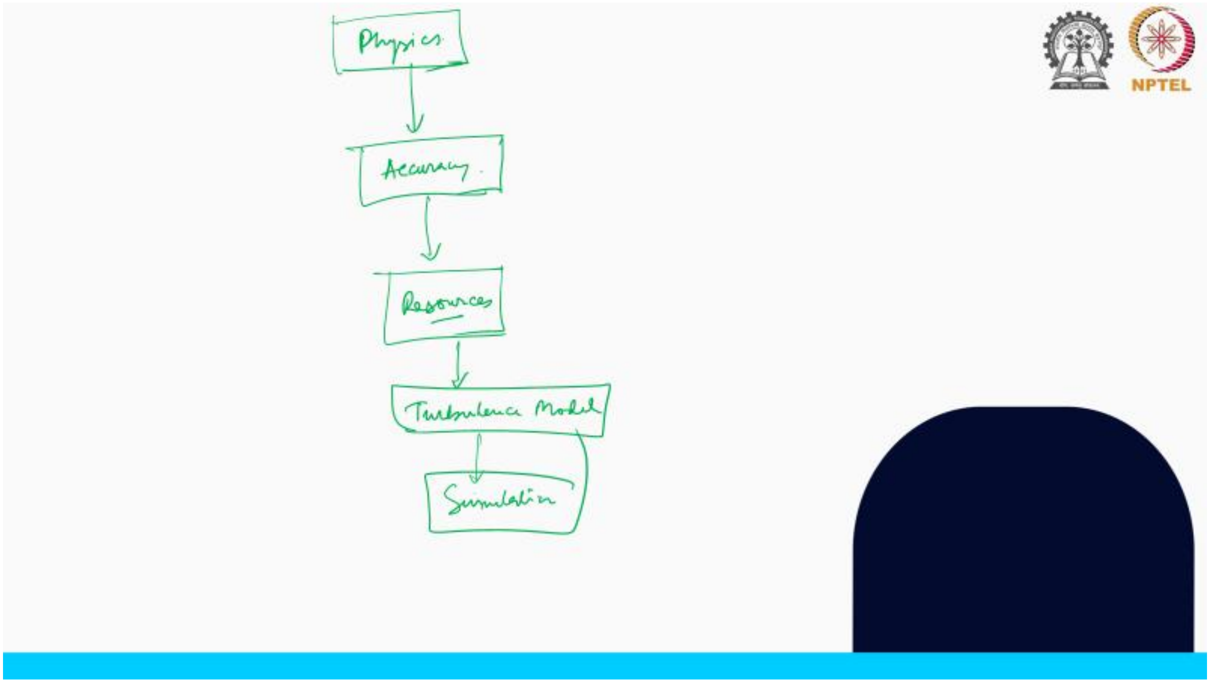
So, this is the summary of the relations. So, these are the relations of various scales. when we talk about Kolmogorov hypothesis the smallest scale and compared to what happens or what are the time scales, length scale and the velocity scales for the large scale. Now, the reason for this to understand these values is that this based on this time scale and length scale velocity scale we have to realize that how what is the value of say the smallest scale

that you need to resolve if you think of resolving the complete turbulence of a given problem. Say for example, here from this table or from this information the smallest turbulent scale or the Kolmogorov's scales that you have to resolve for say with an example that if you have a problem that happens with 10 watt of a mixer. there is a mixer the power is 100 watt. So, it generates vortices the eddies and the smallest scale that you need to resolve in such scenario how do you estimate that because based on that you have to

create your domain grid, you have to take into account of the different modeling strategies etcetera that we will discuss later. But, in order to estimate or have an understanding of what is the smallest turbulent length scale for which you have to prepare your grid or the model is that that comes from this calculations. Now, what happens is that in this case if it is filled with water that has viscosity which is nu is 10 to the power minus 6 meter square per second and rho is 10 to the power 3 kg per meter cube. then if all energy in ideal scenario which is not the practical case, but consider that in an ideal case this whatever energy is

produced by the 100 watt is actually dissipates homogeneously in the fluid in the water here with the example. Then the watt is essentially joule per second which is essentially again can be converted to meter square kg per second cube because we have a per second here. Though the energy dissipation rate per unit mass which is epsilon is essentially equals to 10 meter square per second cube. because of this wattage that we have here.

So, the smallest turbulent length scale that this mixer create is essentially to the power 1 4 from this expression that we see here. If you put these values it comes out to be in the range of or the value of 10 micron. and the characteristic time scale that you have to progress your simulation or the iterations accordingly that can be estimated by this value which is 0.1 millisecond. So, this gives an estimate that you have the smallest eddy of 10 micron in size for this particular problem with water that you can resolve or you have to resolve if you think of resolving the complete turbulence.



So, that is the utility of this and then we comes to we come to the turbulence modeling process based on this understanding. So, turbulence modeling ranges from, say, extremely large-scale—that is, the exact wind or wind pressure and all these calculations for an aircraft. To the heat exchangers—say, gas turbine engines—these are the extremely large-scale operations that you are considering. Now, and also very high-energy-intensive processes or intensive situations that we have to resolve turbulence. Now, if we think of doing this with high fidelity—that means resolving each and every scale of turbulence that is created—then it is nearly impossible even with the current computational resources.

What we have now understood is that it is not that at a particular point a certain length scale or turbulence can occur, but it is in the range or entire domain for that particular range the turbulence would propagate. So, the choice of turbulence before doing this modeling—we have to understand what is my goal, what is the objective. So, the characteristics of the flow feature in that particular problem—we have to understand, and that we say the physics of the problem—we have to understand what to resolve or how much simulation accuracy we require from there. That means there are different flow features available there—which one you need to capture or what is the overall objective. And that actually dictates the choice of the turbulence model.

So, the flow features that you have to capture versus the turbulence complexity is one of the parameters that dictates the accuracy of the solution. And then also you have to consider your resources that you have—that means available computational power. These two combined dictate the choice of the turbulence model. The choice of the turbulence model and also the near-wall treatment. because the near wall treatment in turbulence is essential because although the turbulence the eddies and all these things that we have discussed are extremely dominant wherever it is creating or it is occurring.

But eventually, it dissipates near the wall. So, near the wall, there is a subtle amount of different considerations that you have to put in because the eddies are getting dissipated there. So, there we have the smallest scales of turbulence, which in turn are going towards the laminar in nature. So, there is an extremely thin layer which has to be taken care of if we require that kind of accurate turbulence modeling, and then eventually we do the simulations.

So, the point is, this turbulent flow is actually characterized by the velocity fluctuating velocity field. In which there coexist, in fact, the small scale as well as the large scale. And these turbulent fluctuations are mainly characterized by the small fluctuations and their high frequency in nature. So, we require an extremely high amount of data if we try to resolve even that smaller scale, the smallest scales that we have understood. Say, for example, if we take the gas turbine or an aircraft design situation, if we try

to do or if we try to simulate even at the Kolmogorov length scales, the amount of data or the amount of simulations or the computational power required is extremely enormous. So, we require something less than this complete time history because turbulence is not local in nature; it has a time history, okay. So, we have to have something that is less than—we require less than the complete time history information of this flow that covers all the spatial coordinates but essentially also gives us an average value. So, while doing so while manipulating such equations then we can understand there will be some closer models that we require

in order to address those small scale high frequency fluctuations because the tendency would be to filter out those. Although their fluctuating influence would be encountered or would be taken care of in the turbulence modeling. So, on this note and with this foundation, I will stop here today because this builds the background for the various turbulence modeling that we will now show in the coming lectures. So, with this, I thank you for your attention, and we will see you in the next lecture soon. Thank you.