CFD APPLICATIONS IN CHEMICAL PROCESSES

Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology Kharagpur

Week-10

Lecture 48: Turbulence Modeling

Hello everyone welcome back with another lecture on turbulence modeling in the course safety applications in chemical processes in the last lecture we discussed about the characteristics of turbulence how do we understand that flow is a turbulent one and there I spoke about the flow instability at higher Reynolds numbers. There, we briefly touched upon the aspect that there is a time-scale difference between viscous damping and convective transport. So, let us take a closer look at it so that we can understand why there is a certain number—or a critical Reynolds number—that exists in certain scenarios.

And it is not just about memorizing those numbers. The point is, the Reynolds number we define is essentially the ratio of inertial force to viscous force. From that force balance, what we usually write is the nu. Here, it represents the relative increase in inertial force in relation to viscous force. Now, this transition from laminar to turbulent

is due to disturbances or eddies, and it is this energy cascade—or its amplification—that exists in the fluid. So, the damping of the small velocity fluctuations in the turbulent flow actually is taken care or it dictates the stability of the flow and the viscosity it actually takes care of that small scale turbulence even in laminar flow and the flow remains stable and laminar. But if those fluctuations are further amplified and the flow becomes turbulent or unstable, it leads to turbulence through the transition. This is why, for pipe flow, Reynolds numbers beyond 2200 or 2100 up to 4000 mark the transition region.

These turbulent eddies are usually created in the near-wall region, where there is insufficient damping by viscosity. So, when flow passes through a pipe near the wall, these fluctuations are typically generated near a solid surface and then propagate. if it is not appropriately damped by the viscosity that propagates and results in flow instability. So, say the necessary time scale that we are say necessary time for damping this if we understand that if I have say U is the eddy viscosity if we consider U is the eddy viscosity. eddy velocity and say it is flowing through a pipe diameter which is L. Then say the maximum distance once a small eddy is formed near the wall.

So, any wall that can be upper or the lower wall the maximum distance it can travel is the length L. which is the domain length which are the characteristics length of this domain that we understand. So, the necessary time for damping this A D that is if I say T c is essentially L by U. So, the it can be damped until it goes till that other end of this pipe it can travel that distance. Now during this time the viscosity acts on a distance that is L is essentially to the power half.

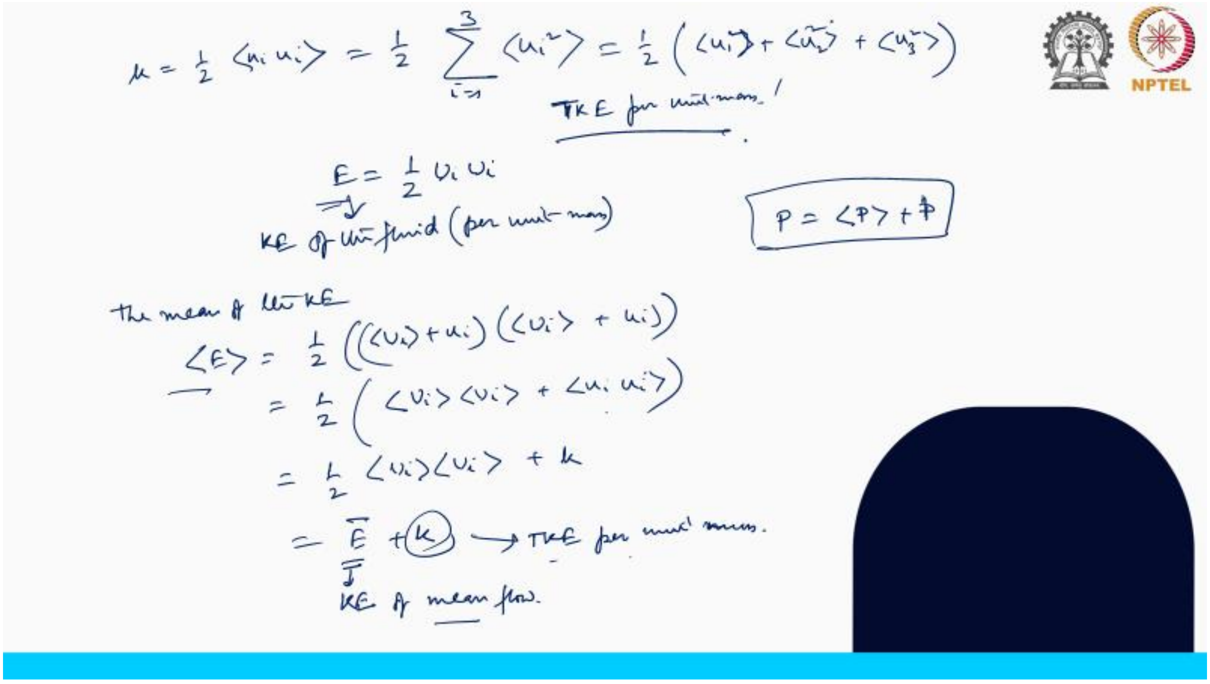
So, the viscosity within this T c eventually acts for a length of small l that is related with the T c by this relations that here we have the kinematic viscosity this time and the length till which it would act the viscosity. So, viscosity will be able to damp that velocity fluctuations if it is smaller than this value, which means L is essentially has to be this or if I write here this is the relation that it requires to damp the viscosity to damp that fluctuations. Now, from this relation what we can see here and this relation that we have if we replace it now here the value of T c. What we find is that

U L by mu has to be much greater than 1. So the velocity fluctuations that we have which is represented by ADs or the small u that is usually much larger than this average velocity that we have. So, the point is once we further replace this here what we see which is Reynolds number is essentially this by nu and we see this has to be much greater than 1. if this is the case then this viscosity would not be able to damp the velocity fluctuations and the flow would become turbulent. So, for high Reynolds number, that is why we have flow instability, which results in turbulent flow.

Now, this is just greater than 1 value. This is the theoretical development, and then Osborne Reynolds actually did the experiment and found that through this pipe, this number. So, this is an indication that beyond which the flow would be turbulent, and then after controlled experiments in the laboratory, he found that. This number is indeed 2100 for the pipe flow, after which these instabilities occur or are dominant in the flow, leading to turbulence. So, the point is now based on controlled experiment on several scenarios like the pipe that can have on external flow that is flow around sphere, flow on a flat surface, boundary layers, etc., which can also be found, and that is why we see different Reynolds numbers, that is, critical Reynolds numbers, that exist for different scenarios.

For external flow, that is, around a sphere, if the Reynolds number is usually more than 350, then there is a transition from laminar to turbulent. So, this is for the flow past, flow around a sphere. For flow along surfaces, the Reynolds number has to be greater than 5 × 10^5. So, different based on different characteristic length scales. Based on different characteristic length scales, different critical Reynolds numbers are found, and this situation would be further complicated if we have surface roughness.

So, based on this understanding, we had that comment on why at higher Reynolds numbers we have flow instability. and the features that we discussed earlier is the irregularity, the energy cascade, the continuum assumption that this flow of this flow instability at higher Reynolds number, the 3D nature of the flow. So, all these we have now understood happen in turbulent flow. Now, at the same time, when we try to model this, We have also seen the instantaneous velocities of different fluctuations, and in which if I try to find out its magnitude.



So, say this is my average component, which is u, that is there. Average velocity and the turbulent velocity is u, in which this is the fluctuating component—the small u, the u, the eddies, the larger eddies, or the smaller eddies. So, we have seen the larger eddies pass with the larger amplitude but with the smaller frequency, and the smaller eddies pass through smaller amplitude but with high frequency. So, Reynolds also introduced several years ago that in fact centuries ago that we can do statistical averaging considering an average quantity and its fluctuating component in turbulence.

So, this is what we call the mean value. It is averaged in space and time, and we have its corresponding fluctuating component, and then the way we write it is that. So, if I try to write it, it is something—its fluctuating component—that we can write like this, which we call the Reynolds decomposition. So, and by this method we can so, if I specifically further write that if I have this instantaneous velocity which means what I would write here the average quantity plus the fluctuating component at that instance.

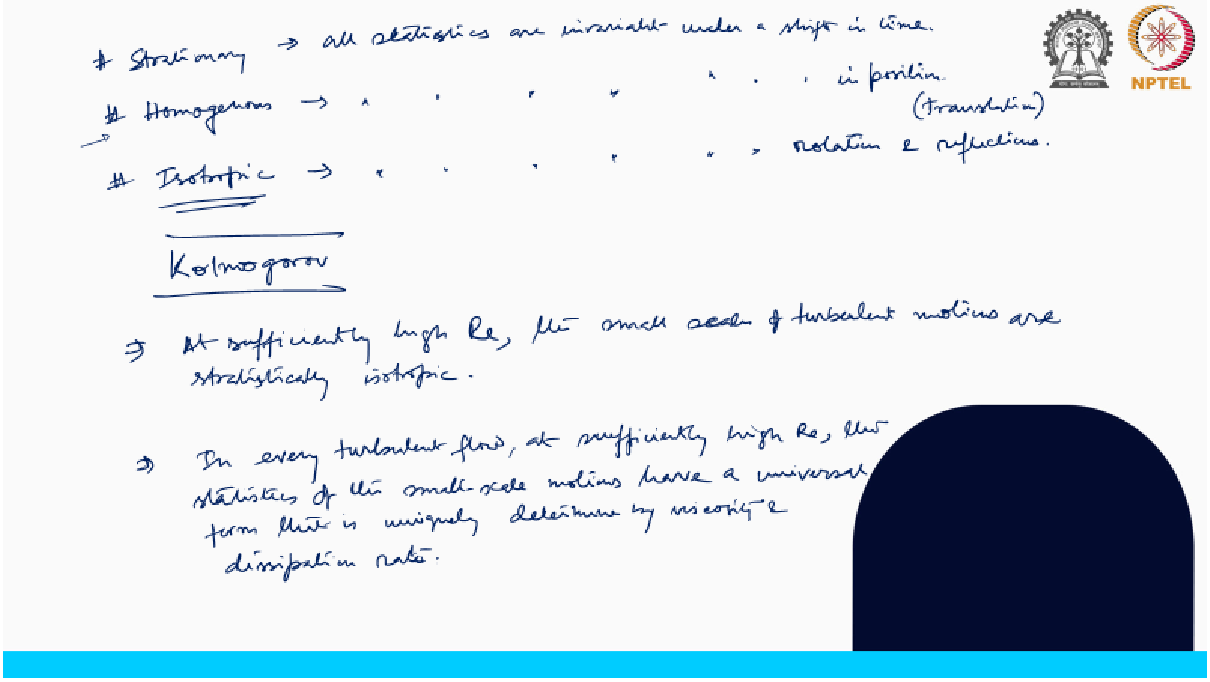
So, the average represents the mean velocity, whereas the fluctuating part is usually interpreted as representing the turbulence. So, this part represents the turbulence, and this is the mean velocity or the mean component of the flow. So, such decomposition—and this is the eventual turbulent velocity of the flow. So, with this Reynolds decomposition, Our situations become easy or easier than if it had to be represented by the full velocity of all scales.

Whereas, this average quantity and this average value is essentially minus t to t u i dt. This is the definition of this averaging quantity. Now, the time scale used in this filtering operation is chosen so that the instantaneous variables are averaged over a period of time that is large compared with the turbulent time scales, but small compared to the time scale of the mean components. So, turbulent flows can be considered as consisting of randomly varying components superimposed on an average or mean quantity.

So, similarly, the intensities and all that now can be measured with the help of this Reynolds decomposition is that, for example, the k value, the turbulent kinetic energy, is in fact represented by So, in three different directions, this can be measured as the turbulent kinetic energy per unit mass. So, turbulent kinetic energy per unit mass. So, now the kinetic energy per mass at a specific point in time of this mean flow is essentially half of this.

Now, this mean of the kinetic energy. So, this is the kinetic energy of the fluid, say, per unit mass at a specific point. Now, the mean of the kinetic energy can be decomposed into two and that is the way that we have plus u i, this is multiplied by which is which means that plus K from this definition, and this we can write as this, where this quantity is the kinetic energy of the mean flow.

and K is the kinetic energy per unit mass, turbulent kinetic energy per unit mass. So, the kinetic energy of the mean flow plus turbulent kinetic energy per unit mass is essentially the mean of the kinetic energy. Similarly, the pressure can also be decomposed and is then replaced in the equation. So, it is some small p, which is the fluctuating component, and this is then replaced in the Navier-Stokes equation and solved once the fluctuating pressure is modeled. Now, for a compressible fluid, the density also similarly has to be decomposed, okay?



So, the point is with the Reynolds decomposition. We eventually decompose the turbulent fluctuating part and the mean component, which are then replaced in the Navier-Stokes equation while solving the corresponding turbulent flow or the turbulent motion of the fluid. Now, while formulating this statistical average or modeling the turbulent 3D flow that is inherent in nature, we perform several statistical averages or find statistical symmetries. Now, this steady or stationary turbulent flow is defined as one in which the spatial properties do not change over time.

So, stationary turbulence is what we call it when all statistics are invariant under a shift in time. Homogeneous turbulence means if all statistics are invariant under a shift in position. Or, say, translation. And then, one mostly used

turbulent modeling we call isotropic turbulence, which means if all statistics are invariant under rotation and reflection. So, which means the assumption of isotropic turbulence is a stricter condition than homogeneous turbulence. So, isotropic is therefore, considered a stricter this stricter condition of the homogeneous that simplifies several things that all statistics are invariant under translation that is rotation as well as the reflection. Now, homogeneous turbulence is rarely encountered in practice, but in laboratory experiments and controlled environments, one can find something close to homogeneity.

So, these are essentially the assumptions that people or researchers make in order to simplify turbulence modeling. So, the point is we have several such examples or several such scenarios where we can apply one of such assumptions that essentially gives us freedom to simplify the turbulence although it is not naturally occurring or actually the real cases. So, stationary so, turbulence would be called stationary or if we consider stationary turbulence which means all statistics are invariant under shift in time homogeneous means if all statistics are invariant under shift in position and isotropic if all statistics are invariant under rotation and reflection.

So, now, regarding this length scale modeling, what we have to understand are the different length scales that we will try to resolve. In order to do that, Kolmogorov's hypotheses are quite important to understand. Because those hypotheses proposed three ideas that are still very prevalent in turbulence modeling today, addressing several key aspects when modeling turbulence. So, the first hypothesis was that at sufficiently high Reynolds numbers, the small-scale turbulent motions are statistically isotropic. So, what it says is that at sufficiently high Reynolds numbers,

the small scales of turbulence or the turbulent motions are statistically isotropic. So, with this, we can now understand that there is a certain level of simplification possible in the generic category of fully turbulent flows. Because at sufficiently high Reynolds numbers, it is assumed that all small-scale turbulent motions are statistically isotropic. So, that gives us leverage

to filter out the effect or to average the effect of those small-scale phenomena. The second hypothesis was in every turbulent flow at sufficiently high Reynolds number, the statistics of the small-scale motions have a universal form that is uniquely determined by viscosity and dissipation rate. So, in every turbulent flow at sufficiently high Reynolds number, the statistics of the small-scale motions have a universal form, which is uniquely determined by viscosity and dissipation rate.

Which means that this So, the issue of considering completely 3D turbulent flows or the simulations is why people go to 2D turbulence. is the fact that they filter out the influences of the 3D fluctuations, specifically at the smaller scales. Now, those are addressed when this hypothesis is that say in this second one it says in every turbulent flow with high Reynolds number the statistics of the small scales motions have a universal form. that can be determined by viscosity and dissipation rate.

So, we need not require any other information to understand the effect of those small-scale fluctuations in a fully turbulent region with a sufficiently high Reynolds number. So, this determines the turbulent viscosity, the eddy viscosity, and the dissipation rate. If we provide these two pieces of information in our modeling, then the influences of those small-scale eddies are usually accounted for. So, essentially this actually helps us in simplifying the turbulence modeling from a stochastic approach to a more averaged with the help of the deterministic modeling that is done by Navier-Stokes equation.

So, we will continue this discussion. We will continue with the third hypothesis and its interpretations in the next class. Meanwhile, I hope you have understood the background, the physics of turbulence, and the preamble. Before we go into the modeling strategy, why the statistical averaging quantities are necessary and how those come, and the critical Kolmogorov hypothesis. These things we will further continue, but I hope up to this point you have been able to grasp the physics of turbulence. So, thank you for your attention.

We will see you in the next class.