CFD APPLICATIONS IN CHEMICAL PROCESSES
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Week-06
Lecture 27: Pressure-velocity coupling

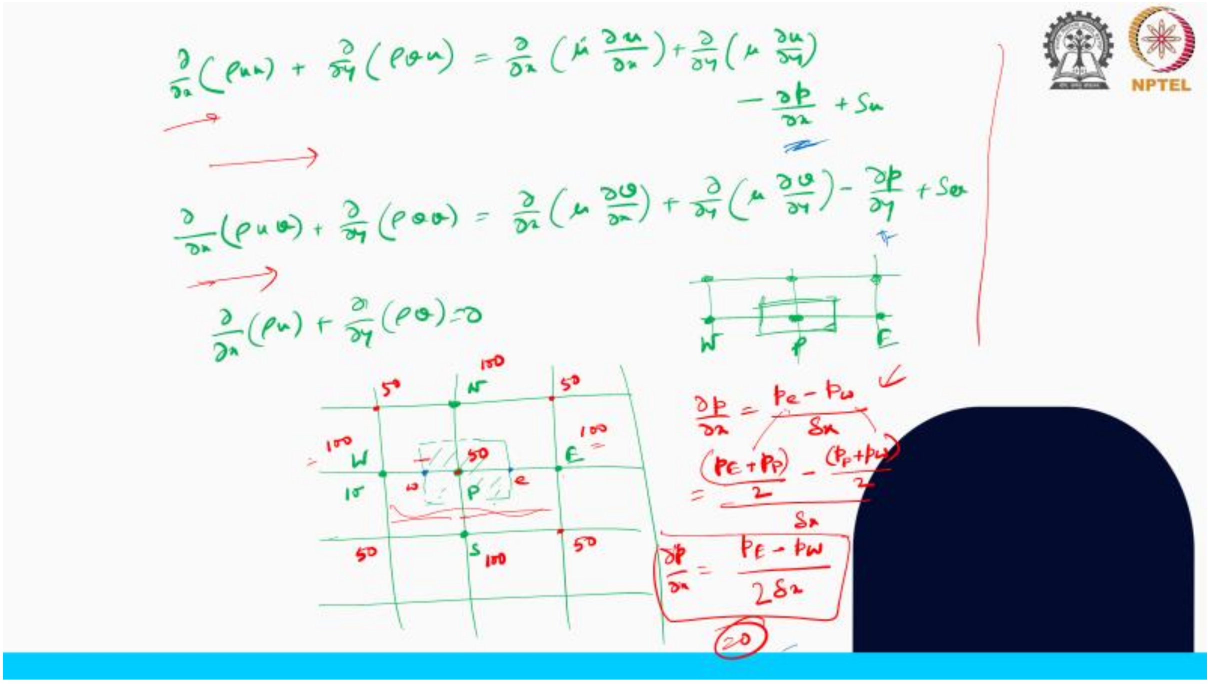
Hello everyone, welcome back to another lecture on CFD applications in chemical processes. We were discussing pressure-velocity coupling. And to do so, I have shown you one example that if I have this equation set, the set of equations, and I have this kind of a checkerboard actually in the domain, based on the idea that ∂P/∂X for the X-momentum and ∂P/∂Y for the Y-momentum, when we try to estimate it from this flow field on the pressure field, using our previous criteria, we see that despite having some pressure

in the domain at point P, this discretization would result in zero values in the equations, which will not accurately capture the physics. So, the flow physics. So, what is the solution? The solution is the staggered grid formation. It is the staggered grid formation. So in the staggered grid formation, what is essentially done is that what is assumed that on this conventional points,

we will have the scalar value stored, but the velocities would be estimated at the phases of the Here, at this point, we will store our velocities. So, how would it look? Let me draw it carefully, and then you will realize So, say I consider this is my I. This is I plus 1, this is I minus 1, and say this is capital J. So, this is capital J minus 1, capital J plus 1. So, this is the IJ point. So, I consider say this is point P.

Now conventionally, what is done is that So, this is small i, this is small i plus 1, this is small i minus 1. Similarly, we have this is say small j, small j plus 1 and this is small j minus 1. So, conventionally what we are doing is that we consider this is the control volume. where this is the IJ point the cross section of the I and J this is the face these are the conventional grid points

So in this case, what happens is that we consider, so in this case, this is my So, all the coordinates of this blue dots would be named by say small i here, this point is small i and capital J. This is small i, small j plus 1. So, accordingly we will have the nomenclature. Now, the point is and for the other case So the location of the arrows here on this case are the points where the velocities would be stored.

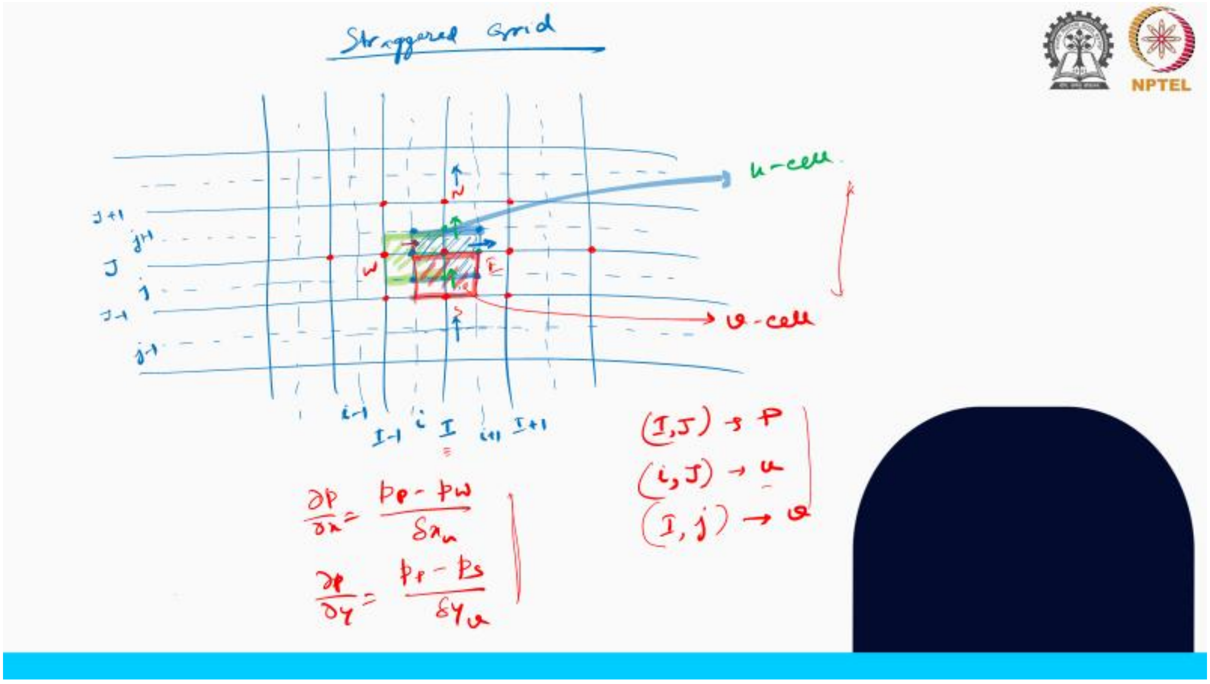


And the red dot points, which are the conventional grid points, here the scalar values would be stored. So in this staggered manner, it is planned that if we do that, then that problem of checkerboard solution could be avoided. So for example here, so here this conventional, this is the conventional control volume where at its centroid, the red dot point which is point P, we are storing all the scalar quantities. But for the velocity, we consider this cell where the arrow is given at its center and we consider that at that center we store the velocities.

We consider such staggered manner and similarly this is for, so this particular we call this as the U cell. And here similar to this, just because it is very close to this drawing for point P if I remove this drawing, so at this junction we also actually store the other directional values and in that case this one becomes my V cell.

This red one, where at its center again, I store the V component values. So I have u cell this is the normal cell that we have this is the u cell and this is my v cell. at point P which I can say is the point that is designated by I capital J is my point P where I store the scalar quantities. This is sorry, this is IJ. At point I capital J I have my U values and at I capital J and capital I and small j I have V values.

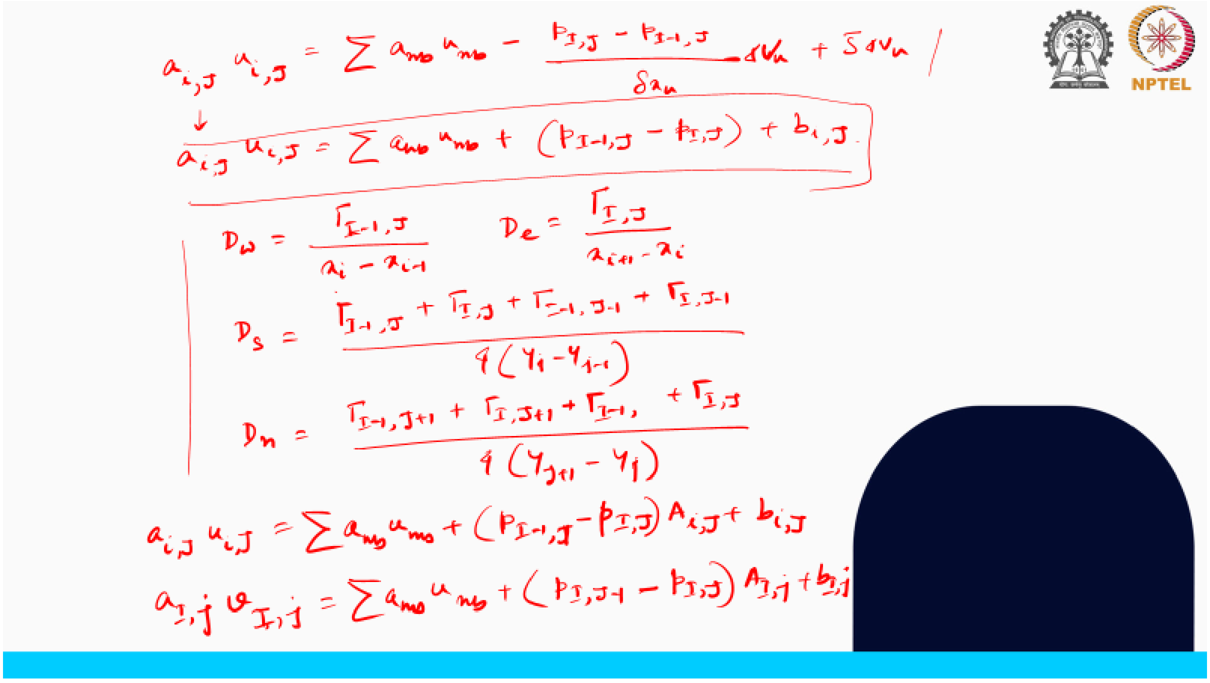
If the grid is such so constructed then what we have At this point, the point where we are trying to find out the value of del P by del X, that we can write as P minus PW divided by delta X, the U location and this minus for the S, S point south, north, this is east, this is west, this is point P divided by delta V in the V direction. Now, if you replace this value in the checkerboard solution, where you had point P and 100, you would not find any zero values to calculate or to be replaced at that point.



It will take care of that oscillatory profile or the checkerboard solution. So this formulation of this P cell, so where we store the scalar quantities, we call that as the P cell and u and v cell are the cells where at the center we store the velocities, the velocity profiles, the velocity values. Accordingly, we have to discretize this previous expression that are here, x and y momentum after the long derivations of replacing this in terms of IJ, now without writing north, south, east, west, because that would complicate the scenario that we would not be able to identify the P, U, V, these cells.

So that is why if we write now in terms of capital I, which is the conventional grid point, so capital I stands for capital I or capital J stands for the conventional grid lines, small i and small j and subsequently all those i minus one, i plus one, small i, these actually represents the control volume faces, face lines. If we go by this convention, there can be different other conventions, but if we go by this process, of designating capital I and capital J for the conventional grid line and small letters for the control volume phases. Then what we would find for this and this equation discretization, we will have an expression that A small i capital J U small i capital J is equals to summation of a neighbor point all the neighbor points minus P capital I capital J minus P I minus 1 capital I minus 1 and J divided by delta x control volume So you clearly see that the scalar quantities at the capital I capital J points, where we know the values, and U is essentially for the small i and capital J points. Similarly, the V cells are at capital I and small J points, and accordingly, that will be. So, if I write it more simply, in a simplified manner, this equation, is essentially P i minus Pij plus Bij.

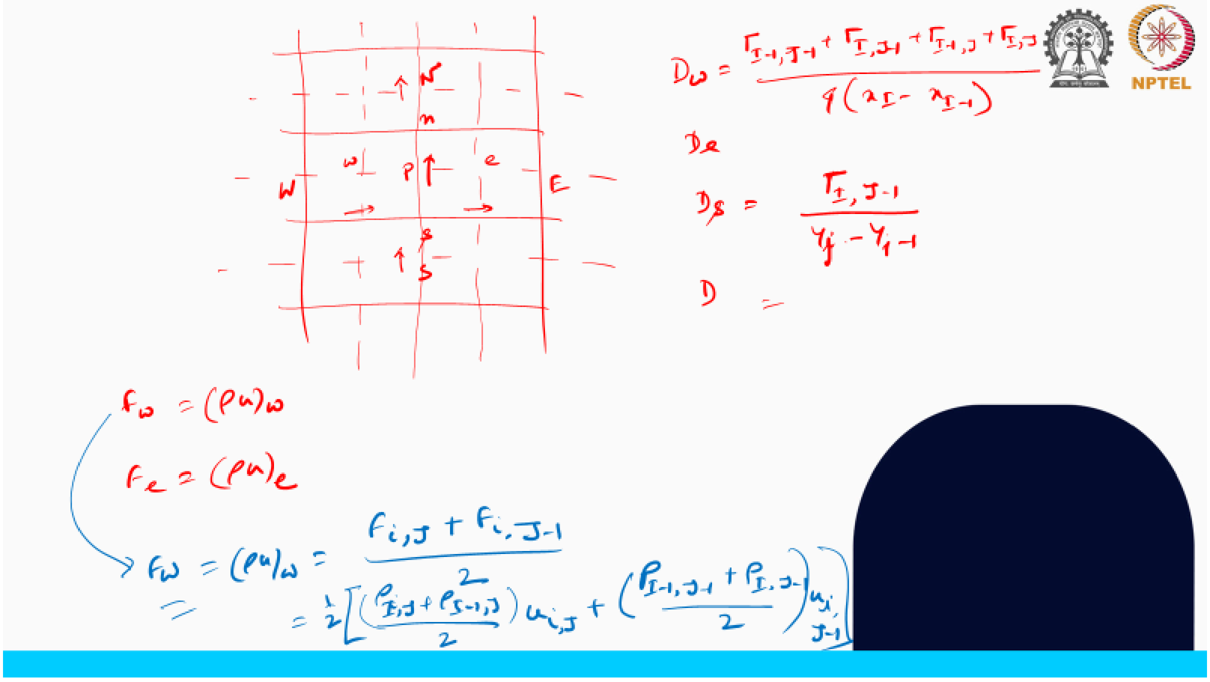
So this is eventually written in the form that we have seen: AP phi P equals the summation of the neighbor terms plus SU SP terms. Similarly, the other one we can also write for the jth, for the y momentum. But then the point is, for all the coefficients that we have to estimate, for example, here, DW would be represented by, say, the gamma capital I minus one J divided by XI minus XI minus one. So the west phase that we have



This west phase of this blue P cell is essentially that we are writing: gamma capital I minus 1 J capital I minus 1j for this phase divided by this xi minus xi minus 1, xi minus xi minus 1. Similarly, D at the east face is the gamma capital IJ divided by xi plus one minus xi. The north and the south cell similarly would be the combination of all the four points where the values are known.

It's the weighted average that you will be covering is I minus one j. You see, whenever I am writing these scalar quantities, it is always with the capital letters because at those junctions, we are aware of the values of those scalar quantities. So, I minus one I, so this would be your I minus one J minus one plus I J minus one divided by four times Y J, Y J minus one. And similarly, we can write for the N also. I minus 1 J plus 1 I for the north point J plus 1 this I minus 1 J.

Plus I J divided by 4 times Y J plus 1 minus Y J. So again, in compact form, the x momentum would look like. Capital J is the summation of. A neighbor point, all the neighbor points, plus P I minus 1 capital J minus P small I J times A small I capital J plus B I J that we have written earlier, and for this capital I small J. It is the V. J minus one minus P I J multiplied by A I J plus B capital I small J. So again, for the yth case, we can find out the dW, dS, dN, all the parameters.



What we have to clearly understand. Say, for example, this Y cell, what we will have. So this is our point P, this is East face, this is West face, this is basically capital W, this is capital E, this is my South small S, this is small N, this is my capital N, and this is capital S, where again these are. So, these arrow positions are the velocity points where we calculate the velocity, or we estimate the velocity, or store the velocity. So for example here, now if you look at it, here the dW at this would be gamma dW.

I minus 1 J minus 1 plus I J minus 1 I minus 1 J plus I J divided by 4 times X I minus X I minus 1. Similarly, the dE can be written dS J minus 1 divided by Y J minus Y J minus 1. Similarly, Dn can also be found. So what it is doing, or we are doing, is that we are essentially trying to evaluate these terms, which is f i plus 1 capital J plus f i plus 1 capital J minus 1. This is here; it is for the

If this I can write, so if rho u, this is essentially f i j plus f i j minus 1 divided by 2. And we further write it in terms of rho capital Ij plus rho I minus 1. So, all these are capital because rho are the scalar properties, which are known at the scalar location at the P cells. So, i minus 1 j j minus 1 plus i j minus 1 divided by 2, multiplied by u i j minus 1. So, similarly, each and every term would be discretized, would be approximated along this understanding that

on the conventional points, on the conventional grid lines, and its center, we store the scalar properties, but at the control volume center, we store the velocity field. In this staggered manner, if we do it, all the terms are simplified. It may look complicated, but if you start doing it on your own, you would realize the rationale behind it, and you would find out a discretized form of this expression, this equation, which then would be solved by processes that, again, we will discuss in the next class. There are different strategies to solve this set because these are all coupled.

But this is the process of how we discretize. So, what are the processes by which we solve this discretized set? We'll discuss this in the next class. So, until then, thank you.